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Challenges of steel fibre reinforced concrete in load bearing structures

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Summary. This article focuses on concrete composite reinforced by short metal fibres, where the main role of fibres is to carry the tensile stresses, while the concrete matrix transfers and distributes the loads to the fibres. The efficiency of load transferring from matrix to fibres depends on both the bonding interface between matrix and fibres and the anchorage length of fibres. The effect of fibre orientation in matrix on the strength of composite is also introduced. In the paper the assumptions made in a cross-sectional dimensioning concerning the behaviour of ordinary reinforced or fibre reinforced cross-sections are discussed and compared.

Key words: fibre reinforced composites, orientation of fibres, crack propagation

Description of steel fibre reinforced concrete (SFRC)

Today fibre reinforced concrete is perhaps one of the most realistic possibilities to develop the use of concrete in load bearing structures. Even if the properties of fibre reinforced concrete have not been thoroughly explored, it is already widely used in the construction industry but not usually as a load bearing structure. Its applications are concentrating on floors resting on soil and less extent on floor slabs, walls and foundations. Interest in using fibre reinforced concrete widely in various structural components is high, as its use is expected to improve quality of concrete structures. Its use may also increase the effectiveness of designers' and constructors' work.

Theoretical background in terms of microstructure

Most materials are heterogeneous not only on the microscale but also on the meso- and macroscales due to manufacturing or formation processes. While the properties of such materials vary point-wise in the material space, the concept of nonlocality might be of use to describe the material properties on average. In physical terms, nonlocality means that a field variable at a point X at time t depends on the values of field variables at points of the body other than the point X. Nonlocality in the frame of SFRC can be explained more precisely in the view of the behaviour of a single fibre as follows: if one end of the fibre is influenced by some stress then the other end of the fibre is also affected. In meso-scale nonlocality with SFRC indicates the presence of the interactions between separate constituents of the material (steel fibres, aggregate, binder etc). The SFRC requires an approach, which takes into account the presence of a complex system, which is composed of parts interrelated in a way, which is challenging to be described. As opposite to nonlocal behaviour, the local behaviour occurs when the stress in one point cannot easily be explained by the deformation occurring at a different point. For instance, the stress near the tip of a crack may not be explained by a global average stress field.

The concept of microstructured materials is quite wide. Examples are media with regular or stochastic distributions of voids (dislocations), fibres (inclusions), cracks (dislocations), etc. Steel fibre reinforced concrete is a kind of microstructured material belonging to cement-matrix composites. This material has a basic matrix made of concrete, which includes embedded short metal fibres. All microstructured materials are characterised by the existence of intrinsic space-scales as the size of grain or a crystallite, the distance between the microcracks, etc. that introduces scale dependence into governing equations.

According to these factors two main questions are under consideration: the first question is heterogeneity / inhomogeneity and respectively the concept of nonlocality; the second question is the distribution of stresses in concrete steel fibre composite.

Start up with the concept of homogeneous solids, which has been successfully applied to many technical problems. Nonlocality is actually introduced to validate a *homogeneous* model for a *heterogeneous* material. This could be explained by a simple example, as follows.

Assuming that a block consists of periodically alternating layers of two different elastic materials, the gross material is elastic in the usual sense, but with the elastic module varying in a discontinuous manner throughout the body. The material can be *homogenized* by describing its gross behaviour with a constitutive equation involving only a single *constant* effective module. This crude homogenization may be acceptable for static problems.

Research hypotheses and methods

The investigation of SFRC might be divided into three stages. The first stage concerns uncracked material in the sense of macrocracks. In this stage a crucial point is to determine orientation distribution function of fibres in a continuum element. This can be done by the mesoscopic theory, where the domain of the field quantities is enlarged by an additional variable characterizing the internal degree of freedom connected to the internal structure of the material [2, 3]. The orientation of fibres in the considered cross section can be characterized by an average vector *n*, which is composed from the respective vector field. The vector field refers to the arrangement of all fibres in considered cross section. The vector *n* should be defined using the spherical coordinate system, as in 3D case the position of the vector *n* can be unambiguously determined only by taking into account both angles, i.e. the inclination angle (between the vector *n* and the surface normal) and the in-plane angle (azimuth angle). This parameter will be taken as an additional variable in the mesoscopic theory. The orientation distribution of fibres is relevant, because only the component of the stress vector parallel to the main orientation vector *n* causes fibres to work, i.e., in a sample where fibres are mostly parallel to the direction of applied stress, the work of fibres is more pronounced than in a sample where fibres are mostly perpendicular to the stress direction.

