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Optimal design of stiffened plate using metamodeling techniques

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Summary. In this article mass minimization of a stiffened plate is reported. From the actual finite element model of the plate, surrogate models are constructed using response surface methodology and the Kriging method. Estimation of the structural response is carried out using three different design of experiment models. As a numerical example a typical off-shore structure is optimized with respect to stress constraint equations. The optimization procedure is based on the standard NLPQL algorithm with iteratively moving response estimation window.

Key words: design of experiments, response surface methodology, Kriging method, metamodeling, optimization

Introduction

In the design and optimization of practical load-carrying structures, the search for global optimum configuration requires usually huge computer resources and fine-tuned optimization algorithms and software. From this point-of-view it seems to be well reasoned to concentrate on optimization methods that can find at least local optimum solution with relatively small computational costs. Especially this is true in industrial applications, where fast design process does not usually allow any detailed optimization studies of the product. Moreover, in industrial applications, any enhancement (e.g. lower mass or lower costs) achieved by using optimization is appreciated. For example, moderate mass saving (i.e. finding a local optimum) achieved using amount c of computational costs is more attractive choice than somewhat larger mass saving (i.e. finding the global optimum) with costs of αc, in which α is usually considerably larger than unity. Approximate optimization appears to be very suitable tool also for preliminary design of structures e.g. for sales and tendering purposes.

Metamodeling, or in the other words the use of surrogate models, is an approximation methodology in which the original problem is expressed globally in more simple form using e.g. polynomial representation of structural responses as in [1]. If the basis functions for the response approximations are chosen a priori, the approach is called response surface methodology (RSM) [1-3]. If the problem contains remarkable non-linearity, restriction to a priori chosen basis functions may lead to large estimation errors. In such cases the Kriging method is more attractive choice [4,5]. In the Kriging method, the response estimation is expressed as a sum of a polynomial global function and a local stochastic function [6].

In this paper, we apply the concept of metamodeling using the response surface methodology and the Kriging method for the single-objective optimization of a stiffened plate structure. Such structures are widely used e.g. in off-shore engineering [7]. The proposed sequential optimization procedure is however, not restricted to such structures, and it can applied generally.

The basic idea of the paper is to demonstrate the mass minimization process using three different response estimations. First, quadratic polynomials are used to construct the global response surfaces using design of experiments data generated with the so called central composite design method (CCD) [8,9]. In this method, the design variables obtain five separate value levels as depicted in figure 1. Secondly, linear polynomials (two separate value levels) are fitted to the generated design of experiments data. Obviously, linear polynomials cannot be used for global estimation, but the response is estimated in a smaller window of the entire design variable space. In this case the optimization process must be iterative in nature, and the estimation window moves in the design variable space while the optimization proceeds. Finally, a compromise between linear and quadratic models is tested. We adopt the Kriging method to the set of design of experiments data generated using Hadamard test matrices [10]. Since the Hadamard test matrix is three-level, the estimation can be done for larger window than in the case of linear polynomials. This method shows some advantages, e.g. smooth convergence to (at least) local optimum as reported in the section of numerical example.

The first step in defining the response estimates is to define how many experiments we will use, and what their locations in the design space are. When metamodeling is applied to structural optimization, the “experiment” means typically a FEM-analysis in which the design parameters are fixed to some chosen values and the required responses (stress, displacement, natural frequency, etc) are evaluated. The next section is devoted to highlight different strategies to the definition of design of experiments.

When the required responses are evaluated in m locations in the design space, the regression analysis and other statistical methods are obtained for the construction of the metamodel that replaces the original structural problem. In this step, the primary goal is to define such models, which contain minimal estimation errors. Metamodel construction using either response surface method or the Kriging method is explained concisely in the surrogate models section.

Structural optimization of the estimated problem is much easier than the optimization of the original problem. The cost we have to pay for this is the estimation error in response surfaces. Nevertheless, an iterative optimization strategy to cope with this error is tested in the context of a stiffened plate example problem. The standard NLPQL-algorithm is adopted for the mass minimization problem itself and ANSYS/WB v.13 is utilized for response evaluations and metamodel computations.

