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Stochastic fracture analysis of systems with moving material

Maria Tirronen

Summary. This paper considers the probability of fracture in a system in which a material travels supported by rollers. The moving material is subjected to longitudinal tension for which deterministic and stochastic models are studied. In the stochastic model, the tension is described by a multi-dimensional Ornstein-Uhlenbeck process. The material is assumed to have initial cracks perpendicular to the travelling direction, and a stochastic counting process describes the occurrence of cracks in the longitudinal direction of the material. The material is modelled as isotropic and elastic, and LEFM is applied. For a general counting process, when there is no fluctuation in tension, the reliability of the system can be simulated by applying conditional sampling. With the stochastic tension model, considering fracture of the material leads to a first passage time problem, the solution of which is estimated by simulation. As an example, the probability of fracture is computed for periodically occurring cracks with parameters typical to printing presses and paper material. The numerical results suggest that small cracks are not likely to affect the pressroom runnability. The results also show that tension variations may significantly increase the probability of fracture.

Key words: moving material, fracture, stochastic model, first passage time, multi-dimensional Ornstein-Uhlenbeck process, simulation, paper industry

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Introduction

In many industrial processes there are stages at which a material travels in a system of rollers. Examples of such processes can be found in the print industry and in the manufacturing of different kinds of materials, such as textiles, plastic films, aluminium foils and paper. In this kind of a system, the material moves between rollers unsupportedly under longitudinal tension. The tension is essential for the transport of the material, and in paper machines and printing presses, it is created by velocity differences of the rollers.

The mechanical behaviour of the moving unsupported material has gained interest in research. For example, vibration charateristics and the mechanical stability of the moving material is widely investigated (see literature reviews in [18, 41, 20]). From the studies of instability it is known that increasing tension has a stabilizing effect. However, when tension is increased, the probability of fracture increases and thus, it is of interest to study the behaviour of the moving material from the view point of fracture.

In pressrooms, web breaks are an important runnability issue [4], and the effect of cracks on web breaks has gained interest among researchers. Recently, researchers have approached the question of possible causes of web breaks by conducting data analysis on press room data. Deng et al. [4] gathered data from several press rooms and found that cracks were actually a minor cause of web breaks. Also, Ferahi and Uesaka [5] found, by using special optics and a web inspection system, that most of the web breaks in pressrooms are not uniquely related to the presence of obvious defects. According to Uesaka [38], the concept that has begun to be accepted in the industry is that a web break is a combined probabilistic event of high tension and low strength. However, earlier studies found through pilot-scale experiments defects to be the major causes for web breaks in pressrooms [38]. Recently, it has also been suggested that the lowest values of tensile strength may be caused by defects [27].

As web breaks are statistically rare events, a large number of rolls is required to determine the causes of web breaks with a reasonably high confidence level [4]. Thus, mathematical modelling may provide an efficient tool to study the causes of web breaks. Previously, Swinehart and Broek [32] studied the effect of cracks on web breaks by a model, based on fracture mechanics, which included the number and the size distribution of flaws, web strength and web tension. In [32], the tension was regarded as constant. Uesaka and Ferahi [39] proposed a break rate model based on the weakest link theory of fracture. The number of breaks per one roll during a run was derived by considering the strength of characteristic elements of the web. In [39] it was assumed that there is a single crack in every roll and the tension in the system was regarded as constant. Moreover, Hristopulos and Uesaka [9] presented a dynamic model of web transport derived from fundamental physical laws, and in conjunction with the weakest link fracture model, the model allows investigating the impact of tension variations on web break rates.

The break rate model used in [39, 9] predicts the upper estimate of the break frequency. However, considering an upper bound of fracture probability may lead to an over-conservative upper bound for a safe range of tension. The studies of mechanical stability suggest that, when tension is increased, the material can be transported with a higher velocity [2]. From the viewpoint of maximal production, an over-conservative tension is undesirable as it underestimates the maximal safe velocity.

Motivated by paper industry, Banichuk et al. [3] studied the optimal value of set tension for a cracked band travelling in a system of rollers. The band was assumed to have initial cracks of bounded length and to be subjected to constant or cyclic tension. The optimal average tension was sought for the maximum crack length by considering a productivity function which takes into account both instability and fracture. Moreover, cracked moving plates with random parameters were studied by Tirronen et. al [35]. In [35], critical regimes for the tension and velocity of the material in the presence of a crack were obtained by considering fracture and instability. In [35], the tension was assumed to be constant while a crack travels from one roller to another although the constant value was assumed to be random.

Tension in a printing press is known to exhibit random fluctuations [38] and such fluctuations may have a significant impact on web breaks [37]. Tension variations are partly caused by draw (the relative speed difference between two succeeding rollers) variations which contain specific high/low frequency components and white noise [38]. In a printing press, out-of-round unwind rolls or vibrating machine elements such as unwind stands may cause cyclical tension variations (see [25] and the references therein). In addition to cyclical variations, tension may vary aperiodically due to poorly tuned tension controllers, drives, or unwind brakes ([25] and the references therein). The net effect of such factors cause the tension to fluctuate around the mean value [25].

A continuous-time stochastic model for tension fluctuations was proposed by Tirronen [33], for a system with two rollers. In [33], the tension fluctuations were modelled by a stationary one-dimensional Ornstein-Uhlenbeck process. With such a model, tension has a constant mean value around which it fluctuates temporally. The one-dimensional Ornstein-Uhlenbeck process can be regarded as the continuous-time analogue of the discrete-time AR(n) process. It is a mathematically well-defined continuous-time model for fluctuations of systems whose measurements contain white noise [7, Chapter 4]. The stationary Ornstein-Uhlenbeck process can be regarded as a simplified model of tension variations in a printing press. Moreover, in [33], the fracture probability of the moving material was studied in the case in which there continually exists a crack in the material that occurs between the rollers. Furthermore, Tirronen [34] studied the fracture probability of a moving band when cracks occur in the material according to a stochastic counting process. The models proposed in [33, 34] differ from the ones presented in [35] by allowing investigation of the system longevity, which is of practical interest.

