

## Steel building optimization applying metamodel techniques

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**Summary.** The paper demonstrates the use of metamodels in a cost optimization of single story industrial or commercial steel building including one span symmetric tubular steel roof trusses. A cost optimal solution for truss and wall combination is presented by varying the unit cost of the wall. One span truss metamodel is used. The main conclusions are: Optimize larger compositions, not only one product; cost optimal span/height ratio for one span Warren-type truss is 10; for low cost walls, say 50 - 70 €/m<sup>2</sup>, the optimal span/height ratio is 12; for more expensive walls the span/height ratio should be more than 12; careful building of metamodels enables their use in many kinds of optimization problems to quickly get results for preliminary design; the metamodels are essential tools in optimization still a long time, although software and hardware are getting more and more efficient.

*Key words:* metamodel, tubular steel truss cost optimization, truss+wall optimization

### Introduction

In building projects the fabricators of single products want to optimize the costs of their products to get a contract. The sum of optimized products is not necessary the cost optimum for the client. This is shown to be true using a simple example. The paper demonstrates the use of metamodels in a cost optimization of single story industrial or commercial steel building including one span symmetric tubular steel roof trusses. The section of the building is shown in Fig. 1.

Metamodels are used to build an approximation of a problem which has a high computational cost (Diaz et al., 2012). Metamodels are used in optimization and similar problems to avoid evaluations using computational heavy tasks, such as non-linear finite element analyses. The process of developing a metamodel consists of three steps (Diaz et al., 2012): design of experiments (DoE), metamodel building and validation. In this paper the focus is in demonstration of the use of a metamodel in an optimization problem.

### Building the metamodel

There are plenty of metamodeling techniques and optimization approaches using metamodels (Wang and Shan, 2007). The one used in this paper is as follows. Tubular steel trusses have been optimized with different spans, heights and loading widths before final optimization of the building in the roof of which the trusses are placed. Optimal truss solutions are stored in a table. Using interpolation functions, these solutions are used to approximate the mass and cost of trusses with arbitrary span, height and loading width. The sample points are gotten with a rather heavy topological, sizing and geometry optimization of tubular steel trusses (DoE). Theoretical background for sample point calculations is given by Bzdawka and Heinisuo (2012).

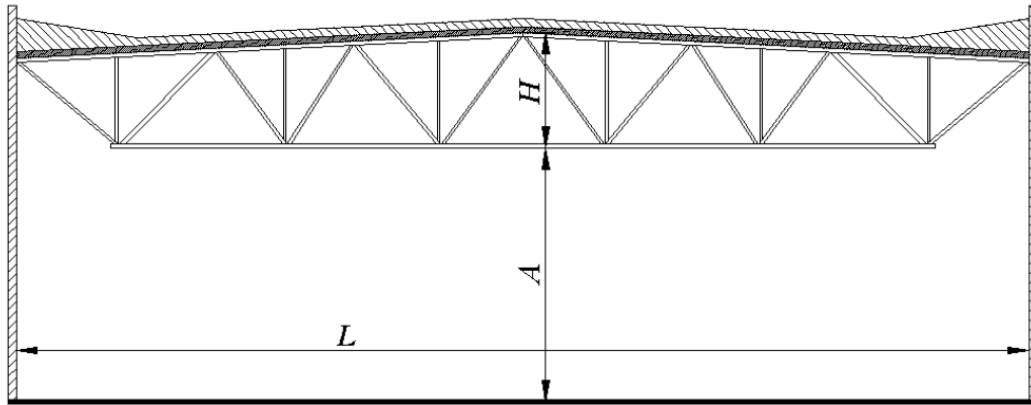


Figure 1. Section of the building.

