

An approximate analytical method for design of thin-walled Z-purlins supported by sandwich panels

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Summary. In this paper the features of design of thin-walled metal structures are considered. Particularly the analysis of Z-cross section cold-formed purlins is represented. The method of calculation the value of spring stiffness C_{DA} for case of using sandwich panels is proposed. The possibility of applying proposed method is verified by experimental tests and numerical analysis done with the use of NASTRAN program.

Key words: metal structures, thin-walled structures, sandwich panel

Introduction

Nowadays buildings based on lightweight steel skeletons are widespread in civil and industrial engineering. The trapezoidal sheet or the sandwich panels fastened to thin-walled cold-formed purlins can be used to cover the building envelopes. Due to the loading of the sheeting by distributed load (dead load, snow load, etc.) the thin-walled purlin undergoes a restrained torsion (Figure 1).

This restraining can be represented as a rotational stiffness of the connection between the purlin and sheeting. In case of using sandwich panels as sheeting this rotational stiffness is not sufficiently studied. The aim of the research is investigation of the rotational stiffness of the connection “purlin-sandwich panel”.

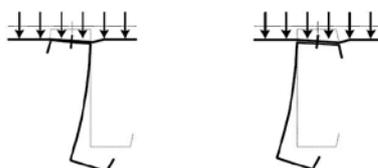


Figure 1. Purlin undergoes restrained torsion

Method proposed by Eurocode

Let us see the recommendations of Eurocode EN 1993-1-3 for the design of thin-walled purlins, which can be considered as laterally restrained beam. In Eurocode [1] this restraining is modeled as spring with rotational stiffness C_D (Figure 2).

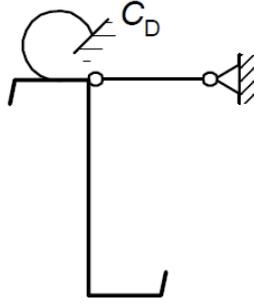


Figure 2. Restraint modeling by spring with rotational stiffness $C_{D,A}$

For simplification the rotational spring C_D can be replaced by a lateral spring stiffness K (Figure 3). Then the purlin can be regarded as a beam on an elastic foundation.

If the value of C_D and, respectively, K are known then, using the theory of beams on elastic Winkler foundation, the coefficient R can be calculated to determine the value of the correction factors for parameters of stress-strain state

$$R = \frac{KL_a^4}{\pi^4 EI_{fz}}$$

where I_{fz} is the second moment of area of the gross cross-section of the free flange plus the contributing part of the web for bending about the x-x axis; K is the lateral spring stiffness per unit length; L_a is the distance between anti-sag bars, or if none are present, the span L of the purlin.

In this case the lateral spring stiffness K can be determined as follows:

$$\frac{1}{K} = \frac{1}{K_A} + \frac{1}{K_B},$$

where K_A is the lateral spring stiffness corresponding to the rotational stiffness of the connection between the sheeting and the purlin and K_B the lateral stiffness due to the distortion of the cross-section of the purlin.

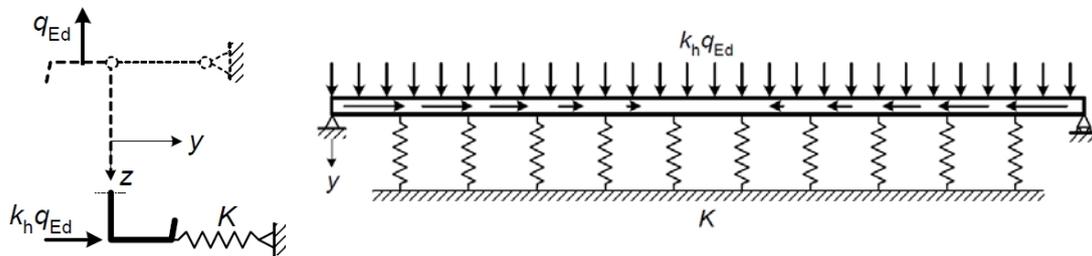


Figure 3. Purlin as a beam on an elastic foundation

The lateral spring stiffness K per unit length may be determined by calculating [1]:

$$\frac{1}{K} = \frac{4(1-\nu^2)h^2(h_d + b_{mod})}{Et^3} + \frac{h^2}{C_D}, \quad (1)$$

where a is the distance from the sheet-purlin fastener to the purlin web; b is the width of the purlin flange connected to the sheeting; C_D is the total rotational spring stiffness; h is the overall height of the purlin; h_d is the developed height of the purlin web; b_{mod} is determined as follows: (i) for cases where the equivalent lateral force bringing the purlin into contact with the sheeting at the purlin web: (ii) $b_{mod} = a$; for cases where the equivalent lateral force bringing the purlin into contact with the sheeting at the tip of the purlin flange: $b_{mod} = 2a + b$.

