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Screw dislocation in functionally graded layers with arbitrary gradation

S. M. Mousavi, S. J. Fariborz, and J. Paavola

Summary. Dislocation solutions were utilized in the fracture analysis of materials via the distributed dislocation technique (DDT). Recently, DDT was utilized in static and elastodynamic analysis of different functionally graded (FG) domains. The authors applied DDT for the analysis of a cracked FG layer, taking into account the energy dissipation in the material (Mousavi et al., 2011; Mousavi et al., 2012). The material properties in the above mentioned investigations were assumed to vary exponentially with the same rate. The assumption considerably simplifies the solution of ensuing differential equation. Nonetheless, it may cause a significant inaccuracy of the results. In this article, the assumption has been relaxed for static analysis allowing the solution of screw dislocation in functionally graded material (FGM) layers with arbitrary exponential gradation. The solution to the governing equation is carried out by utilizing complex Fourier transform.

Key words: screw dislocation, functionally graded material, antiplane analysis, distributed dislocation technique

Introduction

Functionally graded materials are the subject of many investigations in fracture mechanics. Different material properties such as modules of elasticity in different direction, density and damping are space-dependent in nonhomogeneous materials. Since the FGMs are utilized in different situation, it is necessary to analyze the fracture behavior of these materials in static and dynamic loading. Inplane and antiplane fracture modes have been the topic of many recent investigations (Ma et al., 2005; Chen et al., 2005; Guo et al., 2005; Ma et al., 2004; Sladek et al., 2005; Hongmin et al., 2008).

In many methods in fracture mechanics, very few functions can be utilized for the variation of material properties to obtain analytical solutions. It is quite common to assume the variation of material properties to be as an exponential function (Ma et al., 2005; Chen et al., 2005). For the sake of reducing mathematics complexity, the exponent coefficients for these parameters are usually assumed to be identical. Having this assumption, the governing partially differential equations can be simplified to PDEs with constant multipliers for which the analytical solutions are easy to obtain. This is common in most recent investigations about fracture of graded materials (Guo et al., 2005; Ma et al., 2004; Sladek et al., 2005). However; the material properties may have gradation with different exponent coefficients. Therefore it is necessary to consider some general case for the material properties in functionally graded materials.

The analysis of the FGMs with the assumption of exponential properties can be extended via approximated methods to FGMs with properties arbitrarily distributed in spatial position. For example, Wang et al. (Wang et al., 2000; Wang et al., 2002) and Itou (2001) presented a homogenous multi-layered model (HM model) in which a FGM layer is divided into a number of homogeneous layers. Employing the continuity condition between the layers yields approximate solution for the problem. Also Wang and Gross (2000) and Huang et al. (2004) suggested a linear multi-layered model (LM model) in which the nonhomogeneous materials are divided into some sub-layers along the thickness direction with the material properties varying linearly in each layer. Gue and Noda (2007) presented a piecewise-exponential model (PE model) to investigate the crack problem of the functionally graded materials (FGMs) with arbitrary properties.

The latter approximated method for exponential model (Gue and Noda, 2007) is applied to exponential properties with identical exponent for different properties. Once we have the solution for FGMs with different exponential coefficients, these approximate methods can be generalized to cover FGMs with complete arbitrary properties.

The Distributed Dislocation Technique (DDT) is a useful method in the fracture analysis of domains containing multi-cracks (Faal et al., 2006; Fotuhi et al., 2006; Mousavi et al., 2011; Mousavi et al., 2012). In this method, the solution of the domain in the presence of a dislocation should be determined. This solution may be utilized in DDT to form various configurations of cracks and determine the stress intensity factors. It should be mentioned that in order to utilize the dislocation solution in crack formation, we should have the exact solution for the dislocation. Therefore it is important to find the analytic solution for the dislocation problem.

In the present article, the antiplane dislocation solution is sought in a more general case for FGM. Various gradation parameters for material properties are under consideration. These solutions may be utilized in the distributed dislocation technique to determine fracture behavior of the domain containing multi-cracks.

Dislocation solution

A layer with thickness h made up of an orthotropic FGM wherein material properties vary exponentially in the thickness direction is under consideration, figure 1.