The trusses are optimized using the cost as criterion and then the respective structural masses were calculated for cost optimal trusses. The truss costs are calculated based on used materials and features of fabrication with following actions (Jármai and Farkas, 1999; Jalkanen, 2007): preparation, assembly and tack welding; joint welding and additional tasks such as electrode changing, deslagging, chipping; surface preparation; painting; cutting and edge grinding. The feasibilities of trusses are checked based on relevant Eurocode standards (EN 1993-1-1, 2005; EN 1993-1-8, 2005) meaning resistance checks of members and joints. The linear beam FE structural analysis model includes semi-rigid joints (Snijder et al., 2011) with corresponding eccentricities between chords and braces. Particle swarm optimization (PSO) algorithm (Kennedy and Eberhart, 1995) was used with variations of parameters. The design variables were member sizes of chords and braces, and locations of nodes along the chords of the truss.

Cost of a truss structure is specific to manufacturing methods used as well as labour and material costs at given time and place. Figure 2 illustrates typical cost breakdown of Warren-type one span tubular truss (Bzdawka and Heinisuo, 2012) made in Western Europe (material cost of 1.5 €/kg (Haapio, 2012) and labour cost, manufacturing methods and parameters as in (Bzdawka and Heinisuo, 2012)). It shows that, even though being the most important factor, material only represents a little over one third of the total cost.

Fig.3 shows example of the layouts of optimal trusses at different spans. Contrary to widely used practises it seems that in optimal geometry of a truss the distances of connections are unequal and that regardless of the span the layouts look very similar.

The sample points have been calculated with 15 PSO runs with the population 50 and with 125 iterations and 10 PSO runs with the population 70 and with 175 iterations were done keeping other parameters of PSO as in (Bzdawka and Heinisuo, 2012). PSO is a stochastic method so many runs are required to ensure convergence. CPU time to perform one truss (fixed span  $L$  and height  $H$ ) calculation was about 36 hours. The characteristic dead load of the roof is 1 kN/m<sup>2</sup> and the characteristic snow load on the roof is 2 kN/m<sup>2</sup>. Finnish NAs are used for Eurocode design. The steel material grade is S420. Roof slope is 1:20.

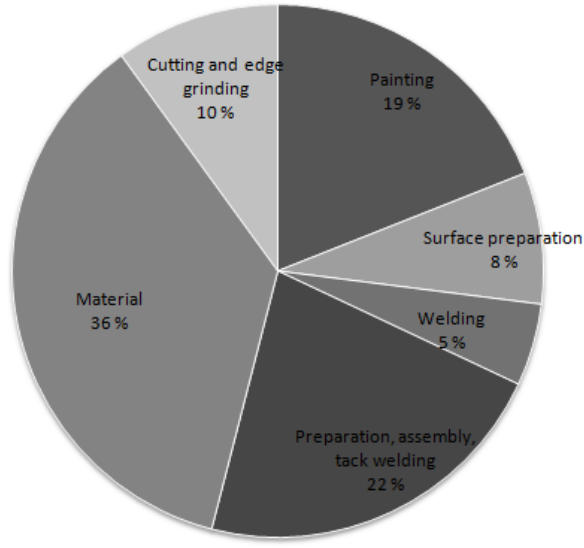


Figure 2. Cost breakdown of a typical warren-type truss (Bzdawka and Heinisuo, 2012).

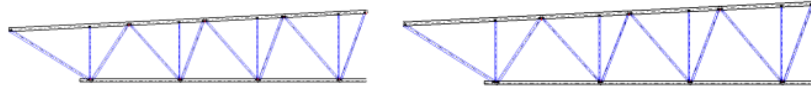


Figure 3. Layouts of optimal trusses.

### Data from metamodel and validation

The optimal tubular steel trusses with different heights  $H$ , spans  $L$  and loading widths  $C/C$ <sup>1</sup> are shown in Appendix A. The values often needed in optimization of a building are the weight or cost of a (nearly) optimal truss with arbitrary span, height and loading width. Therefore, following interpolation approach was adopted.

