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A Bayesian approach to vibration based structural health monitoring with experimental verification

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Summary. In this paper we present a damage identification method for use in structural health monitoring. Our method uses Bayesian inference to solve the inverse problem of locating structural damage based on measurements of vibrational parameters of the structure. Based on a structure model, a damage model and a measurement model we can estimate measured parameters for a given damage state. Using statistical assumptions of the measurement noise and the possible damage states we derive an estimate of the damage state. Finally, the method is experimentally verified to correctly identify damage on a steel cantilever beam.

Key words: structural health monitoring, damage identification, inverse problem, Bayesian inference

Introduction

In structural health monitoring, damage sensitive mechanical parameters (for example vibration amplitudes and frequencies) of an operational structure are continuously measured to assess its health and to verify its safety. This measuring is done using a network of sensors attached to the structure. Currently information of a structure's health, i.e. its ability to perform its function, is often only obtained by scheduled maintenance and periodic inspections, which are expensive because of the amount of labor and downtime involved.

Methods of structural health monitoring have been under active research over the past 30 years [7, 15, 6, 5, 9]. Many of the proposed methods are based on the fact that vibration characteristics of a structure change when it receives mechanical damage. This change is not only related to the extent of damage, but also to its locations [15, 5, 14, 12]. As the vibration modes of a structure can be measured even when the excitation to the structure is not exactly known [13, 1, 3, 4], for instance the vibrations induced by the wind, they are well suited for damage identification.

Methods which seek also to locate the damage, in addition to just detecting its presence, are often based on a model of the structure and on an assumption of the way damage changes that model. The simulated modal parameters of the structure are then compared against those measured from the structure, and a damage state is sought in such a way that the simulated modal parameters equal the measured ones. This, however, is an ill-conditioned problem and can only be solved approximately under regularizing assumptions.

As measurement noise and modeling error are always present in a realistic case, the approach presented in this paper is a statistical one. The ability to take these uncertainties directly into account is a benefit of the proposed method. Given prior information of the probability of damage and the different types of damage, Bayesian inference can be used to update this information based on a set of measured modal parameters.

Finally, the presented method is experimentally verified by using measurements done on a cantilever beam.

Structural dynamics

Structure model

We assume that we have a suitably accurate discretized linear model of the dynamic behaviour of our structure. Such a model can for instance be obtained by creating a rough finite element model and using one of several model updating procedures to refine its parameters [8].

The free vibrations (without damping) of the structure can then be characterized as the solutions of a differential equation of the form

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ is the mass matrix, $\boldsymbol{K} \in \mathbb{R}^{n\times n}$ is the stiffness matrix and $\boldsymbol{u} \in \mathbb{R}^n$ is a vector of the degrees of freedom in the model. Assuming that the discretization is obtained using the finite element method for an elasticity problem, both $\boldsymbol{M}$ and $\boldsymbol{K}$ are known to be symmetric positive definite.

One approach to solve equation (1) is to first consider the generalized eigenvalue problem given by

$$\boldsymbol{K}\boldsymbol{x}_i = \omega_i^2\boldsymbol{M}\boldsymbol{x}_i, \tag{2}$$

where the eigenvectors $\boldsymbol{x}_i$ are called the mode shapes of the vibration and the eigenvalues ω_i^2 are the squares of modal frequencies. Collecting the eigenvectors into columns of a matrix $\boldsymbol{X}$ and the corresponding eigenvalues into a diagonal matrix $\boldsymbol{\Omega}^2$, then equation (2) can be written as

$$\boldsymbol{K}\boldsymbol{X} = \boldsymbol{M}\boldsymbol{X}\boldsymbol{\Omega}^2. \tag{3}$$

The eigenvectors are commonly chosen to be mass normalized, i.e.,

$$\boldsymbol{X}^T\boldsymbol{M}\boldsymbol{X} = \boldsymbol{I}. \tag{4}$$

Multiplying equation (3) by $\boldsymbol{X}^T$ from the left then gives

$$\boldsymbol{X}^T\boldsymbol{K}\boldsymbol{X} = \boldsymbol{X}^T\boldsymbol{M}\boldsymbol{X}\boldsymbol{\Omega}^2 = \boldsymbol{\Omega}^2. \tag{5}$$

Taking a change in variables

$$\boldsymbol{u} = \boldsymbol{X}\boldsymbol{\eta} \tag{6}$$

and inserting it to equation (1) gives

$$\boldsymbol{M}\boldsymbol{X}\ddot{\boldsymbol{\eta}} + \boldsymbol{K}\boldsymbol{X}\boldsymbol{\eta} = \boldsymbol{0}. \tag{7}$$

Multiplying this by $\boldsymbol{X}^T$ from the left and using equations (4) and (5) then gives

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{\Omega}^2\boldsymbol{\eta} = \boldsymbol{0}, \tag{8}$$

which just describes n uncoupled harmonic oscillators. The solution to each one is given by

$$\eta_i(t) = A_i\cos(\omega_i t) + B_i\sin(\omega_i t), \tag{9}$$

where A_i and B_i depend on appropriate initial conditions specified for $\boldsymbol{u}$ and $\dot{\boldsymbol{u}}$. The solution to equation (1) is then given by

$$\boldsymbol{u}(t) = \boldsymbol{X}\boldsymbol{\eta}(t) = \sum_{i=1}^{n} \eta_i(t)\boldsymbol{x}_i. \tag{10}$$

The free vibrations of a damped structure, on the other hand, can be characterized by the solutions of the equation

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0}. \tag{11}$$

Now the analysis becomes much more convoluted since in general one ends up with non-normal operators. A special case, however, is the so called modal damping. In this case, the damping matrix $\boldsymbol{C}$ is such that

$$\boldsymbol{X}^T\boldsymbol{C}\boldsymbol{X} = \boldsymbol{D}, \tag{12}$$

where $\boldsymbol{D}$ is a diagonal matrix with $D_{ii} = 2\zeta_i\omega_i$.