Design of Experiments

Let us consider a general problem having n factors that are chosen to represent the responses of the structure. When metamodeling is used in order to optimize the structure, the factors are usually also the design variables. Let us define m points (called as design points) in the design space, in which the required responses of the structure are computed. Usually, and also in this study, the finite element method is utilized for this purpose. It is obvious, that using more design points gives us more information on the structural responses, and thus, the estimation errors reduce when the number of design points increases. In practice, we must make a compromise between the used computing time and required accuracy. The most simple way to estimate the structural response is to use $m = n + 1$ design points. In this case each design variable obtains values from two different levels. Using this minimal set of data, a multi-linear surface including all the design point values can be defined in a unique manner. This set of design of experiments is called linear model.

Linear model can also be used if the number of design points $m > n + 1$. In this case the regression analysis (e.g. method of least squares) can be adopted to fit the multi-linear surface in optimal way to the available data.

If linear model cannot predict the responses accurately enough, we can increase the number of design points so that each design variable can get values from three different levels. In this case the class of Hadamard test matrices is one way to define minimal number of design points in a systematic and optimal way to the design space. It can be shown, that there are certain similarities between Hadamard and the well known Taguchi test matrices [10]. Since the number of design points is larger than in the linear model, it is not an efficient approach to span multi-linear response surfaces. More attractive choice is to use the Kriging method, in which the global polynomial surface is augmented by local stochastic variations. The Kriging method is explained concisely in the next chapter.

When the number of design points is increased so that each design variable obtains three different values, it is also possible to construct full quadratic response surface that goes through each design point. These kind of *full factorial* designs are not favored, because the number of design points increases drastically with the number of design variables. Hence, fractional design of experiments are usually used. In fractional method, only a half or one-fourth of the all factor combinations are taken into account [8]. Usually the three value levels are chosen to be the lower and upper limits and the center of the design variable value range.

One widely used design of experiments method is the so called central composite design (CCD), in which each factor achieves values from five different levels as depicted in figure 1. Total number of design points consists of at most 2^n corner points, $2n$ axial points and one central point. The central composite design method is discussed in detail in e.g. [8,9].

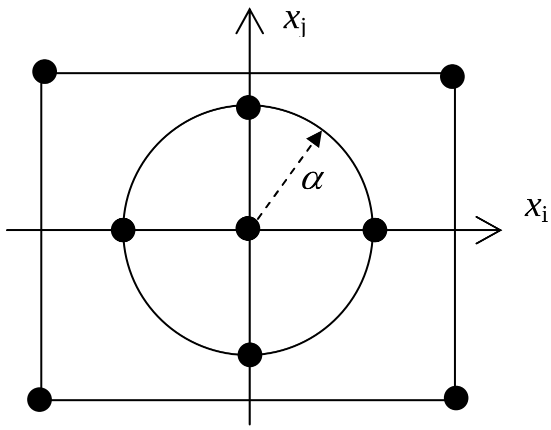

Figure 1. Central Composite Design test points in the case n = 2. Radius α, that defines the axial points, can be chosen according to various optimality criteria [8,9].

Surrogate models

Response surface method, RSM

In the RSM the unknown function is expressed using usually linear or quadratic polynomials as in (1) and (2), respectively

$$y(\mathbf{x}) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \varepsilon \tag{1}$$

$$y(\mathbf{x}) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n} \sum_{j \geq i}^{n} \beta_{ij} x_i x_j + \varepsilon \tag{2}$$

in which y is the unknown estimated function of factors x_i collected in vector $\mathbf{x}$, and ε is a random error. If the number of design points is larger than the number of coefficients β_0, β_i, β_{ii} and β_{ij}, regression analysis can be utilized in order to fine-tune the surface so that certain error measure is minimized. The quadratic polynomial is full if all the cross-terms $x_i x_j$ are included in the surface expression. In this case, the minimum number of design points is [9]

$$m = 1 + \frac{3}{2} n + \frac{1}{2} n^2 \tag{3}$$

If the cross-terms are omitted, the model is called pure quadratic model, and the minimum number of design point evaluations is

$$m = 2n + 1 \tag{4}$$

The coefficients β_0, β_i, β_{ii} and β_{ij} can be listed in vector **β** that can be solved using e.g. method of least squares leading to the equation [11]

$$\boldsymbol{\beta} = \left[\mathbf{X}^{\mathrm{T}}\mathbf{X}\right]^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y} \tag{5}$$

in which **X** is a matrix containing the design points and **y** is vector containing the estimated function values at the design points.