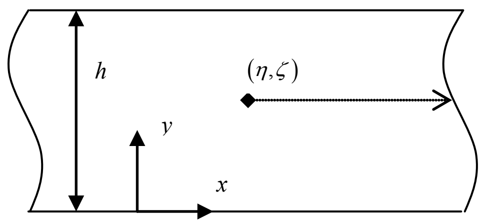

Figure.1. Schematic view of the FGP layer weakened by a screw dislocation

The coordinate axes are taken as directions of principal material orthotropy. A Volterra-type screw dislocation where the line of dislocation is parallel with the y-axis is located at (η,ζ). The displacement components in anti-plane deformation are

$$u = 0, \quad \mathrm{v} = 0, \quad w = w(x, y) \tag{1}$$

Utilizing strain-deformation relationships in linear elasticity, the non-vanishing strain components become

$$\gamma_{zx} = \frac{\partial w}{\partial x}, \quad \gamma_{zy} = \frac{\partial w}{\partial y} \tag{2}$$

Substituting equation (2) into Hooke's law for the orthotropic FGM strip, leads to the stress components in terms of displacement field as

$$\sigma_{zx} = \mu_x(y)\frac{\partial w}{\partial x}, \quad \sigma_{zy} = \mu_y(y)\frac{\partial w}{\partial y} \tag{3}$$

where $\mu_x(y)$ and $\mu_y(y)$ are the shear moduli of elasticity of FGM in the x- and y-directions, respectively. The equation for anti-plane deformation of a body reads

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = 0 \tag{4}$$

Equation (4) in view of equations (3) becomes

$$\frac{\mu_x(y)}{\mu_y(y)}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\mu_y'(y)}{\mu_y(y)}\frac{\partial w}{\partial y} = 0 \tag{5}$$

For the layer, the boundary, continuity and limiting conditions may be expressed as

$$\begin{aligned}
&\sigma_{yz}(x, 0) = 0, \quad \sigma_{yz}(x, h) = 0 \\
&w(x, \zeta^-) - w(x, \zeta^+) = b_z H(x - \eta) \\
&\sigma_{yz}(x, \zeta^-) = \sigma_{yz}(x, \zeta^+) \\
&\lim_{|x| \to \infty} w = 0
\end{aligned} \tag{6}$$

where b_z is the dislocation Burgers vector and $H(..)$ is the Heaviside step-function. We assume that the material properties of the FG layer vary exponentially in the y-direction.

In contrast to a common practice in fracture mechanics community, we let material properties of FG layer vary with different rates. Therefore,

$$\begin{aligned}\mu_x(y) &= \mu_{0x} e^{2\kappa_1 y}\\ \mu_y(y) &= \mu_{0y} e^{2\kappa_2 y}\end{aligned} \tag{7}$$

where μ_{ox} and μ_{0y} are the shear moduli of elasticity of the layer at lower edge ($y=0$) in the x- and y-directions, respectively. Also κ_1 and κ_2 are gradation exponentials in x- and y-directions, respectively. Equation (5) in view of (7) reduces to

$$f_\mu e^{2(\kappa_1-\kappa_2)y}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+2\kappa_2\frac{\partial w}{\partial y}=0 \tag{8}$$

while $f_\mu=\mu_{0x}/\mu_{0y}$. Application of the complex Fourier transform (Appendix A) on x-variable to equations (6) and (8) results in an ordinary differential equation

$$\frac{d^2 w^*}{dy^2}+2\kappa_2\frac{dw^*}{dy}-f_\mu s^2 e^{2(\kappa_1-\kappa_2)y}w^*=0 \tag{9}$$

and

$$\begin{aligned}&\frac{\partial w^*(s,y)}{\partial y}\Big|_{y=0}=0\\ &\frac{\partial w^*(s,y)}{\partial y}\Big|_{y=h}=0\\ &w^*(x,\zeta^-)-w^*(x,\zeta^+)=b_z e^{-is\eta}\left(\pi\delta(s)-\frac{i}{s}\right)\\ &\frac{\partial w^*(s,y)}{\partial y}\Big|_{y=\zeta^-}=\frac{\partial w^*(s,y)}{\partial y}\Big|_{y=\zeta^+}\end{aligned} \tag{10}$$

where $w^*(s,y)$ is the complex Fourier transform of displacement field and $\delta(s)$ is the Dirac delta function. After solving this differential equation, $w(x,y)$ is determined by applying inverse complex Fourier transform (Appendix A).

In a special case, if $\kappa_1=\kappa_2=\kappa$ and $f_\mu=1$, the solution to equations (9) and (10) is

$$\begin{aligned}w_1(x,y)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{(\lambda-\kappa)y}+A_2 e^{-(\lambda+\kappa)y}\right]e^{isx}ds \qquad 0\le y\le\zeta\\ w_2(x,y)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{(\lambda-\kappa)y}+B_2 e^{-(\lambda+\kappa)y}\right]e^{isx}ds \qquad \zeta\le y\le h\end{aligned} \tag{11}$$

while $\lambda=(\kappa^2+s^2)^{1/2}$. The coefficients A_i and B_i, i=1,2 are introduced in Appendix B. This case has been reported by Fotuhi and Fariborz (2006). They have utilized this solution to analyze multi-cracked layers via the DDT.