Local solution for aligned fibres according to Taya and Arsenault

As a simplification, it can be considered, that all fibres in a volume element are uniformly distributed. In modelling a crucial point is the choice of theory. A possible model can be a simplified description of microstructure (empirical or semi-empirical theories) obtained by assuming that a model developed for a *unit cell* is distributed uniformly throughout the material [1]. This is basis for the shear lag model, which describes the gross behaviour of a composite material consisting of a matrix with embedded short fibres distributed uniformly and aligned in loading direction. The unit cell consists of matrix material with a single representative fibre of length *2l* and diameter *2r*. The width *2R* of the *unit cell* is taken as the mean lateral separation of neighbouring fibres (Figure 1).

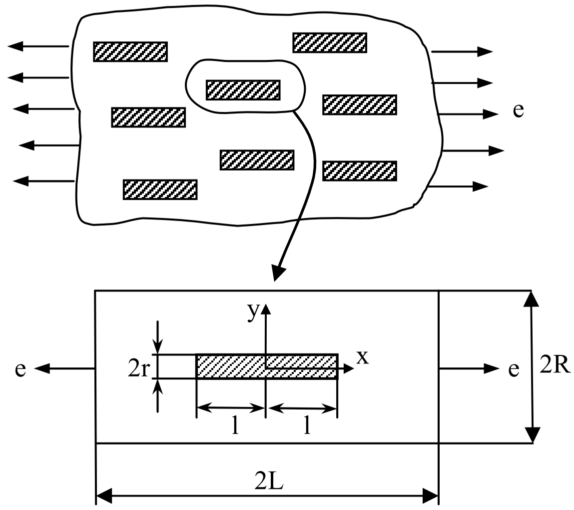

Figure 1. Shear lag model for aligned short fibre after Taya and Arsenault (1989)

If the unit cell is elongated with uniaxial strain *e* along *x* direction, the matrix will exert the shear stress τ_0 or $d\sigma_f/dx$ at the matrix fibre interface, which is proportional to the difference, $(u-v)$, if the axial displacements in the fibre and the matrix on the boundary of the unit cell are denoted by *u* and *v* respectively [1].

$$\frac{d\sigma_f}{dx} = -\frac{4\tau_0}{(2r)} = h\cdot(u-v) \Rightarrow \tau_0 = r\frac{h\cdot(v-u)}{2}, \tag{1}$$

where σ_f is the axial stress in the fibre, h is a constant, which will be determined later, and the local coordinate x is measured from the midpoint of the cell. In the fibre, one dimensional Hooke's law is valid

$$\sigma_f = E_f \frac{du}{dx}, \tag{2}$$

where *Ef* denotes elastic modulus of the fibre. The applied composite strain e is equal to dv/dx. Hence, from eqs. (1) and (2) the ordinary differential equation can be obtained

$$\frac{d^2\sigma_f}{dx^2} = h\left(\frac{du}{dx} - \frac{dv}{dx}\right) = h\left(\frac{\sigma_f}{E_f} - e\right). \tag{3}$$

The general solution to eq. (3) is given by

$$\sigma_f = E_f e + C_1 \cosh \beta x + C_2 \sinh \beta x, \tag{4}$$

where

$$\beta = \sqrt{\frac{h}{E_f}}, \tag{5}$$

and C_1 and C_2 are unknown constants. Applying boundary conditions, $\sigma_f =$ constant (σ_0) at $x = l$ and $d\sigma_f/dx$=0 at $x = 0$, the stress of the fibre is

$$\sigma_f = E_f e \cdot \left\{ 1 + \frac{\left(\frac{\sigma_0}{E_f e} - 1\right)\cosh \beta x}{\cosh \beta l} \right\}. \tag{6}$$

It is noted in eq. (6) that $\sigma_0 = 0$ was used in the original derivation by H.L.Cox, implying the absence of additional anchoring at the end of the fibre. σ_0 may not be zero, if anchoring affects around fibre ends are strong as it can be with hooked ends. The value of σ_0 in this case will be proportional to the method of anchoring of fibre ends (mechanical, chemical etc.) and the stress field σ_f in the fibre will not be uniform.