Kriging method

In the Kriging method, the response is estimated as

$$y(\mathbf{x}) = f(\mathbf{x}) + Z(\mathbf{x}) \tag{6}$$

in which $f(\mathbf{x})$ is global known polynomial part, that is usually taken as constant β, and $Z(\mathbf{x})$ is a realization of a stochastic process with zero mean value, σ^2 variance and non-zero covariance [11]. The local part $Z(\mathbf{x})$ of the estimation takes into account local variations of the response. The details of the method are not repeated here, because they can be found in literature, see e.g. [4,5,11]. Usually, the correlation function R between any two design points is chosen to be Gaussian type as

$$R(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\sum_{k=1}^{n}\theta_k \left|x_{i,k} - x_{j,k}\right|^2\right] \tag{7}$$

in which $x_{i,k}$ is k^{th} component of design point $\mathbf{x}_i$ and θ_k is correlation parameter, the value of which determined to fit the model. Correlation functions can be collected to a positive definite matrix **R** whose component (i, j) is $R(x_i,x_j)$. Diagonal elements of this square matrix have the value of 1. This leads to the following formula for the estimation

$$y(\mathbf{x}) = \hat{\beta} + \mathbf{r}^{\mathrm{T}}(\mathbf{x})\mathbf{R}^{-1}\left(\mathbf{y} - \mathbf{f}\hat{\beta}\right) \tag{8}$$

In which **y** is the column vector containing response values at the design points, **f** is unit vector and **r** is a correlation vector between current value **x** and the design points as

$$\mathbf{r}(\mathbf{x}) = \left[R(\mathbf{x}, \mathbf{x}_1) \quad R(\mathbf{x}, \mathbf{x}_2) \quad \ldots \quad R(\mathbf{x}, \mathbf{x}_n)\right]^{\mathrm{T}} \tag{9}$$

and estimation $\hat{\beta}$ of β is

$$\hat{\beta} = \left(\mathbf{f}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{f}\right)^{-1}\mathbf{f}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{y} \tag{10}$$

Numerical example for structural optimization

Problem description

As a numerical problem we consider the stiffened plate depicted in figure 2. Uniform pressure 0,1 MPa is acting on the top of the plate and the vertical z-directional displacement at the end edges (at $y = 0$ and at $y = L$) are restricted. The design variables are thickness of the top skin, thicknesses of the stiffener webs and flanges as well as the widths of the stiffener flanges as shown in figure 3. Total mass of the structure is minimized with respect to the following constraints: The von Mises stress must not exceed 200 MPa anywhere in the structure, absolute value of shear stress in the stiffener webs must not exceed 70 MPa and the compressive stress σ_y in the top skin must not exceed 120 MPa. The design variables are not restricted to any range and all the design variables are considered as continuous variables.

