

## Comparing passive walker simulators in Matlab and Adams

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**Summary.** This article presents a few different methods to present a passive walker model in Adams MD R3 software and compares their simulation results to a walker model built in Matlab software. The discontinuous Matlab simulator applies manually entered mathematical equations to describe system dynamics while the Adams simulation software creates the dynamic models automatically. In the Adams simulator we apply a few different methods to present the walker models. In the first case the Adams model is built with continuous equations to describe contact events. This is in contrast to the Matlab model where these events are discontinuous. In the second case the Adams model is discontinuous applying a scripted simulation and discontinuous motion constraints. A special attention in this article is paid for modeling of knee-locks and foot contacts in the Adams simulation environment. In further experiments the Adams model combines continuous and discontinuous models for the contact events. The simulation results show that each model behaves differently and parameter tuning is needed to make each of them walk. The Matlab model and the Adams models seem to have a different dynamic response to the knee-lock event. It is not possible to completely repeat the Matlab simulation results in Adams. Reason for this is not currently known and deserves further studies. During the heel-strike event either a two-link model or a three-link model of the walker can be applied, although the general convention applies the two-link model only. The three link model at the time of heel-strike event can be made to walk passively but its applicability in practice requires further studies.

*Key words:* modeling and simulation, bipedal passive limit-cycle walking, contact modeling, scripted simulation, Adams MD R3

### Introduction

A mechanical model of a bipedal walker is being built in the Department of Automation and Systems Technology in the School of Electrical Engineering of Aalto University. The walker will apply theory of passive walking for energy efficient powered walking. Early simulation results will give help in development of a proper mechanical design and suitable control methods. Simulations have been conducted with two different simulation platforms which are Matlab and Adams MD R3.

The model in Matlab follows a general convention and similar simulators have been presented frequently in literature. These models similar to our Matlab model have been used for numerical analysis of passive limit cycle walkers. The analyses have provided information about boundary parameter values for the limit cycle, robustness of the cycle, existence and shape of several different cycles, initial conditions for the cycle and other similar. The Matlab model is therefore an efficient tool for mathematical study of the passive walker.

On the contrary, literature does not reveal many passive 2D walker models created with Adams software. Models in Adams tend to present often active and controlled 3D walkers, often carrying also a torso. Our main motivation for transferring the passive 2D limit cycle walker model from Matlab to Adams is in parametrization and extended contact modelling capabilities provided by Adams. Through parameterisation we wish to study effects of parameter variation (varying link lengths, link masses and cog. locations) on walking performance. Extended contact modelling capabilities of Adams allow us to bring the walker model closer to real world application, since the Matlab model is very ideal regarding the contacts and joint properties. Further the Adams model can be equipped with actuator models to make it an active walker coupled with a control system running in Matlab Simulink software. Later the model can be easily extended into 3D. Thus the Adams model is good for more realistic presentation of physical walker structure and properties, and it can be used for testing and development of control software for an active walker, that is our ultimate goal.

The first simulation model of a passive walker is written with Matlab R2007b software applying the theory by Mochon and McMahon (1980) and McGeer (1990a and 1990b), in a similar way as presented by Chen (2007). The model written in Matlab, nominated as 'Matlab model' in this article, describes the walker dynamics with manually entered mathematical equations. The model is discontinuous, as will be explained later, and it is limited with respect to contact properties, joint friction, and locations of link masses. Because of these limitations we wish to transfer the Matlab model into Adams MD R3 simulation software where we would have more freedom to test and adjust the parameters.

The basic difference between the Matlab model and the Adams model is in creation of the dynamic model; For the Matlab model we have hand-written the dynamic equations according to the convention described in the references. The Adams simulation software, however, creates the dynamic model automatically on basis of model geometry and mass properties as given when creating the model. We have paid a close attention to create a walker model identical to that used for the Matlab model.

The mathematical equations used for the Matlab model are commonly known and regularly applied for simulation and analysis of passive bipedal walkers. Therefore this model presents a starting point to be modified for studies on different (more realistic) contact parameters, joint properties and mass distribution. In order to understand the effect of the modifications, and to be consistent with other simulation results performed elsewhere, behaviour of the Matlab simulator first needs to replicate in the Adams environment. Only after that modifications and further studies on system details in Adams may start. The results from the Matlab and Adams simulations, to be presented in this paper, indicate that these models give a different response on the knee-lock event, reason for which is a subject for further studies. This paper describes our efforts in transferring the Matlab model into the Adams environment, the challenges we met and the differences we found in behaviour in these different simulation environments. A special attention is on modelling options for foot contacts and knee-locks of the passive bipedal walker in Adams simulation software. Currently we have not yet available any measurement data from real hardware and thus our aim at this moment is not yet to

discuss how the models reflect reality, but we limit to comparing the Adams model to the Matlab model.

Since we eventually aim to build the actual energy efficient powered bipedal walker, we are interested to study such constructions where reasonable link masses would allow placement of real electric actuators in each link to facilitate active walking. Because of this the overall weight of our walker becomes quite high, compared to most common passive walker models presented in literature, and relation between the thigh mass and the shank mass is not as favourable as would be desirable. See Chen (2007) for some examples of favourable simulation models and also an example of a mechanical model. As a consequence from its mass properties our simulation model becomes extremely sensitive to parameter variations, which emphasizes any minor differences that might occur between the models or the simulation environments.

## Paper structure

This paper discusses findings from comparison of simulation results from the Matlab-model and different walker models in the Adams-environment.

Section ‘Simulation models’ describes in its first subsection the passive walker model created in Matlab software applying manually entered functions to describe the walker dynamics, and the following five subsections describe the same walker model constructed in five different ways in Adams MD R3 simulation software.

Section ‘Results and discussion’ presents simulation results of the Adams models, each in its own subsection, and compares those to the simulation results of the Matlab model.

‘Conclusions’ section collects the findings from the preceding sections.

## Nomenclature

2-link model	Walker model with two rigid legs without knees or both knees locked to be rigid.
3-link model	Walker model with one rigid leg (usually the stance leg), and other leg (usually the swing leg) comprising of a thigh and a shank and a free moving knee.
Matlab model	Model of the passive walker written in the Matlab-environment applying mathematical functions to describe dynamics of the mechanical system.
Continuous Adams model	Model of the passive walker running in the Adams-environment and applying CONTACT-statements to model foot contacts and BISTOP-functions to model knee-stops.

Scripted Adams model	Model of the passive walker running in the Adams-environment and applying MOTION-constraints to model foot contacts and/or knee-stops. Use of a simulation script allows the simulation to stop at each contact event and to modify the dynamic model of the walker.
BISTOP	Adams function that presents zero value inside a specified range of selected variable values and spring-damper-like response at the limits of the range. BISTOP-function describes the torque in knee-joints and models knee-locks in the continuous Adams model.
CONTACT	Adams statement that creates contact forces between two objects. CONTACT-statement models foot contacts in the continuous Adams model.
MOTION	Adams constraint that defines location, acceleration or velocity of a marker or body. MOTION-constraint here defines zero velocity for locked knee-joint or stance leg contact point in the scripted Adams model.
JOINT	Adams function that measures force acting on a joint. JOINT function measures here stance foot support force on the knee joint in the continuous Adams model.
STEP	Adams function that creates continuous transfer from a numeric value $y_0$ (here 0) to a value $y_1$ (here 1) when defined condition changes from a numeric value $x_0$ to $x_1$ . STEP function here disables or enables knee-lock BISTOP-function on the basis of selected input states (joint angles and support force) in the continuous Adams model.
cog.	Center of gravity
$q_1, q_2, q_3$	Angle measured in radians clockwise from the horizontal plane to the stance foot, the swinging thigh and the swinging shank.
$q_1\_d$ $q_2\_d$ $q_3\_d$	Angular velocities in radians/second of respective angles.

## Simulation models

The following subsections describe and discuss separately one passive bipedal walker model in Matlab and five different passive walker models in Adams, which are:

1. Matlab model with mathematically described dynamics.
2. Continuous Adams model with continuous force functions in knee-locks and foot contacts.
3. Scripted Adams model with discontinuous motion constraints in knee-locks and foot contacts.
4. Scripted Adams model with a continuous force function in knee-locks.
5. Scripted Adams model with a continuous force function in foot contacts.
6. Scripted Adams model with momentum preservation calculation in a three-link mode.

The Matlab model describes walker dynamics with manually entered mathematical equations following the convention presented in the references. This model is discontinuous since simulation stops at each contact event and replaces the dynamic equations with new ones describing the model's new state. At the time of contact event the model adopts discontinuous changes in link velocities. On the contrary the continuous Adams model applies continuous force functions in the knee-locks and foot contacts. The first scripted Adams model is a modification of the continuous Adams model replacing the continuous force functions with discontinuous motion constraints. The two other scripted models combine the continuous and discontinuous elements of the previous ones and they are used to point out the origins for different model behaviour. The last scripted Adams model studies the model behaviour in the case where the knee-lock is released before calculation of the system dynamics after a heel-strike event (instead of having the knee-lock activated during that calculation).

### *Passive walker model in Matlab*

Our limit cycle walker model has its basis in the theory of passive walking by Mochon and McMahon (1980) and McGeer (1990a and 1990b).

Mochon and McMahon (1980, p. 50-52) presents the mathematical model of the passive walker with equations 1-10 of the said article. McGeer (1990a) discusses passive walking in wide context and relates that to rolling of a wheel. It also presents the dynamics with notation to be applied in McGeer (1990b). McGeer (1990b) concentrates more on passive walking with knees and presents the methodology for deriving the walking trajectories (p. 1642, bullets 1-4). It also explains how to derive the proper initial conditions through iteration (p. 1642, last paragraph on the leftmost column), and discusses foot clearance, stability and perturbations.

We apply the theory in Matlab software using mathematical expressions describing the model dynamics with manually entered mathematical functions similar to those presented by Chen (2007). Chen (2007) presents the walker geometry in Figure 3-1 on p. 21 of the said reference. Equations 3.1 through 3.5 (on p. 22-26 of the said reference) describe the walker dynamics in the same form as we have adopted for our Matlab model. Appendix A of Chen (2007, p. 53-56) explains the inelastic contact events in a similar way that we have applied.