All parameters except C_D are known for the case of Z-purlin connected to sandwich panels. Total rotational spring stiffness C_D should be determined from:

$$\frac{1}{C_D} = \frac{1}{C_{D,A}} + \frac{1}{C_{D,C}},$$

where $C_{D,A}$ is the rotational stiffness of the connection between the sheeting and the purlin; $C_{D,C}$ is the rotational stiffness corresponding to the flexural stiffness of the sheeting.

The aim of the research is to study the rotational stiffness $C_{D,A}$. Its values can be obtained from a combination of laboratory tests and calculations.

The research

Experimental model

The test arrangement proposed in Eurocode for determining all required characteristics by testing is shown on Figure 4. In our case it consists of a purlin Z200x2 length 1 m, a sandwich panel SPC120/80PU, which are connected together by screws GT6 175-5.5/6.3 mm [2].

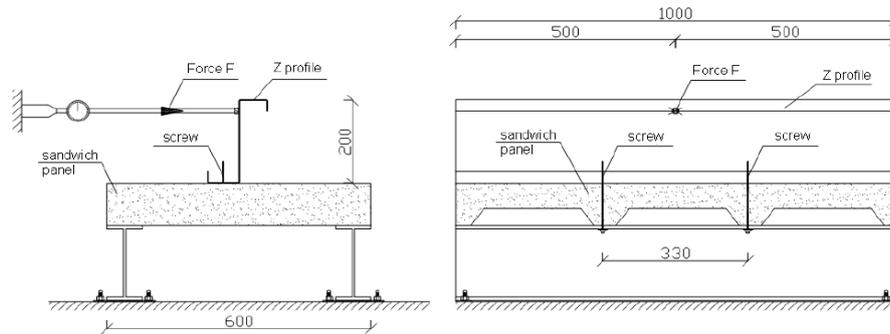


Figure 4. Scheme of laboratory stand

Numerical analyses

The same model is made in program NASTRAN (Figure 5). Using the test results [2] numerical analyses have been done. Purlins are modeled with using PLATE finite elements ($E=210000$ MPa, $\nu=0.33$), sandwich panels are modeled as three layer plates – flat and trapezoidal metal sheets (PLATE finite element, thickness 0.5 mm ($E=210000$ MPa, $\nu=0.33$)) and polyurethane core (SOLID finite element, $E=3.5$ MPa, $\nu=0.15$). The thickness of the polyurethane core of panel is in general 80 mm and in the crests 120 mm. Connection between the purlin and sandwich panel are modeled with using RIGID finite element, which connects one row nodes of purlin with nodes on the panel (independent node – on purlin, connecting DOF – all linear displacements). Screws are modeled as BEAM elements, on the external side of sandwich panel screw-nut it is modeled as 4 intersect beam element to distribute stresses and to realize more real behavior of construction.