Equation (11) is now transformed to

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{D}\dot{\boldsymbol{\eta}} + \boldsymbol{\Omega}^2\boldsymbol{\eta} = \boldsymbol{0}, \tag{13}$$

which describes n uncoupled damped harmonic oscillators. The solution to each one is given by

$$\eta_i(t) = \exp(-\zeta_i\omega_i t)\left(A_i\cos(\sqrt{1-\zeta_i}\;\omega_i t) + B_i\sin(\sqrt{1-\zeta_i}\;\omega_i t)\right), \tag{14}$$

where A_i and B_i again depend on the intial conditions. The damping ratios ζ_i for steel structures are typically in the range $\zeta_i = 0.02\ldots0.05$ [2]. This translates to a change in frequency of 0.02% to 0.15%, which is quite insignificant compared to measurement errors. As it is possible to show a similar result even without assuming modal damping, the use of the undamped model is quite sufficient for structures with light damping.

Damage model

Ultimately we want to estimate damage based on measured changes in the behaviour of a structure. In order to do that, we first need a model of how damage affects the structure. In this article we will present only a very simple damage model, which nevertheless is adequate for our experimental setup. More elaborate models are described in the literature [17, 16].

The first assumption made is that the damage does not affect the mass of the structure, i.e., the mass matrix of the structure model stays constant under damage. Common types of damage, such as fatigue and corrosion cause no relevant change in mass. The stiffness of the structure, however, is affected. In this simple model, we begin with the fact that the stiffness matrix of a finite element model can be expressed as

$$\boldsymbol{K} = \sum_{i=1}^{N} \boldsymbol{K}_{T_i}, \tag{15}$$

where $\boldsymbol{K}_{T_i}$ is the stiffness contribution of element T_i, and N is the total number of elements in the structure model. The stiffness of the structure model with damage is then

$$\boldsymbol{K}_d = \sum_{i=1}^{N} (1 - d_i) \boldsymbol{K}_{T_i}, \tag{16}$$

where d_i is the stiffness reduction in element T_i. The variables d_i get values in the range $[0, 1]$, where 0 indicates no damage and 1 indicates total loss of stiffness in the corresponding element. It is useful to define the damage state vector as

$$\boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix} \in \mathbb{R}^N. \tag{17}$$

For any damage state $\boldsymbol{d}$, the modes of the damaged structure model can be obtained from the generalized eigenvalue problem

$$\boldsymbol{K}_d \boldsymbol{x}_{d,i} = \omega_{d,i}^2 \boldsymbol{M} \boldsymbol{x}_{d,i}. \tag{18}$$

Measurement model

In this article, we assume that we can measure a number of the structure's vibration modes individually. Our measurement then consists of modal frequencies and observed components of mode shapes for these modes. Such data can be extracted also from output-only time-domain data using algorithms described in the literature [13, 1, 3, 4].

Let us say that we can measure p modes of the structure. We will denote the measured modal frequencies by $\hat{\omega}_i$ and the measured mode shapes by $\hat{\boldsymbol{y}}_i$, where $i = 1, \ldots, p$. The measured mode shapes are vectors of the measured components of the actual mode shapes. For example, if the modes of the structure are measured using accelerometers, then each $\hat{\boldsymbol{y}}_i$ contains the acceleration amplitudes (and phases) of vibration mode i measured at the locations of the accelerometers. We will collect all the measurements into a measurement vector as

$$\hat{\boldsymbol{m}} = \begin{pmatrix} \hat{\omega}_1 \\ \hat{\boldsymbol{y}}_1 \\ \vdots \\ \hat{\omega}_p \\ \hat{\boldsymbol{y}}_p \end{pmatrix}. \tag{19}$$

The idea of the measurement model is to estimate what the measurements would be, if we knew the damage state of the structure. In our case we first compute the vibration modes of our structure model, using equation (18). The computed modal frequencies can be directly compared with the measured ones, but for each of the mode shapes, however, we need to simulate the output of the measurement sensors to obtain data comparable with the measured mode shapes.

Let $\omega_{d,i}$ and $\boldsymbol{x}_{d,i}$ be the modal frequencies and mode shapes of the structure model under damage. We assume that our measurement sensors have a linear response. In this case, the output of each sensor can be modeled as a linear functional of the mode shape. We can thus simulate the measured mode shapes as

$$\boldsymbol{y}_{i,d} = \boldsymbol{H} \boldsymbol{x}_{i,d}. \tag{20}$$

Let the modes be indexed so that each simulated mode $(\omega_{i,d}, \boldsymbol{y}_{i,d})$ corresponds to the measured mode $(\hat{\omega}_i, \hat{\boldsymbol{y}}_i)$ for $i = 1, \ldots, p$. The simulated measurements are then collected into a vector as

$$\boldsymbol{m}_d = \begin{pmatrix} \omega_{d,1} \\ \boldsymbol{y}_{d,1} \\ \vdots \\ \omega_{d,p} \\ \boldsymbol{y}_{d,p} \end{pmatrix}. \tag{21}$$

For simplicity of notation, we define a function $\boldsymbol{f}$ such that the simulated measurement $\boldsymbol{m}_d$ is expressed as

$$\boldsymbol{m}_d = \boldsymbol{f}(\boldsymbol{d}), \tag{22}$$

where $\boldsymbol{d} \in \mathbb{R}^N$ is any valid damage state of the structure.