The walker has two legs with knees and a hip, which makes a total of five bodies and three joints. Figure 1 presents the walker model in Adams sharing the same

geometry with the model in Matlab. In the figure the five spherical objects are geometrical illustrations only to indicate locations and relative magnitudes of the link masses. The illustrated spring-damper constraints relate only to the Adams model knee-lock, they are not present in the Matlab model.

The model takes into consideration link lengths, link masses as mass points, and the location of that mass point along the link. The inertia of the links or friction in the joints is not present in the equations. The off-set of the link center of gravity (cog.) from the link center line is zero. The contacts between the legs and the ground, and also in the knee-lock event, present ideal fully inelastic contact events. The equations of motion are calculated only on preservation of momentum while the friction or physical contact properties do not play any role. The knee-lock remains also artificially locked during the entire stance-phase.

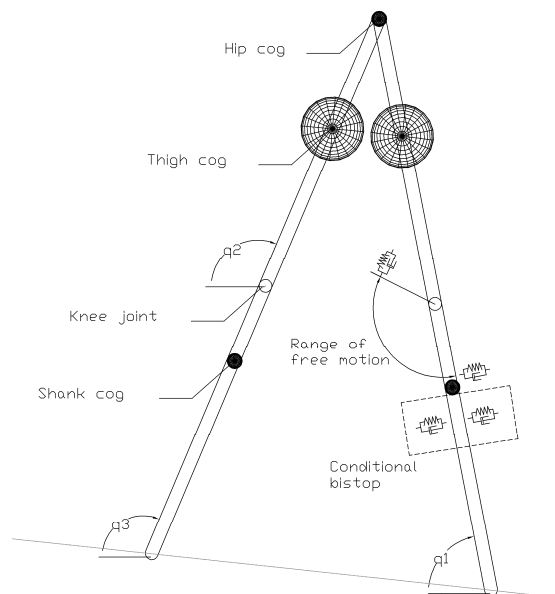


Figure. 1. The passive walker model in Adams.  $q_1$ ,  $q_2$  and  $q_3$  are initial joint angles. The right leg shows spring-damper symbols where a BISTOP-function is used to model the knee-locks. Similar knee-locks are also present on the left leg. The spherical objects present locations of link cogs. and relative masses of the links.

The simulation of the passive walker proceeds in discontinuous phases changing its dynamic model at contact incidents. Figure 2 presents a typical walking sequence. For convenience we identify for the model a left leg and a right leg. Due to symmetry the ideal Matlab model does not make any difference between the left and the right, but only the stance leg and the swing leg are of importance. In the Adams models, however, the left side and the right side may adopt different behaviours and the graphs to be presented later identify those as ‘Left’ and ‘Right’.

The sequence starts when taking the first step with the right foot. The right knee is locked and the right leg (the stance leg) is carrying the weight of the walker. The left knee (the knee of the swing leg) is free to move. Usually the left thigh swings forward at

this time due to its initial velocity and the left shank starts to move upwards. The walker is then said to be in a ‘three-link mode’, since the right leg presents one rigid link due to the locked knee, and the left thigh and shank present two more links. As the walker moves over the right stance leg, motion of the left thigh and shank continue until they pass the right leg. After passing the stance leg the swinging leg straightens completely and also the left knee locks, which is called a ‘knee-lock event’. Now the walker is said to be in a ‘two-link mode’. Both the right and left legs are completely straight and both knees are locked. The sequence continues until the next heel-strike event where the left heel touches the ground. Now the two-link model is standing on both legs, both knees still locked. Momentum of the system keeps it moving and when the weight has moved on the left leg the right knee is released to return the system back to the three-link mode. The sequence repeats then as a mirror of the previous step.

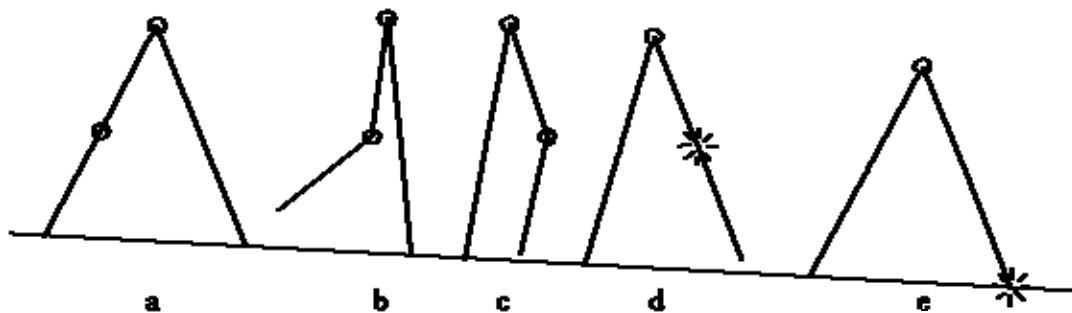


Figure. 2. The passive walking sequence. a) Start in three-link mode, b) Rear knee bends, c) Rear foot passed the stance foot, d) The knee-lock event, the walker enters into the two-link mode, e) The heel-strike event

The simulation of the Matlab model is a discontinuous process where the contact events divide the walking cycle into different phases. Each phase is described by a different dynamic model depending on the state of the knee-locks and whether the legs are swinging free or touching the ground. The contact events are the ‘knee-lock’-event at the moment when the swinging leg becomes fully extended and its knee locks, and the ‘heel-strike’-event at the moment when the forward swinging leg hits the ground. Between the events the simulator calculates the link trajectories and velocities by numerically integrating the mathematical equations describing the current dynamic model of the walker. At the moment of the contact event the integrator stops and the simulator calculates instantaneous changes in the link velocities. The model equations describe fully inelastic collisions and assume preservation of momentum. After the event an updated dynamic model is applied for further integration until next contact event. With properly defined initial conditions, which are joint angles and link velocities, the walker returns after each step back to the state where the joint angles and velocities are the same as in the end of the previous step. Then the walker is performing a limit cycle that can continue forever. Walker geometry, mass distribution, slope angle

and gravity define the proper initial conditions, the values of which may be found through a numerical iteration process as described by McGeer (1990b, p. 1642).

For our own biped model we have created more than 30 different variants where the link lengths and mass locations are the same, but mass distribution between the shank, thigh and hip change. For each of these variants the iteration process has produced a unique set of initial conditions that allows the biped to walk in a limit cycle. Out of the 30 models the one giving the best performance in the Adams environment was then selected as a baseline for further studies with Adams models.

### *Continuous Adams model*

Knowing that the current simulation model in Matlab ignores some dynamical properties of real hardware, -like inertia, mass-offset from the link center line, joint friction and contact parameters; we were interested to transfer the simulation model into another environment that would give us more freedom to apply and modify these parameters. Adams-software provides one possible platform for such further studies.

Building a walker model in Adams to present the existing Matlab model requires plenty of attention to many details to maintain consistency between the two models. Especially the definition of the link initial velocities is very important because the model simulation starts in motion. The detailed description of all modelling techniques being applied for the walker model is out of the scope of this paper, but this article draws attention to some details of interest. Figure 1 presents the Adams model. Table 1 presents the common geometrical parameters that are the same for all models described in this paper. Table 2 presents the initial joint angles same for all models; the initial velocities which have been modified to gain better performance; and the simulation results with the achieved walking distance, the walking time and the number of steps.

The first approach in modelling of the limit cycle walker in Adams was to aim for a continuous model using continuous expressions of spring-like and damper-like forces in the knee-locks and foot contacts. Any discontinuous events would be then absent and the integrator should run smoothly during model solving without interrupts or restarts. Table 3 presents the Adams/Solver settings compared to Matlab integrator parameters used for simulations. The integrator settings used by Adams/Solver (Fortran version) are explained in details in Adams/Solver Documentation – INTEGRATOR-statement (Simcompanion, 2010a, p. 967-994). The applied GSTIFF-integrator works best with this specific walker model and the Adams solver does not create any errors with any of the model variants either. (The model variants include the continuous model and scripted models.) Also other integrators available in the Adams software were experimented, including Runge-Kutta-Fehlberg Method (RKF45), but they did not work as well, or not at all with some model variants. Also different solver parameters were experimented, but no better walking performance was found. In Table 3 ‘Error’ means mean-square error of all model parameters being solved. In this walker model the allowed error for a single parameter becomes  $1E-5$ , as announced by the Adams solver. Absolute tolerance ‘abs\_tol’  $1E-20$  was used for the Matlab simulator. Later experiments with Matlab indicated that absolute tolerance  $1E-6$  produced similar results. Adams solver was not able to converge into solution with smaller Error values and Matlab integrator does not allow any larger tolerance.



Table 1. Common parameters for all walker models.

Link lengths (shank and thigh bones) (mm)	450.00
Shank cog. distance to knee-joint, along shank bone (mm)	224.25
Thigh cog. distance to hip-joint, along thigh bone (mm)	184.90
Shank Mass (kg)	2.50
Thigh Mass (kg)	9.81
Hip Mass (kg)	2.36
Slope angle (deg.)	6.70*

\* For the experiment 'Heel-strike with a three-link model' the slope angle is 8.47 deg.

Table 2. Parameters, initial conditions and walking performance of the walker models. Initial velocities for all scripted Adams models have been modified from those of the Matlab model.

Model	Matlab model	Continuous Adams model	Scripted Adams model	Scripted Adams model with BISTOPS	Scripted Adams model with CONTACTS	Scripted Adams model in 3-link mode at heel-strike event
Parameter*						
q1 (rad)	1.3623					
q2 (rad)	2.0130					
q3 (rad)	2.0130					
q1_d (rad/s)	1.3705		1.2195	1.3842	1.2480	1.4
q2_d (rad/s)	-0.0951		-0.0944	-0.09513	0.00566	1.0
q3_d (rad/s)	-0.0951		-0.0944	-0.09513	0.00566	1.0
Walking dist. (m)	∞	195	14.8	>118	12.5	13
Walking time (s)	∞	215	17	>123	14	13
Steps taken	∞	>200	24	>200	19	20

\*Blank cell indicates the same value as in the Matlab model

Table 3. Adams and Matlab Solver parameters.