Consider four points  $\mathbf{x}_i = (L_i, H_i, C/C_i)$ ,  $i = 1, 2, 3, 4$ . The costs  $c_i$  and masses  $m_i$  of optimum trusses (metamodel sample points) are known. By using the four values, the mass and cost related to given point  $\mathbf{x}^* = (L^*, H^*, C/C^*)$  must be acquired. One way is to weigh the four points with weights  $w_i$  and take weighted sum of points  $\mathbf{x}_i$  to get the mass and cost of point  $\mathbf{x}^*$ .

This leads to following equations:

$$w_1 L_1 + w_2 L_2 + w_3 L_3 + w_4 L_4 = L^* \quad (1)$$

$$w_1 H_1 + w_2 H_2 + w_3 H_3 + w_4 H_4 = H^* \quad (2)$$

$$w_1 C/C_1 + w_2 C/C_2 + w_3 C/C_3 + w_4 C/C_4 = C/C^* \quad (3)$$

The sum of weights is required to be one:

$$w_1 + w_2 + w_3 + w_4 = 1 \quad (4)$$

By solving the equations the coefficients  $w_i$  can be acquired and they can be used to calculate cost  $c^*$ :

$$c^* = w_1 c_1 + w_2 c_2 + w_3 c_3 + w_4 c_4 \quad (5)$$

<sup>1</sup>The notation  $C/C$  refers to center to center distance of two trusses, a typical loading width for a single truss

For mass, respectively:

$$m^* = w_1m_1 + w_2m_2 + w_3m_3 + w_4m_4 \quad (6)$$

The four points are chosen by first normalizing the sample point values and then taking the closest four by Euclidian distance. Then if the four points are in the same plane in the three dimensional variable space, the Equations 1–4, cannot be solved. Then in this procedure, the second closest point is replaced with fifth closest.

When this approach is used, the design variables (member sizes, geometry, topology et cetera) of the actual truss remain unknown. In preliminary design the cost information is often enough. If in the later phases the design variable values are needed truss optimization with suitable parameters has to be run.

To validate the interpolation procedure the values were calculated at span length  $L = 33.3$  m and optimized with respective values. The results can be seen in Table 1. Based on this, the interpolated results seem to be on the safe side compared to optimized results, meaning larger costs and weights in this case. The difference compared to values acquired with PSO varies from 5.6 % to - 3.2 % the mean of absolute value of relative differences being 4.475 % at mass and 2.75 % at cost.

Table 1. Masses and costs of optimal trusses and the values predicted by metamodel at span  $L = 33.3$  m.

| Span [m] | $L/H$ | $C/C$ [m] | Interpolated |          | Optimized |          | Relative difference |          |
|----------|-------|-----------|--------------|----------|-----------|----------|---------------------|----------|
|          |       |           | Mass [kg]    | Cost [€] | Mass [kg] | Cost [€] | Mass [%]            | Cost [%] |
| 33.3     | 8     | 6         | 2919.2       | 10944    | 2763.8    | 10404.6  | 5.6                 | 5.2      |
| 33.3     | 10    | 6         | 2718.3       | 9944.5   | 2591.6    | 9562     | 4.9                 | 4.0      |
| 33.3     | 12    | 6         | 2845.5       | 10159    | 2940      | 10174.2  | -3.2                | -0.1     |
| 33.3     | 14    | 6         | 3260         | 11020    | 3129      | 10832.4  | 4.2                 | 1.7      |

## Optimal truss and wall combination

In a building there are usually also other parts than just trusses. It is clear that in building seen in Fig. 1 the amount of wall area and thus the cost related to walls is dependent of the truss height. The height  $H$  of the symmetric one span roof truss has effect to the one storey building's wall cost, if free height  $A$ , span  $L$  and distances of frames  $C/C$  are fixed. Note that height  $H$  is distance between topmost point of the top chord and bottom surface of the bottom chord. The truss has its optimal height with respect to the cost of the truss.