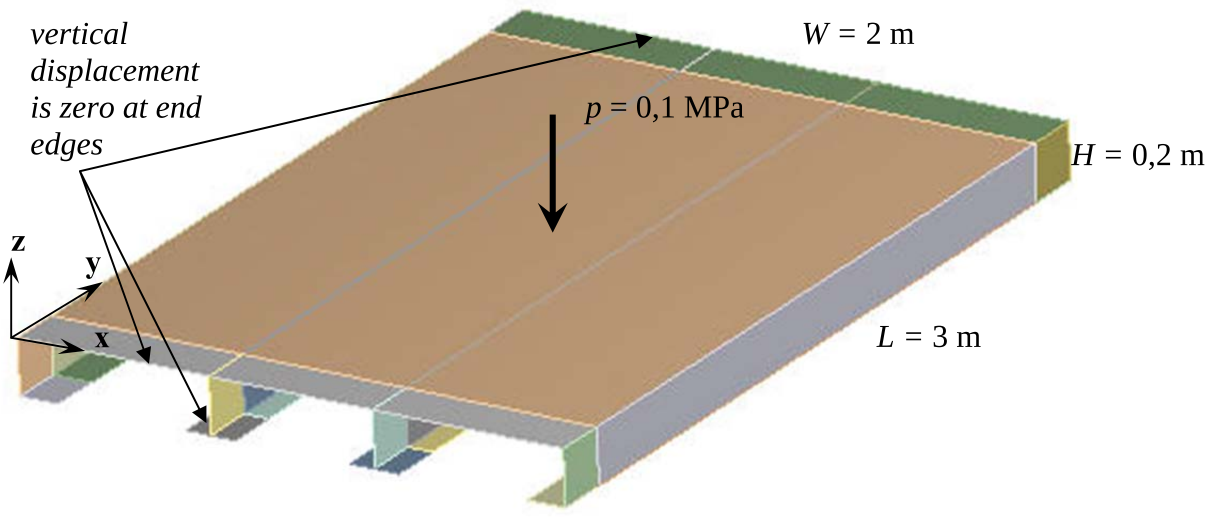

Figure 2. Loading, boundary conditions and dimensions of the stiffened plate.

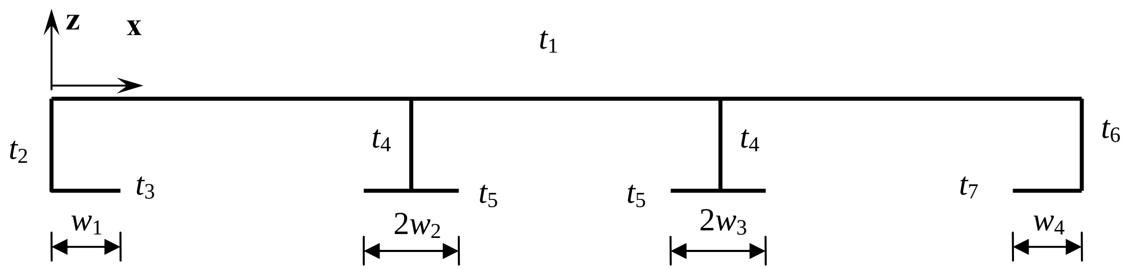

Figure 3. Design variables – seven thicknesses and four flange widths.

Results of the RSM utilizing linear surfaces

First the plate was modeled using linear response surfaces for each output variable (stresses and mass) appearing in the minimization problem. Altogether 12 FEM analyses were used to span the response surfaces. The surfaces were constructed in the design variable space in a window the size of which is 10% from the design variable values. The window size was chosen to be so that the estimation errors remain rather small (less than 1%).

In the optimization procedure, the estimation window at iteration step i+1 is placed so that the center of the window is in the result point of iteration step i. The optimization iteration results are depicted in figure 4 and in Table 1. The initial configuration was chosen from feasible range of the design space.

Since the multilinear response surfaces were constructed with minimal number of design points, the surface goes through all of these points and the estimation errors in these points are zero. The optimal structure shown in figures 5 and 6 seems also to have only minor estimation errors. In the optimum configuration, the active constraint equations are the global von Mises stress (limit 200 MPa) and the shear stress in the two central stiffeners (absolute limit value 70 MPa). The shear stress constraint in the outermost stiffeners and the compressive stress constraint in the top skin are not active. Convergence of the constraint equation values is shown in figure 4.

