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Performance-based seismic optimization design

Qimao Liu[1] and Juha Paavola

Summary. Performance-based engineering is to design, evaluate and construct, as economically as possible, the engineering facilities that can meet the uncertain future demands of owner-users and natural hazards. The performance-based design is believed to be the promising method in earthquake engineering, wind engineering and fire engineering. The paper takes the performance-based seismic optimization design as an example to describe the philosophy of the performance-based design method. First, the basic concepts of performance-based seismic design are introduced. Second, how to quantify the uncertain future hazard levels, i.e., to obtain the future demand diagrams, is presented. Third, how to quantify the capability of the structures to resist the future hazard, i.e., to achieve the capability diagram, is in detail. Fourth, how to evaluate the performance of the structures at different future hazard levels, i.e., to catch the performance points, is described. Finally, the optimization modelling is proposed for the performance-based design. The performance-based design of steel frame is demonstrated. The limitations of the current performance-based seismic design method are also discussed.

Key words: performance-based design, earthquake engineering, capability diagram, demand diagrams, performance points

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Introduction

Performance-based engineering is to design, evaluate and construct, as economically as possible, the engineering facilities that can meet the uncertain future demands of owner-users and different natural hazards. The performance-based design is broadly believed to be the promising and advanced design method in earthquake engineering, wind engineering and fire engineering. The philosophy of the performance-based design is that the design criteria are expressed using a set of achieving stated performance objectives when the structure is subjected to the different stated hazard levels. For

[1] Corresponding author. qimao.liu@aalto.fi

example, in Figure 1, there are three stated performance objectives [1], i.e., Basic Objective, Enhanced Objective 1 and Enhanced Objective 2, when a structure subjected to three different stated hazard levels, i.e., Frequent Earthquake, Occasional Earthquake and Rare Earthquake. The design criteria are: For Basic Objective, the performances of the structure are in Serviceability, Life safety and Collapse prevention when it is subjected to Frequent, Occasional and Rare earthquakes, respectively. For Enhanced Objective 1, the performances of the structure are in Serviceability and Life safety when it is subjected to Occasional and Rare earthquakes, respectively. For Enhanced Objective 2, the performance of the structure is in Serviceability when it is subjected to Rare earthquake. It is obviously that the construction cost of the structure: Enhanced Objective 2 > Enhanced Objective 1 > Basic Objective. Traditional design method actually is a kind of simple performance-based design method, for example, a limit state is a performance objective. In fact, performance-based design is a new powerful approach including traditional design methods with significant upgrades. The performance-based design uses nonlinear static/dynamic analysis where an attempt is made to capture the real behaviours of the structure by explicitly modelling and evaluating post-yield ductility and energy dissipation when subjected to actual earthquake ground motions. However, in traditional structural design, the structural engineers employ elastic (linear) static/dynamic analysis under the frequent or occasional earthquakes to get things done. Then nonlinear responses under the rare earthquake, of the done structures have to be checked.

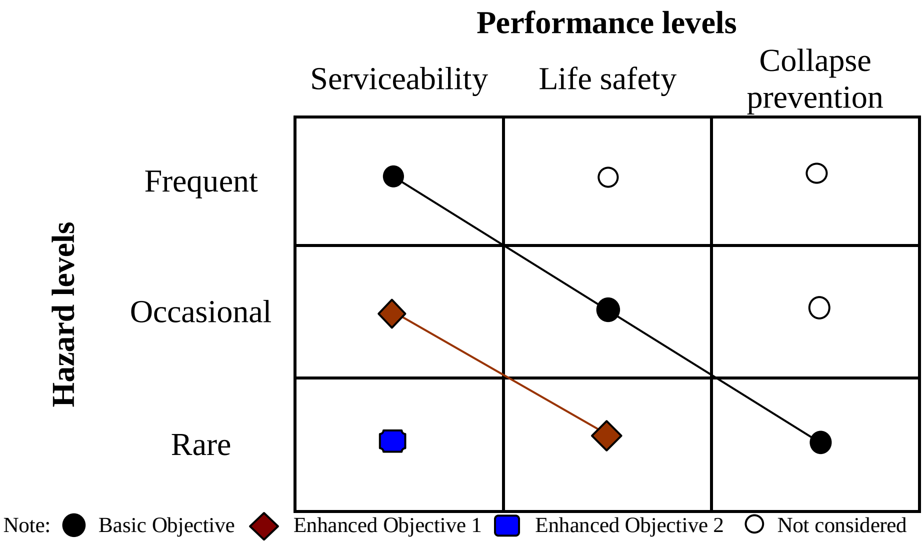

Figure 1. Performance objectives

The performance-based design methods have been widely researched and applied in earthquake engineering, wind engineering and fire engineering. Some performance-based design methods have also been used in design codes. For example, the N2 methods [2, 3] have been inserted in the Eurocode 8 [4]. The capacity spectrum method in different procedures has been proposed in ATC 40 [5]. Many Japanese design codes adopt the performance-based design concepts, for example, Principles for Foundation Designs Grounded on a Performance-based Design Concept (Japanese Geotechnical Society) [6] and Principles, Guidelines and Terminologies for drafting design codes founded on performance based design concept (Japan Society of Civil Engineers) [7].

The paper takes the performance-based seismic optimization design as an example to depict the philosophy of the performance-based design method in engineers' language. First, the basic concepts of performance-based seismic design are explained. Second, how to quantify the uncertain future hazard levels, i.e., to obtain the future demand diagrams, is presented. Third, how to quantify the capability of the new design structures to resist the future hazard, i.e., to achieve the capability diagram, is in detail. Fourth, how to evaluate the performance of the structures at different future hazard levels, i.e., to catch the performance points, is described. Finally, the optimization modelling is proposed for the performance-based seismic design. The performance-based design of steel frame is demonstrated. The limitations and further development of the current performance-based seismic design method are also discussed.