	ADAMS	Matlab
Integrator	GSTIFF (Newton-Raphson)	Ode45 (Runge-Kutta (4,5) Dormand-Prince pair)
Formulation	I3	
Output time step	0.001 s	0.001 s
Units	mm-kg-s	mm-kg-s
Error	1E-3	
abs_tol		1e-20
Hmax	(none, default output time step)	
Hmin	(none)	
Hinit	(none, default of 1/20th of the output step)	
Adaptivity	off	
Interpolate	off	
Kmax	6	
Maxit	10	

CONTACT-statement and BISTOP-function in Adams software allow continuous description of the foot contacts and knee-stops respectively. Both expressions create a continuous action force between the contacting bodies. The CONTACT-statement, MOTION-statement, BISTOP-function and related IMPACT-function are explained in details in Adams/Solver Documentation (Simcompanion, 2010a, p. 833, 1051, 1207 and 1249). Parameter selection for contacts attempts here to imitate a fully inelastic event where the spring and damping constants must be in the correct relation. In addition, the contact friction must be sufficient to prevent any slipping of the foot. The walker model is not insensitive of these parameters, and some models work better with one set of values, while another model favours another different set of values. More robust models are less sensitive to parameter variations. Out of the 30 walker models created for the Matlab, for this study we selected the most robust model that was already from the beginning behaving well both in the Matlab-simulation and in the continuous Adams-simulation.

Simulation experiments reveal quite easily suitable pairs of spring constant and damping constant for CONTACT-parameters. For successful simulation, the walker model spring stiffness may vary from 12 to 12000 N/mm and damping constant from 1.2 to 12000 Ns/mm. A low spring ratio (12 N/mm) with a high damping coefficient (120 Ns/mm) was chosen to replicate results from the earlier Matlab simulations as closely as possible. However, these values may not be suitable for further studies since the low spring ratio is only 10% of what could be achieved with a soft rubber contact at the end of the shank, while the high damping constant is 600 times that of a typical car shock absorber ( $c \sim 0.2$  Ns/mm, Rao and Gruenberg 2002). Later studies indicate that the spring ratio 120 N/mm, presenting a soft rubber contact, with a 12 Ns/mm damping still works well. A higher spring constant or a lower damping in the foot contact often would cause vibration and bouncing and so a reduced friction force leading into slipping of the foot in the Adams model.

The BISTOP-function in the Adams model creates continuous torques in the knee-joints to present the knee-locks. When applied as a force function, the BISTOP creates a

zero force when the joint angle is inside a range of free motion, while outside the limits of the free motion a spring force tends to push it back while a damping force resists all motion. A proper selection of limit values, spring constant and damping constant provides the desired behavior of the knee-lock. Here we wish to imitate a fully inelastic collision. A single BISTOP-function is sufficient to create a simple knee-lock that allows no over-extension of the knee but a natural bending backwards over a suitable angle. In many passive walker models this would be a sufficient approach since a favorable mass distribution and geometry keep the knee-joint locked automatically over the entire stance phase. However, our model has an unfavorable distribution of mass that causes the knee to often collapse prematurely. Therefore we need an active knee lock that is forced to remain locked over the entire stance phase. An additional conditional BISTOP-function limits also the backward motion of the knee during the stance phase. The condition for the active knee-locking measures the stance leg force, i.e. the knee remains locked as long as the stance leg is carrying the weight of the walker, or at least a fraction of it exceeding 100 N, which value was found suitable through simulation experiments.

The Adams/View command script presented below defines the double BISTOP-function for the knee-locks. The first BISTOP-function sets limits for over-extension and backward motion, while the second BISTOP-function keeps the knee locked while the leg is loaded. A JOINT-function measures the knee load and a STEP-function creates a smooth continuous activation or deactivation of the second BISTOP-function. The JOINT-function and STEP-function are explained in details in Adams/Solver Documentation (Simcompanion, 2010a, p. 1258, and 1288). Adams/View command language and especially the force definition below are explained in details in Adams/View Documentation (Simcompanion, 2010b, p. 1266). The knee-lock requires a high stiffness 1e6 mNm/rad (17.5 Nm/deg) to remain locked during the stance phase. As for comparison a steel plate 20 mm long, 30 mm wide and 2.4 mm thick provides a similar bending stiffness. The knee-lock damping coefficient is 5e7 mNms/rad at 1 radians displacement. The BISTOP-function aims to keep the knee angle displacement below 0.01 radians where the damping coefficient has still a high value of 5e5 mNms/rad (8.7 Nms/deg.)

```
force modify direct single_component_force &
single_component_force_name = Left_Bistop &
function = "-BISTOP(AZ(Shank, Thigh), WZ(Shank, Thigh) ,
-0.01, 2, 1e6, 1.1, 5e7, 1 )-BISTOP(AZ(Shank, Thigh), WZ(Shank,
Thigh, Thigh), -0.01, 0.01, 1e6, 1.1, 5e7, 1 )
*STEP(JOINT(Knee-joint, 1, 1, 0), 100, 0, 101, 1)"
```

Figure 1 in the beginning of the first chapter shows a schematic presentation of both BISTOP-functions. The angle between the upper spring-damper pair shows the free motion range of the knee, the lower pair shows the very limited motion range that is activated when the conditions are as desired. Both knees have similar knee-locks although the figure presents only one for clarity. Figure 3a presents the knee angle, BISTOP-torque and CONTACT-force for the right leg during one step period. Zero

contact force and large knee-joint angle values are clearly visible during the swing phase, and BISTOP-torque peaks are clear at the knee-lock and heel-strike events. The high BISTOP-torque peak on the right knee at the time  $\sim 0.35$  seconds is a reflection from the left knee-lock event.

The step sequence starts with the right leg stance phase with a sufficient support force to allow the BISTOP-torque to keep the right knee angle close to zero. In the end of the stance phase the left foot takes the step and the support force disappears from the right leg. Then the right knee BISTOP-torque becomes zero and sets the knee-joint free to move. In the three-link mode the left leg is the stance leg and the right shank swings freely around the knee-joint. In the end of the three-link mode the right knee straightens again which activates the first term of the BISTOP-function. This prevents the knee from over-extension and the walker enters into two-link mode where both legs are straight. During the two-link phase the contact force is still zero, but the conditional part of the BISTOP-function is not needed anyway since momentum of the swinging shank keeps the knee extended. Upon the right heel-strike event the contact force increases and activates the conditional BISTOP-torque again and the cycle repeats. The high peaks in the BISTOP-torque reflect the knee-lock and heel-strike events on either leg. Figure 3b shows in a detail how the BISTOP-torque vanishes when the contact force drops below 100N.

### *Scripted Adams models*

The continuous Adams model did not produce results that were fully comparable with the ones from the Matlab model. Most of the 30 walker models that walk in the Matlab simulator fail during the first few steps in Adams. Reasons for falling are various including foot scuffing (premature contact to the ground), knee collapse, foot slipping and too long steps. The major difference between the Matlab model and the continuous Adams model is in the presentation of the knee-lock and heel-strike events. In the Matlab model those are presented as discontinuous events where the link velocities change in a discontinuous manner (infinite accelerations), whereas the Adams model applies continuous functions to describe the contact forces. In order to maintain consistency of the Adams model with general convention and walker modelling done elsewhere, we want replicate behaviour of the Matlab model in Adams as closely as possible. In an attempt to do that we modified the Adams model to have similar discontinuous contact events as the Matlab model has.

The scripted Adams model returns back to a discontinuous walker model similar to that in the Matlab model. This Adams model uses sensors to detect the foot steps and knee extension, stops the simulation at the event, modifies the model dynamics and then continues integration until the next event. Adams MOTION-constraints define the foot motion or knee motion after each event. When a foot step takes place a MOTION-constraint sets the shank's velocities to zero at the marker located at the foot contact point. Then the foot can only rotate around the contact point, but it cannot bounce or slide. Thus the contact resembles a fully inelastic contact event similar to that in the Matlab model. When a knee-lock event occurs the MOTION-constraint sets the knee-joint velocity to zero. Release of the foot contact or knee-lock deactivates the respective MOTION-constraint

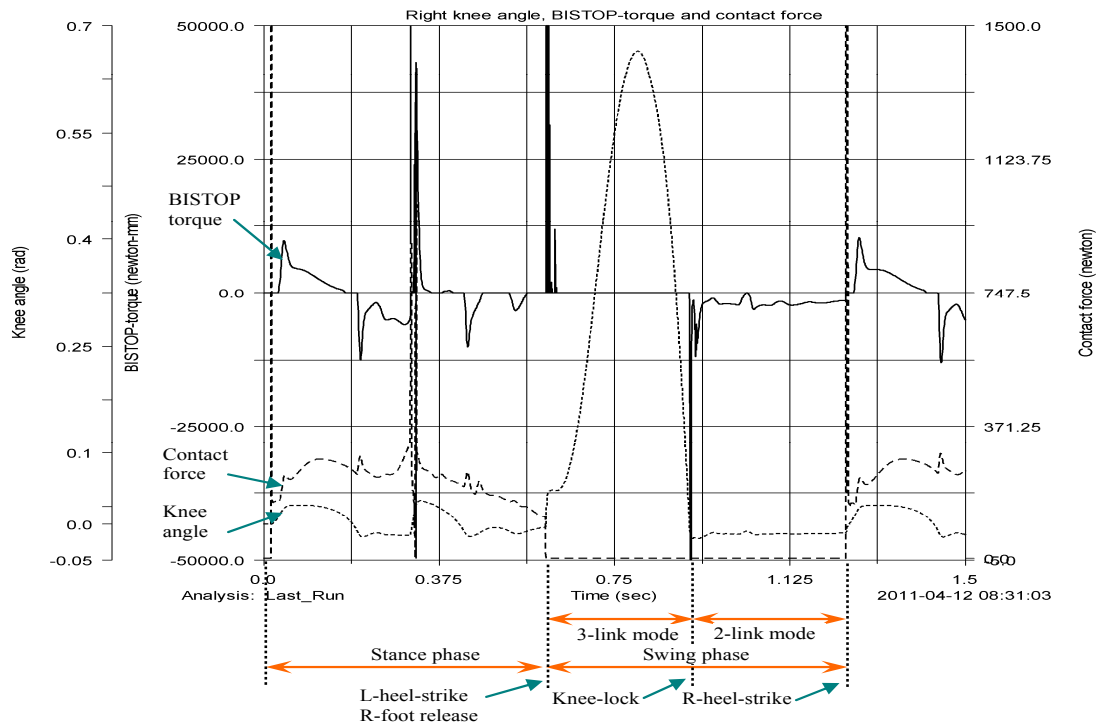


Figure 3a. The right knee angle, BISTOP-torque and CONTACT-force.