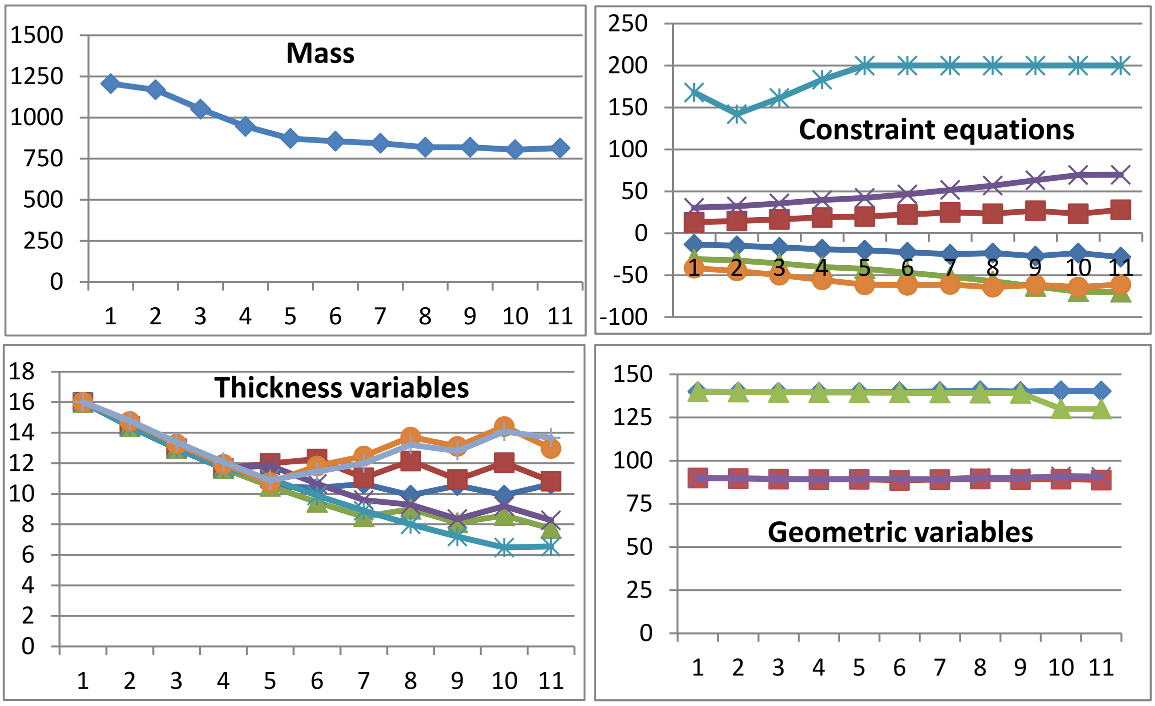

Figure 4. Convergence of the optimization iteration using RSM and linear surfaces.

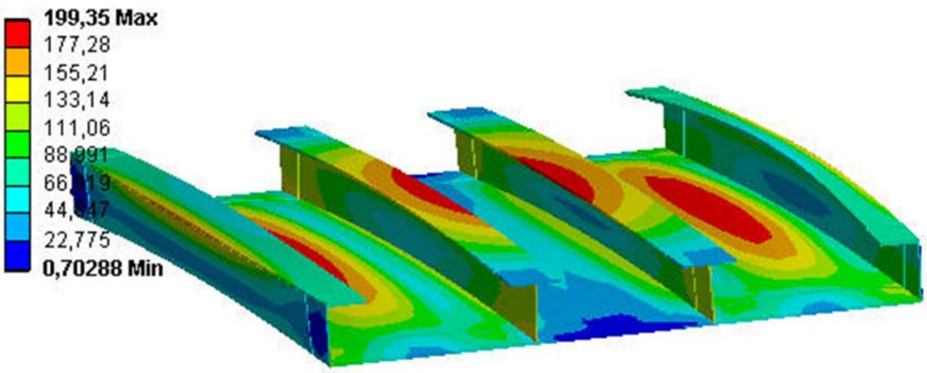

Figure 5. Von Mises stress distribution in the optimized structure. Maximum value 199.35 MPa contains only 0.3% estimation error compared to the target value of 200 MPa.