We then assume that the actual measurement $\boldsymbol{m}$ may be decomposed as

$$\boldsymbol{m} = \boldsymbol{f}(\boldsymbol{d}) + \boldsymbol{\epsilon}, \tag{23}$$

with some unknown damage state $\boldsymbol{d}$ and additive error $\boldsymbol{\epsilon}$.

Statistical inverse

Bayesian inference

As the measurement process always contains some random measurement error, it is quite natural to consider the measuring as a statistical process. It turns out that the Bayesian framework is very useful for problems of this sort. In this framework all variables are considered random variables [10, 11].

Let the measurement outcome be denoted by $\boldsymbol{M}$, the damage state by $\boldsymbol{D}$ and errors by $\boldsymbol{E}$. We then assume, that these random variables are related as

$$\boldsymbol{M} = \boldsymbol{f}(\boldsymbol{D}) + \boldsymbol{E}, \tag{24}$$

i.e. the measurement can be explained by our measurement model with some damage state and an additive error term. We go on to assume that the error term $\boldsymbol{E}$ is independent of the damage state $\boldsymbol{D}$ and that it is normally distributed so that

$$\boldsymbol{E} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}). \tag{25}$$

The probability density of such a distribution is given by

$$p(\boldsymbol{E} = \boldsymbol{\epsilon}) = C \exp\left(-\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{\epsilon} - \boldsymbol{\mu})\|^2\right), \tag{26}$$

where $\boldsymbol{S}$ is such that $\boldsymbol{S}^T\boldsymbol{S} = \boldsymbol{\Sigma}^{-1}$, for instance the Cholesky decomposition of $\boldsymbol{\Sigma}^{-1}$, and C is a normalization constant.

Now, assume that the damage variable has a fixed value, i.e. $\boldsymbol{D} = \boldsymbol{d}$. Under this condition equation (24) gives

$$\boldsymbol{M} = \boldsymbol{f}(\boldsymbol{d}) + \boldsymbol{E}, \tag{27}$$

which means that the measurement $\boldsymbol{M}$ has the same distribution as the error $\boldsymbol{E}$, but only shifted by the constant $\boldsymbol{f}(\boldsymbol{d})$. Hence under the condition $\boldsymbol{D} = \boldsymbol{d}$ we have

$$\boldsymbol{M} \sim \mathcal{N}(\boldsymbol{f}(\boldsymbol{d}) + \boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{28}$$

and thus

$$p(\boldsymbol{M} = \boldsymbol{m}|\boldsymbol{D} = \boldsymbol{d}) = C \exp\left(-\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{d}) - \boldsymbol{\mu})\|^2\right). \tag{29}$$

This is the probability density of seeing measurement $\boldsymbol{m}$, when we know the damage is $\boldsymbol{d}$. When the structure is undamaged, this can be used to estimate the statistical parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ of the error variable.

For damage estimation we would like to know the probability density of having damage $\boldsymbol{d}$ when we have obtained measurement $\boldsymbol{m}$. For this purpose, we can use Bayes' formula [10, 11], which states

$$p(\boldsymbol{D} = \boldsymbol{d}|\boldsymbol{M} = \boldsymbol{m}) = \frac{p(\boldsymbol{M} = \boldsymbol{m}|\boldsymbol{D} = \boldsymbol{d})p(\boldsymbol{D} = \boldsymbol{d})}{p(\boldsymbol{M} = \boldsymbol{m})}. \tag{30}$$

Bayes' formula thus switches the roles of the random variables in the conditional probability. It does this with the aid of additional information contained in the distribution $p(\boldsymbol{D} = \boldsymbol{d})$. This is the so-called prior distribution, which does not yet take into account the additional information provided by the measurement, and relies only on a prior assessment of the likelihood of finding different damage states $\boldsymbol{d}$. Bayes' formula effectively takes this prior information and combines it with our measurement model to produce what is called the posterior probability distribution. The factor $p(\boldsymbol{M} = \boldsymbol{m})$ in the formula is the probability density of obtaining measurement $\boldsymbol{m}$. However, since $\boldsymbol{m}$ has actually already been obtained, this factor can be interpreted just as a normalizing constant.

Prior distribution

The prior distribution describes for each possible damage state what we already know (or assume) before processing the additional information obtained from measurements. It is thus the probability of having damage state $\boldsymbol{d}$ in general. Since there are strict physical limits on the values of the damage parameters d_i, then at the very least the distribution should be such that there is zero probability of $d_i < 0$ or $d_i > 1$ in any element T_i. While monitoring the structure, it is most probable that the structure is in fact not damaged (all $d_i = 0$). Even if there is damage it is probable that the damage is small. It is hence desirable that the prior distribution gives higher probability to states of low damage compared to states of high damage. A very simple statistical model for this is the truncated normal distribution. This model assumes that

$$p(\boldsymbol{D} = \boldsymbol{d}) = C\chi_{[0,1]}(\boldsymbol{d}) \exp\left(-\frac{1}{2\sigma^2}\|\boldsymbol{d}\|^2\right), \tag{31}$$

where

$$\chi_{[0,1]}(\boldsymbol{d}) = \begin{cases} 1, & \text{if } 0 \le d_i \le 1 \quad \forall i = 1, \dots, N \\ 0, & \text{otherwise} \end{cases}, \tag{32}$$

σ^2 is the variance of the underlying normal distribution and C is a normalizing constant. Many other distributions could be used as well. In this paper, this distribution was chosen because of its simplicity and because it was sufficient for our test case.