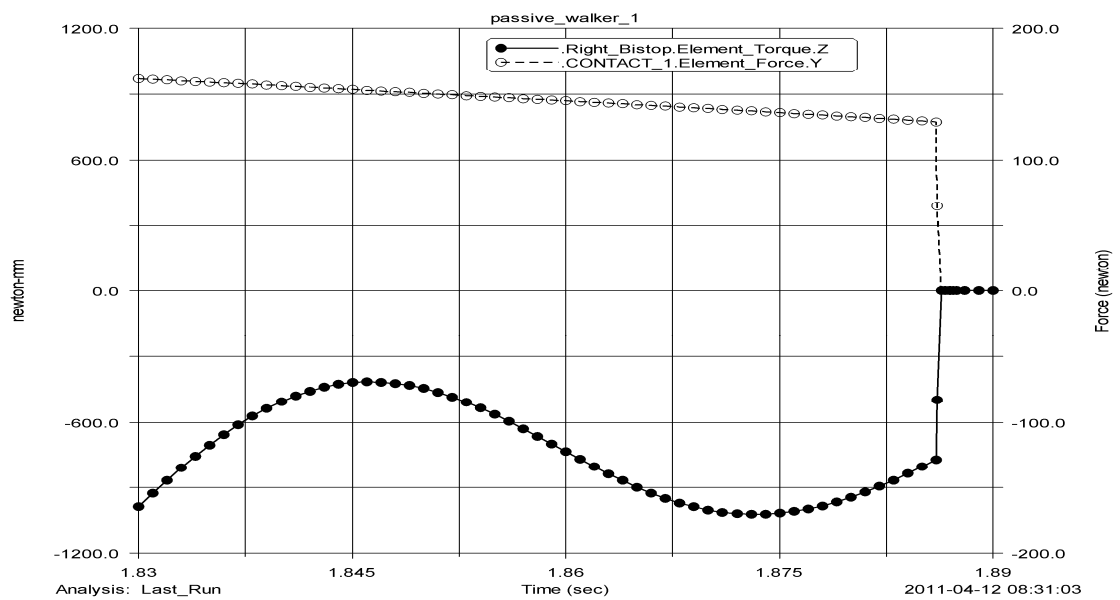


Figure 3b. The right knee BISTOP-torque and CONTACT-force. As soon as the CONTACT-force drops below 100 N the BISTOP-torque goes to zero.

Using a scripted simulation with sensors requires that the order of the events is known beforehand and it does not change. Use of two scripts, one describing a full walking cycle, and another repeating the first one in a loop allows performing a simulation lasting the desired number of steps. The two Adams/View simulation scripts are presented below. Adams/View scripting language is explained in details in Adams/View Documentation (Simcompanion, 2010b, p. 369-373).

The script must activate and de-activate the sensors so that they will be operational when needed, and disabled at the times when they would repeatedly produce unwanted triggering signals halting the simulation. It is important to realize that the convention presented by Mochon and McMahon (1980), McGeer (1990a and 1990b) and Chen (2007), and many more, applies a two-link model when calculating momentum preservation and related velocities at the time of the heel-strike event. This means that the knee of the swinging leg remains locked during the event, as does the knee of the stance leg. To realize this in the scripted simulation the heel-strike event must be followed by a short simulation in the two-link mode before releasing the swinging knee into the three-link mode.

In order to compare and better understand the differences and similarities between the Matlab model, the continuous Adams model and the scripted Adams model, the last one was still modified for further experiments. First the familiar BISTOP-force functions replace the knee-lock MOTION-constraints while the foot contacts remain modelled with the MOTION-constraints. In the second experiment the CONTACT-definitions similar to the continuous Adams model replace the MOTION-constraints related to the heel-strike event, while the MOTION-constraints model the knee-locks.

```
!Simulation script 1 - loop
!  

!Runs Simulation script 2 as many times as indicated by the loop
!Start loop for steps
for variable_name=tempreal start_value=1 end_value=20
  simulation single_run scripted sim_script_name=SIM_SCRIPT_2
  reset_before_and_after=no
end
```

```
!Simulation script 2 - two steps
!  

!Simulates one complete cycle of 2 steps starting with right
  stance leg  
  

!In the beginning deactivate all MOTION-constraints and sensors
DEACTIVATE/MOTION, ID=1,2,3,4,9,10
DEACTIVATE/SENSOR, ID=1,5,6,7,8  
  

!Set right knee lock
ACTIVATE/MOTION, ID=1
MOTION/1, VELOCITY, FUNC=0  
  

!set right foot motion restriction
ACTIVATE/MOTION, ID=9,10
MOTION/9, VELOCITY, FUNC=0
```

```

MOTION/10, VELOCITY, FUNC=0

! simulate a short time in 2-link-mode before switching into 3-
  link-mode
! set left knee lock
ACTIVATE/MOTION, ID=2
MOTION/2, VELOCITY, FUNC=0
SIMULATE/DYNAMIC, DURATION=0.001, DTOUT=1.0E-003

! switch to 3-link-mode and release left knee lock
DEACTIVATE/MOTION, ID=2

! simulate a short time to allow left knee bend
SIMULATE/DYNAMIC, DURATION=0.1, DTOUT=1.0E-003

! prepare to detect left knee extension and simulate until left
  knee straightens
ACTIVATE/SENSOR, ID=6
SIMULATE/DYNAMIC, DURATION=0.5, DTOUT=1.0E-003

! upon left knee extension set left knee lock
ACTIVATE/MOTION, ID=2
MOTION/2, VELOCITY, FUNC=0
DEACTIVATE/SENSOR, ID=6

! prepare to detect left foot step and simulate until left foot
  step
ACTIVATE/SENSOR, ID=8
SIMULATE/DYNAMIC, DURATION=0.5, DTOUT=1.0E-003

!upon left foot step
DEACTIVATE/SENSOR, ID=8

!release right foot motion
DEACTIVATE/MOTION, ID=9,10

!set left foot motion restriction
ACTIVATE/MOTION, ID=3, 4
MOTION/3, VELOCITY, FUNC=0
MOTION/4, VELOCITY, FUNC=0

! simulate a short time in 2-link-mode before switching into 3-
  link-mode
! (remove this step for momentum calculation in 3-link model at
  heel-strike event)
SIMULATE/DYNAMIC, DURATION=0.001, DTOUT=1.0E-003

! switch to 3-link-mode and release right knee lock
DEACTIVATE/MOTION, ID=1

! simulate a short time to allow right knee bend
SIMULATE/DYNAMIC, DURATION=0.1, DTOUT=1.0E-003

! prepare to detect right knee extension and simulate until right
  knee straightens
ACTIVATE/SENSOR, ID=5
SIMULATE/DYNAMIC, DURATION=0.5, DTOUT=1.0E-003

! right knee extension set right knee lock

```

```
ACTIVATE/MOTION, ID=1  
MOTION/1, VELOCITY, FUNC=0  
DEACTIVATE/SENSOR, ID=5
```

```
! prepare to detect right foot step and simulate until right foot  
step  
ACTIVATE/SENSOR, ID=7  
SIMULATE/DYNAMIC, DURATION=0.5, DTOUT=1.0E-0
```

### *Heel-strike with a three-link model*

The last experiment applies a slightly modified version of the scripted Adams model. The ‘three-link model’ refers here to the walker model at the time of the heel-strike event. Here we tested the behaviour of the model when momentum calculation at the heel-strike event is performed with a three-link model instead of the two-link model.

The convention presented by Mochon and McMahon (1980), McGeer (1990a and 1990b) and Chen (2007) applies the two-link model for the incident. The authors did not manage to find from the reference literature any argumentation for this convention. The two-link model can be assumed to be effective if the knee-lock is being actively locked, as for example, with an electromagnet or other actuator. In addition, if the physical construction of the knee-joint allows a slight over-extension, the support force on the rear leg may also keep the rear knee locked during the heel-strike event. However, a simple knee-cap similar to that of a human knee allows the knee-joint motion during the heel-strike event. Thus different physical constructions of the knee-joint would lead to different behavior and we wish to examine these two options, especially since the general convention is limited only to one of those. In the scripted Adams model the three-link mode activates when we remove the MOTION-constraint from the rear knee at the same time as when we apply the front foot MOTION-constraints.

## **Results and Discussion**

We have created 30 different sets of walker models each with a different mass distribution. An iteration process using the Matlab model produced initial conditions that lead each model into limit-cycle walking in Matlab. We transferred all 30 models into the Adams-environment and tested them using the continuous Adams model. From all the models we selected the best behaving model for further studies to apply different contact event modelling methods.