The average fibre stress σ_f is computed as

$$\overline{\sigma_f} = \frac{1}{l}\int_0^l \sigma_f dx = E_f e \left\{ 1 + \frac{\left(\frac{\sigma_0}{E_f e} - 1\right)\tanh \beta l}{\beta l} \right\}. \tag{7}$$

Consider next the displacement along x direction at an arbitrary point $(y = y)$ in the matrix, w, where $w(y = r) = u$, and $w(y = R) = v$. Force equilibrium at $y = r$ and arbitrary point $(y = y)$ provides

$$2\pi y\tau = 2\pi r\tau_0 . \tag{8}$$

The shear strain at $y = y$, γ is related to τ_0 as

$$\gamma = \frac{dw}{dy} = \frac{\tau}{G_m} = \frac{\tau_0}{G_m}\frac{r}{y}, \tag{9}$$

where τ is the shear stress in the matrix at $y = y$, and G_m is the shear modulus of the matrix. Integrating eq. (9) from $y = r$ to $y = R$, it can be obtained

$$v - u = \frac{\tau_0 r}{G_m}\ln\left(\frac{R}{r}\right) \Rightarrow \tau_0 = \frac{G_m \cdot (v-u)}{r\ln\left(R/r\right)}. \tag{10}$$

From eqs. (1) and (10) constant h is solved as

$$h = \frac{2G_m}{r^2 \ln(R/r)}. \tag{11}$$

From eqs. (5) and (11) β is found as

$$\beta = \frac{\sqrt{2}}{r}\sqrt{\frac{G_m / E_f}{\ln(R/r)}}. \tag{12}$$

The factor β reflects the relative rigidity of the surrounding matrix in respect to the fibre. With β given by eq. (12) the average stress σ_f in the fibre can be calculated from eq. (7). In order to describe the gross behaviour of the compound material along loading x direction, the mean stress value σ_c can be estimated by using the law of mixtures

$$\sigma_c = \sum_{i=0}^{N} V_i \sigma_i , \tag{13}$$

i.e.

$$\sigma_c = \left(1 - V_f\right)\overline{\sigma}_m + V_f \overline{\sigma}_f , \tag{14}$$

where $\overline{\sigma}_m$ and $\overline{\sigma}_f$ are interpreted as the average quantities in the relevant domain and V_f is the volume fraction of fibres. For a given applied strain e, one can assume, that

$$\overline{\sigma}_m = E_m e\,, \tag{15}$$

$$\sigma_c = E_c e\,. \tag{16}$$

A substitution of eqs. (7), (16) and (15) into eq. (14) yields Young's modulus of the composite E_c

$$E_c = \left(1 - V_f\right) E_m + V_f E_f \left\{ 1 + \frac{\left(\frac{\sigma_0}{E_f} - 1 \right) \tanh \beta l}{\beta l} \right\}, \tag{17}$$

where σ_0 can be equal to zero, which means there is no load transfer at the fibre ends.
The shear lag model presented by Taya and Arsenault (1989) *excludes nonlocal effects*. It is a usual effective-modulus theory, which describes the composite material by the classical Hooke's law

$$\sigma_c = E_c e\,. \tag{18}$$

The whole theory is about the determination of the effective Young's modulus *Ec* from the geometric and material data. Certainly, a nonlocal model can be generated using some ideas from Taya and Arsenault's approach, but in the original form it is an example for *localization*.

Nonlocal solution for aligned fibres according to Becker and Bürger

Another approach includes *nonlocality* [4]. Becker and Bürger (1975) have studied a similar problem assuming the fibres being in contact with the matrix at their endpoints only. Consider the uniaxial stretching over elastic material (elastisity modulus *Em*) in which parallel elastic fibres (elastisity modulus *Ef*, cross-section area is *Af*, length *2l*) are embedded. X and X^* are, respectively, the position of the cross-section and the right endpoint of the fibre from the origin in the reference configuration. The fibre at position X^* extends from $X^* - 2l$ to X^* (Figure 2).