Following the statistical framework, the parameter σ should be estimated using statistical data of actual structural damage from similar structures. In practice, however, such information is not available. Data driven parameter estimation methods can be used instead, such as Morozov's discrepancy principle or the L-curve method [11]. To get correct qualitative results, it is often sufficient to just have σ set to the correct order of magnitude.

Using this distribution as our prior in equation (30), we can write the posterior probability distribution as

$$p(\boldsymbol{D} = \boldsymbol{d}|\boldsymbol{M} = \boldsymbol{m}) = C\chi_{[0,1]}(\boldsymbol{d}) \exp\left(-\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{d}) - \boldsymbol{\mu})\|^2 - \frac{1}{2\sigma^2}\|\boldsymbol{d}\|^2\right), \quad (33)$$

where all the normalizing constants are merged into C.

Maximum a posteriori estimate

Although the posterior probability density contains all the information of the inverse problem solution, and sometimes is considered the solution of a statistical inverse problem, it is often still not very useful as it is. Instead, in many cases some representative point estimate is desired. Common point estimates of probability distributions are for instance the mean and the median. However, the computation of these estimates for non-linear problems like the one presented here is computationally very intensive requiring sampling schemes such as Markov Chain Monte Carlo (MCMC) methods. We therefore turn to a third commonly used point estimate, which is the mode of the distribution. In the vocabulary of statistical inference, the mode of the posterior distribution is called the maximum a posteriori (MAP) estimator. It is defined as

$$\begin{aligned}
\boldsymbol{d}_{MAP} &= \underset{\boldsymbol{d}}{\operatorname{argmax}}\, p(\boldsymbol{D} = \boldsymbol{d}|\boldsymbol{M} = \boldsymbol{m}) \\
&= \underset{\boldsymbol{d}}{\operatorname{argmax}}\, C\chi_{[0,1]}(\boldsymbol{d}) \exp\left(-\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{d}) - \boldsymbol{\mu})\|^2 - \frac{1}{2\sigma^2}\|\boldsymbol{d}\|^2\right) \\
&= \underset{0 \le d_i \le 1}{\operatorname{argmax}} \exp\left(-\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{d}) - \boldsymbol{\mu})\|^2 - \frac{1}{2\sigma^2}\|\boldsymbol{d}\|^2\right) \\
&= \underset{0 \le d_i \le 1}{\operatorname{argmin}} \left\{\frac{1}{2}\|\boldsymbol{S}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{d}) - \boldsymbol{\mu})\|^2 + \frac{1}{2\sigma^2}\|\boldsymbol{d}\|^2\right\}.
\end{aligned} \quad (34)$$

For our choice of distributions in the exponential family, this corresponds to generalized Tikhonov regularization. This is a problem of non-linear least squares, which may be solved for instance by using the methods of Gauss-Newton or Levenberg-Marquardt.

As this is a non-linear problem, it is not known if the optimization methods will converge to a global optimum. However, in the prior distribution it was assumed that there is high probability of low damage levels. This limits the amount of non-linearity that can be present. The optima are thus expected to be clustered close to each other, and that any one of them is qualitatively representative of the distribution.

Experimental verification

Experimental setup

A steel cantilever beam of length 1400 millimeters (x direction) and of rectangular cross-section 5 millimeters thick (y direction) and 60 millimeters wide (z direction) was measured with 7 accelerometers. Initial measurements were taken when the beam was still undamaged. Using a hacksaw an approximately 1 millimeter wide slot was then sawn into the beam, at approximately 260 millimeters from the fixed end. The orientation of the

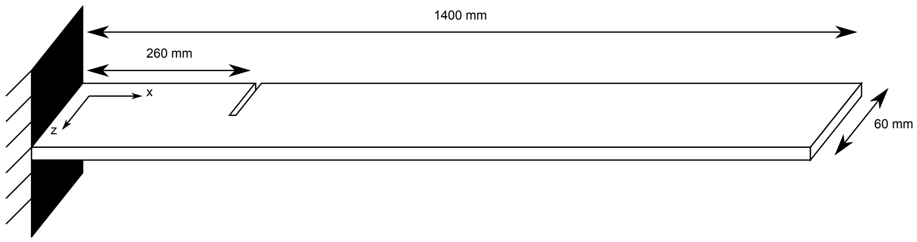

Figure 1. A schematic view of the cantilever under experiment.

slot was such that it made a rectangular hole in the xz-plane. This is the direction where the beam stiffness is least sensitive to the depth of the slot. See figure 1 for an illustration of the beam. Measurements were taken at 4 different slot depths: 5 millimeters, 10 millimeters, 15 millimeters and 20 millimeters. Photographs of the experiment are presented in figure 2.