Basic concepts of performance-based seismic design

The basic concepts of performance-based seismic design are included in Figure 2.

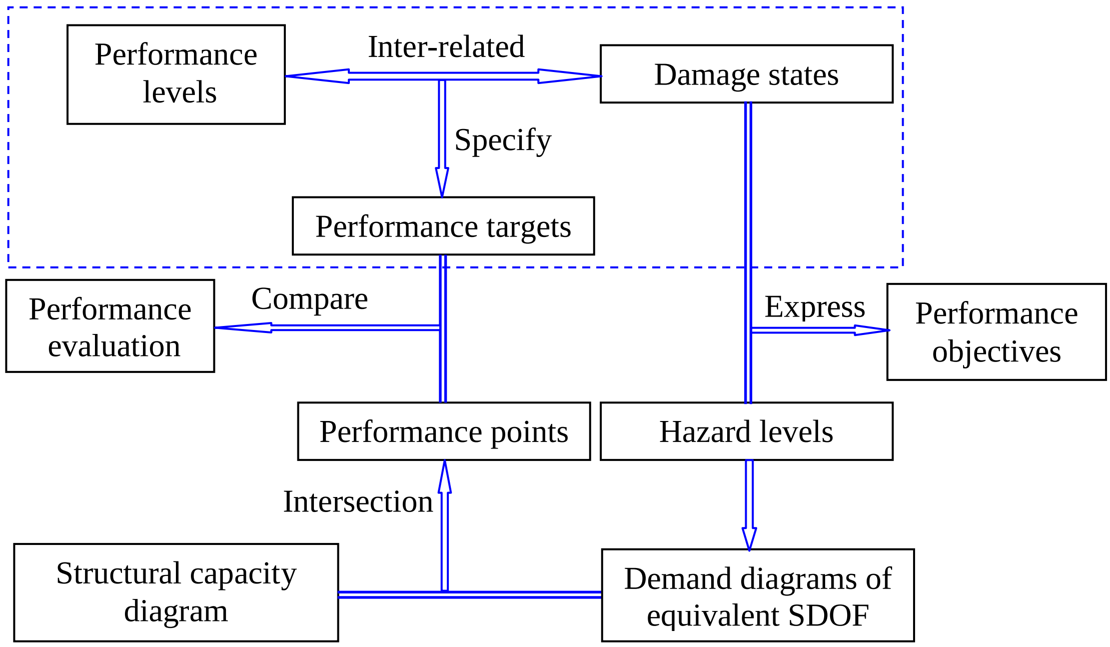

Figure 2. Performance-based seismic design concepts

As shown in the dashed box in Figure 2, the performance levels, targets and damage states are actually the same category for performance-based seismic design. As described in Table 1, the performance of a structure is divided into 3 levels, i.e., serviceability, life safety and collapse prevention. Their relevant damage states are no damage, moderate damage and repairable, and severe damage, respectively. Their relevant performance targets are the different values of the top displacement. The performance targets are used to quantify the damage states and performance levels. The performance targets can be used in structural design because they are quantitative. The hazard levels are described by the peak ground accelerations and response spectrum shapes. As described in Table 2, three earthquake hazard levels are defined according to peak ground accelerations and spectrum shape (EC8) [4]. The targeted damage states (performance levels) under the different hazard levels can express the performance objectives of the structure, as shown in Figure 1. A set of demand diagrams are plotted into AD format (Acceleration Displacement format) by using response spectra related to hazard levels. The response spectra are defined in Acceleration Period format in the different seismic design codes. The demand diagrams will be introduced in detail in Section 3. The capacity diagram represents the structural capability to resist the natural hazards and is achieved by structural analysis. The capacity diagram is also plotted in AD format. When the capacity diagram and a set of demand diagrams are plotted in the same coordinate system (horizontal ordinate is Displacement and ordinate is Acceleration). A set of intersections of the structural capacity diagram and demand diagrams are called performance points of the structure under the different hazard levels. The top displacement of the structure at the performance points can be calculated. Comparing the top displacement of the structure at the performance points with the performance targets, we can evaluate the performance (good or bad) of the structure under the different hazard levels.

Table 1. Three performance levels, corresponding damage states and performance targets

Performance levels	Damage states	Performance targets (limits on the top displacements)
Serviceability	No damage	$\bar{D}_{tp}^{(1)}$
Life safety	Moderate damage and repairable	$\bar{D}_{tp}^{(2)}$
Collapse prevention	Severe damage	$\bar{D}_{tp}^{(3)}$

Table 2. Three earthquake hazard levels (represented by elastic acceleration spectrum)

Earthquake frequency	Return period for years	Probability of exceedance	Peak ground accelerations and spectrum shape
Frequent	43	50% in 30 years	a_{1g} (=0.15g) and Design codes (EC8)
Occasional	72	50% in 50 years	a_{2g} (=0.40g) and Design codes (EC8)
Rare	475	10% in 50 years	a_{3g} (=0.60g) and Design codes (EC8)

Demand diagrams

The seismic demand diagrams are the elastic response spectrum at serviceability level, and the inelastic response spectrum at life-safety and collapse prevention levels. It means that the response of a structure is in the elastic scope (no damage) at serviceability level, in the elastoplastic scope (damage happens) at life-safety and collapse prevention levels.