The number of fibres per volume (number density) n in the cross-section A at X composes the volume fraction, which is equal to following

$$\int_X^{X+2l} nA_f dX^* = nA_f 2l\,. \tag{19}$$

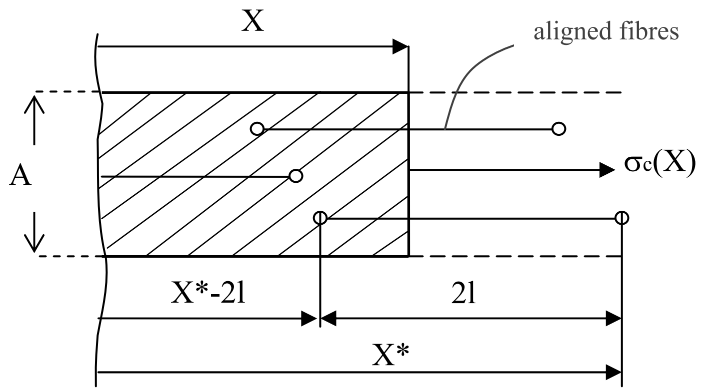

Figure 2. Fibre reinforced material, after Becker and Bürger (1975)

The fibres are glued to the matrix material at their endpoints only. Therefore, the stress in one fibre, which passes through the cross-section at X is equal to $E_f(x(X^*)-x(X^*-2l)-2l)/2l$. The relation $x=x(X)$ or $x=x(X^*)$ defines the motion expressed by Lagrangian description according to which the material point X or X^* is transferring to the spatial point x. For the average stress $\sigma_c(X)$ within the whole cross-section area A follows

$$\sigma_c(X)A = A(1-nA_f 2l)E_m e_x(X) + \int_X^{X+2l} E_f \cdot \frac{x(X^*)-x(X^*-2l)-2l}{2l} AnA_f dX^* . \quad (20)$$

From the eq. (20), taking into account the integral (19) and the following substitutions: $(1-nA_f 2l)\cdot E_m = (1-V_f)E_m$ and $nA_f 2l \cdot E_f = V_f E_f$ as well as $e_x = dx/dX - 1$, the next expression can be obtained:

$$\sigma_c(X) = \left(1-V_f\right)E_m\left(\frac{dx}{dX}-1\right) + V_f E_f\left\{\frac{1}{(2l)^2}\int_{X-2l}^{X+2l}\left|x\left(X^*\right)-x\left(X\right)\right|dX^* - 1\right\} \quad (21)$$

As opposed to eq. (18), Becker and Bürger's eq. (21) exhibits true nonlocal behaviour.

The second stage of investigation of considered material could be concentrated on the analysis of cracked solid. The crucial points of this stage are separation of the whole stress between fibres and matrix and cracks propagation and growth. To solve the problem with stress distribution Becker and Bürger's (1975) approach could be employed.

Cracks propagation and growth

Crack resistance of material (viscosity of fracture) characterizes the ability of material to resist the propagation of pre existing cracks. In case of SFRC propagation of cracks is also under research. In connection with this, it is useful to recall A.A. Griffith's theory about rupture in solids. It is well known, that theoretical strength of solids, calculated by some atomistic model, is much higher than the real one [6]. A.A. Griffith was the first who suggested that this phenomenon occurs due to the presence of microscopic flaws (discontinuities) in the real material. Griffith's theory gives excellent agreement for brittle materials, but not applicable for ductile ones. A group led by G. R. Irwin assumed, that plasticity should play an important role in the fracture of ductile materials. In such materials, a plastic zone d develops near the tip of a crack (see Figure 3). As the stresses are increasing in the material, the value of plastic zone is increasing as well until the crack is growing and the material behind the crack tip is unloading. The ability of material to resist crack propagation is expressed by the stress intensity factor K (SIF) and the energy release rate G, i.e. the work for plastic deformation per unit of newly created crack surface [6]. Assume an infinite body, uniaxial stress field and the case of plane deformation. In considered instance, the critical value of the stress intensity factor K_{Ic} (fracture toughness) will be equal to following:

$$K_{Ic} = \sigma\sqrt{\pi c}\,, \tag{22}$$

where σ denotes a uniform stress field, subscript I denotes mode I loading, i.e. plane strain (a tensile stress is normal to crack plane), c denotes a half of the length of the crack and d denotes the length of the plastic zone in front of the crack (Figures 3, 4).

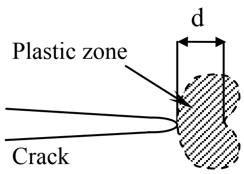

Figure 3. Plastic zone in front of the crack

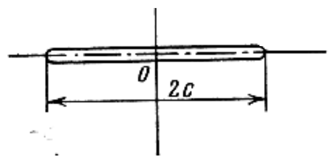

Figure 4. The length of a crack

In real materials, the stress near the tip of a crack is very high and exceeds the yield strength of the material, i.e. the local plastic yielding occurs. Hence, plastic yielding plays a significant role in the fracture process of materials.