This paper extends [33, 34] by considering a system with several spans. For the tension, we study deterministic and stochastic models. In the deterministic models, the tension is described by a vector with constant values. The stochastic model describes the tension as a multi-dimensional Ornstein-Uhlenbeck process. With the latter model, the tension in each span has a constant mean value around which it fluctuates. Similar to the one-dimensional Ornstein-Uhlenbeck process, the multidimensinal Ornstein-Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time vector autoregressive (VAR(n)) process. Moreover, in this study, the material is assumed to have straight line initial cracks perpendicular to the travelling direction, and the crack occurence is modelled by a stochastic counting process as in [34]. The crack geometries are described by i.i.d. random vectors.

In this study, the travelling material is modelled as elastic and isotropic, and linear elastic fracture mechanics (LEFM) is applied. According to the literature review in [13], Balodis [1] was the first to apply LEFM to paper material. Other fracture mechanics theories have also been suggested for paper material. For example, Uesaka et al. [40] proposed the use of the J-integral to paper. However, Swineheart and Broek [31] advocated the use of LEFM to paper due to its simplicity [13]. They argued that in most cases the paper is sufficiently elastic in the machine direction to justify the use of LEFM [13]. Other proposed methods for predicting the fracture of paper include the essential work of fracture [30] and the cohesive zone model [36]. Fracture mechanics literature for paper is reviewed more extensively in [13, 17].

When the tension in the system is constant, the nonfracture probability can be simulated by applying conditional Monte Carlo sampling. For conditional sampling, see [26, Section 5]. When the tension exhibits random fluctuations, considering the probability of fracture leads to a first passage time problem. When there is only one span in the system, a series representation for the first passage time distribution of the one-dimensional Ornstein-Uhlenbeck process to a fixed boundary (see, e.g., [15]) can be exploited in estimating the fracture probability [33, 34]. In this study, we focus on a system with more than one span and approximate the reliability of the system with tension fluctuations by simulating sample paths of the tension process and the crack model.

Examples are computed for a system with three spans and a material that has central through thickness cracks of varying length that occur in the material (almost) periodically. For example, in paper making, a condensation drip in pressing or drying section or a lump on press rolls or press felt can cause holes in the paper web which occur in a fixed pattern. The material parameters used in computing the examples are typical of dry paper (newsprint).

The paper is outlined as follows. In the following section, we present a mathematical model for a band moving in a series of rollers. In the subsections, models for tension and cracks are proposed, and an example of a system with three spans and periodically occuring cracks is presented. In the next section, we first formulate the nonfracture criterion for the material, after which the nonfracture probability is formulated. In the last subsection, techniques for simulating the nonfracture probability are proposed. In the following section, examples are computed for a system with three spans and periodically occuring cracks by using parameters typical to paper. In addition, limitations of the model are discussed. In the last section, the model presented in this study and the numerical results obtained by the model are summarized.

Problem setup

This study considers an elastic and isotropic band that travels in a system in which there are stages at which the material moves unsupportedly from one support (roller) to another. The material has initial defects, and the band travels between the rollers under a longitudinal tension. Below, a mathematical model for the moving cracked band travelling in a system of rollers is presented. The model is similar to the one presented in [35, 33, 34]. As an example, we consider a system with three spans and cracks occurring (almost) periodically in the material.

Moving band

Consider a system of $k+1$, $k \geq 2$, rollers located at $x = \ell_0, \ell_1, \ldots, \ell_k$ in x, y coordinates, see figure 1. For simplicity, we set $\ell_0 = 0$. Let us study the behaviour of a band that travels supported by the rollers in the x direction. For this, we consider a rectangular part of the band that occurs momentarily between and on the supports at $x = \ell_{i-1}, \ell_i$:

$$\mathcal{D}_i = \{(x, y) : \ \ell_{i-1} \leq x \leq \ell_i, \ -b \leq y \leq b\}. \tag{1}$$

The part $\mathcal{D}_i$ is modelled as a plate which has simply supported sides at

$$\{x = \ell_{i-1}, \ -b \leq y \leq b\} \text{ and } \{x = \ell_i, \ -b \leq y \leq b\} \tag{2}$$

and sides free of tractions at

$$\{y = -b, \ \ell_{i-1} \leq x \leq \ell_i\} \text{ and } \{y = b, \ \ell_{i-1} \leq x \leq \ell_i\}. \tag{3}$$

Moreover, we assume that the band has constant thickness h and Young modulus E. The width of the band is $2b$.

Tension

The plate element in (1) is subjected to tension acting in the x direction. It is assumed that the tension profile is homogeneous, that is, the tension is constant in the y direction. For the time behaviour of tension, we consider different models. The simplest model describes the values of tension in the considered k spans as constants:

$$\boldsymbol{T} = \boldsymbol{T}_0 = \left(T_{0_1}, \ldots, T_{0_k}\right)^\top. \tag{4}$$

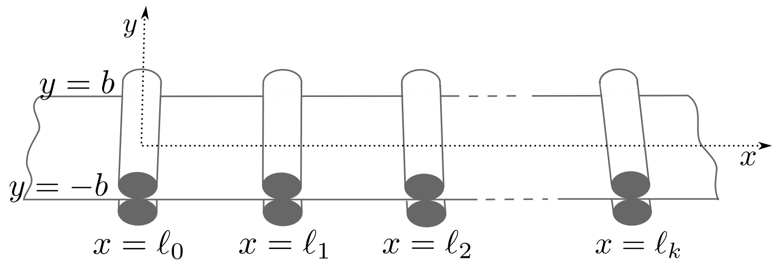

Figure 1. A band travelling in a system of rollers.

Moreover, we consider the case in which the value of tension changes randomly with respect to time. Random fluctuations of tension are described by a multi-dimensional continuous-time stochastic process

$$\boldsymbol{T} = \left\{ \big(T_1(s), \ldots, T_k(s)\big)^\top, \;\; s \geq 0 \right\} \tag{5}$$

in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In (5), s denotes the length of the band that has travelled past the first support at $x = \ell_0$, see figure 2. Futhermore, we describe the tension in the system by a multi-dimensional Gaussian Markov process. That is, $\boldsymbol{T}$ satisfies the stochastic differential equation (Langevin equation)

$$d\boldsymbol{T}(s) = \boldsymbol{C}(\boldsymbol{T}_0 - \boldsymbol{T}(s))ds + \boldsymbol{D}d\boldsymbol{W}(s) \tag{6}$$

with $\boldsymbol{T}(0)$ Gaussian or constant. Above, the factors $\boldsymbol{C}$ and $\boldsymbol{D}$ are deterministic $k \times k$ and $k \times p$ matrices, respectively, and $\boldsymbol{W}$ is a standard p-dimensional Brownian motion. In the following, we assume that $p = k$ so that there are as many sources of random fluctuations as there are spans in the system.