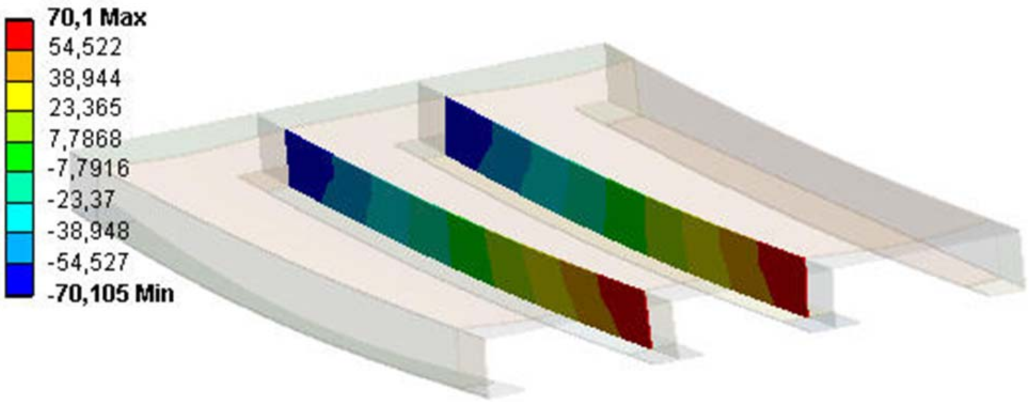

Figure 6. Shear stress distribution in the optimized structure at the central stiffeners. Maximum value 70.1 MPa contains only 0.1% estimation error compared to the target value of 70 MPa.

Table 1. Convergence of the optimization iteration using RSM and linear surfaces. The window size is reported only for the first iteration.

		Iteration # 1		
Des. Variable	Initial value	lower limit = 0.9*inital value	upper limit = 1.1*inital value	Optimum
t_1	16	14,4	17,6	10,63
t_2	16	14,4	17,6	7,72
t_3	16	14,4	17,6	13,67
t_4	16	14,4	17,6	6,55
t_5	16	14,4	17,6	10,83
t_6	16	14,4	17,6	8,26
t_7	16	14,4	17,6	12,97
w_1	140	126	154,0	130,04
w_2	90	81	99,0	90,68
w_3	90	81	99,0	88,79
w_4	140	126	154,0	140,29

Despite the fact, that the initial structure, loading and support are symmetric, the computed optimal configuration is not. This drawback is due to the estimation errors in multi-linear response surfaces. Since the symmetry is broken already at the first iteration step, the asymmetries accumulate when the iteration proceeds. Another and more severe drawback of the linear method is that the values of the thickness variables $t_1 - t_7$ do not converge to certain values, but they tend to oscillate between lower and upper limit values.

It is also worth noting, that the geometric design variables (flange widths) $w_1 - w_4$ remain almost constant, and small changes are present only in the three last iterations. This is due to the fact, that their influence to stresses is much smaller than the influence of thickness variables. Based on the results, it seems that the geometric variables could be even fixed to their initial values.

Results of the Kriging method with Hadamard-model

When the Kriging method is used with the Hadamard test matrices, the number of analyses for this case is 27. The correlation between the number of factors and number of test evaluations is discussed in detail in [10]. Window size is chosen to be ±20% from the current design variable values as shown in Table 2. The estimation window is moved in the design space as in the case of the RSM utilizing linear surfaces, but for the last iteration the window size is decreased to cover only ± 5% variations from the previous iteration step result. This adaptive window size adjustment is an attempt to fine-tune the response estimates in the vicinity of the local optimum. The initial values for the design variables are the same as in the previous model as shown in Table 2. The optimization convergence is depicted in figure 7 whereas figure 8 contains the von Mises stress distribution of the optimum configuration. In this method, the von Mises global stress constraint is the only active constraint equation. The optimization iteration was stopped after fifth iteration, after which the design variables were converged to certain final values. There was no need to the use of *a priori* defined mathematical convergence criterion, because such a criterion could not be adopted for the linear method at all.

Table 2. Convergence of the optimization iteration using Kriging method. The window size is reported only for the first iteration.

		Iteration # 1		
Des. Variable	Initial value	lower limit = 0.8*inital value	upper limit = 1.2*inital value	Optimum
t_1	16	12,8	19,2	10,34
t_2	16	12,8	19,2	8,31
t_3	16	12,8	19,2	8,69
t_4	16	12,8	19,2	8,19
t_5	16	12,8	19,2	13,71
t_6	16	12,8	19,2	9,17
t_7	16	12,8	19,2	9,00
w_1	140	112	168,0	138,93
w_2	90	72	108,0	89,35
w_3	90	72	108,0	89,39
w_4	140	112	168,0	138,90