In each damage state of the beam 5 individual measurements were taken. Each measurement contained 30 seconds of acceleration data following a tap with a rubber hammer. The modal parameters were identified from each of the individual measurements independently. As the structure was simple and the excitation was approximately an impulse, the identification of 6 modes (modal frequencies and measured mode shapes) was done by hand from the spectral magnitude response of the measured accelerations (figure 3). Such a task becomes excessively difficult for more complex structures under more general excitation. Methods described in the literature [13, 1, 3, 4] should be used for such cases.

A finite element model of the vibrations in the y direction of the beam was created using the measured dimensions and material parameters $E = 210$ GPa, $\rho = 7860$ kg/m^3. The modes of the beam in the other two directions were not considered as they fell out of the measured range of frequencies. The beam was modeled as an Euler-Bernoulli beam using 50 standard C^1-elements along the length. The modeled beam had a constant flexural rigidity $EI = 131.25$ Nm^2 and mass of unit length $\mu = 2.358$ kg/m along its length. Boundary conditions were set so that the beam was rigidly fixed at one end ($x = 0$ m) and free at the other ($x = 1.4$ m).

The data from the accelerometers were assumed as point values of the second time derivative of the displacement field, i.e. point values of the acceleration field. The measured mode shapes are thus point values of the acceleration mode shapes. The accelerometers were measured to be located at $350, 520, 700, 875, 1045, 1225$ and 1390 millimeters from the fixed end.

The differences between the finite element model and the measurements, which are caused by measurement noise and model error, were expressed as an error term, as defined in equation (24). In the initial state, the damage state is known to be zero. Hence the measurements in this state were used to estimate the statistics of the error term using equation (27). For this purpose, we assumed that the individual components of the error term are independent. The variance of each component was then estimated as the sample variance of the computed errors. The mean of the error term was estimated as the sample mean of the computed errors. As we only had 5 samples, we did not test our statistical assumption of normality.

Allowing the error term to have a non-zero mean value removes any constant modeling error, but does not affect non-constant modeling error (i.e. error depending on the damage state). This remaining part of the error could be made smaller by performing model

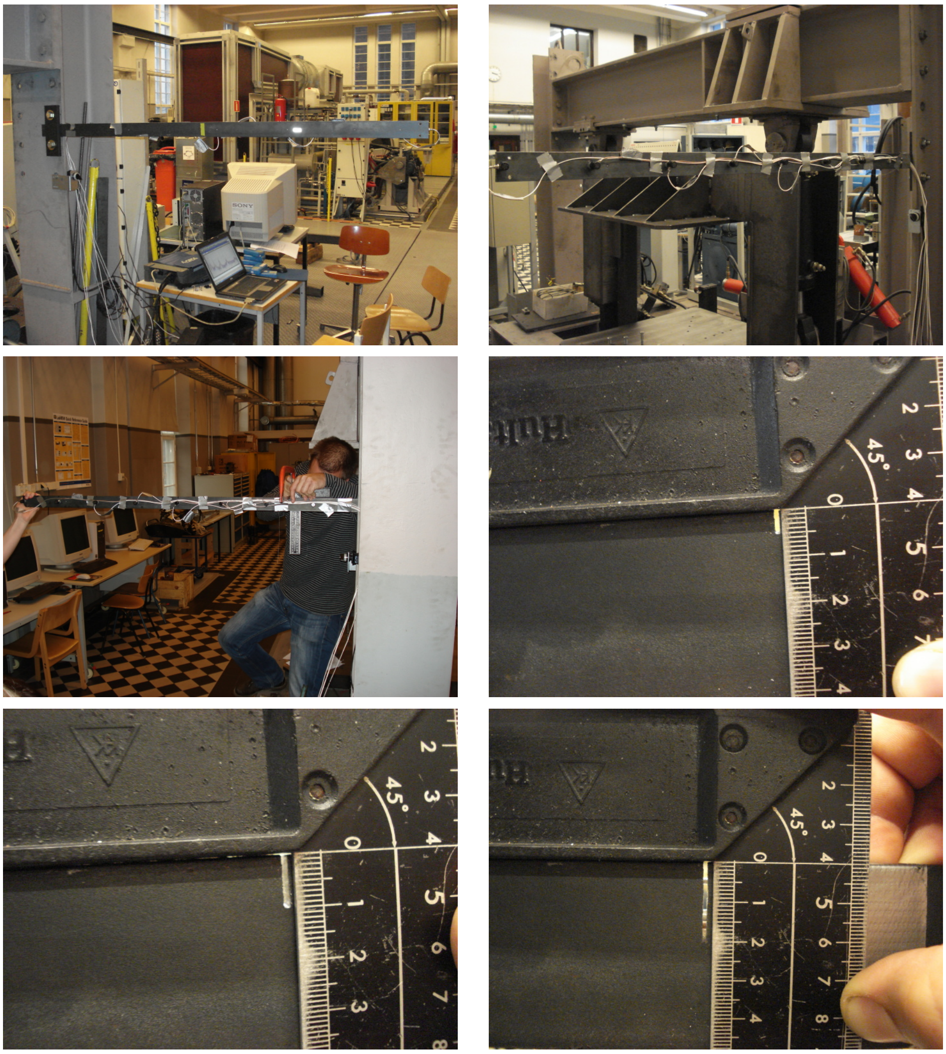

Figure 2. Photographs of the cantilever under experiment.

updating [8]. In our model, the modeling error of the modal frequencies in the initial measurements was in the order of 5%.