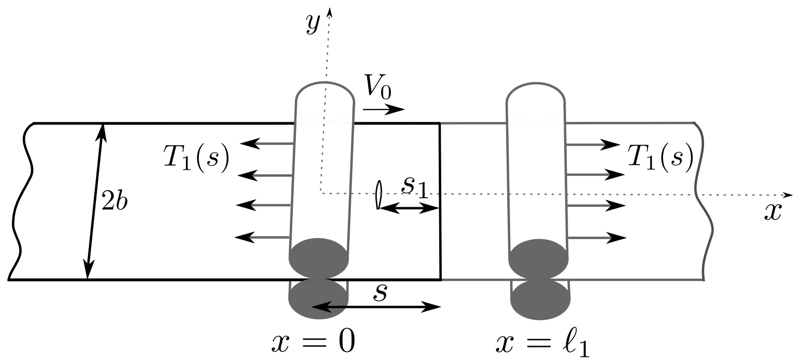

Figure 2. A cracked band travelling through the first open draw, in which it is subjected to tension T_1. The drawing is adapted from figure 1 in [35].

The analytical solution of (6), the multi-dimensional Ornstein-Uhlenbeck process, reads as

$$\boldsymbol{T}(t) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{T}(s) + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 + \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}d\boldsymbol{W}(u) \tag{7}$$

for $t > s \geq 0$. The matrix exponential $e^{\boldsymbol{C}t}$ in (7) is the $k \times k$ matrix given by the power series

$$e^{\boldsymbol{C}t} = \sum_{i=0}^{\infty} \frac{t^i}{i!}(\boldsymbol{C})^i. \tag{8}$$

The solution (7) can be obtained by introducing the integrator (similarly as in [7, Section 4.4.4])

$$\boldsymbol{X}(t) = e^{\boldsymbol{C}t}\big(\boldsymbol{T}(t) - \boldsymbol{T}_0\big) \tag{9}$$

and by applying the multi-dimensional Itô formula [21, Thm 4.2.1] to $\boldsymbol{X}$. For this, note that

$$\frac{d}{dt}e^{\boldsymbol{C}t} = \boldsymbol{C}e^{\boldsymbol{C}t}. \tag{10}$$

When $\boldsymbol{T}_0$ depends on s, the solution of (6) is obtained similarly and reads as [8, Section 3.3.3]

$$\boldsymbol{T}(t) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{T}(s) + \int_s^t \boldsymbol{C}e^{-\boldsymbol{C}(t-u)}\boldsymbol{T}_0 du + \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}d\boldsymbol{W}(u). \tag{11}$$

In this study, we consider a system that exhibits only random variations. However, the stochastic differential equation (6) can also describe, e.g., deterministic cyclic variations of tension when $\boldsymbol{T}_0$ is made time-dependent. The process remains Gaussian and Markovian if the vector $\boldsymbol{T}_0$ and the matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ are made time-varying but deterministic [8, Section 3.3.3].

From (7) we see that the expected value of $\boldsymbol{T}(t)$ reads as

$$\boldsymbol{\mu}(t) = e^{-\boldsymbol{C}t}\mathbb{E}\big[\boldsymbol{T}(0)\big] + (\boldsymbol{I} - e^{-\boldsymbol{C}t})\boldsymbol{T}_0. \tag{12}$$

For the covariance matrix of $\boldsymbol{T}(t)$, denoted by $\boldsymbol{\Sigma}(t)$, it holds that [7, Section 4.4.]

$$\boldsymbol{\Sigma}(t) = e^{-\boldsymbol{C}t}\boldsymbol{\Sigma}(0)e^{-\boldsymbol{C}^\top t} + \int_0^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(t-u)}du. \tag{13}$$

Especially, we notice that for the distribution of $\boldsymbol{T}(t)$ conditional to $\boldsymbol{T}(s)$, it holds

$$\boldsymbol{T}(t)\big|_{\boldsymbol{T}(s)=\boldsymbol{x}} \sim \mathcal{N}(\tilde{\boldsymbol{\mu}}(t,s), \tilde{\boldsymbol{\Sigma}}(t,s)) \tag{14}$$

with the deterministic drift

$$\tilde{\boldsymbol{\mu}}(t,s) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{x} + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 \tag{15}$$

and the covariance matrix

$$\tilde{\boldsymbol{\Sigma}}(t,s) = \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(t-u)}du. \tag{16}$$

When $\boldsymbol{C} \oplus \boldsymbol{C}$ is invertible, the matrix (16) can be expressed as [19]

$$\mathrm{vec}\big(\tilde{\boldsymbol{\Sigma}}(t,s) = (\boldsymbol{C} \oplus \boldsymbol{C})^{-1}(\boldsymbol{I} - e^{-(\boldsymbol{C}\oplus\boldsymbol{C})(t-s)})\mathrm{vec}(\boldsymbol{D}\boldsymbol{D}^\top). \tag{17}$$

Above, vec and $\oplus$ denote the stack operator and the Kronecker sum, respectively.

Although the stochastic differential equation (6) has a solution for a general matrix $\boldsymbol{C}$, the process is not stationary in all cases. According to [28, Thm 4.1], the stochastic process defined by (6) is stationary if the eigenvalues of $\boldsymbol{C}$ have positive real parts. In this case, the tension process has the long-term mean

$$\lim_{s\to\infty} \boldsymbol{\mu}(s) = \boldsymbol{T}_0. \tag{18}$$

Moreover, when the eigenvalues of $\boldsymbol{C}$ have positive real parts, it holds [19]

$$\lim_{s\to\infty} \boldsymbol{\Sigma}(s) = \boldsymbol{\Sigma}_\infty \tag{19}$$

with

$$\operatorname{vec}\big(\boldsymbol{\Sigma}_\infty\big) = \big(\boldsymbol{C} \oplus \boldsymbol{C}\big)^{-1} \operatorname{vec}\big(\boldsymbol{D}\boldsymbol{D}^\top\big). \tag{20}$$

For (18) and (19), first notice that the matrix $\boldsymbol{C}$ and its transpose $\boldsymbol{C}^\top$ share the same eigenvalues. Moreover, if all the eigenvalues of $\boldsymbol{C}$ have positive real parts, also the eigenvalues of the Kronecker sum $\boldsymbol{C} \oplus \boldsymbol{C}$ have positive real parts [this follows, e.g., from Thm 13.16 in 14]. Now, (18)–(20) are obtained by applying Thm 2.49 in [11]. When all the eigenvalues of $\boldsymbol{C} \oplus \boldsymbol{C}$ are nonzero, $\boldsymbol{C} \oplus \boldsymbol{C}$ is invertible.