Expression of contact events presents a clear theoretical difference between the Matlab model and the continuous Adams model. While in the Matlab model the contacts are fully inelastic events where momentum remains, in the continuous Adams model the CONTACT-statement and BISTOP-function used to define contacts apply a spring-damper –type contact behaviour where penetration between contacting bodies and energy loss through damping will happen. Even bouncing and slipping may be present. Although this kind of contact model may represent reality better than the fully inelastic contact of the Matlab model, when compared to contact definition in the Matlab model it clearly loses energy and violates the contact constraint. Objective of the second Adams model, the scripted Adams model, is to get rid of this difference to



the Matlab model. In the scripted Adams model the contact events are modelled with the aid of additional MOTION-constraints that immediately change motion and freedoms of the links, without using any springs or dampers. There any penetration, bouncing or slipping may not happen. Thus the scripted Adams model was expected to replicate the Matlab model results very well, but the outcome was different, as will be explained later.

### *Continuous Adams model*

Out of the 30 limit cycle walkers only three were able to walk longer than 12 seconds applying the continuous Adams model with spring/damper-like contact models; one reached 18 meters, one 65 meters and the best one 195 meters. The rest of the models failed after a few steps. We selected the best behaving model for further studies.

Figures 4a, 4b and 4c present the joint angles, velocities and cycles for the selected continuous Adams model and the Matlab model. The time span of the figures covers the first two steps. Figure 4a indicates that models behave in quite a similar way until the first step (with left foot), after which the differences start to develop more clearly. Figure 4b reveals high joint velocity peaks at the time of the contact events. These were expected knowing that the knee-joint and foot contacts have springs and dampers present. The flexibility of the knee-locks and foot contacts is visible in the joint velocity graphs. Detail A in Figure 4b shows the velocity peaks in hip-joints at the time of the knee-lock event. The peaks present a clearly different behaviour from the Matlab model, but the difference attenuates away fast. The peaks at the moment of the heel-strike event are even more prominent, and they attenuate away even faster. The overall behaviour seems to be mostly unaffected by the peaks, but the reasons causing the different walking performance are more subtle and develop during the sequence. Figure 4c shows that the cycle of the continuous Adams model is neither symmetric nor stable; the cycles are oscillating within moving bounds and the right-side cycles are clearly different from the left-side cycles. Yet this Adams model is able to walk for 195 meters in 215 seconds before falling.

The reason for non-symmetric cycles can be assumed to be in the initial conditions of the continuous Adams model; It starts with the same joint angles and velocities as the Matlab model, but having a zero foot penetration into the ground. The Matlab model does not allow any ground penetration but the Adams model creates some during the first simulation steps due to the elastic foot contact. Upon the first step the Adams model has acquired a different state than in the beginning and the initial state will not repeat again, which causes non-symmetry and difference from the Matlab model.

Origin of the high velocity peaks is in the contact forces at the moment of a heel-strike event when the walker is in two-link mode. Front leg motion changes rapidly during the event. As a consequence, a high torque develops in the rear knee-lock. The spring coefficient of the knee-lock allows some motion which is, however, rapidly damped away by the damping coefficient. This velocity peak can be seen in all joints, and also a respective knee-lock torque and foot contact forces can be monitored with an Adams post-processor.

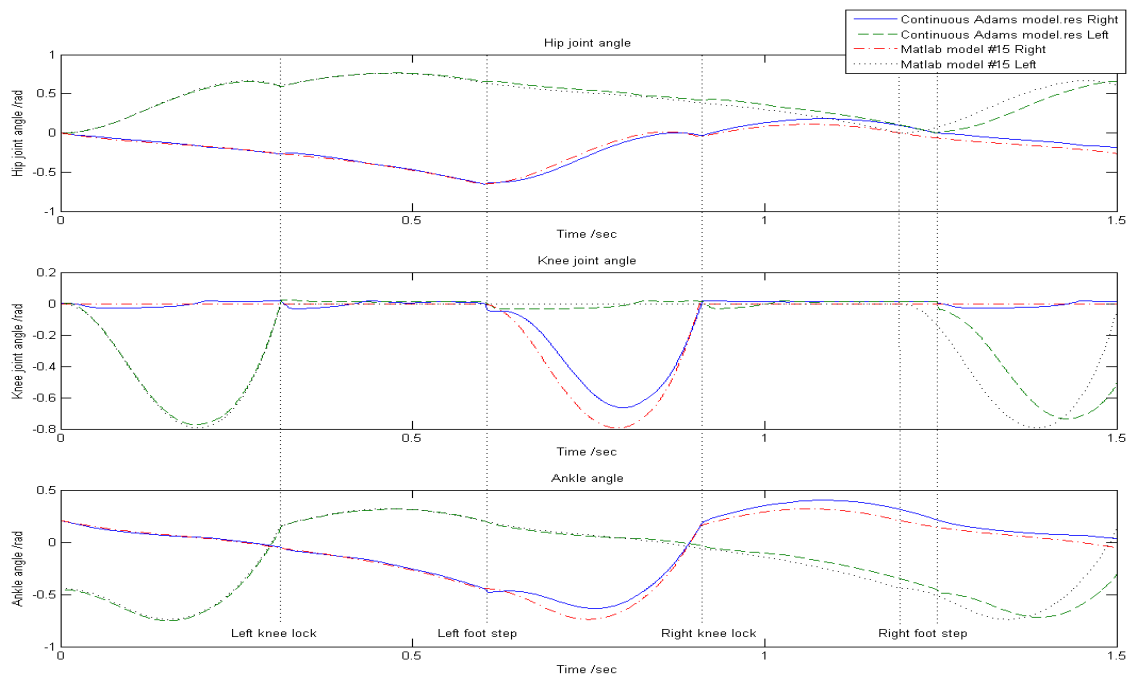


Figure 4a. Joint angles for the continuous Adams model and the Matlab model.

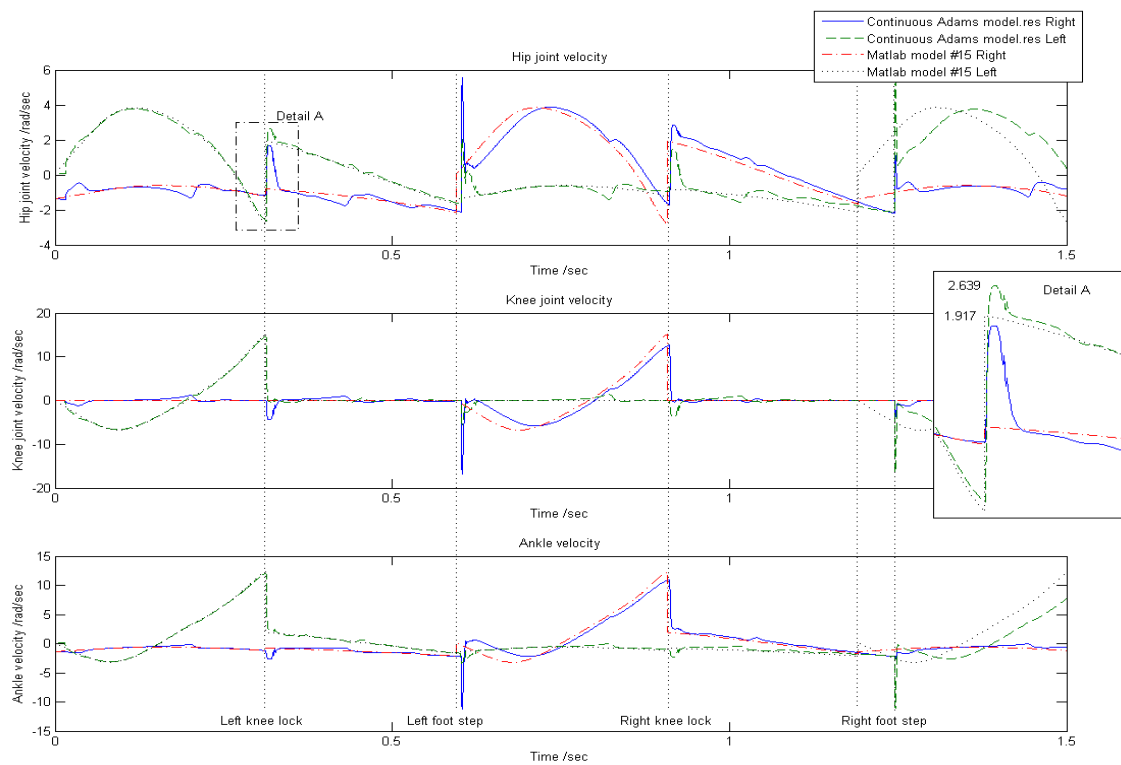


Figure 4b. Joint velocities for the continuous Adams model and the Matlab model.