Elastic response spectrum in AD format

The EC8 (European Standard, 2004) [4] defines the elastic response acceleration spectrum $S_{ae}(T)$ for the horizontal components of the ground motion as,

$$\begin{aligned} &0 \le T \le T_B: \quad S_{ae}(T) = a_g S\left[1 + \frac{T}{T_B}(2.5\eta - 1)\right]; \quad T_B \le T \le T_C: \quad S_{ae}(T) = 2.5 a_g S\eta \\ &T_C \le T \le T_D: \quad S_{ae}(T) = 2.5 a_g S\eta \frac{T_C}{T}; \qquad T_D \le T \le 4\text{s}: \quad S_{ae}(T) = 2.5 a_g S\eta \frac{T_C T_D}{T^2} \end{aligned} \tag{1}$$

where $S_{ae}(T)$ is the elastic response spectrum; T is the vibration period of a linear single-degree-of-freedom system; a_g is the design ground acceleration on type A ground; T_B is the lower limit of the period of the constant spectral acceleration branch; T_C is the upper limit of the period of the constant spectral acceleration branch; T_D is the value defining the beginning of the constant displacement response range of the spectrum; S is the soil factor; η is the damping correction factor with a reference value of $\eta = 1$ for 5% viscous damping ratio. For ground type A, $S = 1.0$, $T_B = 0.15$ s, $T_C = 0.4$ s, $T_D = 2.0$ s.

For an elastic SDOF system, the displacement response spectrum [8] is given as

$$S_{de}(T) = \frac{T^2}{4\pi^2} S_{ae}(T) \tag{2}$$

where $S_{de}(T)$ is the value in displacement spectrum corresponding to the period T and a fixed viscous damping ratio. Eq. (2) is the elastic response spectrum in AD format.

Inelastic response spectrum in AD format

The relationship between inelastic response spectrum and elastic response spectrum [9] is

$$S_{ae}(T) = R_\mu S_a(T) \tag{3}$$

$$S_{de}(T) = S_d(T)\frac{R_\mu}{\mu} \tag{4}$$

where R_μ is the reduction factor and μ is ductility factor. $S_d(T)$ and $S_a(T)$ are the values in displacement and acceleration spectra, respectively, corresponding to the period T and a fixed viscous damping ratio (5%).

Substituting Eqs. (3) and (4) into Eq. (2), we obtain the inelastic response spectrum function in AD format,

$$S_d(T) = \mu \frac{T^2}{4\pi^2} S_a(T) \tag{5}$$

The factors R_μ and μ are relative to both the structural response and elastic acceleration response spectrum. The elastic spectrum AD format and inelastic spectrum AD format in Figure 3 are called demand diagrams. We can only obtain one elastic spectrum AD format ($\mu = 1$). However, we can achieve many inelastic spectrum AD format (with different $\mu > 1$).

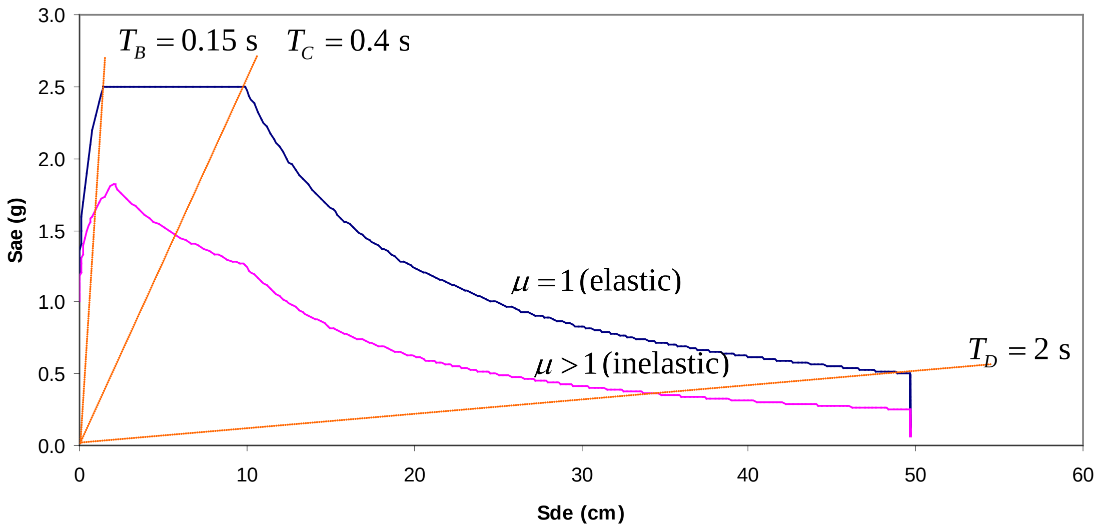

Figure 3. Elastic spectrum AD format and inelastic spectrum AD format

Capability diagram

The capability diagram can be achieved by the nonlinear time history analysis or nonlinear static analysis methods in seismic design. The nonlinear time history analysis is believed to be more accurate method than nonlinear static analysis methods and has solid physical foundation. In this paper, the nonlinear static analysis method (also called pushover analysis method) is employed to demonstrate how to plot the capability diagram. First, the relationship of the base shear force and top displacement of the frame structure (MDOF system) is obtained by using nonlinear static analysis. Second, the relationship of the base shear force and top displacement of the MDOF system is converted into the shear force and displacement relationship for the equivalent SDOF system. Third, the capability diagram is plotted.

Base shear-Top displacement diagram (MDOF system)

A monotonically increasing pattern of lateral forces is applied to structures in pushover analysis. For example, a planar steel frame shown in Figure 4 is assumed that the number of the story is n, the height of the i th story is h_i and the mass of the i th story is m_i.