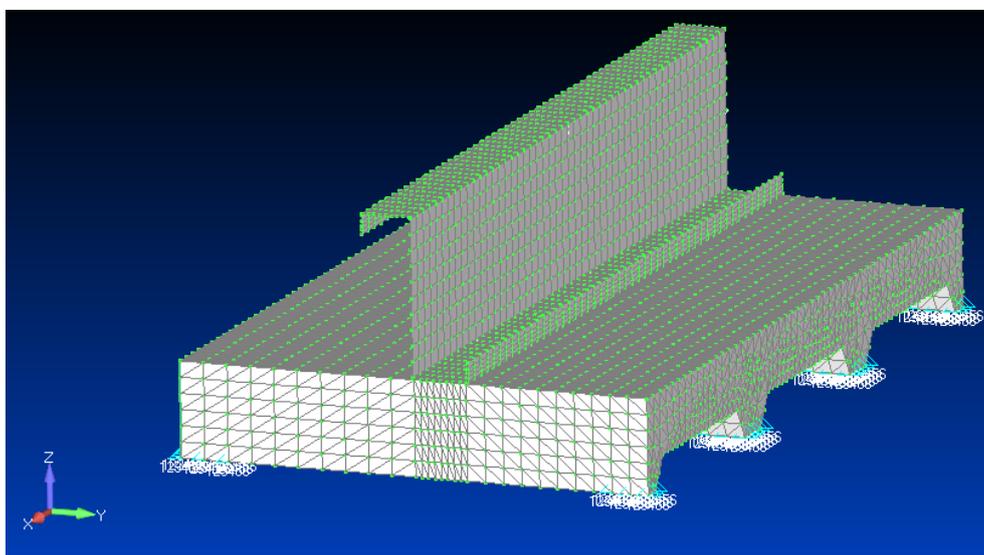


Figure 5. Model for numerical analysis

The values of force applied to the free flange are obtained by tests. The force causes the displacement of the free flange along Y-axis at $h/10$ (h – the height of the purlin). The linear analysis has been done. Analyses has been done for different directions of the load (uplift – T1S, T2S, gravity – T3P, T4P), different positions of the purlin and different positions of the screws on the sandwich panel (Figure 6). The results are presented in Table 1.

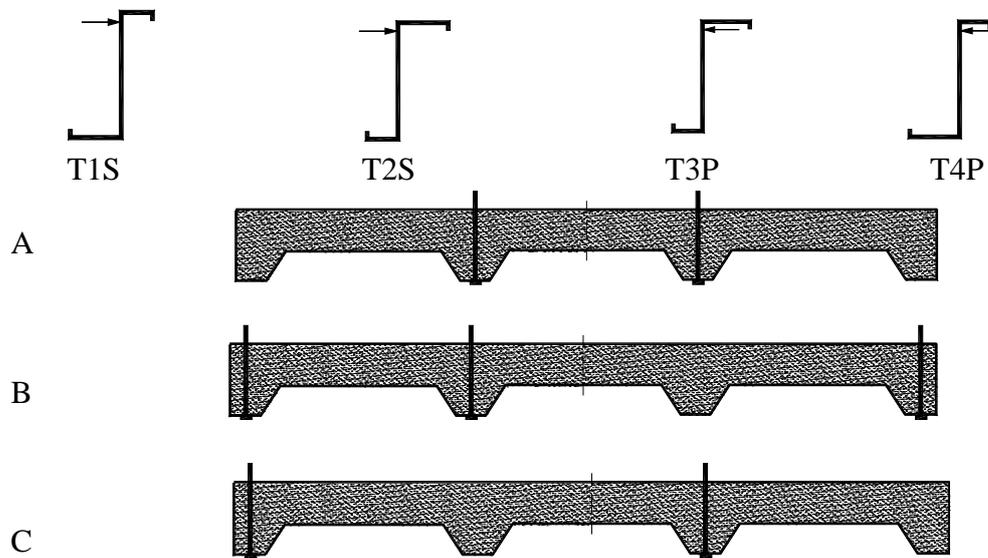


Figure 6. Schemes used in analysis.

Table 1. Displacement of free flange and error % of numerical analysis as compared to laboratory tests results

	Case	Force, N	Displacement , mm	Error, %
A	T1S	323.96	17.7	-11.5
	T2S	348.10	20.5	+2.5
	T3P	308.09	18.6	-7
	T4P	393.42	22.4	+12
B	T1S	319.39	17.7	-11.5
	T2S	307.64	18.5	-7.5
	T3P	346.38	20.2	+1
	T4P	344.91	17.4	-13
C	T1S	233.02	15.9	-20.5
	T2S	211.2	15.3	-23.5
	T3P	244.16	18	-10
	T4P	278.52	19.8	-1