Dugdale and Barenblatt have offered a model to find the extent of plastic zone. They considered a long, slender plastic zone at the crack tip in plane stress. The strip model is founded on a crack of length $2a = 2c + 2d$, where d is the length of the plastic zone with a closure stress equal to σ_{ys}, applied at each crack tip, see Figure 5. The size of d is chosen that the stress singularity vanishes at the end of the effective crack [11], i.e.

$$K_\sigma + K_d = 0\,, \tag{23}$$

where the definition of K_σ and K_d will be given below.

The estimation of stress intensity due to the closure stress may be implemented assuming a normal force P acting on the crack at a distance x from the centre line of the crack, see Figure 6.

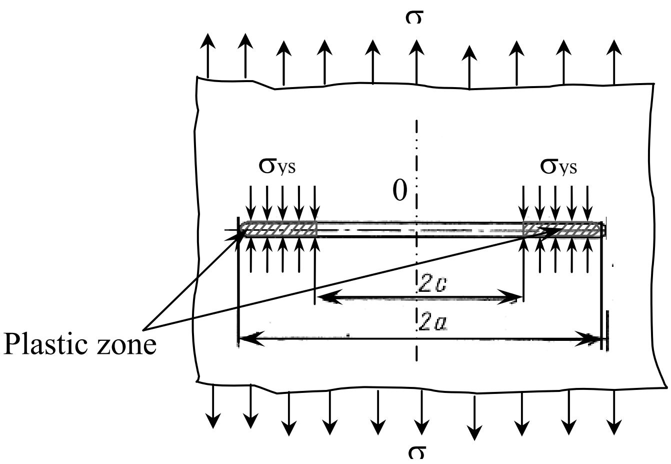

Figure 5. Stresses near the tips of a crack

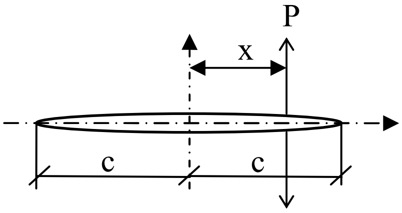

Figure 6. Applied load in a distance x

The resultant stress intensity factors at the right and respectively at the left crack tips are [11]:

$$K_{left} = \frac{P}{\sqrt{\pi c}}\sqrt{\frac{c+x}{c-x}},\ K_{right} = \frac{P}{\sqrt{\pi c}}\sqrt{\frac{c-x}{c+x}}, \tag{24}$$

The closure force within the plastic zone is:

$$P = -\sigma_{ys} dx\,. \tag{25}$$

Hence, the total stress intensity at each crack tip resulting from the closure stress is obtained by replacing c with $c + d = a$ [11], i.e.

$$K_d = \frac{-\sigma_{ys}}{\sqrt{\pi a}}\int_c^a \left\{\sqrt{\frac{a-x}{a+x}} + \sqrt{\frac{a+x}{a-x}}\right\} dx = \frac{-2\sigma_{ys} a}{\sqrt{\pi}}\int_c^a \frac{dx}{\sqrt{a^2 - x^2}} = -\frac{-2}{\pi}\sigma_{ys}\sqrt{\pi a}\cos^{-1}\left(\frac{c}{a}\right). \tag{26}$$

The stress intensity from the remote tensile stress is $K_\sigma = \sigma_\infty \sqrt{\pi a}$ and accordingly eq. (23) leads to

$$\frac{c}{a} = \cos\frac{\pi\sigma_\infty}{2\sigma_{ys}}. \tag{27}$$

The formula (27) allows to determine the length of plastic zone in cases, when linear fracture mechanics is powerless [6]. The value $d = a - c$ can be quite large and it aims to infinity when $\sigma_\infty \to \sigma_{ys}$. On the contrary, for $\sigma_\infty \ll \sigma_{ys}$ neglecting the higher order terms in the series development of the cosine, d is found as [6]

$$d \sim \frac{\pi^2}{8}\frac{\sigma_\infty^2}{\sigma_{ys}^2}c = \frac{\pi}{8}\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2. \tag{28}$$