The distribution defined in equation (31) with $\sigma = 0.1$ was chosen as the prior distribution. The choice was based on a qualitative assessment comparing the clearness of the reconstruction with the numerical stability of the optimization problem. The reconstructions were not exceedingly sensitive to the exact value of this parameter. Qualitatively similar results would still be obtained even with an order of magnitude higher or lower value.

Experimental results

The measured changes in the modal parameters were quite small. The largest absolute change in modal frequencies between the undamaged case and the most damaged case was just over 5 rad/s. The largest relative change on the other hand was just under 1.5%. However, due to the large amount of parameters and the low noise levels, the measured changes had high statistical significance.

Using the Levenberg-Marquardt method for equation (34), we computed the maximum a posteriori estimates of the damage for each of the 5 individual measurements in each

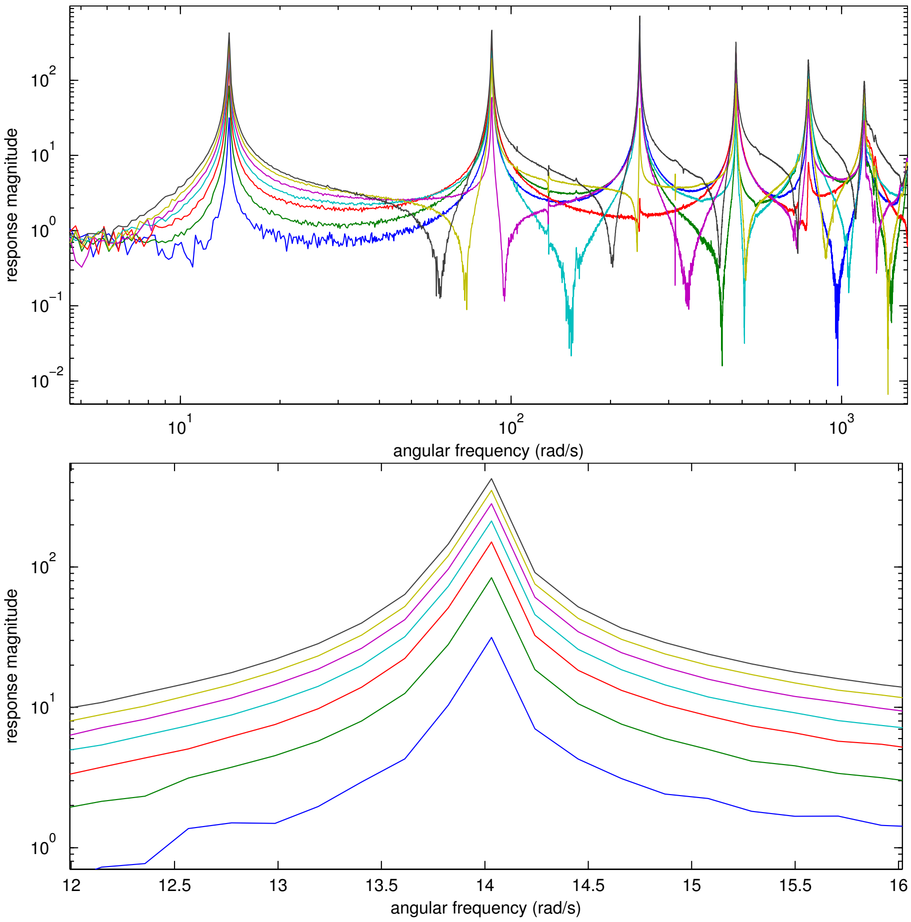

Figure 3. Upper: frequency response of the undamaged beam when excited with a tap of a rubber hammer. Lower: close-up on first spike, where one can clearly see the responses of individual sensors. Sensor 7, which was located near the free end, detected the largest deflection, whereas sensor 1, which was closest to the fixed end, detected the smallest deflection.

damage case. Instead of looking at the individual MAP estimates in each damage case, we chose to compute the first, second and third quartiles of the damage parameter values over the individual MAP estimates. The first quartile is the value for which 75% of the estimates give a higher value. The second quartile is the median value of the estimates, and the third quartile is the value for which 75% of the estimates give a lower value.

No damage

The damage state estimation was also run on the measurements taken of the undamaged structure. This was done as a sanity check for the method. A characterization of the MAP estimates is visualized in figure 4. As expected, the estimates show damage parameter values which are close to zero. No damage is detected.

Slot depth 5mm

A characterization of the MAP estimates is visualized in figure 5. Two areas in the estimates show a few percent loss of stiffness. However, changes of a few percent in stiffness are possible by environmental factors. No damage is detected.

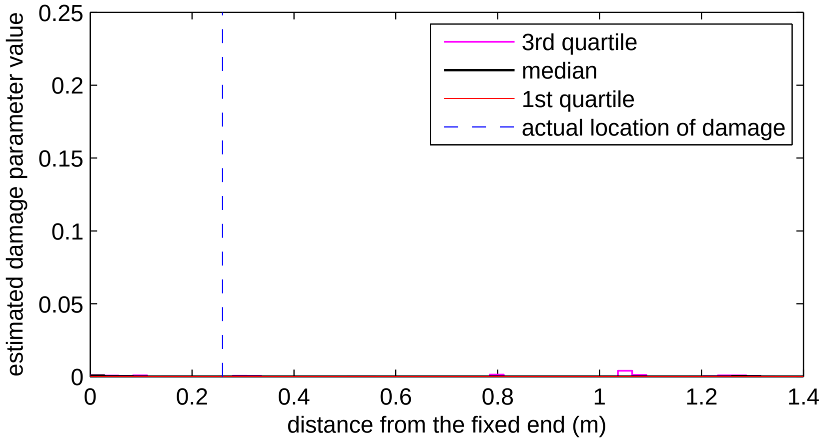

Figure 4. Characterization of the maximum a posteriori estimates in the case with no damage. No damage is detected.