The cost optimum  $L/H = 10$  is old "rule of thumb" which has been used by designers for a long time. Looking at Tables 1 and 2 it could be said that in most cases it is valid. But there are exceptions. Consider a truss of span  $L = 30$  m and  $c/c = 14.4$  m. At  $L/H = 8$  the cost is 15425.2 € and at  $L/H = 10$  the cost is 15851.8 €. The higher truss costs only 2.8 % more so  $L/H = 10$  is a pretty good guess in this case, too. But is this optimal height also optimal for truss and wall combination?

To find out this, cost optimal solution for truss and wall combination is sought after with cases varying the unit cost (€/m<sup>2</sup>) of the wall. One span secondary truss metamodel is used. The size of the building is 600 m<sup>2</sup> and two cases are considered: truss span 20 m and 33.3 m keeping the frame distance as 6 m. The wall costs are varying between

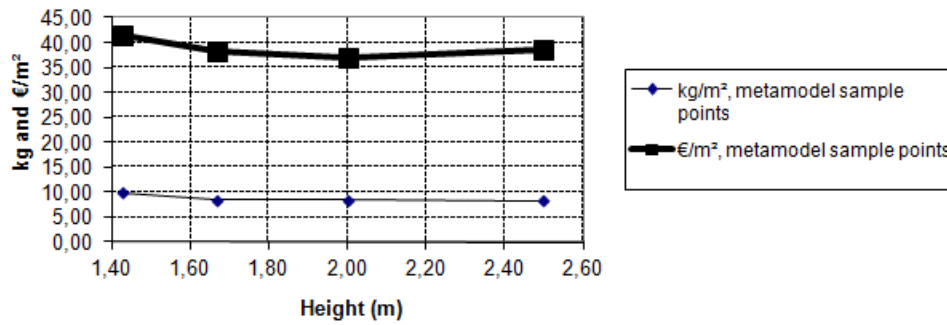


Figure 4. The weights and costs of optimal trusses for the span 20 m and loading width 6 m.

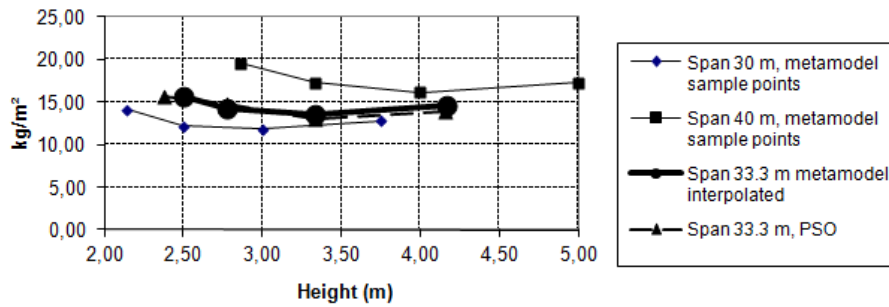


Figure 5. Weight of the trusses of different spans per floor area.

50 – 150 €/m<sup>2</sup> meaning realistic minimum and maximum costs of the day in Finland. Steel sandwich panel cost is about 50–70 €/m<sup>2</sup> and concrete wall is about 100–120 €/m<sup>2</sup> (Mittaviiva Oy, 2010).

The capital costs of the structures are calculated only for trusses and walls at truss height  $H$ . It is supposed that the trusses cover the entire floor area so the costs of the trusses are  $600 \cdot E_t$  where  $E_t$  is the truss cost per m<sup>2</sup> got from the metamodel. The wall cost for 20 m span building is  $100 \cdot H \cdot E_w$  and for 33.3 m span  $102.7 \cdot H \cdot E_w$  where  $E_w$  is the unit cost of the wall. The weights and costs of optimal trusses for the span 20 m are shown in Fig. 4. It can be seen that the weight optimum is at  $H = 2.5$  m but the cost optimum is at  $H = 2$  m.

In Fig. 5 it can be seen that the weight optima are in all cases  $L = 30, 33.3$  and 40 m at  $L/H = 10$ . The costs for the same spans are shown in Fig. 6.