Solution of the singularity at the end of a crack, i.e. J-integral

Consider W to be the strain energy density (the density of elastic energy), which depends on the infinitesimal strain tensor $e_{ij} = \overline{e}_{ij} = u_{(i,j)}$, where $u_{(i,j)} = 1/2\left(u_{i,j} + u_{j,i}\right)$ is the symmetric part of the displacement gradient [6]. It is approved that the integral,

$$J = \int_A^B \left(W dy - \sigma_{ij}\frac{\partial u_i}{\partial x} n_j ds \right), \tag{29}$$

where

$$W = \int_0^{\bar{e}} \sigma_{ij} d\overline{e}_{ij}, \tag{30}$$

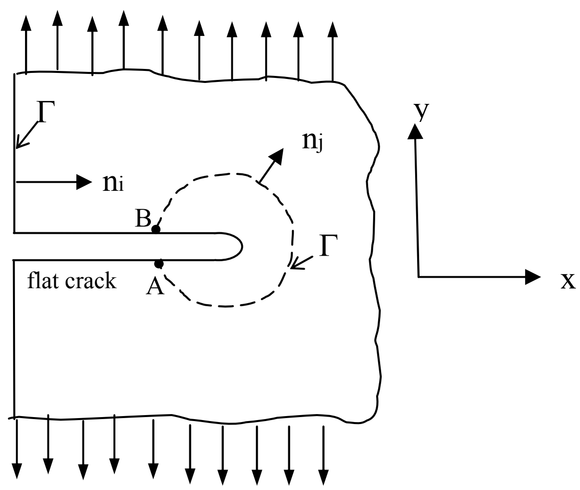

Figure 7. J-integral around a crack in two dimensional deformation field

$\sigma_{ij} n_i = \sigma_i$ is the traction vector defined according to the outward normal n_i along Γ, $\partial u_i / \partial x$ is the displacement vector and ds is an element of arc length along Γ, does not depend on the path of integration between points A and B (Figure 7).

For the proof, it is necessary to show, that integral (29) vanishes for any closed path. On the other hand, it can be proved, that if points A and B are taken on different edges of a crack, then

$$J = -G, \tag{31}$$

where $G = \partial U / \partial \Delta$ is the strain energy release rate, which is equal to the ratio of change in the total elastic energy ∂U, contained in the domain S with boundary Γ, to the propagation of crack end to a distance $\partial \Delta$ [6] (Figure 8).

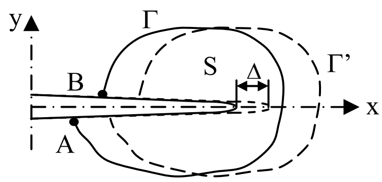

Figure 8. Propagation of a crack

Crack propagation is accompanied by the work of plastic deformation G (strain energy release rate), which is also called resistance force of crack propagation [6]. If G does not reach the critical value of G_c for a given material, the crack is stable. Crack becomes unstable when G reaches or exceeds the critical value of G_c. In case of plane strain (mode I loading), connection between the critical value of strain energy release rate (toughness) and the critical stress intensity factor (fracture toughness) is

$$G_{Ic} = \frac{1-\nu^2}{E_c} K_{Ic}^2, \tag{32}$$

where ν is Poisson's coefficient. In general, to assess the stability of a crack it is possible to compare K (the actual value of stress intensity factor) with K_c (the critical value of stress intensity factor in considered loading mode and material). For stabilized crack $K \le K_c$ must be.

Crack growth begins when crack opening is critical and equal to certain value δ_c [6]. Let's consider $\delta = u^+(c,0) - u^-(c,0)$, where indexes "+" and "–" denote opposite crack surfaces. By choosing the way of integration as in Figure 9, the following can be received: $dy = 0$, $\sigma_{ij} n_j n_i = \sigma_*$ is the stress applied in normal direction, and further

$$G = -J = \sigma_* (u_y^+ - u_y^-) = \sigma_* \delta_c. \tag{33}$$

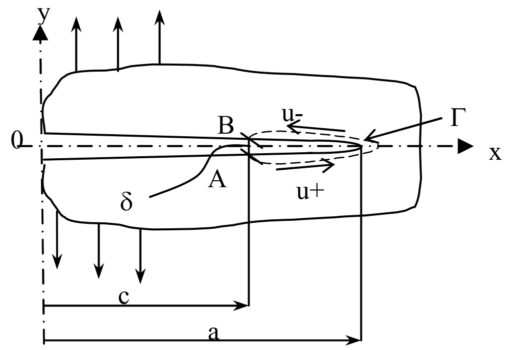

Figure 9. The way of integration

For determination of the critical value of K_{Ic} it is necessary to measure δ_c in a moment when the crack begins to grow

$$\delta_c = \frac{1}{\sigma_*} \cdot \frac{1-\nu^2}{E_c} K_{Ic}^2 \Rightarrow K_{Ic} = \sqrt{\delta_c \cdot \frac{E_c \sigma_*}{(1-\nu^2)}} . \tag{34}$$