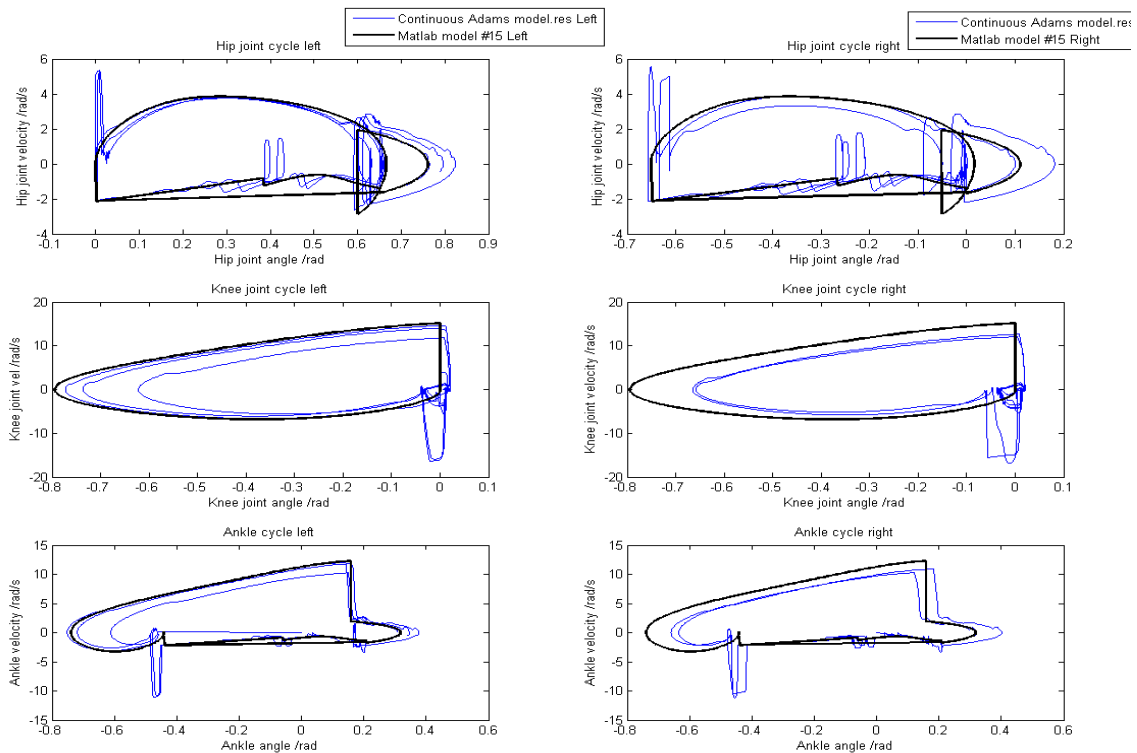


Figure 4c. Joint cycles for the continuous Adams model and Matlab model

### *Scripted Adams model*

The continuous Adams model did not produce results that were fully comparable with the ones from the Matlab model. The major difference between the Matlab model and the Adams model is in presentation of the knee-lock and heel-strike events. In order to maintain consistency of the Adams model with the general convention and walker modelling done elsewhere, we wish to replicate the behaviour of the Matlab model in Adams as closely as possible. For this reason the scripted Adams model attempts to replicate the Matlab simulation results in Adams through inelastic contact modelling applying a scripted simulation, discontinuous joint constraints and discontinuous link velocities. Despite the expected more similar contact modelling with the Matlab model, the scripted Adams model falls already on the fourth step.

Figures 5a and 5b present for comparison the joint angles and velocities for the Matlab model, the continuous Adams model and the scripted Adams model. The joint angle trajectories show how the continuous Adams model and the Matlab model have a similar behaviour until the first heel-strike event. On the contrary, the scripted Adams model starts to deviate from the others already at the moment of the first knee-lock event. The scripted Adams model knee-joint angle follows after the heel-strike event a significantly larger curve than the Matlab model. Due to the larger shank trajectory the knee-lock event of the right foot takes place slightly later with the scripted Adams model than with the Matlab model. The continuous Adams model takes the right foot step a little later than the Matlab model, while the scripted Adams model takes it much later, which eventually results in loss of balance.

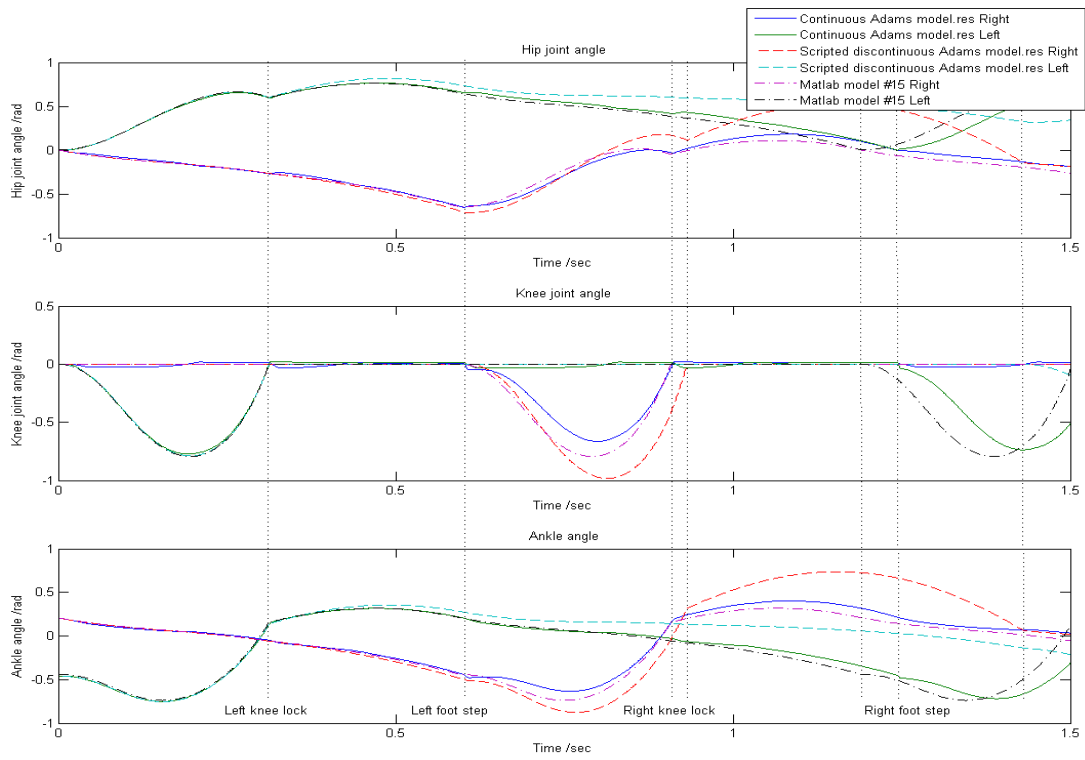


Figure 5a. Joint angles for the continuous and scripted Adams models and for the Matlab model.

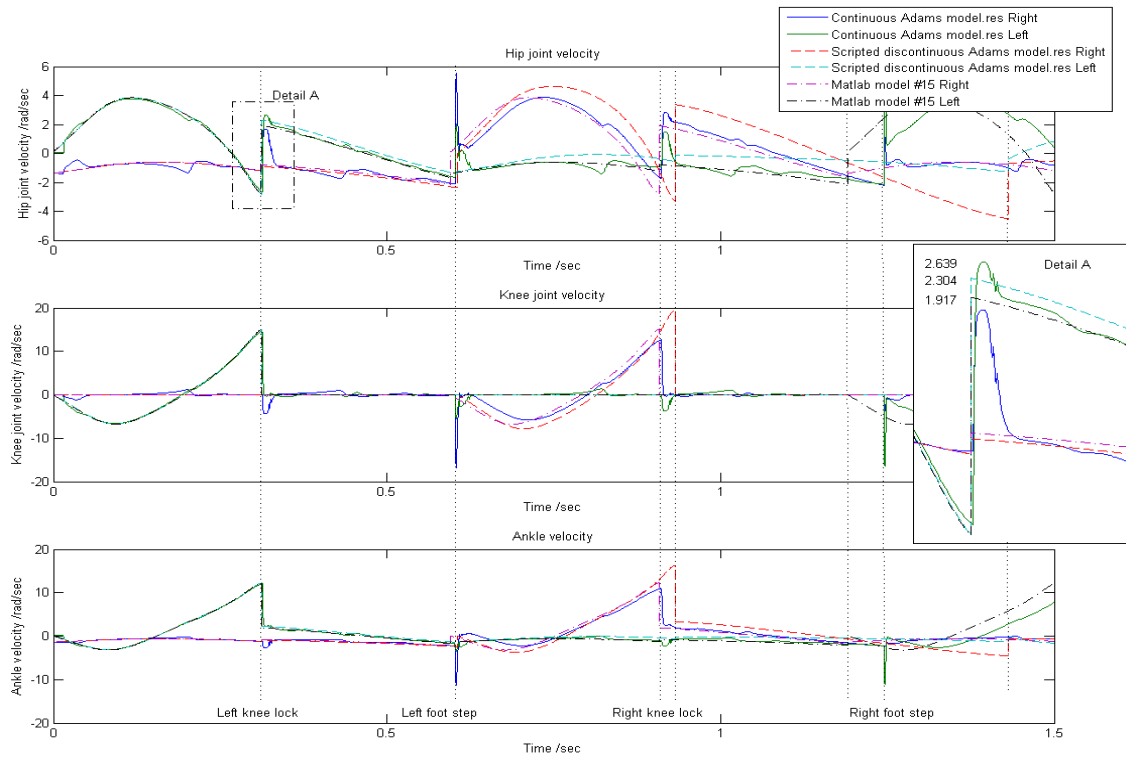


Figure 5b. Joint velocities for the continuous and scripted Adams models and for the Matlab model.

The velocity peaks we have seen in the continuous Adams model are not present in the scripted Adams model due to lack of the elastic knee-locks. During the two-link phase the knee-joint angles and velocities in the scripted model remain zero as expected. Detail A in Figure 5b reveals that at the moment of the first knee-lock event the left hip-joint velocity of the scripted Adams model jumps higher than in the Matlab model, the difference being 0.387 rad/s (~20%) remains. Also the continuous Adams model shows jump in hip-joint velocity but the difference to the Matlab model decays away quickly.

The Matlab model and both Adams models seem to give different dynamic responses for the knee-lock event, regardless of the way the event has been modelled in the Adams model. It is not possible to draw conclusions about similarity or dissimilarity of the heel-strike event modelling since the model behaviour is already affected by the preceding knee-lock event. The reason for the different behaviour between the Matlab and Adams models is not currently known and deserves attention in future studies. At the moment of contact events, the mathematical equations being applied in the Matlab model may calculate the link velocities in a different way than takes place in the Adams software.

Because the original scripted Adams model is performing poorly, a systematic search applying Adams' Design of Experiments –feature for better initial conditions reveals a set of new initial link velocities that allows the model to walk for 24 steps. These values are presented in Table 2.

### *Scripted Adams model with BISTOP-knee locks*

In order to separate the effects of the different heel-strike model from the different knee-stop model we tested each of those separately with the scripted Adams model. Replacing in the scripted Adams model the MOTION-constraint of the knee-lock with the BISTOP-force function brings back the velocity peaks upon the heel-strike events, as shown in Figures 6a and 6b. The knee-stop events are similar with the continuous Adams model, as can be expected. The joint angles and velocities are also similar until the first heel-strike event at which moment the different foot contact model starts to take effect. The scripted Adams model with the BISTOPS appears to be more similar to the Matlab model than to either of the other versions of the Adams model.