Fig. 7 shows unit costs of steel at different spans. The unit cost ranges from 3.2 to 4.6

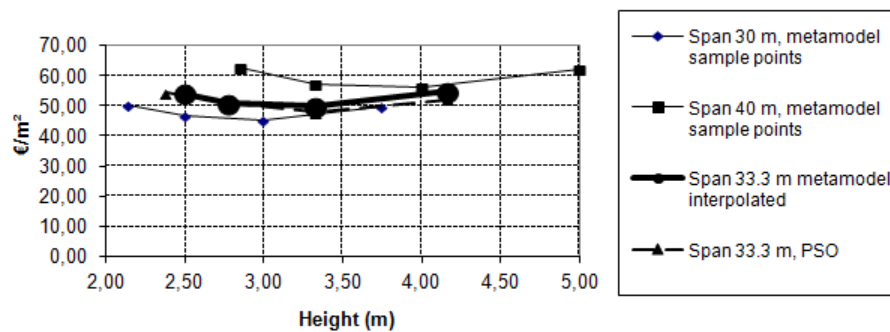


Figure 6. Optimal costs of trusses at different spans.

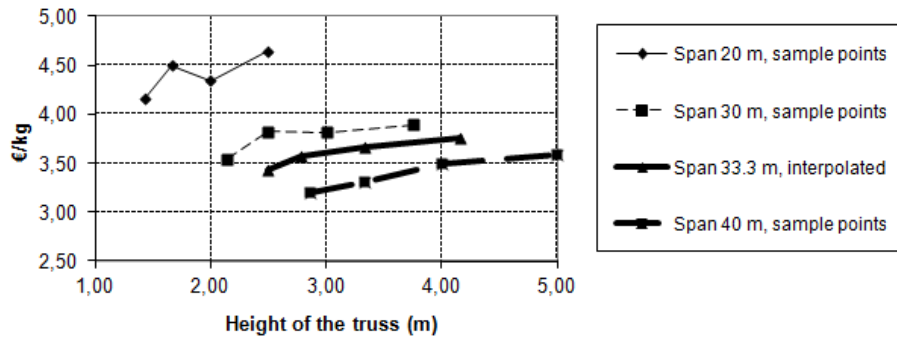


Figure 7. Unit costs of steel in trusses at different spans.

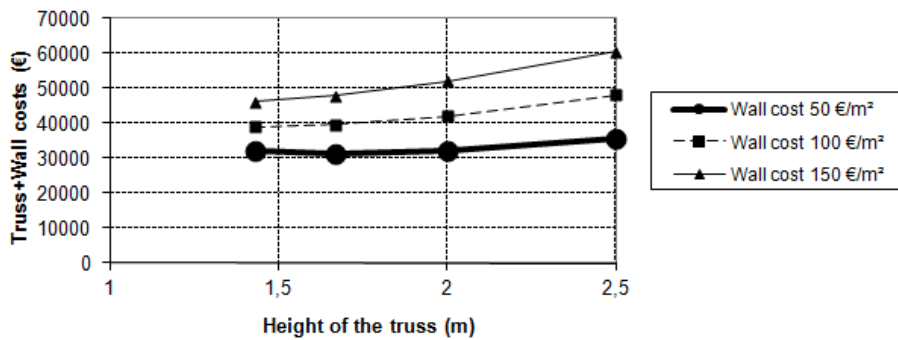


Figure 8. Truss and wall costs for 20 m span and loading width of 6 m.

€/kg so traditional structural optimization approaches where the weight of the structure is used as criterion can be considered more or less inaccurate.

Fig. 8 presents the truss and wall cost for 20 m span. It can be seen that when the wall unit cost is 50 €/m<sup>2</sup> then the optimal truss height is about 1.67 m ( $L/H = 12$ ), not at 2 m which is the truss optimum. If the wall cost is larger then the best truss height is 1.43 m and smaller. Fig. 9 shows truss and wall costs for the building with truss span 33.3 m.