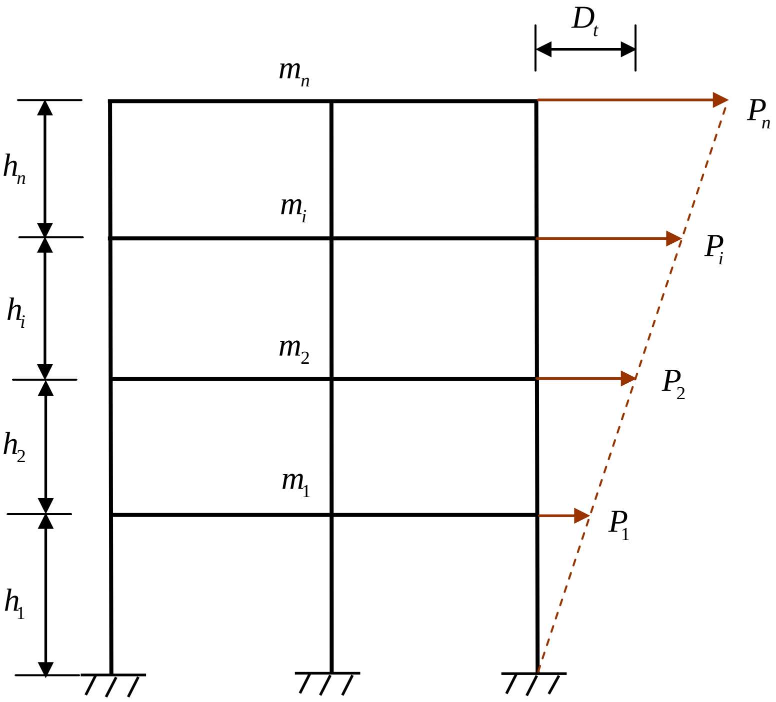

Figure 4. n -story steel frame

The inverted triangular load pattern with maximum loading at top and zero loading at the ground level is often employed and actually represents the first order mode shape. It is assumed that the lateral force at the i th story shown in Figure 4 is proportional to the component of the assumed displacement shape Φ_i weighted by the story mass m_i,

$$P_i = pm_i\Phi_i \tag{6}$$

where the component of the assumed displacement shape Φ_i is

$$\Phi_i = \frac{\sum_{k=1}^{i} h_k}{\sum_{k=1}^{n} h_k} \tag{7}$$

where p is used to control the magnitude of the lateral loads.

Therefore, the base shear force V can be calculated as

$$V = \sum_{i=1}^{n} P_i = p\sum_{i=1}^{n} m_i\Phi_i = pm^* \tag{8}$$

where $m^* = \sum_{i=1}^{n} m_i\Phi_i$ is called the equivalent mass of the SDOF system.

The base shear force (V) - top displacement (D_t) diagram, as shown in Figure 5, is obtained using the nonlinear static analysis.

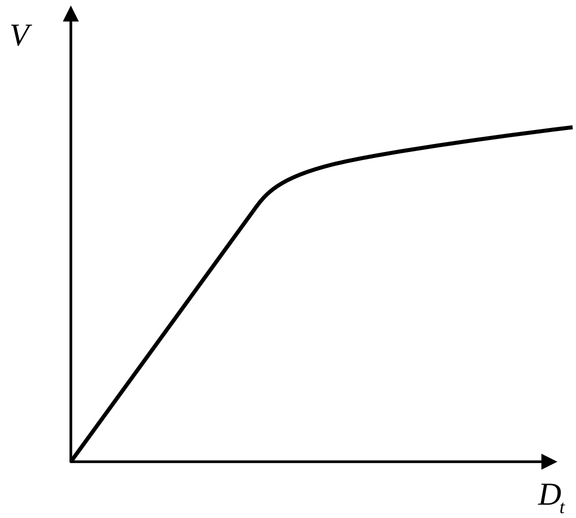

Figure 5. V - D_t diagram

The F^*-D^* diagram (equivalent SDOF system)

The displacement of the equivalent SDOF system is defined as [9]

$$D^* = \frac{D_t}{\Gamma} \tag{9}$$

And the base shear force of the equivalent SDOF system is defined as

$$F^* = \frac{V}{\Gamma} \tag{10}$$

where Γ is a constant and calculated as

$$\Gamma = \frac{\sum_{i=1}^{n} m_i \Phi_i}{\sum_{i=1}^{n} m_i \Phi_i^2} = \frac{m^*}{\sum_{i=1}^{n} m_i \Phi_i^2} \tag{11}$$

Provided that both shear force (V) and top displacement (D_t) are divided by Γ, the force - displacement relationship determined for the MDOF system, i.e., V - D_t diagram, becomes the shear force and displacement relationship for the equivalent SDOF system, i.e., the shear force F^* and displacement D^* diagram shown in Figure 6.

In practice, the F^* - D^* diagram is often simplified into a bilinear model, as shown in Figure 6, so that the performance points are easy to compute. It should be known that the yield point in Figure 6 is determined by using the engineers' experience. The elastic period of the SDOF system can be calculated as

$$T^* = 2\pi \sqrt{\frac{m^* D_y^*}{F_y^*}} \tag{12}$$

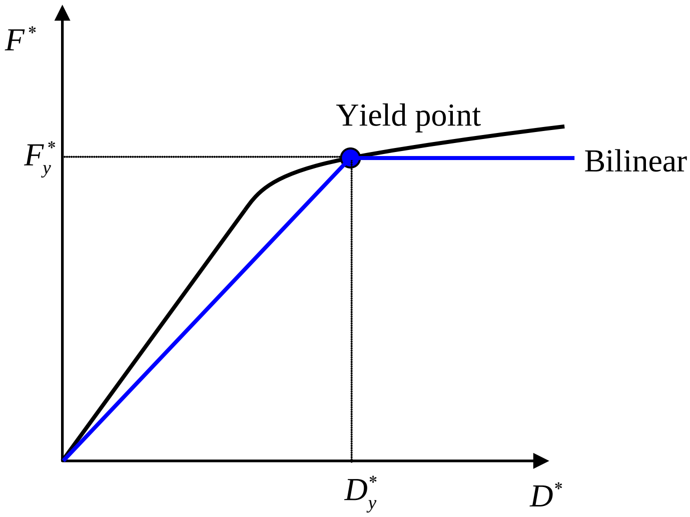

Figure 6. F^* - D^* diagram and its bilinear model

Capacity diagram in AD format

The capacity diagram in AD format shown in Figure 7 is obtained by dividing the forces, i.e., the ordinate of the bilinear F^* - D^* diagram (Figure 6), by the equivalent mass m^*.