The Adams knee-stop model applying continuous BISTOP-functions imitates better the Matlab model than the scripted discontinuous model. Detail A in Figure 4b supports the finding as the difference between the models decays away quickly. The result is a bit surprising and indicates that there exist some differences in knee-lock dynamics between the Matlab and Adams models.

Despite the similarities to the Matlab model and continuous Adams model this model does not manage to perform a long walk. With the same initial conditions this model fails on the third step due to a collapsing knee. The knee-lock fails because the stance leg support force decreases quickly after the heel-strike event down to 86 Newtons which disables the knee-lock and causes falling. During earlier experiments we had already learned that the models are very sensitive to all parameters, including CONTACT-parameters and BISTOP-force definition; for the optimal performance the parameters need to be tuned for each model separately. Therefore parameter tuning is

justified also for this model and re-defining the BISTOP-force function to be active above 80 Newtons instead of 100 Newtons support force prevents the knee collapse.

Having the BISTOP-force function in this scripted model with the discontinuous MOTION-constraint causes additional difficulties for the Adams solver; Around discontinuities the rapidly changing forces in the knee-locks make it difficult to find solutions for the equations of motion. Adjusting integrator error tolerances and integration step size around the contact incidents helps the walker to take a few more steps. A good walking result emerges with modified initial velocities as shown in Table 2. The model was simulated for 200 steps during which it walked for 118 meters in 123 seconds, and it would have been able to continue further. The simulation test did not attempt to find the time when the model would eventually fail.

### *Scripted Adams model with spring-damper-like foot CONTACT model*

The second modification of the scripted Adams model returns the spring/damper-like CONTACT-model between the foot and the ground. The MOTION-constraints remain to present the knee-locks. Contrary to the scripted model with the BISTOPS in the knee-locks, this model does not seem to cause any trouble for the solver and the integrator and the simulation run fluently without warnings or errors. However, with the original initial conditions this model fails after the first step. Modifying the initial velocities as presented in Table 2 allows the model to walk better, now for 19 steps in 14 seconds that yields into a 12.5 meters walk. Performance of this model is similar to that of the original scripted Adams model and so the plots for this model are not included here.

A conclusion can be made that the modelling method of the knee-lock event is the driving factor between the different Adams models. The heel-strike modelling with continuous CONTACTS or discontinuous MOTION-constraints produces mostly similar results, the latter being perhaps closer to the Matlab model.

This model, like all the models presented here, is very sensitive to initial conditions and other model parameters. Figure 7 presents the effect of the initial velocities on the travelled distance before falling. From the figure we can see that a 0.01% change in link velocity  $q1\_d$  (corresponding to a change of 0.12 milliradians per second) reduces the walked distance from 12.5 m to 4.5 m (64% reduction) or below. A 0.0088% change in link velocity  $q2\_d$  reduces the walking distance from 12.5 m to 11.5 m (8% reduction).

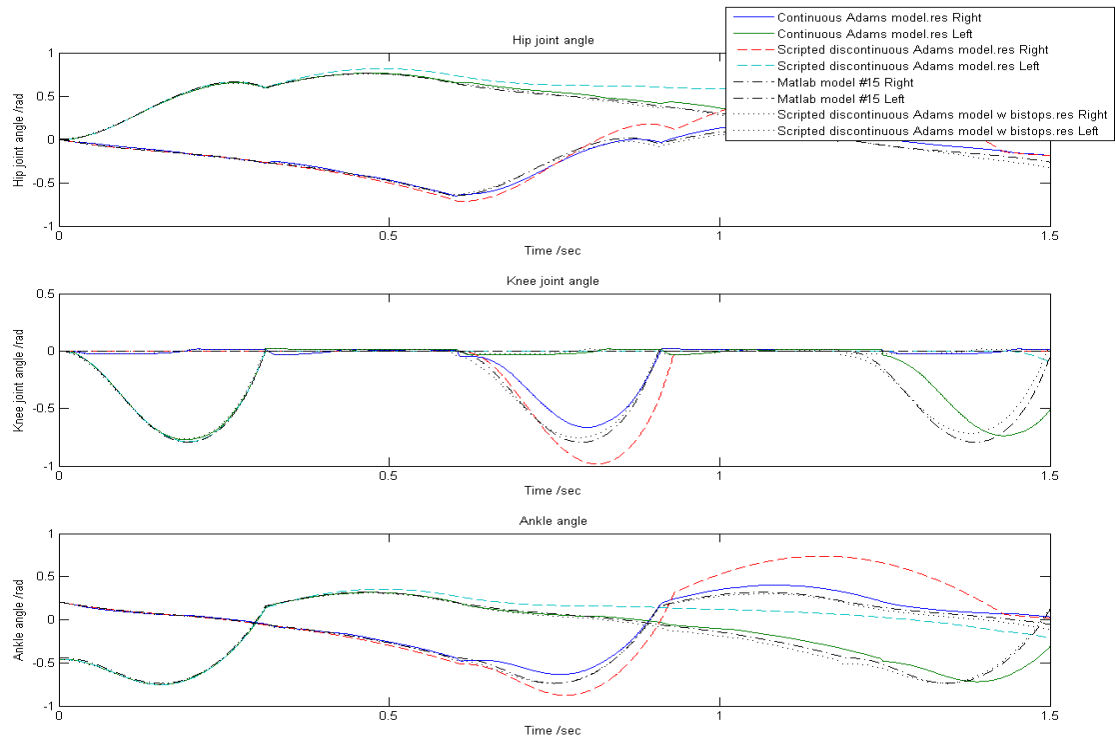


Figure 6a. Joint angles for the continuous and scripted Adams models, for the Matlab model and for the scripted Adams models with BISTOPS.

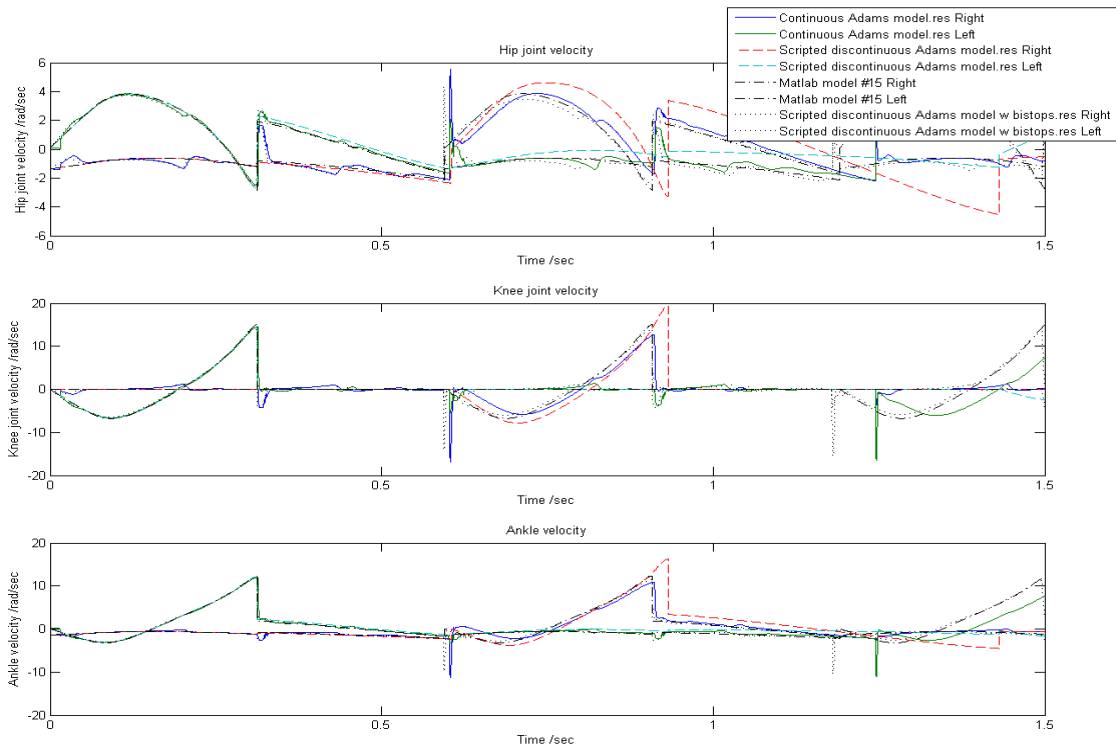


Figure 6b. Joint velocities for the continuous and scripted Adams models, for the Matlab model and for the scripted Adams models with BISTOPS.

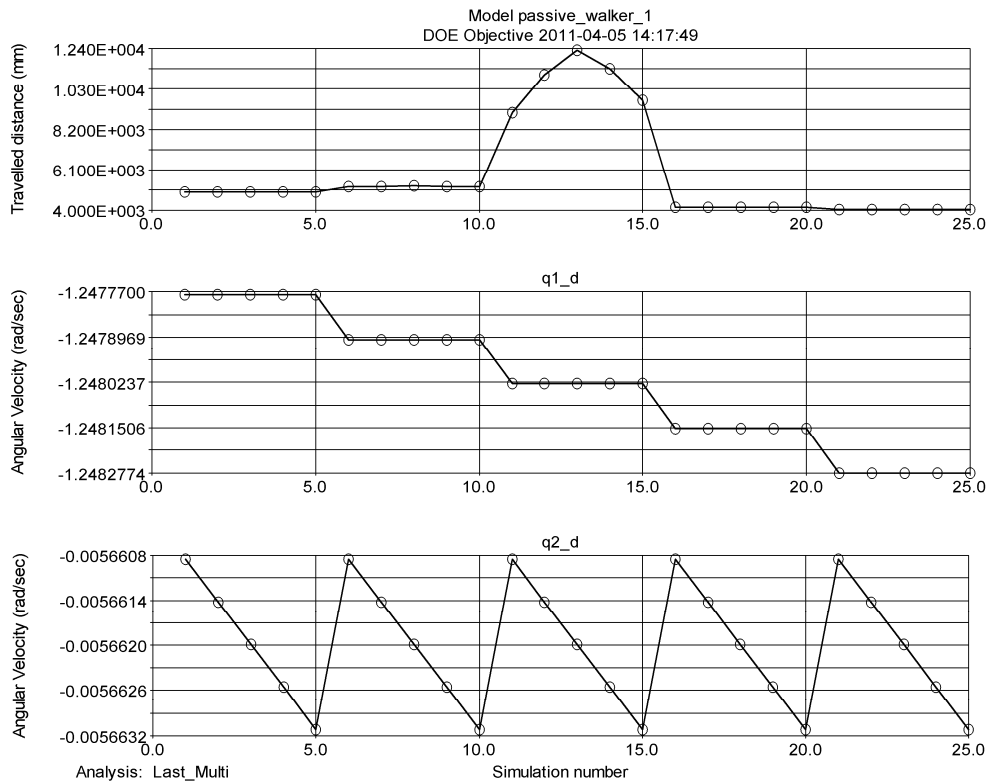


Figure 7. Walking distance sensitivity to q1\_d and q2\_d on the scripted Adams model with foot CONTACTS.