The optimum can be found for the lowest wall cost 50 €/m<sup>2</sup> and the optimal truss height is 2.78 m ( $L/H = 12$ ), again not the same as for the trusses only ( $H = 3.33$  m). When using optimal truss (3.33 m) for truss and wall combination for span 33.3 m, then the total costs are 5, 10 and 14 % more than smallest in Fig. 9 for wall cost 50, 100 and

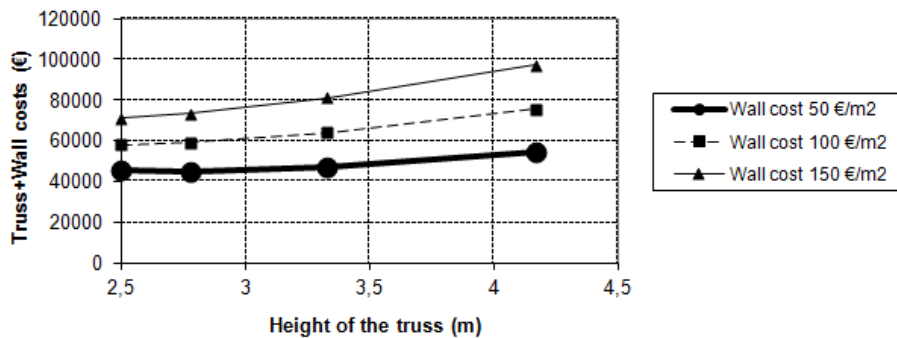


Figure 9. Truss and wall costs for 33.3 m span and loading width of 6 m.

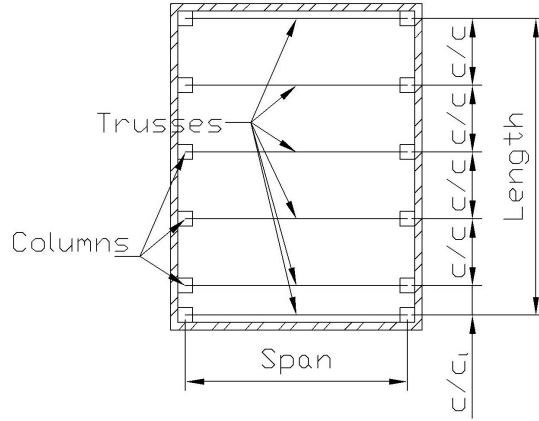


Figure 10. Building layout and dimensions.

150 €/m<sup>2</sup>, respectively.

### Preliminary design of a hall

The metamodel was created for optimization purposes. The example showed in the earlier section was fairly simple manual optimization without optimization algorithms. In this section a more realistic but still simple optimization problem example using metamodels is presented. Consider 1200 m<sup>2</sup> hall using Warren type roof trusses shown in Figures 1 and 10. The building is rectangular and is constructed of columns, roof trusses and walls. Other structures such as roof or floor are omitted. Only vertical loads are taken into account.

By using design variables distance  $C/C$ , span  $L$  and the height ratio  $L/H$  of the truss, a simple optimization problem for cost optimal building could be written as:

$$\begin{aligned} \min f(\mathbf{x}) &= n_c c_c + n_t c_t + A_w c_w & (7) \\ \text{s.t. } & 6 \text{ m} \leq C/C \leq 14.4 \text{ m} \\ & 8 \leq L/H \leq 14.4 \\ & 10 \text{ m} \leq L \leq 40 \text{ m} \end{aligned}$$

where  $n_c$  is the number columns (twice the number of trusses),  $c_c$  cost of column,  $n_t$  the number of trusses,  $c_t$  cost of a truss,  $A_w$  is the area of the wall, and  $c_w$  is the unit cost of the wall. Given a free height of  $H_f$ , area of the wall can be calculated by

$$A_w = 2(L + A_f/L)(H_f + H) \quad (8)$$

where  $A_f$  is the floor area. Cost  $c_c$  is acquired from another metamodel for steel column cost. Fig. 11 shows the relation between cost and resistance for axial resistance for steel column at length 6 m and buckling lengths at both directions 12 m (cantilever columns supposed).