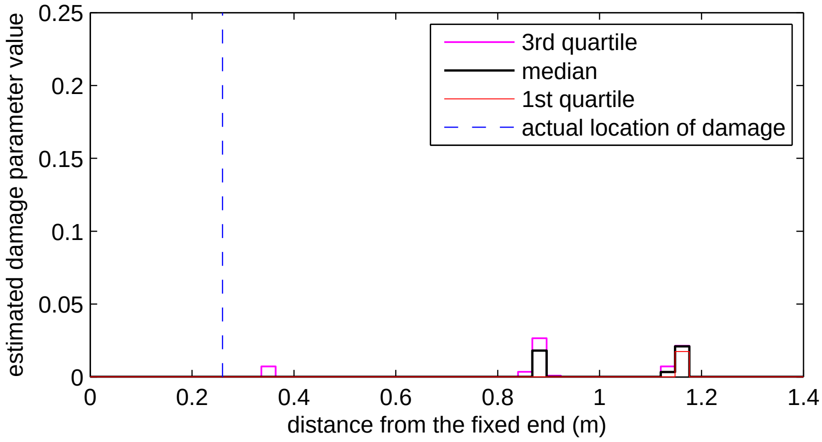

Figure 5. Characterization of the maximum a posteriori estimates in the case of a 5mm deep slot. No damage is detected.

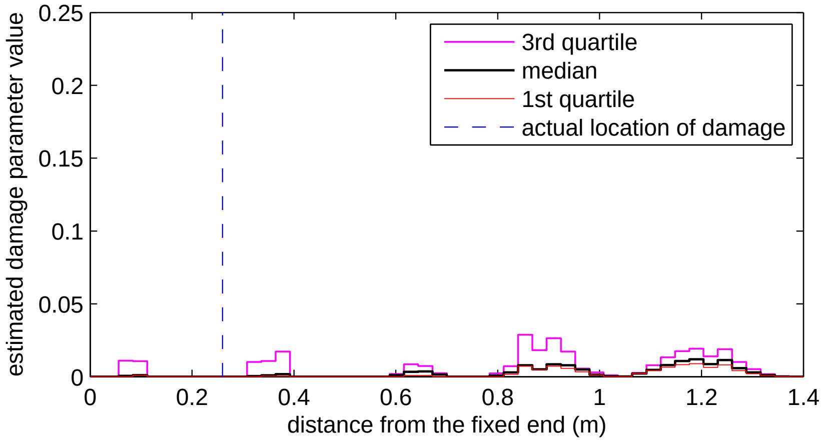

Figure 6. Characterization of the maximum a posteriori estimates in the case of a 10mm deep slot. No damage is detected.

Slot depth 10mm

This set of measurements contained spurious values, which are either due to some environmental factors which were not modeled or some error that happened when taking the measurements. A characterization of the MAP estimates is visualized in figure 6. As with the 5mm slot depth case, areas of a few percent loss of stiffness are seen. No damage is detected.

Slot depth 15mm

A characterization of the MAP estimates is visualized in figure 7. There are two areas where significant localized loss of stiffness is estimated. One of them is localized near 0.3 m from the fixed end. As the actual slot was located at 0.26 m from the fixed end, we conclude that the method detected the damage and that it was localized to within two possible locations.

Slot depth 20mm

A characterization of the MAP estimates is visualized in figure 8. There are four areas where significant localized loss of stiffness is estimated, with one area much larger than the others. This area of heavy loss of stiffness is seen located near 0.3 m from the fixed end. As the actual slot was located at 0.26 m from the fixed end, we conclude that the damage is detected and localized.

Conclusions

At the very least, the presented method correctly detected the location of the damage when the slot depth was 20 mm. For the slot depth of 15 mm, the detection is not quite as clear. However, if a similar case would come up in an actual structure, one would inspect it at the indicated locations and find the damage. The presented method can hence detect and localize damage in simple structures, given that the damage levels are high enough to cause changes in measurement which exceed modeling errors and noise. In this simple case a sufficiently good model was easily obtained. For complex structures this is no longer the case, and we expect that the need to use model updating schemes is in practice unavoidable.

The direction of the slot was deliberately chosen to be such that the stiffness of the beam was not much affected by it. Hence, only small changes in the modal parameters were expected. This was also observed. It would provide an interesting comparison, if the same experiment were carried out with the slot sawn in the direction where the beam is most sensitive to it.

Even though the presented method does not handle real-time data as such, it could still be used for almost real-time structural health monitoring. New data could be collected while the previous set is processed. Assuming the damage does not occur faster than, say, in the order of a day, it should be possible to identify it in essentially real time.