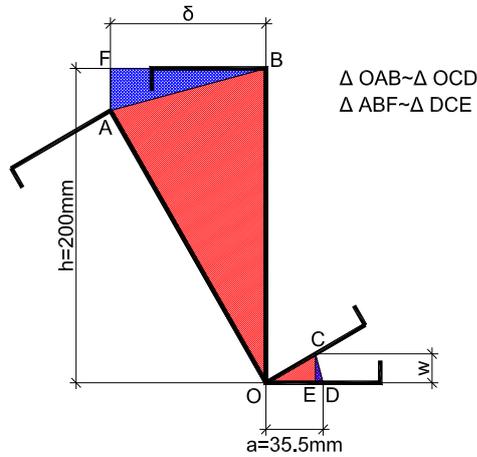


Figure 7. The deformations occurred in system (the triangles marked by the same colors are similar)

In Figure 7 the schematic deformations of the purlin are shown. These results are obtained if a screw is in the middle of the purlin flange, but in practice it is difficult to perform. And then to show how results change if analyses with different positions of the screw on the flange have been done.

The Eurocode 3 [1] was used to obtain the values of $C_{D,A}$ in each case. The lateral spring stiffness K may be determined from:

$$\frac{1}{K} = \frac{1}{K_A} + \frac{1}{K_B} = \frac{\delta}{F}, \quad (2)$$

where F is the load per unit length of the test purlin to produce a lateral deflection of $h/10$; δ is the displacement of the top flange in the direction of load F . Then the normative values of $C_{D,A}$ can be determined from equation (1):

$$C_{D,A} = \frac{h^2 / l_A}{(1 / K_A + 1 / K_B) - 4(1 - \nu^2)h^2(h_d + b_{\text{mod}}) / (Et^3l_B)}, \quad (3)$$

where l_A is the modular width of tested sandwich panel and l_B is the length of tested purlin.

Analytical investigation

To derive the analytic equation for $C_{D,A}$ we consider deformations occurred in system. If the screws are symmetrically situated along the length of purlin then it is possible to present this system like it is shown on Figure 8.

The purlin is in equilibrium with the external load and the force, occurred in the screw. The following equations can be derived:

$$\begin{aligned} \text{case a)} \quad \Sigma M_o &= Fh - nT \cdot b / 2 = 0 \\ \text{case b)} \quad \Sigma M_o &= Fh - nTa \cdot \cos \alpha = 0 \end{aligned} \quad (4)$$

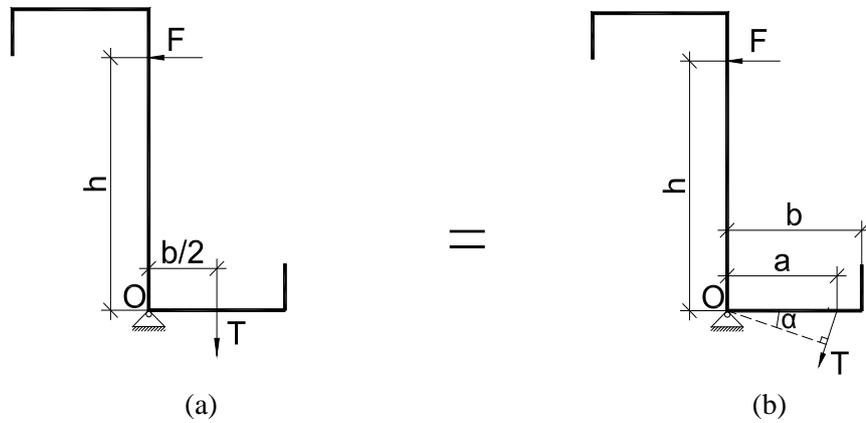


Figure 8. Models of purlin with respect of applied external force and axial force at the screws: a) the screw is in the middle of flange, b) the screw is not in the middle of flange.

where F is external lateral force, applied to the free flange; T is the axial force in the screw; n is number of screws per unit length; α is the angle between direction of perpendicular to the axial force in the screw and the flange of purlin (see Figure 9):

$$\alpha = \arctan \frac{|b/2 - a|}{t_p}$$

Further, to simplify calculation the case when the screw is in the middle of purlin flange (case a) will be considered. This system is equivalent to the similar one with spring having stiffness $C_{D,A}$ (Figure 10).