In this study, we assume that the initial value satisfies

$$\boldsymbol{T}(0) \sim \mathcal{N}(\boldsymbol{T}_0, \boldsymbol{\Sigma}_\infty). \tag{21}$$

Consequently, since the limiting matrix satisfies [7, Section 4.4.6]

$$\boldsymbol{C}\boldsymbol{\Sigma}_\infty + \boldsymbol{\Sigma}_\infty \boldsymbol{C}^\top = \boldsymbol{D}\boldsymbol{D}^\top, \tag{22}$$

we see from (13) that the covariance matrix of the tension process do not change with respect to s. Thus, with (21), the tension process is strictly stationary.

Cracks

We consider a band that contains straight line cracks perpendicular to the travelling direction. The positions of the cracks in the longitudinal direction of the band are described by a counting process

$$N_\xi = \{N_\xi(s), \;\; s \geq 0\}. \tag{23}$$

Let s_j denote the distance between the first end of the band and the jth crack that arrives to the system of rollers (see figure 2). We assume that

$$s_j - s_{j-1} > \max_{i=1,\ldots,k} \ell_i - \ell_{i-1}, \tag{24}$$

so that no more than one crack occurs in a single span simultaneously. Moreover, the crack geometry of the jth crack is described by the random vector $\boldsymbol{\xi}_j$. We assume that $\boldsymbol{\xi}_j$, $j = 1, 2, \ldots$ are i.i.d and independent of N_ξ and $\boldsymbol{T}$, and that the process N_ξ is independent of $\boldsymbol{T}$.

The performance of the system is considered during the transition of a band of length S through the system of supports. In this, the initial and last states of the system are regarded as the states at which the first and last ends of the band are located at the supports at $x = \ell_0$ and $x = \ell_k$, respectively (see figure 3). It is assumed that before and after the band the material continues and remains similar. For simplicity, cracks that may occur in the open draws in the initial and last states are not considered in terms of fracture.

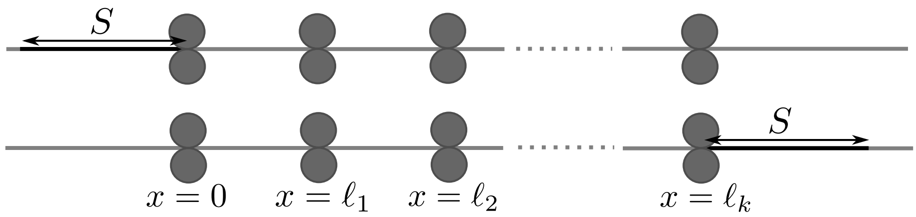

Figure 3. The initial and last states of the system.

Periodically occurring cracks in a system of three spans

As an example we study a system with three spans for which we assume that random fluctuations of tension in one of the spans occur as fluctuations of opposite value in the span(s) next to it. Moreover, we assume that fluctuations in tension in other spans than the ones next to the considered span do not affect directly the tension fluctuations in it. That is, the reliability of the system is studied with

$$\boldsymbol{D} = d \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{25}$$

where $d > 0$ determines the size of random variations in tension. Furthermore, the drifts in the spans towards $\boldsymbol{T}_0$ are assumed to be independent. That is, we set

$$\boldsymbol{C} = c\boldsymbol{I}, \tag{26}$$

where $c > 0$ and $\boldsymbol{I}$ is the identity matrix. With (26), the matrix exponential (8) simplifies to

$$e^{\boldsymbol{C}t} = e^{ct}\boldsymbol{I}. \tag{27}$$

Moreover, we study the reliability of the system in the case in which a failure in the production process causes defects to occur (almost) periodically in some part of the band. Let S be the length of the damage zone, and let the possible crack locations in the longitudinal direction of the band be

$$iL, \ \ i = 1, \dots, \lfloor S/L, \rfloor, \ \ L > \max_{i=1,\dots,k} \ell_i - \ell_{i-1}. \tag{28}$$

We assume that a crack occurs in the location iL with probability p_s independently of other cracks. In this case, the random variable $N_\xi(S)$ follows the binomial distribution with number of trials $\lfloor S/L \rfloor$ and a succes probability p_s in each trial. The crack distance satisfies

$$s_i - s_{i-1} = LX, \tag{29}$$

where X follows the geometric distribution with the success probability p_s and the support $\{1, 2, \dots\}$. The presented crack occurrence model is a renewal process.

Furthermore, we consider through thickness cracks, located at the center of the band in the cross direction. Let the random variable ξ_j describe the length of the jth crack. See figure 4 for the crack geometry.

Reliability

We study the reliability of the system during the time period in which a band of given length travels through the system of rollers. To study fracture of the material, linear

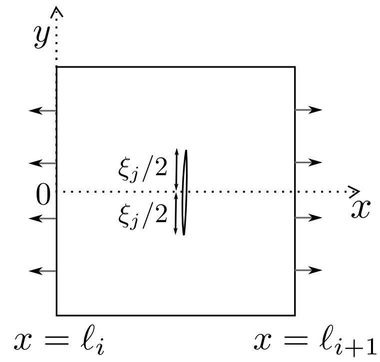

Figure 4. A central crack on a tensioned plate.

elastic fracture mechanics (LEFM) is applied. For constant tension and a general crack occurrence process, the reliability of the system can be simulated by applying conditional sampling. For special crack occurrence models, explicit representations for the nonfracture probability can be derived. When the tension exhibits random fluctuations, the reliability of the system is approximated by simulating sample paths of the tension process and the crack model.

Fracture criterion

The crack occurrence model assumes that more than one crack do not occur in a single span at the same time. Moreover, when studying fracture of the material, we assume that cracks in different spans do not interact. Thus, the nonfracture criterion can be formulated separately for the cracks.