$$S_a = \frac{F^*}{m^*} \tag{13}$$

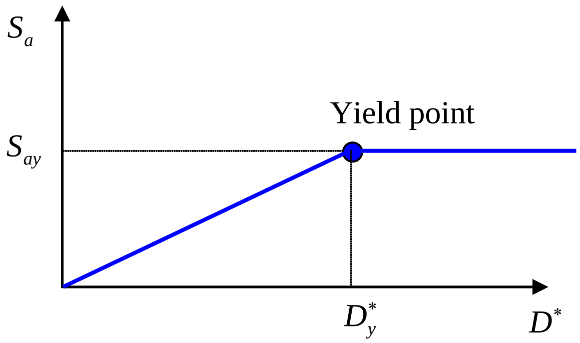

Figure 7. Capacity diagram in AD format

Performance points

The Figure 7 (capability diagram) and Figure 3 (demand diagrams) are plotted in the same coordinate system, as shown in Figure 8 ($T^* \geq T_C$) or Figure 9 ($T^* < T_C$). The intersections are the performance points, i.e., intersection 1 and intersection 2. The performance point 1 is elastic response and performance point 2 is inelastic response. We can see that the pushover analysis is just carried out for one time and a set of performance points can be obtained. The top displacements of the structure at different hazard levels can be calculated using the information of performance points.

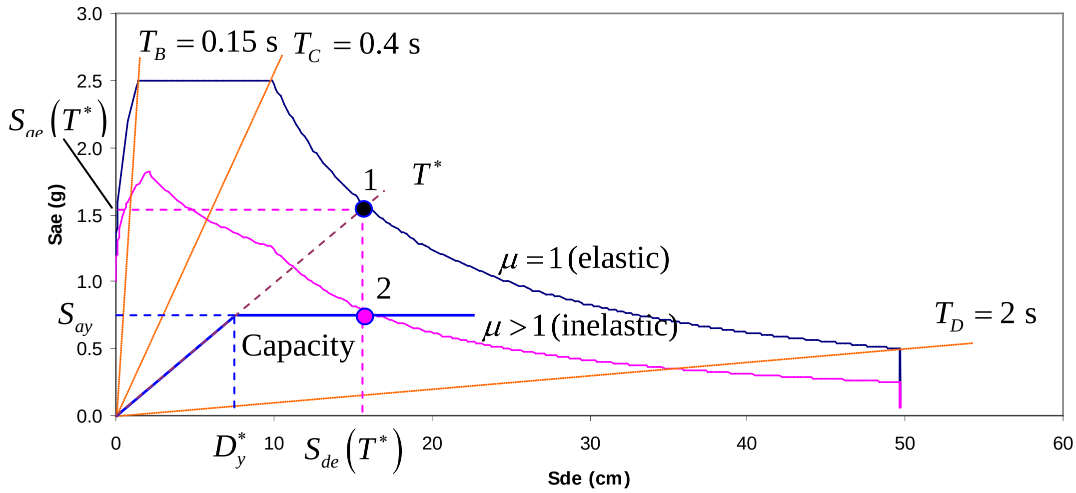

Figure 8. Performance points ($T^* \geq T_C$)

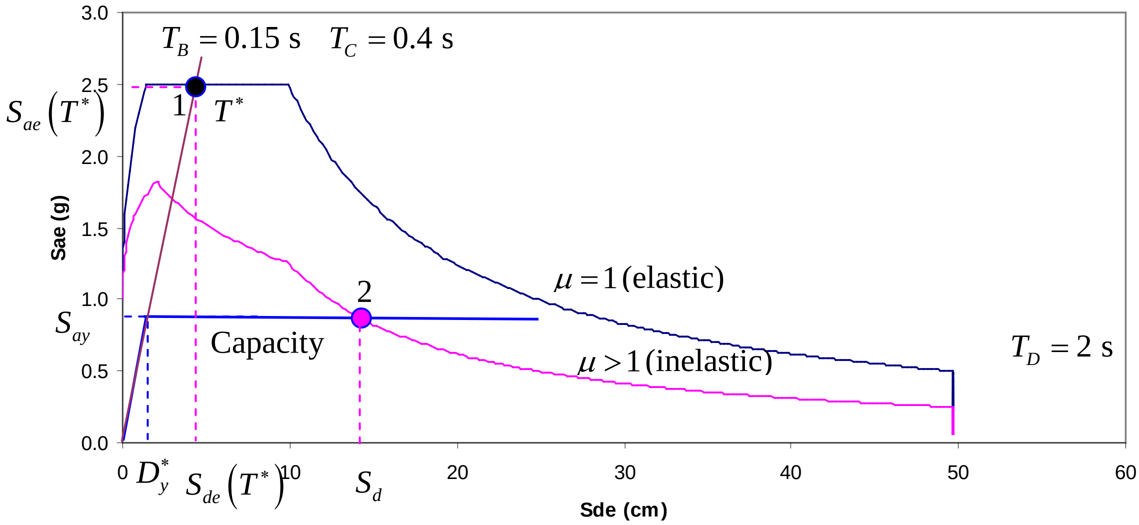

Figure 9. Performance points ($T^* < T_C$)

The ductility factor μ can be calculated as follows:

If $T^* \geq T_C$ (shown in Figure 8),

$$\mu = \frac{S_{de}(T^*)}{D_y^*} = \frac{S_{ae}(T^*)}{S_{ay}} = R_u \tag{14}$$

where $S_{de}(T^*)$ is the displacement value of the elastic spectrum at the period T^* and S_d is displacement value at the intersection of the inelastic spectrum diagram and capacity diagram shown in Figure 8 and Figure 9.