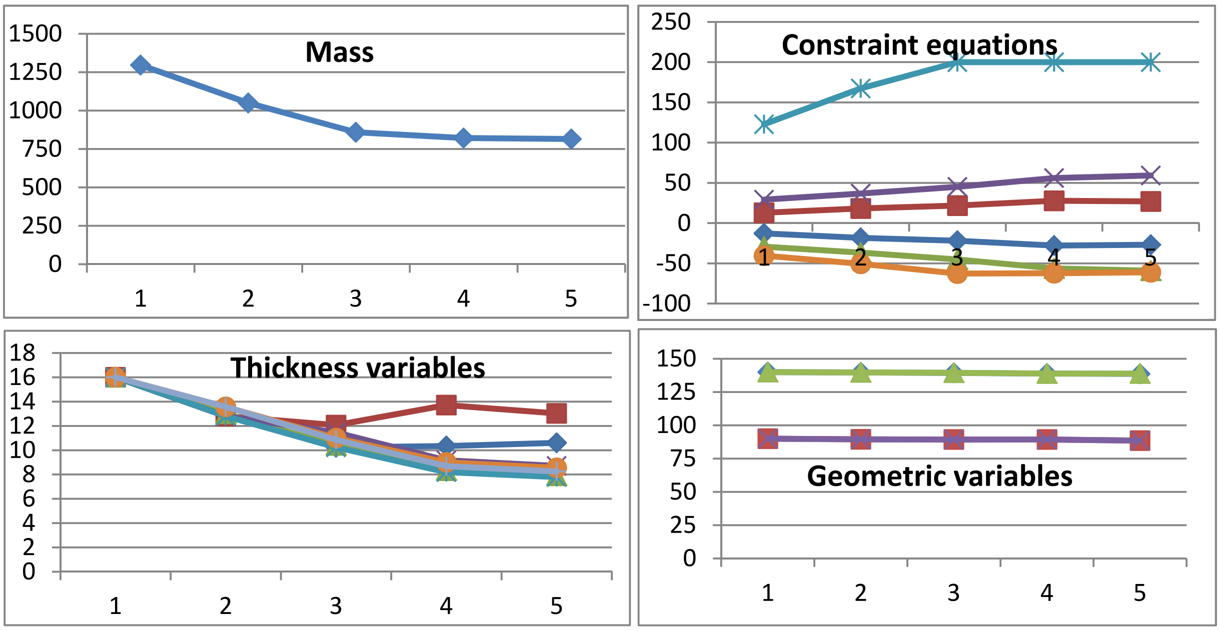

Figure 7. Convergence of the optimization iteration using Kriging method.

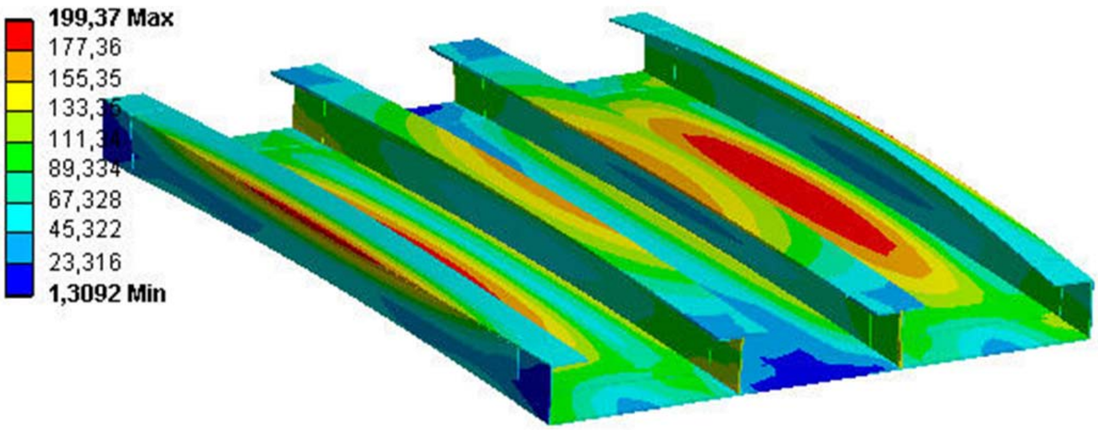

Figure 8. Von Mises stress distribution in the optimized structure. Maximum value 199.37 MPa contains only 0.3% estimation error compared to the target value of 200 MPa.