To study the fracture of the band, we apply linear elastic fracture mechanics (LEFM), which assumes that the inelastic deformation at the crack tip is small compared to the size of the crack. In the following, the fracture criterion is formulated for central through thickness cracks which lengths are described by the random variables ξ_j, $j = 1, \ldots$

Since the moving band is assumed to be subjected only to in-plane tension acting in the travelling direction and the cracks are prependicular to the direction of applied tension, crack loadings in the system are of mode I (opening). When the jth crack travels between the supports at $x = \ell_{i-1}$, ℓ_i, the stress intensity factor related to the crack is a function of the form (see [6])

$$K_i(x, s_j, \xi_j) = \frac{\alpha(x, \xi_j)\ T_i(\ell_{i-1} + s_j + x)\sqrt{\pi\xi_j/2}}{h}, \quad x \in [0, \ell_i - \ell_{i-1}], \tag{30}$$

where α is a weight function related to the geometry of the crack. In this study, we assume that the function α is constant with respect to the location of the crack in x direction and approximate (see [24])

$$\alpha(\xi_j) = \left(\sec\frac{\pi\xi_j}{4b}\right)^{1/2}. \tag{31}$$

In order for the jth crack to travel from the support at $x = \ell_{i-1}$ to the one at $x = \ell_i$ in such a way that the material does not fracture, the stress intensity factor should satisfy

$$K_i(x, s_j, \xi_j) < K_C \ \ \forall \ x \in [0, \ell_i - \ell_{i-1}], \tag{32}$$

where K_C is the fracture toughness of the material. This is equivalent with

$$T_i(\ell_{i-1} + s_j + x) < B(\xi_j) \ \ \forall \ x \in [0, \ell_i - \ell_{i-1}], \tag{33}$$

where

$$B(\xi_j) = \frac{hK_C}{\alpha(\xi_j)\sqrt{\pi\xi_j/2}}. \tag{34}$$

Nonfracture probability

Consequently, by (33), the probability that a band of length S travels through the system of rollers in such a way that fracture does not propagate from any of its cracks reads as

$$\begin{aligned} r =& \mathbb{P}\big[N_\xi(S) = 0\big] && (35)\\ &+ \mathbb{P}\big[N_\xi(S) \geq 1, \\ &\qquad T_i(\ell_{i-1} + s_j + x) < B(\xi_j) \ \ \forall \ x \in [0, \ell_i - \ell_{i-1}] \\ &\qquad \forall \ i = 1, \ldots, k \quad \forall \ j = 1, \ldots, N_\xi(S)\big]. && (36) \end{aligned}$$

The reliability can also be written as

$$r = \mathbb{P}\big[\tau > S\big] \tag{37}$$

with the first passage time

$$\begin{aligned} \tau = \inf \big\{\ell_{i-1} + s_j + x : \ \ & T_i(\ell_{i-1} + s_j + x) = B(\xi_j) \\ & \text{for some } \ x \in [0, \ell_i - \ell_{i-1}] \\ & \text{for some } \ (i, j) \in \{1, \ldots, k\} \times \mathbb{N}\big\}. \end{aligned} \tag{38}$$

When the tension is constant in each span, the reliability of the system simplifies to

$$\begin{aligned} r_1 =& \mathbb{P}\big[N_\xi(S) = 0\big] && (39)\\ &+ \mathbb{P}\big[N_\xi(S) \geq 1, \ \ T_0^{\max} < B(\xi_j) \ \ \forall \ j = 1, \ldots, N_\xi(S)\big] && (40) \end{aligned}$$

with

$$T_0^{\max} = \max_{i=1,\ldots,k} T_{0_i}. \tag{41}$$

Since N_ξ is independent of the crack lengths and the lengths are i.i.d., it holds that

$$r_1 = \mathbb{P}\big[N_\xi(S) = 0\big] + \sum_{j=1}^{\infty} \mathbb{P}\big[N_\xi(S) = j\big]\overline{q}^j \tag{42}$$

with

$$\overline{q} = \mathbb{P}\big[T_0^{\max} < B(\xi_1)\big]. \tag{43}$$

In the example case of periodically occurring cracks, the reliability of the system with constant tension simplifies to

$$\begin{aligned} r &= (1 - p_s)^{\lfloor S/L \rfloor} + \sum_{j=1}^{\lfloor S/L \rfloor} \binom{\lfloor S/L \rfloor}{j} (p_s)^j (1 - p_s)^{\lfloor S/L \rfloor - j} (\overline{q})^j && (44)\\ &= (1 + p_s(\overline{q} - 1))^{\lfloor S/L \rfloor}. && (45) \end{aligned}$$

Simulation

The reliability of the system with constant tension can be estimated by conditional Monte Carlo simulation (for conditional sampling, see [26, Section 5.4]). That is, we may estimate

$$r_1 \approx \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j=0\}} \tag{46}$$

$$+\frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j\neq 0\}} \mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{N_\xi(S)}) \mid N_\xi(S) = k_j\big], \tag{47}$$

where $k_1, \ldots, k_M$ is a sample of size M from the distribution of $N_\xi(S)$. The conditional probability in (47) simplifies to

$$\mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{k_j})\big] = \bar{q}^{k_j}. \tag{48}$$

When the tension exhibits random fluctuations, we estimate the nonfracture probability r by

$$r^{\Delta s} = \mathbb{P}\big[\tau^{\Delta s} > S\big], \tag{49}$$

where $\tau^{\Delta s}$ is a first passage time as in (38) but with a discretized tension process $\boldsymbol{T}^{\Delta s} = (T_1^{\Delta s}, \ldots, T_k^{\Delta s})$.That is, we approximate the process $\boldsymbol{T}$ at points $0 =< x_1 < x_2 < \ldots$ by (see [8, Section 3.1.2])

$$\boldsymbol{T}^{\Delta s}(0) = \boldsymbol{T}_0 + \boldsymbol{y}_0, \tag{50}$$

$$\boldsymbol{T}^{\Delta s}(x_l) = e^{-\boldsymbol{C}(x_l-x_{l-1})}\boldsymbol{T}^{\Delta s}(x_{l-1}) + (\boldsymbol{I} - e^{-\boldsymbol{C}(x_l-x_{l-1})})\boldsymbol{T}_0 + \boldsymbol{y}_l, \quad l = 1, 2, \ldots \tag{51}$$

where $\boldsymbol{y}_0$ is a random variate from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_\infty)$ and $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots$ are independent draws from the distributions $\mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(x_1, x_0)\big), \mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(x_2, x_1)\big), \ldots$, respectively. The initial value (50) follows from (21), and the following values (51) are obtained by exploiting the property (14)–(16). The random variates $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots$ can be obtained by drawing $\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots$ independently from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and then setting

$$\boldsymbol{y}_l = \boldsymbol{\sigma}_l \boldsymbol{z}_l, \tag{52}$$

where the matrix $\boldsymbol{\sigma}_l$ satisfies

$$\boldsymbol{\sigma}_l \boldsymbol{\sigma}_l^\top = \tilde{\boldsymbol{\Sigma}}(x_l, x_{l-1}). \tag{53}$$

Methods for finding such a matrix is discussed in [8, Section 2.3.3.].