If $T^* < T_C$ (shown in Figure 9),

$$\mu = \left(R_\mu - 1\right)\frac{T_C}{T^*} + 1 \tag{15}$$

The displacement demand of the equivalent SDOF system can be determined from the definition of ductility as

$$S_d = \mu D_y^* \tag{16}$$

The displacement demand of the equivalent SDOF system is transformed back to the top displacement of the MDOF system,

$$D_t = \Gamma S_d = \Gamma \mu D_y^* \tag{17}$$

Optimization modelling

The performance-based design method requires that a structure has the different performance levels under the different hazard levels. For example, Basic objective in Figure 1 requires that the performances of the structure are in serviceability, life safety and collapse prevention when it is subjected to frequent, occasional and rare earthquake, respectively. Using traditional trial-and-error design procedures to design a satisfactory structure for the multi-performance and multi-hazard levels is often a tedious task for structural engineers. If the performance-based design is formulated as a structural optimization problem, it is possible to move to fully automated design procedures from the trial-and-error design procedures [10].

The lightweight design is always attractive in vibration environment because lightweight structures attract smaller inertia force than the heavy structures. Therefore, the structural mass is treated as the objective function in this paper. For the Basic objective in Figure 1, the optimization model of minimizing the structural mass and performance well in the frequent, occasional and rare earthquake can be expressed as

$$\begin{aligned} &\text{Find } \boldsymbol{d} \\ &\text{Minimize } M\left(\boldsymbol{d}\right) \\ &\text{Subject to } D_{tp}^{(1)} \le \bar{D}_{tp}^{(1)} \\ &\qquad\qquad\quad D_{tp}^{(2)} \le \bar{D}_{tp}^{(2)} \\ &\qquad\qquad\quad D_{tp}^{(3)} \le \bar{D}_{tp}^{(3)} \\ &\qquad\qquad\quad \underline{d}_i \le d_i \le \bar{d}_i \qquad \left(i = 1, 2, \cdots, N\right) \end{aligned} \tag{18}$$

where $\boldsymbol{d}$ and $M\left(\boldsymbol{d}\right)$ are the design vector and mass of steel frame, respectively. d_i, $\underline{d}_i$, and $\bar{d}_i$ are the ith design variable, its lower and upper boundary, respectively. N is the number of design variables. The top displacements ($D_{tp}^{(1)}$, $D_{tp}^{(2)}$, and $D_{tp}^{(3)}$) are computed using the performance points. $\bar{D}_{tp}^{(1)}$, $\bar{D}_{tp}^{(2)}$ and $\bar{D}_{tp}^{(3)}$ are the performance targets, as shown in Table 1.

Example

A three-story two-bay steel frame is shown in Figure 10. The steel frame consists of 4 groups including B1, C1, C2, and C3. All the members are H-shape section. The design variables are the size of flanges and webs of the H-shape section. The orientations of the column and beam are shown in Figure 10. The properties of material are defined as: Modulus of elasticity $E = 200$ GPa , Poisson's ratio $\nu = 0.3$, Yield stress $\sigma_y = 235$ MPa and Secant modulus of plasticity $E_s = 1.035$ MPa , Density $\rho = 7857$ kg/m^3 . The mass of every floor is assumed to be 1000 kg. The design space, i.e., lower and upper limits, is shown in Table 3. The BEAM189 of ANSYS, based on Timoshenko beam theory, is used to divide the steel frame into 45 elements, as shown in Figure 12.

In this example, the peak ground accelerations are assumed $a_{1g} = 0.15g$, $a_{2g} = 0.4g$ and $a_{3g} = 0.6g$ at frequent, occasional and rare earthquake, respectively. The limits on the top displacements are assumed as $\bar{D}_{tp}^{(1)} = 5$ cm, $\bar{D}_{tp}^{(2)} = 15$ cm, and $\bar{D}_{tp}^{(3)} = 30$ cm. The first-order optimization method [14] of ANSYS is employed to solve the optimization model of Eq. (18). The initial design and optimum design are shown in Table 3.