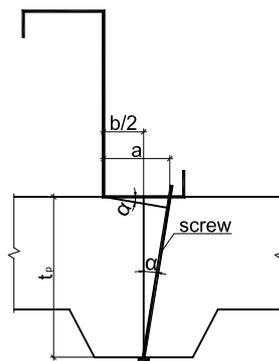


Figure 9. Determination of the angle α

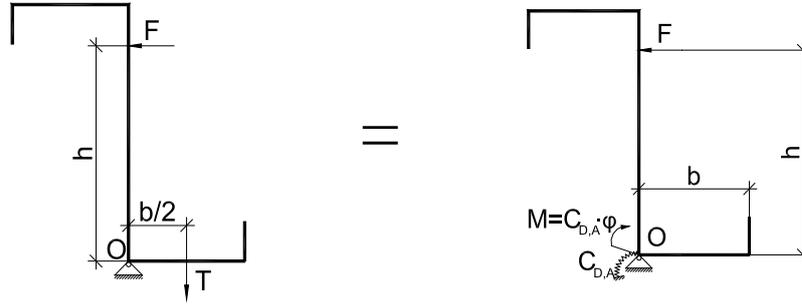


Figure 10. Equivalent models of purlin

The stiffness $C_{D,A}$ can be expressed as follows

$$C_{D,A} = \frac{F \cdot h}{\phi \cdot l_p}, \quad (5a)$$

where ϕ is angle of rotation about X-axis and l_p is the length of purlin. As angle ϕ is very small, it can be expressed as

$$\phi = \frac{\delta}{b/2} = \frac{2\delta}{b}, \quad (5b)$$

where $\delta = \delta_l + \Delta l$ (Figure 11), δ_l is the displacement of the bottom end of the screw in consequence of the deformation of the insulation in the sandwich panel, and Δl is the axial deformation of the screw.

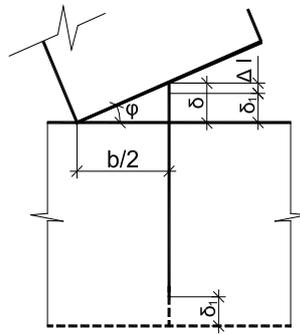


Figure 11. Scheme of screw deformation

Using equation (4) we can express the axial screw force

$$T = \frac{2Fh}{nb}.$$

We can obtain the expression for axial deformation of the screw

$$\Delta l = \frac{T \cdot l}{E_s A_s} = \frac{2Fhl}{nbE_s A_s} = \frac{2Fht_p}{nbE_s A_s}, \quad (7)$$

where E_s is elastic modulus of screw, A_s is the area of screw cross-section, and l is the length of screw, which experiences the deformation. In this case it can be replaced by thickness of the sandwich panel $l=t_p$

To find the value of δ_l it is possible to present the sandwich panel, especially its insulation layer, as a rod with characteristics of the core (E, ν) like polyurethane, and a diameter $d = 190$ mm based on numerical analysis, because only small region of insulation core near the screw undergoes deformations

$$\delta_l = \frac{T \cdot l}{E_p A_p} = \frac{2Fht_p}{nbE_p A_p} = \frac{2Fht_p}{nbE_p A_o}, \quad (8)$$

where

$$A_o = \frac{\pi d^2}{4} = \frac{\pi \cdot 0.19^2}{4} \text{ m}^2 = 0.028353 \text{ m}^2. \quad (9)$$

Then expressions (8) and (9) can be used in equation (5a)

$$C_{D,A} = \frac{nb^2}{4t_p \left(\frac{1}{E_p A_o} + \frac{1}{E_s A_s} \right) \cdot l_p}. \quad (10)$$

Doing the same for case when screw is not in the middle of purlin flange we can obtain the following expression of $C_{D,A}$ [Nm/m]:

$$C_{D,A} = \frac{na^2}{t_p \left(\frac{1}{E_p A_o} + \frac{1}{E_s A_s} \right) l_p}, \quad (11)$$

where t_p is the thickness of sandwich panel, b the width of purlin flange to which sandwich panel is fastened, a the distance between the screw and the purlin web, E_s the elastic modulus of screw; A_s the cross-sectional area of screw, E_p the elastic modulus of insulation in the sandwich panel, A_o the area of substituting polyurethane rod, and n the number of screws per unit length.

Comparison of the results

Analytic and numerical analyses have been done and results of the comparison of $C_{D,A}$ are presented in Table 2. The results, calculated using the proposed formula (11) ($C_{D,AI}$), obtained from the numerical analysis ($C_{D,A}$) and from the laboratory test (Test $C_{D,A}$) with the results given by the Eurocode recommendations (2) - are shown in the Table 2.