The counting process N_ξ can be simulated by generating crack distances. When N_ξ is a renewal process, the crack distances are drawn from their common distribution. Similarly, the crack lengths are simulated by generating random variates from the common distribution of the crack lengths.

When simulating a sample path of the system, the discretization points are chosen in the following way: When there is at least one crack in the band, we choose x_1 to be the location of the first crack. The following discretization points x_2, $x_3, \ldots$ are chosen such that while there is at least one crack travelling between rollers, the value of tension is computed at equidistant points with a distance $\Delta s > 0$. When the distance between two succeeding cracks is more than ℓ_k, we simulate the tension at equidistant points until the first crack exits the system and then compute the value of tension directly at the location of the second crack.

The probability (49) is estimated by

$$\hat{r}^{\Delta s} = \frac{1}{M} \sum_{n=1}^{M} \chi_{\{\tau_n^{\Delta s} > S\}}, \tag{54}$$

where M denotes the number of simulated paths of the system and $\tau_n^{\Delta s}$ denotes the first passage time in the nth such path. This approximation contains both statistical and discretization errors. As usual, the statistical error is estimated by the standard error

$$\frac{\hat{\sigma}_M^{\Delta s}}{\sqrt{M}}, \tag{55}$$

where $\hat{\sigma}_M^{\Delta s}$ is the sample standard deviation

$$\hat{\sigma}_M^{\Delta s} = \sqrt{\frac{1}{M-1} \sum_{n=1}^{M} \left(\chi_{\{\tau_n^{\Delta s} > S\}} - \hat{r}^{\Delta s} \right)^2}. \tag{56}$$

The discretization error is approximated by comparing the estimates obtained by a step size Δs and its double. That is, we consider

$$\left| \hat{r}^{\Delta s} - \hat{r}^{2\Delta s} \right|. \tag{57}$$

In (57), the estimates should be obtained with sufficiently small standard errors. If the absolute difference above is sufficiently small, $\hat{r}^{\Delta s}$ is regarded as being close enough to the real value.

As depicted by (55), the convergence rate of Monte Carlo simulation is $\mathcal{O}(\sqrt{M})$. However, the computational cost of the reliability estimate (54) depends remarkably on the time taken to compute the random variates $\chi_{\{\tau_n^{\Delta s} > S\}}$, $n = 1, \ldots, M$. The time required to compute $\chi_{\{\tau_n^{\Delta s} > S\}}$ depends on the number and the lengths of the spans, the length of the damage zone and the distribution of crack occurrence.

Numerical results for a printing press and discussion

As an example, we consider the reliability of a system with three spans and (almost) periodically occurring cracks. The values of the material parameters are typical of paper.

Periodic cracks in a printing press

As an example we consider a system with three spans, each of them of length ℓ. The values of the material parameters used in the examples are typical of dry paper (newsprint), for which the strain energy release rate G_C was obtained from the results in [29]. The fracture toughness of the material was set to

$$K_C = \sqrt{G_C E}. \tag{58}$$

The values of the deterministic parameters used in computing the examples of this section are listed in Table 1.

Table 1. Parameters.

ℓ	1 (m)
b	0.6 (m)
h	$8 \cdot 10^{-5}$ (m)
E	4 (GPa)
G_{C}	6500 (J/m^2)

In the examples we set $L = 2$ and $p_s = 0.9$. The crack lengths were assumed to be lognormally distributed with the coefficient of variation 0.1.

Moreover, in the computations, we let the (average) tension to be the same in all of the spans. The reliability of the system was studied with different values of the average tension, denoted by T_0. The coefficients in (26) and (25) were set to $c = 1$ and $d = T_0/10$, $T_0/5$. With these parameters, the correlation matrix of $\boldsymbol{T}(s)$ was

$$\boldsymbol{\rho}_T = \begin{pmatrix} 1 & -0.82 & 0.5 \\ -0.82 & 1.0 & -0.82 \\ 0.5 & -0.82 & 1.0 \end{pmatrix}, \tag{59}$$

independent of T_0. Figure 5 shows a sample path of the tension process with $T_0 = 1500$ (N/m) with different coefficients of variation of $\boldsymbol{T}(s)$, denoted by $\boldsymbol{c}_T$.

The reliability of the system was simulated with $\Delta s = 0.001$ and $\Delta s = 0.002$. First, the sample size $M = 300$ was used. If the obtained reliability estimate was not 0 or 1, the sample size was increased to $M = 10000$. With this sample size, the standard error of the reliability estimate was less than 0.005 for all the considered parameter values. The difference between the estimates obtained by different discretizations was less than 0.01 for all the computed estimates.

Figure 6 shows the reliability of the system with respect to the average tension with different values of mean crack length and damage zone length. According to [16, 37], tension values usually applied in printing presses are in the range $[0.2, 0.5]$ (kN/m). From figure 6 we see that, when tension is constant or $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$ and $T_0 \leq 1000$ (N/m), the nonfracture probability is one. Thus, the results suggest that, with the considered crack geometries and crack occurences, cracks do not affect the runnability of system, unless the variation in tension is very large. Moreover, the results suggest that, in this case, the upper bound of safe set tension is higher than what is usually applied in printing presses.