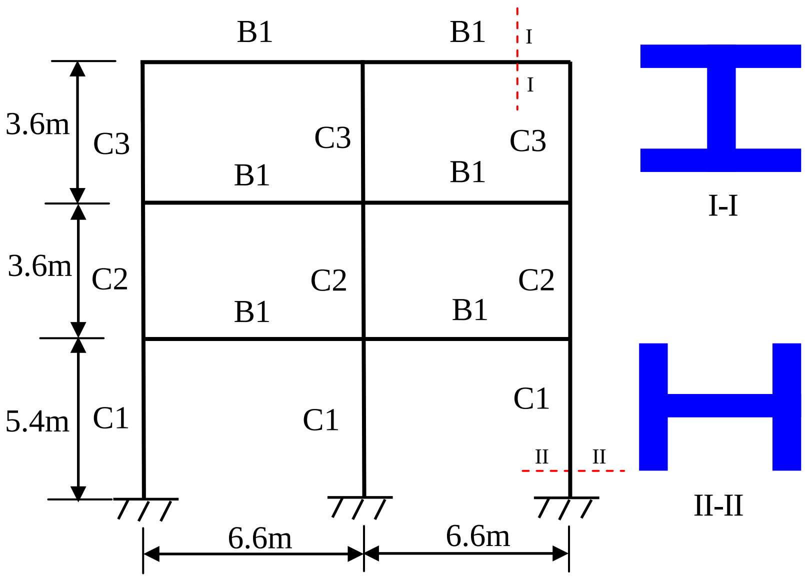

Figure 10. Three-story steel frame with H-shape sections of members

Table 3. Design space, initial and optimum designs

		Initial design	Optimum design	Design space	
				Lower limits	Upper limits
Beam **B1** (mm)	Flange width	150	130	100	180
	Flange thickness	17	12	8	22
	Web depth	400	319	205	600
	Web thickness	14	7	6	20
Column **C1** (mm)	Flange width	150	147	100	180
	Flange thickness	17	16	5	22
	Web depth	400	431	250	600
	Web thickness	14	11	4	20
Column **C2** (mm)	Flange width	150	143	100	180
	Flange thickness	17	14	5	22
	Web depth	400	355	250	600
	Web thickness	14	11	4	20
Column **C3** (mm)	Flange width	150	143	100	180
	Flange thickness	17	14	5	22
	Web depth	400	349	250	600
	Web thickness	14	11	4	20
Structural mass (kg)		6218	4014		
Top displacement D_t (cm)	Frequent	3.10	4.42		
	Occasional	8.25	14.59		
	Rare	13.57	27.88		

Since the optimization mathematical model only includes the top displacement constraints, the other performances (strength, stiffness and buckling) of the optimum structure need to be studied. Take the performance of the optimum design under the rare earthquake as an example. The pushover analysis should be demonstrated once again. According to the top displacements shown in Table 3, the pushover analysis stops if the top displacement is equal or greater than 27.88 cm at collapse prevention level. The Displacement - Base shear curve obtained by pushover analysis is shown in Figure 11. The 1st, 2nd and 3rd inter-story drifts are 13.53 cm, 9.14 cm and 5.25 cm, respectively.

When the top displacement is 28.06 cm, the deformation of the optimum design is shown in Figure 12. The maximum displacement is at the top, i.e., 28.0618 cm. No buckling presents. The von Mises stress of the optimum design is shown in Figure 13. The maximum von Mises stress is 0.237 GPa at the joint of beam and column of the third floor. The first plastic hinge presents at the joint of beam and column of the third floor, not present at the columns.

Discussion

The aforementioned performance-based seismic optimization design method indicates that it is very important to predict the future hazard levels and evaluate the capability of a structure. The performance-based seismic design concepts have been widely used in earthquake engineering and coded in some building design codes. However, it may lead to unreasonable design in the current development status. For example: The pushover analysis to evaluate the capability of a structure actually only includes the contribution of the fundamental mode. It may yield the unreasonable results when the higher modes have obvious contribution to the dynamic response of a structure. Therefore, the modal pushover analysis method is being developed to cover this shortcoming [11]. The nonlinear time history analysis methods are believed to be the correct approach to predict nonlinear seismic responses and evaluate the capability of a structure. However, the nonlinear time history analysis method is thought not practical for everyday design because it involves computational and modeling effort, convergence problem and complexity [12, 13]. In the pushover analysis method, the MDOF system is idealized to be an equivalent SDOF system. It is broadly believed that this equivalent replacement has no solid physical foundation although it becomes more and more popular in the performance-based seismic design. The real force-displacement diagram is simplified to a bilinear force-displacement diagram. This simplicity also depends on the engineers' experience and may also lead to unreasonable evaluation of the capability of a structure. The lateral loading mode and loading path have great influence on the structural nonlinear responses although it is the common character of elastic-plastic analysis. Although the performance-based design is believed to be the promising and powerful method in earthquake engineering, wind engineering and fire engineering, much effort needs to do to improve it the in the future. Although the N2 method is used to demonstrate pushover analysis, any other nonlinear static analysis methods can also be used to implement the pushover analysis. Since the multi performance levels of a structure under multi hazard levels are required to be satisfied, the optimization method may be the best procedure to implement the performance-based seismic design.

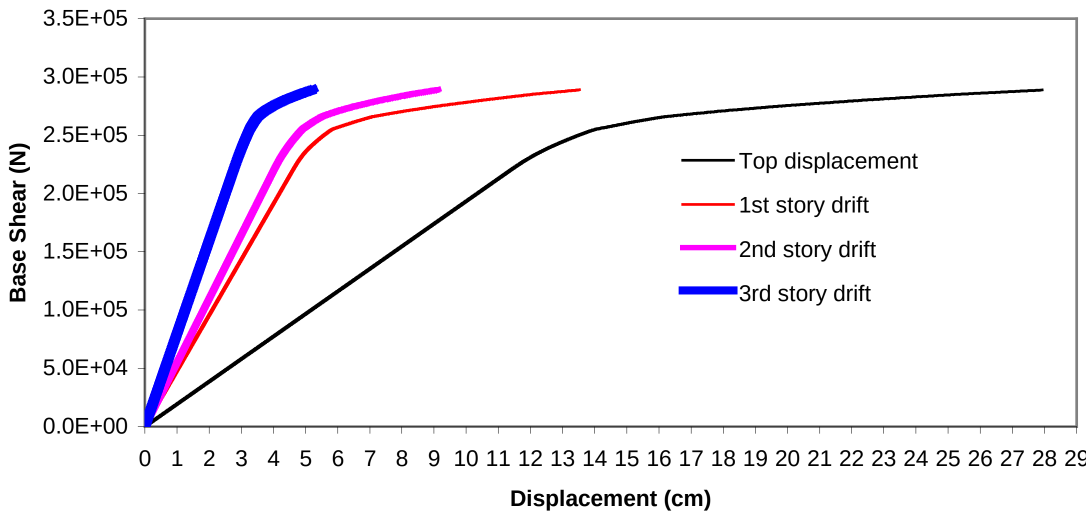

Figure 11. Pushover analysis to top displacement 28.06 cm ($\geq$27.88 cm) for optimum design