In the general case ($\kappa_1\neq\kappa_2$), the exact solution for the problem is

$$w^*(s,y)=e^{-\kappa_2 y}\left[C_1 J_{\gamma_1}(\gamma_2)+C_2 Y_{\gamma_1}(\gamma_2)\right] \tag{12}$$

in which

$$\gamma_1=\kappa_2/(\kappa_1-\kappa_2),\quad \gamma_2=is\sqrt{f_\mu}\,\mathrm{e}^{y(\kappa_1-\kappa_2)}/(\kappa_1-\kappa_2) \tag{13}$$

and $J_v(z)$ and $Y_v(z)$ are Bessel functions of First and Second kind, respectively. To apply boundary conditions (10) to the above solution, it is convenient to divide the layer into two regions $0\leq y\leq\zeta$ and $\zeta\leq y\leq h$.

$$\begin{aligned} w^*(s,y)&=e^{-\kappa_2 y}\left[C_1 J_{\gamma_1}(\gamma_2)+C_2 Y_{\gamma_1}(\gamma_2)\right] && 0\leq y\leq\zeta \\ w^*(s,y)&=e^{-\kappa_2 y}\left[D_1 J_{\gamma_1}(\gamma_2)+D_2 Y_{\gamma_1}(\gamma_2)\right] && \zeta\leq y\leq h \end{aligned} \tag{14}$$

The coefficients C_1, C_2, D_1 and D_2 may be determined utilizing four conditions in equation (10). Results are presented in Appendix C. Utilizing inverse complex Fourier transform (Appendix A), the displacement field may be written as

$$\begin{aligned} w_1(x,y)&=\frac{2e^{-\kappa_2 y}}{\pi}\int_0^{+\infty}\left[C_1 J_{\gamma_1}(\gamma_2)+C_2 Y_{\gamma_1}(\gamma_2)\right]\sin(sx)ds && 0\leq y\leq\zeta \\ w_2(x,y)&=\frac{2e^{-\kappa_2 y}}{\pi}\int_0^{+\infty}\left[D_1 J_{\gamma_1}(\gamma_2)+D_2 Y_{\gamma_1}(\gamma_2)\right]\sin(sx)ds && \zeta\leq y\leq h \end{aligned} \tag{15}$$

This solution fulfils the conditions in equations (6). Utilizing $w_i(x,y)$, the stress components can be obtained via equations (3, 7)

$$\begin{aligned} \sigma_{zx}(x,y)&=\frac{2}{\pi}\mu_{0x}e^{(2\kappa_1-\kappa_2)y}\int_0^{+\infty}\left[C_1 J_{\gamma_1}(\gamma_2)+C_2 Y_{\gamma_1}(\gamma_2)\right]s\cos(sx)ds && 0\leq y\leq\zeta \\ \sigma_{zx}(x,y)&=\frac{2}{\pi}\mu_{0x}e^{(2\kappa_1-\kappa_2)y}\int_0^{+\infty}\left[D_1 J_{\gamma_1}(\gamma_2)+D_2 Y_{\gamma_1}(\gamma_2)\right]s\cos(sx)ds && \zeta\leq y\leq h \\ \sigma_{zy}(x,y)&=-\frac{2i}{\pi}\sqrt{f_\mu}\mu_{0y}e^{\kappa_1 y}\int_0^{+\infty}\left[C_1 J_{\gamma_1+1}(\gamma_2)+C_2 Y_{\gamma_1+1}(\gamma_2)\right]s\sin(sx)ds && 0\leq y\leq\zeta \\ \sigma_{zy}(x,y)&=-\frac{2i}{\pi}\sqrt{f_\mu}\mu_{0y}e^{\kappa_1 y}\int_0^{+\infty}\left[D_1 J_{\gamma_1+1}(\gamma_2)+D_2 Y_{\gamma_1+1}(\gamma_2)\right]s\sin(sx)ds && \zeta\leq y\leq h \end{aligned} \tag{16}$$

The stress components are ready to be used in the DDT to form and analyze the cracked layer. It should be mentioned that, as expected, the behavior of the integrals in stress component depicts that they have singularity in the vicinity of dislocation.