### Heel-strike with a three-link model

In the previous simulations the dynamic behavior of the walker was calculated applying the two-link model at the moment of the heel-strike event, which is the general convention. However, this requires the rear knee to be locked during the contact event. Different physical construction of the knee-joint would lead to a different behavior and we wish to examine what would happen if the knee was free to move during the event. To allow the three-link model at the moment of the heel-strike event, we removed from the simulation script the one simulation step after the heel-strike event before the release of the knee-lock. Figure 8 compares trajectories of the joint angles where the large values of the three-link model are pronounced. The model fails after the first step. Modifying the initial velocities and allowing a slightly steeper slope angle as presented in Table 2 allows the model make a walk of 13 meters with 20 steps in 13 seconds before falling.

It is evident that a model with an unlocked rear knee at the moment of the heel-strike event would be able to walk. Further studies are needed to clarify whether this is possible in practice and whether it can provide any benefit. The result presented here indicates a need for a steeper slope which may reflect weaker energy efficiency.



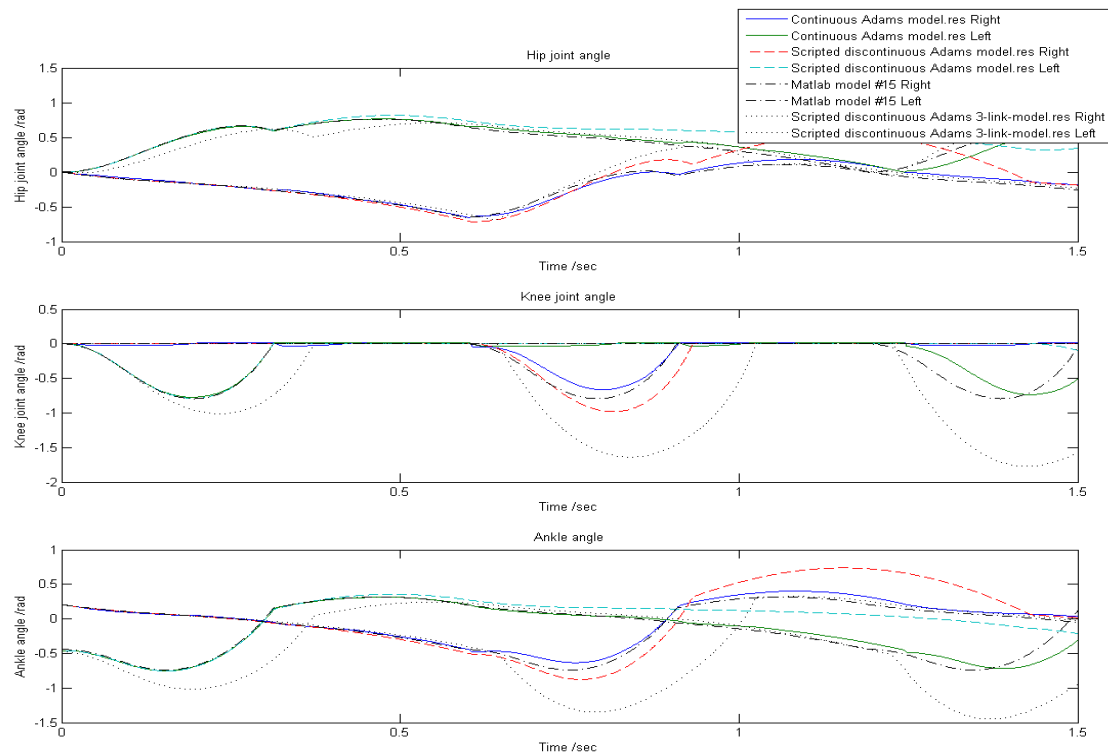


Figure 8. Joint angles of the Matlab model and the continuous and scripted Adams models compared to the scripted Adams model in 3-link mode at the time of heel-strike event.

### *More about model sensitivity*

Analysis of Figure 7 shows that parameter changes in 6<sup>th</sup> decimal – or in link joint a velocity change of a fraction of a milliradian per second- can affect greatly in walker performance. The extreme model sensitivity became very clear when all the 30 different walker models –not only the one presented in this article- were tested in different computer hardware; In the first test the models were transferred from a 32-bit Windows platform running AMD Athlon 64 processor into a 64-bit Unix-platform running AMD Opteron 8360 SE processor. In this case the models were exactly the same, but Adams software version and computer system accuracy were changed. As a result also performance of walker models changed and they provided different walking distances before falling, -some walked longer, some shorter. The most robust models, including the one presented in this article, did not show much –if any- change in walking distance while the ones with most unfavourable mass distribution showed large differences. In the second test the walker models were transferred from one Windows machine (running AMD Athlon 64 processor) to another (running Intel core 2 Quad processor), both having the same Windows version, the same service pack, the same updates, the same Adams software version and identical walker models: the walking performance was altered again, in a similar manner as when transferring to Unix hardware. Reason for this remains unclear; Adams Solver settings were double checked and it was made sure that all the model, solver and integrator parameters were identical. For double

checking the simulations were repeated again in the original Windows computer and the results were identical with the original ones, as may have been expected.

This high sensitivity of the model to any parameter variations was expected since we have chosen such a mass distribution along the walker links that sets the operational parameter space very narrow. The walker model has a tendency for a chaotic behaviour where tiny changes in parameters cause large effects in overall performance.

## Conclusions

This article has described and compared the properties and performance of different passive walker simulation models in Matlab- and Adams-environments. The discontinuous Matlab simulator applies manually entered mathematical equations to describe the system dynamics while the Adams simulation software creates the dynamic models automatically. The dynamic equations in the Matlab model follow the general convention presented in the references. The continuous Adams models apply for knee-lock and heel-strike events continuous CONTACT and BISTOP-statements. The discontinuous scripted Adams models apply MOTION-constraints to present those events. Specific attention has been paid to modelling of the knee-lock and heel-strike events.

Walking distance before falling (failure to take another step) has been selected as for a measure for evaluating walker model performance. Although quite obvious measure, this can not be used for any accurate numerical evaluation, and it also allows minor errors to accumulate on the initial cycle until the walker eventually falls. Therefore, even if the walker manages to walk a long time without falling, we can not tell whether it is performing a stable limit cycle or not. An assumption has been made that if the walker is able to walk 20 steps or more before falling, it has been fairly well defined and can be considered as a possible configuration for a passive walker. Another assumption is, that if one walker can walk further than another, then the former one can be considered being better and its behaviour is closer to an actual limit cycle walker than that of the latter.

The simulation results show that each model behaves differently and parameter tuning is needed separately to make each of them to walk. The Matlab model and the Adams models have a different dynamic response to the knee-lock event and it is not possible to completely repeat the Matlab simulation results in Adams. Reason for this is not currently known and deserves further studies.

The discontinuous scripted Adams model with BISTOP-functions in knee-locks are more similar with the Matlab model than with any of the other Adams models. A conclusion can be made that the Adams knee-lock model applying the continuous BISTOP-functions imitates better the Matlab model than the scripted discontinuous model with MOTION-constraints in the knee-locks. The result is different from what was expected. The heel-strike event modelling with continuous CONTACT-statements or discontinuous MOTION-constraints produces mostly similar results.

To limit object's free motion the BISTOP-function creates contact events with spring and damper in a similar manner as does the CONTACT-statement applying IMPACT-function. Nature of the contacts is discussed in Simcompanion (2010c,d) and

in Mechanical Dynamics (1987). These state that instead of the IMPACT –function a coefficient of restitution method (POISSON) can be used to model the normal force. In contrast to IMPACT-function POISSON is based on impulse momentum theory where the velocities are discontinuous. This appears to be similar approach with the Matlab model and further testing of the Adams models could be considered utilizing POISSON-statement for the knee-stops and/or foot contacts.

During a heel-strike event either a two-link model or a three-link model of the walker can be applied, although the general convention applies the two-link model only. The example simulation result indicates worse energy efficiency when applying the three-link model, and its applicability in practice requires further studies.

The mass distribution and geometric properties of the selected walker model make it very sensitive to parameter variations. Very small differences in model properties yield into large differences in the simulation results, which are emphasized by the selected measure, being the walking distance before fall, which accumulates minor differences during time. With a more robust model having a more favourable mass distribution the differences in walking performance of the Matlab model and Adams models might not have been detected at all.

## Ongoing activities

During the time of writing of this article the simulation project has progressed into a co-simulation phase. A control algorithm is running in Matlab Simulink software controlling in a closed loop the joint torques of the walker model in Adams. The model applied in Adams is the same as the continuous Adams model presented in this paper.

Despite the Adams model is not able to reach the limit cycle in passive walking, under the closed loop control it performs well and walks almost as long as desired. Now we take advantage of the versatility of the Adams model and modify the mass distribution and cog. locations of the links for dozens of different simulation experiments. The modified models would not be able to take a single step in a passive mode, but under the control they are able to walk.

The simulation results show effect of the model variations on the controller performance, walking capability and overall energy efficiency. With the information we acquire we are able to outline the design space to be applied for the real hardware to be built.

## Acknowledgements

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