In Table 2 the values of stiffness $C_{D,A}$ and displacement δ are shown for different cases when screw is located in 7 points along all width of purlin flange. But in reality so big deviation of screw from the middle of purlin flange unacceptable. And during the mounting of the construction such deviations will be avoided. It is only theoretical values. The area of real acceptable in reality deviations is about 10-15 mm from the middle of the purlin flange (for T1SA and T4PA $a=20-50$ mm, for T2SA and T3PA $a=15-45$ mm).

Table 2. Comparison the values of stiffness $C_{D,A}$

Case	F, N	$\delta,$ mm	$1/K,$ $m^2/N \cdot 10^{-5}$	$C_{D,A}$ (3), Nm/m	$C_{D,AI}$ (11), Nm/m	a, mm	Error %	Test $C_{D,A},$ Nm/m
T1SA	323.96	30.1	9.29127	901.591	173.922	10.375	80.71	1646.04
		26.6	8.21089	1068.64	568.041	18.75	46.84	
		21.4	6.60575	1462.70	1188.82	27.125	18.72	
		17.7	5.46364	1995.91	2036.26	35.5	2.02	
		15	4.6302	2742.13	3110.36	43.875	13.43	
		13.3	4.10545	3641.15	4411.13	52.25	21.15	
		12.1	3.73503	4813.51	5938.55	60.625	23.37	
T2SA	348.10	32.8	9.42258	884.800	142.010	9.375	83.95	1840.61
		29.2	8.38839	1036.16	453.322	16.75	56.25	
		24.6	7.06693	1321.17	940.399	24.125	28.82	
		20.5	5.88911	1753.60	1603.24	31.5	8.57	
		17.6	5.05602	2297.18	2441.84	38.875	6.30	
		15.6	4.48147	2948.48	3456.21	46.25	17.22	
		14.1	4.05056	3780.06	4646.35	53.625	22.92	
T3PA	308.09	11.3	3.66776	3745.65	4646.35	9.375	24.05	1526.20
		13.1	4.252	2901.59	3456.21	16.75	19.11	
		15.5	5.031	2214.78	2441.84	24.125	10.25	
		18.6	6.0372	1687.85	1603.24	31.5	5.01	
		22.8	7.40043	1270.68	940.399	38.875	25.99	
		27.7	8.99088	984.670	453.322	46.25	53.96	
		32.1	10.419	819.795	142.010	53.625	82.68	
T4PA	393.42	13.3	3.38061	4493.35	5938.55	10.375	32.16	2252.14
		15.5	3.93981	3388.64	4411.13	18.75	30.17	
		18.3	4.65152	2560.61	3110.36	27.125	21.47	
		22.4	5.69366	1867.32	2036.26	35.5	9.05	
		27.4	6.96457	1398.92	1188.82	43.875	15.02	
		34.3	8.71842	1034.57	568.041	52.25	45.09	
		41.2	10.472	820.793	173.921	60.625	78.81	

The results are also represented in graphs (Figure 12). In graphs the values of alternative $C_{D,A}$ are also shown.

The Alternative $C_{D,A}$ is now proposed by Eurocode [1]. It can be taken as equal to $130p$ [Nm/m], where p – is the number of sheet fasteners to purlin per meter length of purlin. In the case A $p = 2$ and alternative $C_{D,A} = 260$ Nm/m.

In graphs there are shown the boundaries (red vertical lines) of the values of screw deviation from the middle of flange which are possible in reality.