Conclusions and future work

In the present study, the solution of an antiplane dislocation is sought for an FG layer with various gradation parameters for material properties. Since identical exponent factors for different properties is a limiting assumption, in order to model the behavior of FGM, it is necessary to consider more general form for properties gradation.

In the case of identical gradation parameters ($\kappa_1=\kappa_2$), this solution is coincident with those reported by Fotuhi and Fariborz (2006). The general assumption ($\kappa_1\neq\kappa_2$) results in the solution of dislocation in terms of Bessel functions. These solutions can be utilized in Dislocation Distributed Technique to analyze multi-cracked domains which will be carried out in a future work by the authors.

Appendix A

The complex Fourier transform is defined by

$$f^*(s)=\int_{-\infty}^{\infty} f(x)e^{-isx}dx$$

The inversion of the transform is

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f^*(s)e^{isx}ds$$

Appendix B

The coefficients A_i and B_i, $i=1,2$ in equation (11) are

$$A_1=\frac{\lambda+\kappa}{2\lambda\sinh(\lambda h)}e^{-is\eta}\left(\pi\delta(s)-i/s\right)b_z e^{\zeta\kappa}\sinh(\lambda(h-\zeta))$$

$$A_2=\frac{\lambda-\kappa}{2\lambda\sinh(\lambda h)}e^{-is\eta}\left(\pi\delta(s)-i/s\right)b_z e^{\zeta\kappa}\sinh(\lambda(h-\zeta))$$

$$B_1=-\frac{\lambda+\kappa}{2\lambda\sinh(\lambda h)}e^{-is\eta}\left(\pi\delta(s)-i/s\right)b_z e^{\zeta\kappa-\lambda h}\sinh(\lambda\zeta)$$

$$B_2=-\frac{\lambda-\kappa}{2\lambda\sinh(\lambda h)}e^{-is\eta}\left(\pi\delta(s)-i/s\right)b_z e^{\zeta\kappa+\lambda h}\sinh(\lambda\zeta)$$

Appendix C

The coefficients in Eq. (14) are

$$
\begin{aligned}
C_1 = & -\left[\Pi e^{\kappa_2\zeta} Y_{\gamma_1+1}(\gamma_3)\left\{-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)\right\}\right]/ \\
& \left[J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)\right. \\
& \left.-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)Y_{\gamma_1}(k_1\gamma_3)_{\gamma_1+1}J(\gamma_3)-Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)J_{\gamma_1}(k_1\gamma_3)_{\gamma_1+1}Y(\gamma_3)\right] \\
C_2 = & \left[\Pi e^{\kappa_2\zeta} J_{\gamma_1+1}(\gamma_3)\left\{-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)\right\}\right]/ \\
& \left[J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)\right. \\
& \left.-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)-Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)\right] \\
D_1 = & -\left[\Pi e^{\kappa_2\zeta} Y_{\gamma_1+1}(k_2\gamma_3)\left\{-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)\right\}\right]/ \\
& \left[J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)\right. \\
& \left.-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)-Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)\right] \\
D_2 = & \left[\Pi e^{\kappa_2\zeta} J_{\gamma_1+1}(k_2\gamma_3)\left\{-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)\right\}\right]/ \\
& \left[J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)+Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)\right. \\
& \left.-J_{\gamma_1+1}(k_1\gamma_3)Y_{\gamma_1+1}(k_2\gamma_3)Y_{\gamma_1}(k_1\gamma_3)J_{\gamma_1+1}(\gamma_3)-Y_{\gamma_1+1}(k_1\gamma_3)J_{\gamma_1+1}(k_2\gamma_3)J_{\gamma_1}(k_1\gamma_3)Y_{\gamma_1+1}(\gamma_3)\right]
\end{aligned}
$$

while

$$\gamma_1 = \kappa_2/(\kappa_1-\kappa_2),\quad \gamma_3 = \frac{is\sqrt{f_\mu}}{(\kappa_1-\kappa_2)}, k_1 = e^{(\kappa_1-\kappa_2)\zeta},\quad k_2 = e^{(\kappa_1-\kappa_2)h},\quad \Pi = b_z e^{-is\eta}\left(\pi\delta(s)-\frac{i}{s}\right)$$

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S. M. Mousavi, J. Paavola
Department of Civil and Structural Engineering, Aalto University
P.O. Box 12100, FI-00076 Aalto, Finland
mahmoud.mousavi@aalto.fi, juha.paavola@aalto.fi

S. J. Fariborz
Department of Mechanical Engineering, Amirkabir University of Technology
424, Hafez Avenue, Tehran 158754413, Iran
fariborz@aut.ac.ir