Furthermore, figure 6 shows that tension variations may significantly affect the runnability of the system. This effect becomes stronger when the average crack length, the damage zone length or the average tension increases. For example, with $S = 1$ (km) and $T_0 = 1250$ (N/m), the reliability of the system with constant tension is 1 for $\mathbb{E}[\xi_j] = 0.01$ (m) and $\mathbb{E}[\xi_j] = 0.03$ (m). When $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$, the reliability of the system stays at 1 with $\mathbb{E}[\xi_j] = 0.01$ (m) but decreases to 0.35 with $\mathbb{E}[\xi_j] = 0.03$ (m). See table 2. When $S = 0.1$ (km), the reliability only decreases to 0.9 with $\mathbb{E}[\xi_j] = 0.03$ (m). When $S = 1$ (km), $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$ and the average crack length is 0.03 (m), the reliability of the system decreases to 0 when the average tension is increased to $T_0 = 1500$ (N/m). On the other hand, with the average crack length 0.01 (m), the reliability of the system stays at 1 even with $T_0 = 1750$ (N/m). The computed estimates for the reliability with $S = 1$ (km) are gathered in table 2.

The results obtained in this study agree to some extent with the previous results, in which tension variations were found to be a possible cause of web breaks [39]. The

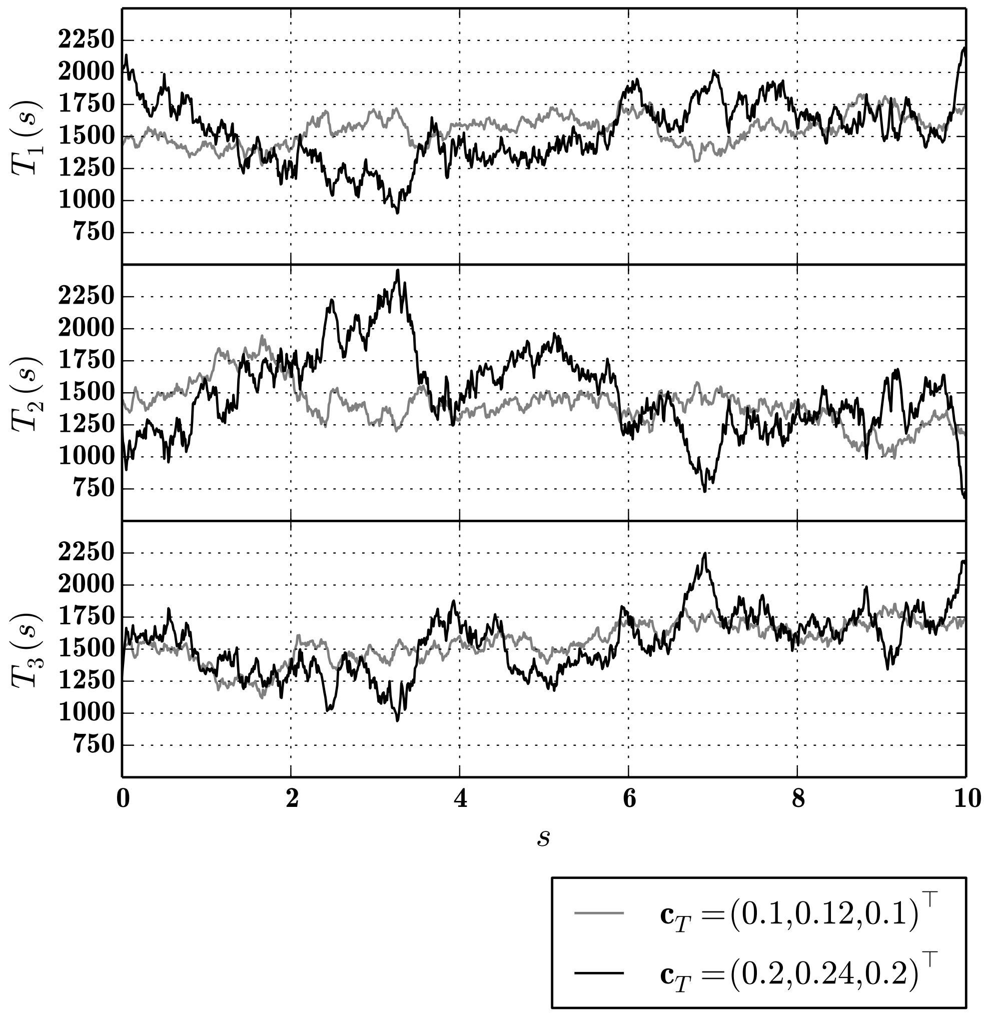

Figure 5. A sample path of tension with different coefficients of variation $\boldsymbol{c}_T$ with $T_0 = 1500$ (N/m) and $\Delta s = 0.01$ (m).

computed examples also suggest that small cracks are not likely to affect the pressroom runnability. Similar results have also been obtained in previous studies of web breaks [39].

Discussion

In this paper, we studied the nonfracture probability of a moving material that travels in a series of open draws and computed numerical examples for material parameters typical of newsprint. However, it should be kept in mind that the numerical results obtained in this study are mainly qualitative. For more rigorous results, data of defects and tension are needed. For printing processes, such data can be obtained by automated inspection systems developed for quality control [10] and devices designed for tension profile measurement [23].

Although the fracture analysis of this paper is carried out for the Ornstein-Uhlenbeck process, a similar analysis can be conducted also for other stochastic processes by applying an appropriate simulation scheme. For simulation of stochastic processes, see [12]. Notice, too, that although the tension in the system was assumed to have a constant mean-