As we can see from graphs (Figure 12) and Table 2, values of $C_{D,A}$ (when the screw is in the middle of flange) calculated by Eurocode equations and by approximate method do not differ more than 10%. But these analytical results are obtained only for case when screws are located symmetrically along the purlin flange (case A)

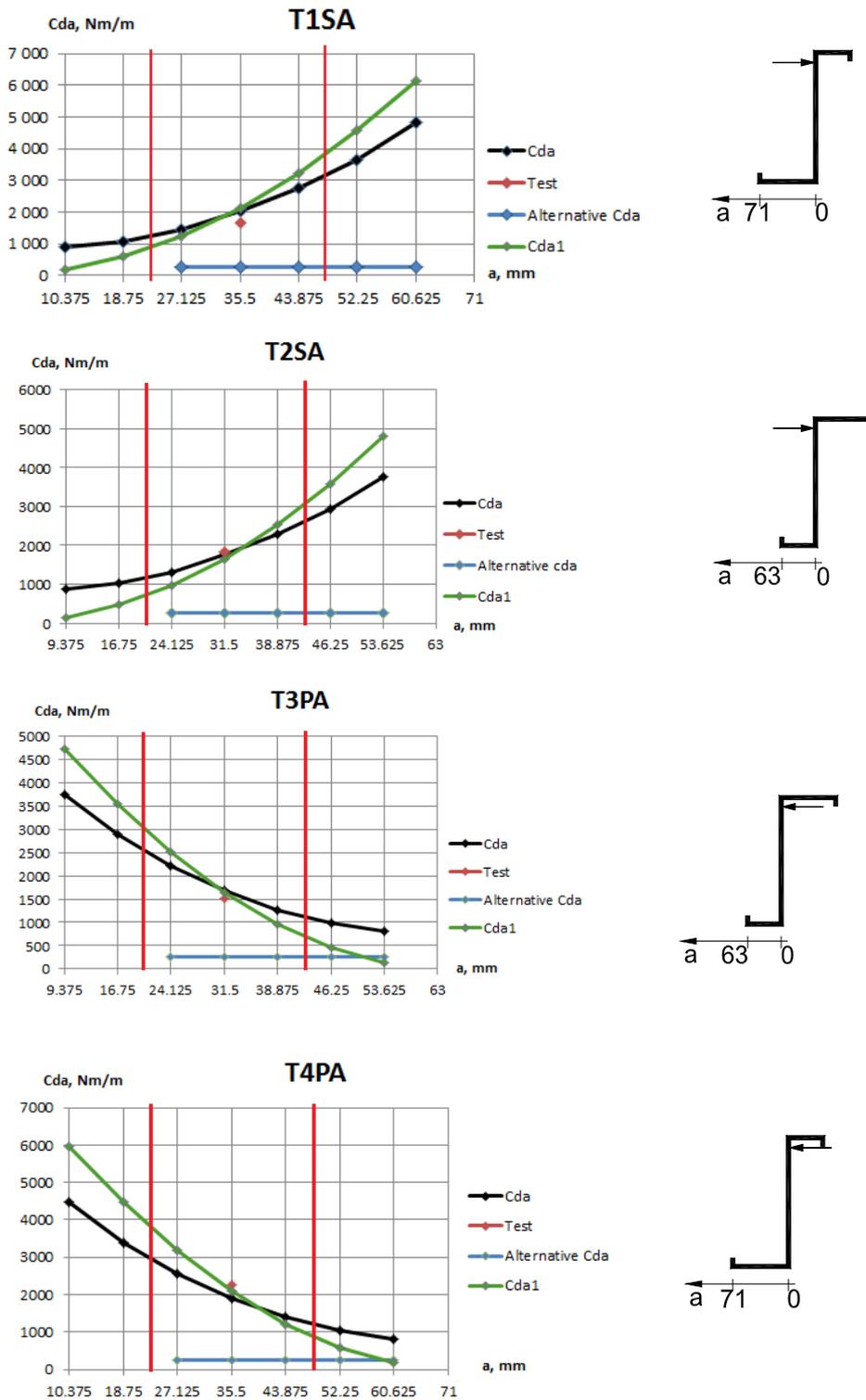


Figure 12. The graphs showing the dependence between stiffness C_{DA} and the position of screw along the purlin flange for case A (between the vertical red lines – the area of the deviation of screw from the middle of the purlin flange possible in reality)