This optimization problem was solved with different wall unit costs with *genetic algorithm* in Global optimization toolbox of Matlab (Mathworks, 2010). A single optimization took about 30 seconds with several runs of genetic algorithm. As opposed to an approach where truss optimization would be included in the problem as whole, the saving in computational time is quite significant. The results presented in Figures 12 and 13 shows the same phenomenon that was found in the first example. The optimal layout of building is

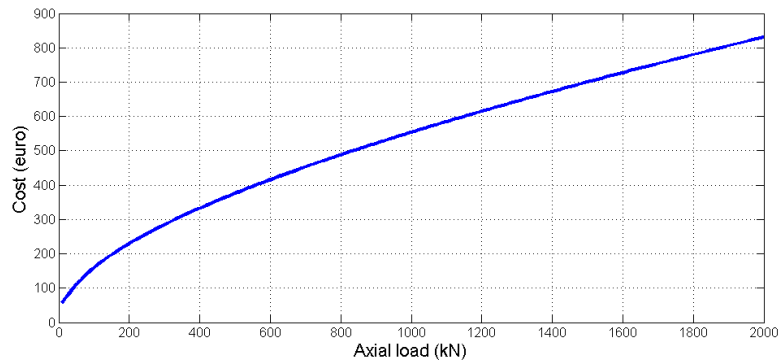


Figure 11. Cost of steel column as function of axial resistance.

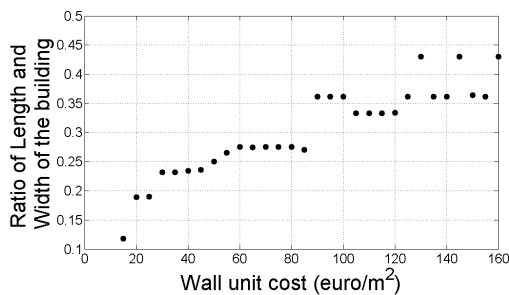


Figure 12. The ratio of length and width of optimal steel frame building.

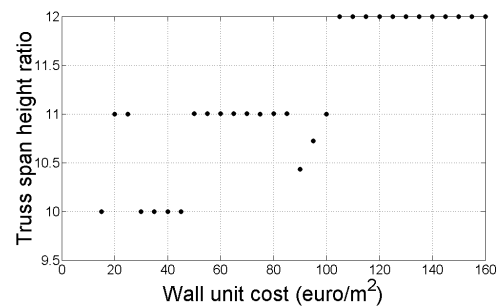


Figure 13. The ratio of span and height of truss used in optimal steel frame building.

dependent of the wall cost and therefore the optimization should consider the all relevant parts of a composition, not only a single product. In this example many simplifications were made. Foundations, wind loads, floor structures, bracing et cetera were omitted. Also, some design variables presented here as continuous might actually be discrete due to product sizes available. To make further conclusions or design guidelines, more realistic optimization problems have to be formulated.

## Conclusions

In this paper a truss metamodel to be used in building preliminary design optimization is presented. The technique seems promising and it cuts down the calculation times remarkably. This comes with a cost that the members sizes, optimal truss topology and geometry are not known. But at the later design phase they are needed, regular optimization techniques can be used to find them.

Optimization of truss only leads to unsatisfactory results when the whole building is considered. Therefore, optimizing larger compositions is recommended. For a Warren-type truss only, cost optimal truss span-height ratio is normally around 10. If wall cost is low, say 50 - 70 €/m<sup>2</sup>, optimal truss span-height ratio is about 12 but with more expensive walls the ratio should be higher.