Results of the RSM utilizing CCD and quadratic surfaces

In the case of quadratic surfaces, the required number of the analyses is 151 even though fractional factorial tests are used with the CCD-method [8]. In this case, the initial point was chosen near the optimum found by the RSM and linear surface estimation as depicted in Table 3 and the whole range of admissible design variable space is estimated globally with no iterations. The results, given in Table 4 show, that the method is not too accurate in this example. After the optimal configuration is found, the estimation error is ca. 7%. This means, that an additional iteration with smaller estimation window would be required, but due to large number of analyses per iteration, this is not efficient. In general, CCD-method could be effective, if the initial point would be rather close to a optimum and the estimation window size would be smaller.

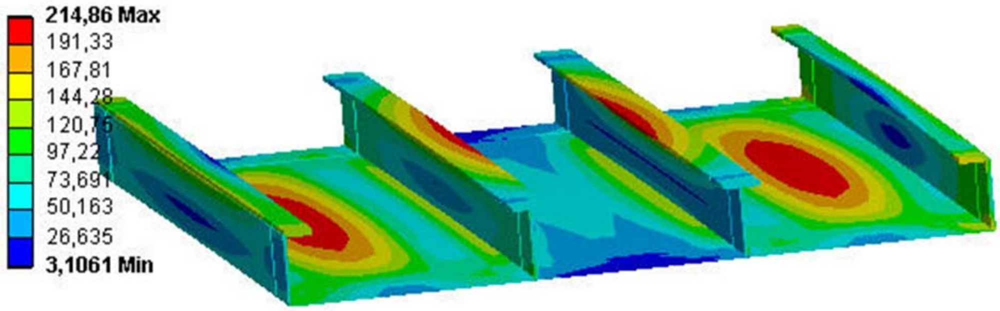

Figure 9. Von Mises stress distribution in the optimized structure. Maximum value 214.86 MPa contains 7,4% estimation error compared to the target value of 200 MPa.

Table 3. Convergence of the optimization iteration using RSM and quadratic surfaces.

Des. Variable	Initial value	lower limit	upper limit	Optimum
t_1	10	6	14,0	9,96
t_2	10	6	14,0	11,17
t_3	14	8	20,0	20,00
t_4	10	6	14,0	13,76
t_5	10	6	14,0	14,00
t_6	10	6	14,0	14,00
t_7	14	8	20,0	14,50
w_1	80	40	160,0	80,05
w_2	50	25	100,0	54,51
w_3	50	25	100,0	53,79
w_4	80	40	160,0	80,12

Conclusion

Based on the results one could conclude, that the first step in the proposed optimization procedure is to find suitable window size for the adopted response estimation scheme. In the considered example, the suitable window sizes were approximately ±10% for the linear model, ±20% for the Hadamard model and less than ±40% for the quadratic model. Actually, the window size should be even smaller for the quadratic model, but due to the large number of required function evaluations, smaller window would not be efficient. The FEM-analyses needed for the estimation surfaces are in this example 12, 27 and 151, respectively. Comparing the required computational work and the accuracy of the results, the use of quadratic model is rather doubtful. Its accuracy is the worst and its computational costs are highest among the three tested methods. Both the RSM with linear surfaces and the Kriging model worked well in the considered example problem, and according to Table 4, both the methods seem to be rather efficient. The Kriging method however, requires less FEM-evaluations and its convergence is much smoother. Thus, the Kriging method seems to be the best choice for the present problem type, and its applicability for other problems must be studied in future. It is also worth noting, that the final optimization problem solution time attached to each metamodel did not play a central role in the total solution time.

Table 4. Results and required number of analyses of the different methods

Method	iterations	analyses per iteration	analyses total	optimum mass
RSM / Linear	10	12	120	813.9 kg
Kriging /Hadamard	4	27	108	814,8 kg
RSM/quadratic	1	151	151	853,7 kg

In general, the adopted methodology of metamodeling and sequential optimization seems to work efficiently in mass minimization. In the literature the accuracy of the surrogate model can be enhanced by adding extra design points to the estimation window [1]. However, in this article such approach is not utilized and the method based on moving estimation window seems to be accurate enough.

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