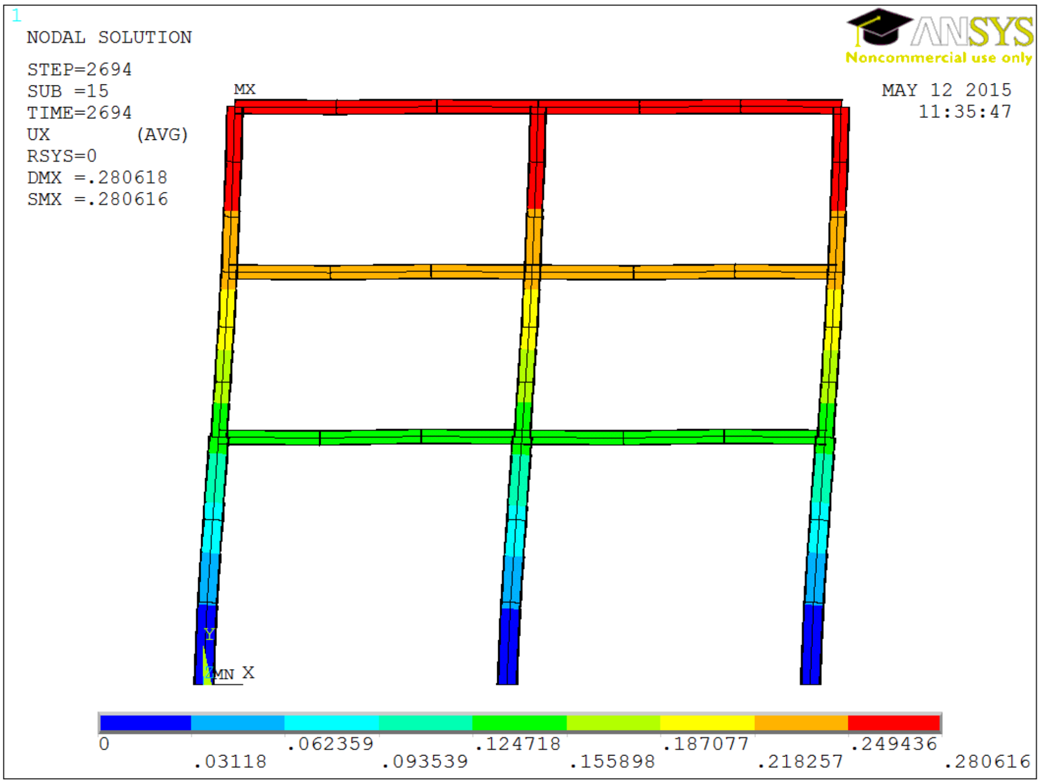

Figure 12. Lateral displacement of the optimum design

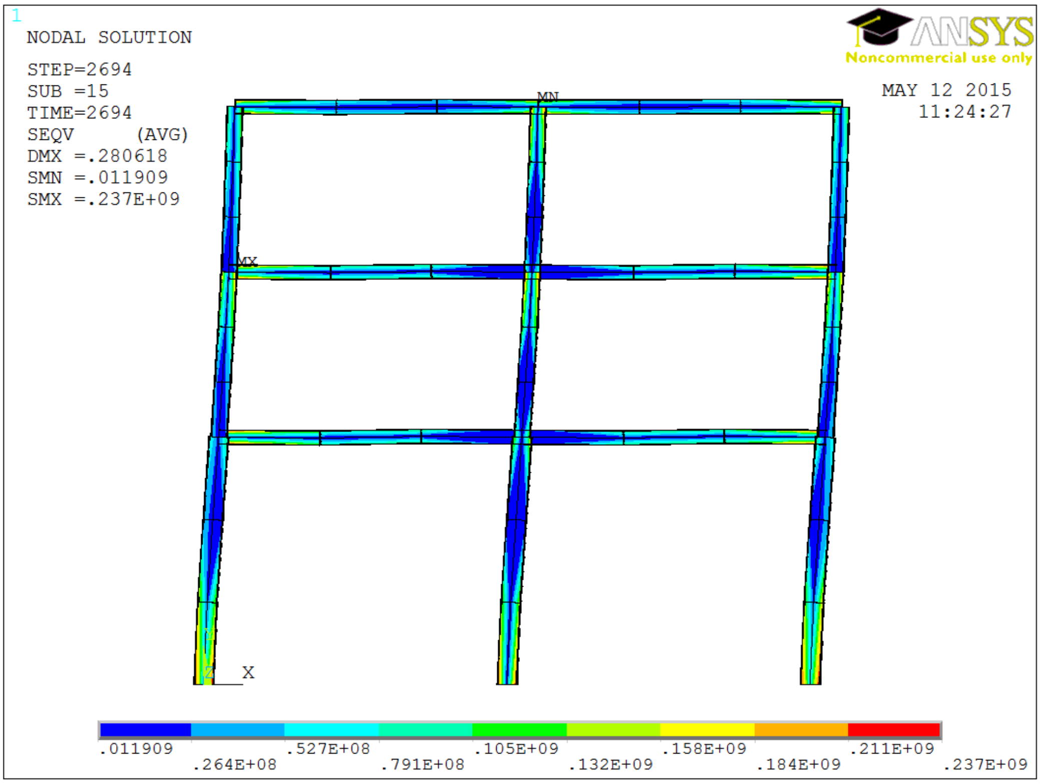

Figure 13. von Mises stress of the optimum design

Conclusion

The performance-based seismic optimization design is demonstrated in this paper to introduce the philosophy of the performance-based design. The basic concept and procedure of the performance based seismic design are introduced by using the engineers' language. The optimization design of a steel frame is demonstrated finally. The limitations of the current performance-based seismic design are discussed. It is pointed out that much effort needs to do to overcome the current limitations in the future.

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Qimao Liu and Juha Paavola
Aalto University
Department of Civil and Structural Engineering
P.O. Box 12100
00076 Aalto, Finland
qimao.liu@aalto.fi, juha.paavola@aalto.fi