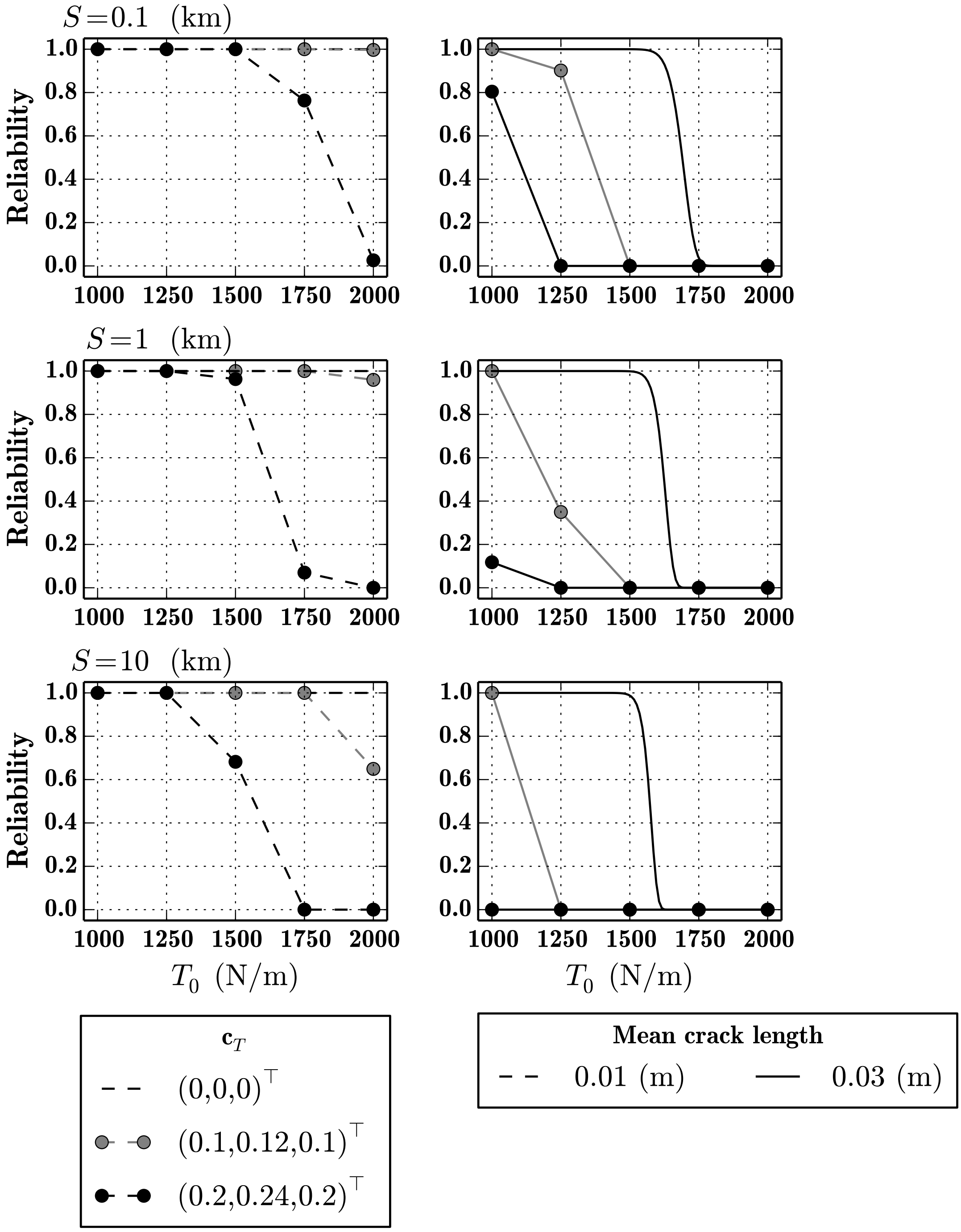

Figure 6. Reliability of the system in terms of fracture.

value, the stochastic differential equation (6) can be adapted to describe also deterministic tension variations by making the average tension $\boldsymbol{T}_0$ time-dependent. In a printing press, deterministic cyclic variations may occur, e.g., as a consequence of an out-of-round (OoR)

Table 2. Reliability of the system with $S = 1$ (km). Upper values are computed with the mean crack length 0.01 (m) while the *lower* values correspond to the mean crack length 0.03 (m).

T_0 (N/m) \ $\boldsymbol{c}_T$	$\mathbf{0}$	$(0.1, 0.12, 0.1)^\top$	$(0.2, 0.24, 0.2)^\top$
1000	1	1	1
	1	*1*	*0.12*
1250	1	1	1
	1	*0.35*	*0*
1500	1	1	0.95
	1	*0*	*0*
1750	1	1	0.07
	0	*0*	*0*
2000	1	0.96	0
	0	*0*	*0*

roll.

It should be kept in mind that the mechanical model presented in this paper is simplified. When studying fracture, it is assumed that cracks in different spans do not interact and the nonfracture criterion is formulated separately for the cracks. Numerical examples are computed for paper modelling the material as isotropic and elastic, although paper is orthotropic and have plastic characteristics. Furthemore, the model represented in this paper describes the tension as constant in the cross-direction of the web, although tension usually varies in the cross-direction of a printing press. Typically, the profile of tension is convex [16]. The model also assumes that the band is subjected to pure tension although, when a material element passes through the pressure area between the rollers (nips), its stress state varies [22]. In addition, the model for fracture does not take into account out-of-plane deformation of the band (see, e.g. [2]) or the air surrounding the material. With the simplified model, crack loadings are of mode I. Including, e.g., the effect of nips in the model may cause crack loadings of mode III (tearing). However, according to [38], tear strength has not been found to predict web breaks in pressrooms. In-plane fracture toughness is relevant for studying the effect of pre-existing macroscopic defects on web breaks [38].

Motivated by the paper and print industry, the aim of this study was to develop a mathematical model for the system that consists of a moving material and a series of open draws, and estimate the reliability of the system in terms of fracture. Compared to computing the break frequency by the formulae proposed in [32, 39], the simulation that was applied in this study to estimate the nonfracture probability may appear time-consuming. However, the model in [32] does not consider tension fluctuations which may significantly decrease the reliability of the system. The break frequency formula in [39] applies the maximal tension and the maximal crack length in a roll of paper, and thus, an upper estimate of the break frequency is obtained. The model and analysis presented in this study aim to take tension variations into account and to directly estimate the fracture probability predicted by the model which is important in optimizing productivity.

Conclusions

In this paper, we studied the reliability of a system in which a cracked material travels under longitudinal tension. Deterministic and stochastic models were considered for tension. The deterministic model described the tension as a constant-valued vector while random fluctuations of tension were modelled by a multi-dimensional Ornstein-Uhlenbeck process. The material was assumed to have initial cracks of random length perpendicular to the travelling direction. The crack occurrence in the longitudinal direction of the material was modelled by a stochastic counting process. The material was assumed to be isotropic and elastic, and LEFM was applied to study fracture of the material.

For constant tension and a general counting process, the reliability of the system can be simulated by applying conditional sampling. For some special crack occurrence models, an explicit representation for the system reliability can be derived. When the tension exhibits random fluctuations, considering fracture of the material leads to a first passage time problem. In this study, we considered a system with more than one span, and the solution of the first passage time problem was estimated by simulating sample paths of the tension process and the crack model.

As an example, the probability of fracture was computed for periodically occurring central through thickness cracks with parameters typical to printing presses and dry paper. With this crack occurrence model, an explicit expression for the reliability of the system with constant tension can be derived. The numerical results suggest that small cracks are not likely to affect the pressroom runnability. The results also showed that tension variations may significantly increase the probability of fracture.

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