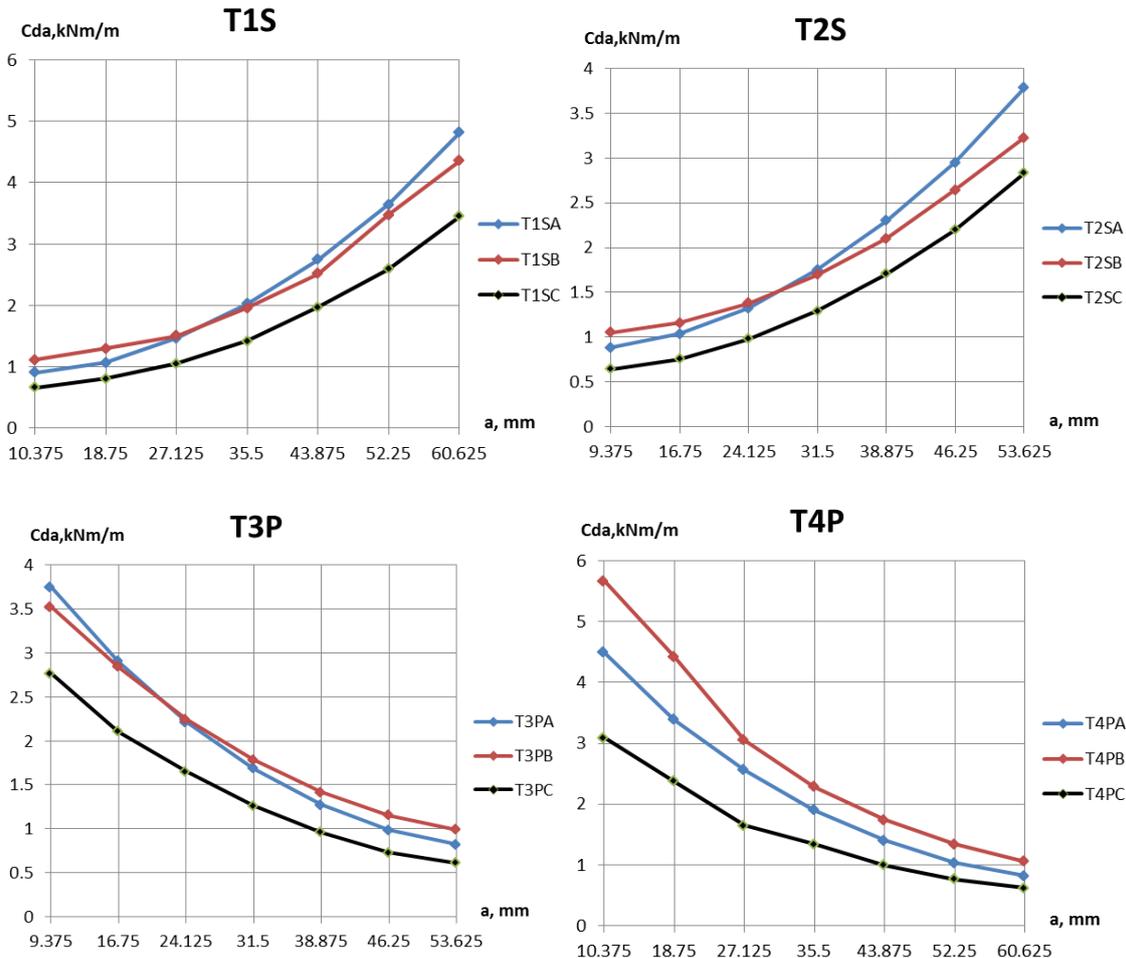


Figure 13. The graphs showing the dependence between stiffness C_{DA} obtained by proposed method and the position of screws along the purlin flange for different directions of load

Also the estimation of the influence of the position of screws along the length of purlin flange on the value of $C_{D,A}$ is done. The results are represented in graphs in Figure 13, where the dependences between stiffness $C_{D,A}$ and distance from screw to purlin web for different cases of screws positions along purlin flange are shown.

As seen from Figure 13 the highest value of $C_{D,A}$ is resulted in the screw configuration A – with symmetrical location of screws and in some cases the configuration B – with 3 screws (Fig. 6).

Conclusions

It can be concluded that proposed method has an error in comparison with results obtained by numerical analysis (recommended by Eurocode) not greater than 10% if the screw is in the middle of purlin flange in case of symmetrical location of screws along purlin flange. It means that proposed method can be used in practice in design. This also

revealed that the stiffest connection in the most case is the one with symmetrical positions of screws along the width of the sandwich panel (case A). Based on the results of the investigation we can recommend to make such connections with symmetrical positions of screws along purlin flange and apply the proposed method (equation (11)).

References

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