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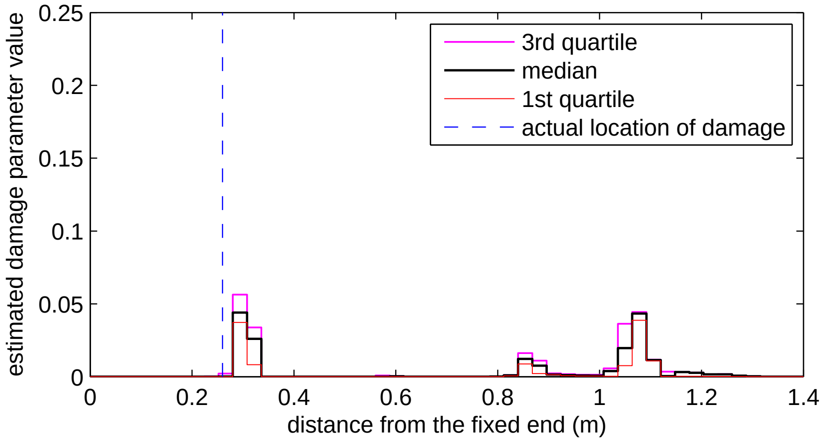

Figure 7. Characterization of the maximum a posteriori estimates in the case of a 15mm deep slot. Two areas with significant loss of stiffness are seen in the estimates. One of them is close to the actual damage location. Damage is thus detected and localized within two possible locations.

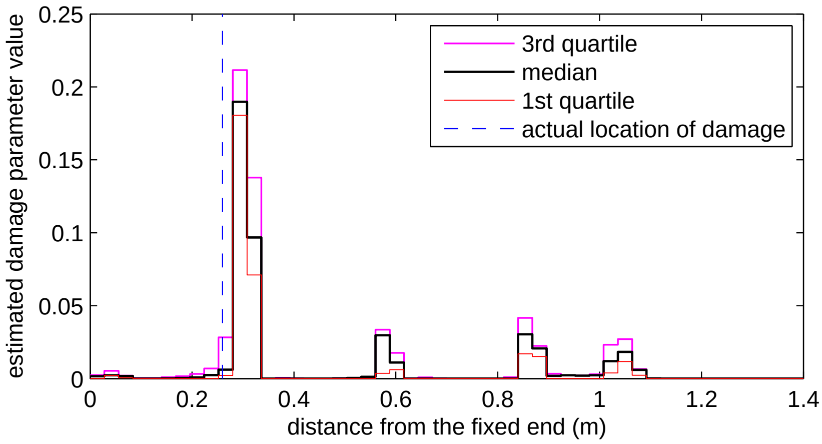

Figure 8. Characterization of the maximum a posteriori estimates in the case of a 20mm deep slot. Four areas with significant loss of stiffness are seen in the estimates. One of them is much larger than the others, and it is also close to the actual damage location. Damage is thus detected and localized.

References

[1] Palle Andersen. *Identification of Civil Engineering Structures using Vector ARMA Models*. PhD thesis, Aalborg University, 1997.

[2] John Argyris and Hans-Peter Mlejnek. *Dynamics of Structures*. North-Holland, 1991.

[3] Albert Beneviste and Fuchs Jean-Jacques. Single sample modal identification of a nonstationary stochastic process. *IEEE Transactions on Automatic Control*, 30(1):66–74, 1985.

[4] Dionisio Bernal. Flexibility-based damage localization from stochastic realization results. *Journal of Engineering Mechanics*, 132(6):651–658, 2006.

[5] P. Cawley and R. D. Adams. The location of defects in structures from measurements of natural frequencies. *Journal of Strain Analysis*, 14(2):49–57, 1979.

[6] P. Cawley and R. Ray. A comparison of the natural frequency changes produced by cracks and slots. *Journal of vibration, acoustics, stress, and reliability in design*, 110(3):366–370, 1988.

[7] Andrew D. Dimarogonas. Vibration of cracked structures: A state of the art review. *Engineering Fracture Mechanics*, 55(5):831 – 857, 1996.

[8] M. I. Friswell and J. E. Mottershead. *Finite Element Model Updating in Structural Dynamics*. Kluwer, 1995.

[9] Michael I. Friswell. Damage identification using inverse methods. *Philosophical Transactions of The Royal Society A*, 365(1851):393–410, 2007.

[10] Andrew Gelman, John B. Carlin, Hal S. Stern, and Donald B. Rubin. *Bayesian Data Analysis*. Chapman & Hall/CRC, 2004.

[11] Jari Kaipio and Erkki Somersalo. *Statistical and Computational Inverse Problems*. Springer, 2004.

[12] A. Messina, Williams E. J., and Contursi T. Structural damage detection by a sensitivity and statistical-based method. *Journal of Sound and Vibration*, 216(5):791–808, 1998.

[13] Laurent Mevel, Albert Beneviste, Michele Basseville, and Maurice Goursat. Blind subspace-based eigenstructure identification under nonstationary excitation using moving sensors. *IEEE Transactions on Signal Processing*, 50(1):41–48, 2002.

[14] A. K. Pandey and M. Biswas. Damage detection in structures using changes in flexibility. *Journal of Sound and Vibration*, 169(1):3–17, 1994.

[15] O. S. Salawu. Detection of structural damage through changes in frequency: a review. *Engineering Structures*, 19(9):718 – 723, 1997.

[16] Branislav Titurus, Michael I. Friswell, and Ladislav Starek. Damage detection using general elements: Part ii. damage detection. *Computers & Structures*, 81(24–25):2287–2299, 2003.

[17] Branislav Titurus, Michael I. Friswell, and Ladislav Starek. Damage detection using generic elements: Part i. model updating. *Computers & Structures*, 81(24–25):2273–2286, 2003.

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