## Acknowledgements

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## Appendix A

Table 2. Calculated optimal trusses.

| Span [m] | $L/H$ [-] | C/C [m] | Mass [kg] | Cost [€] | Span [m] | $L/H$ [-] | C/C [m] | Mass [kg] | Cost [€] |
|----------|-----------|---------|-----------|----------|----------|-----------|---------|-----------|----------|
| 10       | 8         | 6       | 216.26    | 1570.22  | 30       | 8         | 12      | 3918.8    | 13518.4  |
| 10       | 8         | 8.4     | 284.56    | 1851.8   | 30       | 8         | 14.4    | 4687.4    | 15425.2  |
| 10       | 8         | 12      | 385.6     | 2343.4   | 30       | 10        | 6       | 2139      | 8156.8   |
| 10       | 8         | 14.4    | 555       | 2888.4   | 30       | 10        | 8.4     | 2737.6    | 9851.4   |
| 10       | 10        | 6       | 218.62    | 1515.52  | 30       | 10        | 12      | 3766.6    | 13308.6  |
| 10       | 10        | 8.4     | 299.96    | 1944.72  | 30       | 10        | 14.4    | 4967.6    | 15851.8  |
| 10       | 10        | 12      | 494.38    | 2612.8   | 30       | 12        | 6       | 2196.6    | 8394     |
| 10       | 10        | 14.4    | 520.6     | 2846.6   | 30       | 12        | 8.4     | 3082      | 10706.2  |
| 10       | 12        | 6       | 247.44    | 1667.04  | 30       | 12        | 12      | 4326.6    | 14074    |
| 10       | 12        | 8.4     | 327.56    | 2008.8   | 30       | 12        | 14.4    | 5437.8    | 17194.8  |
| 10       | 12        | 12      | 498.08    | 2713     | 30       | 14        | 6       | 2544.4    | 9022.8   |
| 10       | 12        | 14.4    | 601.98    | 3182     | 30       | 14        | 8.4     | 3511.8    | 11948    |
| 20       | 8         | 6       | 996.14    | 4629.6   | 30       | 14        | 12      | 5290      | 16461.6  |
| 20       | 8         | 8.4     | 1176.98   | 5288.8   | 30       | 14        | 14.4    | 6335.2    | 20992    |
| 20       | 8         | 12      | 1823.16   | 7092.6   | 40       | 8         | 6       | 4155.8    | 14918.6  |
| 20       | 8         | 14.4    | 1837.68   | 7451.6   | 40       | 8         | 8.4     | 5276.2    | 17465    |
| 20       | 10        | 6       | 1020.62   | 4431     | 40       | 8         | 12      | 7255.4    | 22288    |
| 20       | 10        | 8.4     | 1184.06   | 5142.8   | 40       | 8         | 14.4    | 8899.8    | 26496    |
| 20       | 10        | 12      | 1720.48   | 6741.2   | 40       | 10        | 6       | 3877      | 13520.4  |
| 20       | 10        | 14.4    | 1846.54   | 7329     | 40       | 10        | 8.4     | 4998.2    | 16373    |
| 20       | 12        | 6       | 1017.6    | 4577.2   | 40       | 10        | 12      | 7227.4    | 22480    |
| 20       | 12        | 8.4     | 1369.38   | 5595     | 40       | 10        | 14.4    | 8843      | 26636    |
| 20       | 12        | 12      | 1913.84   | 7302.4   | 40       | 12        | 6       | 4143.6    | 13690    |
| 20       | 12        | 14.4    | 2277.8    | 8409.6   | 40       | 12        | 8.4     | 5632.2    | 17392    |
| 20       | 14        | 6       | 1193.74   | 4964.4   | 40       | 12        | 12      | 8397.4    | 25032    |
| 20       | 14        | 8.4     | 1503.98   | 6052.4   | 40       | 12        | 14.4    | 9991.8    | 29026    |
| 20       | 14        | 12      | 2249.4    | 8271.6   | 40       | 14        | 6       | 4691.4    | 15013.6  |
| 20       | 14        | 14.4    | 2846.6    | 10381.6  | 40       | 14        | 8.4     | 6657      | 20804    |
| 30       | 8         | 6       | 2301      | 8956.6   | 40       | 14        | 12      | 9762.6    | 28330    |
| 30       | 8         | 8.4     | 2778.2    | 10419    |          |           |         |           |          |

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