Crack propagation in a composite with aligned fibres

In the researched material crack growth is interrupted by metal fibres, which disturb relative displacements of crack edges. The crack can achieve the value of δ_{ic} when fibres will be broken in some distance ξ from the tip of a crack [6] (Figure 10).

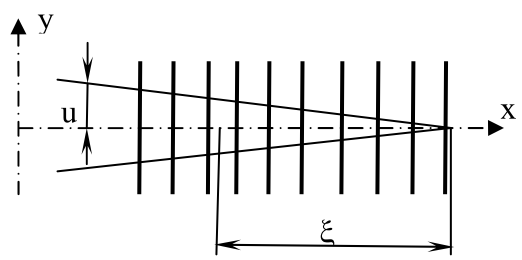

Figure 10. Crack growth in composite with short fibres

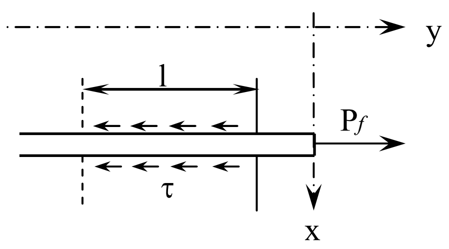

Figure 11. Force applied on a fibre, which is fixed in a matrix

The simplest assumption would be that friction forces τ per unit of surface area balance the tension stresses in fibres and prevent pull out of fibres from the matrix. Let *Pf* be the force, which is applied to the tip of a fibre with radius *r* fixed in the matrix. This force is balanced by the uniform shear stress in a segment with length l.

The equation of balance is

$$P_f = 2\pi r l \tau \Rightarrow l = \frac{P_f}{2\pi r \tau}. \tag{35}$$

Hence, the lengthening of fibre segment with length l in *y* direction considering one half of the crack side (Figure 10) and eq. (35) will be

$$u = \frac{1}{2}\cdot\frac{1}{E_f}\cdot\frac{P_f}{\pi r^2}\cdot l = \frac{1}{4}\cdot\frac{1}{E_f}\cdot\frac{P_f^{\,2}}{\pi^2 r^3 \tau}. \tag{36}$$

If σ_c is the average stress of the composite and the crack opening is controlled only by fibres, then

$$\sigma_c = \frac{P_f V_f}{\pi r^2} \Rightarrow P_f = \pi r^2 \sigma_c \cdot \frac{1}{V_f}, \tag{37}$$

where V_f denotes volume density of fibres content in the material. Considering the correlation (36), it can be noticed, that the relationship between the stress in a zone near the tip of a crack and the relative displacements of crack edges will be

$$u = \frac{1}{4}\cdot\frac{r}{E_f \tau V_f^2}\cdot\sigma_c^{\,2} = \alpha\sigma_c^{\,2}. \tag{38}$$

Assuming that $\sigma_c = \sigma_{c0}\cdot(\xi / L)^n$ (where σ_{c0} is some stress value, *L* is some reference length and ξ denotes the distance to the crack tip), $u = \alpha(\sigma_{c0}\cdot(\xi / L)^n)^2$ and calculating the Rice-Cherepanov's integral the following value of the work for plastic deformation can be obtained

$$\begin{aligned} G_c &= \int \sigma_c \frac{\partial u}{\partial \xi} d\xi = \int \sigma_{c0}\cdot\left(\frac{\xi}{L}\right)^n \cdot 2n\frac{\alpha}{L}\sigma_{c0}^2\left(\frac{\xi}{L}\right)^{2n-1} d\xi = \\ &= \frac{2}{3}\alpha\cdot\sigma_{c0}^3\left(\frac{\xi}{L}\right)^{3n} + C = \frac{1}{6}\cdot\frac{r}{E_f \tau V_f^2}\cdot\sigma_c^{\,3} + C \end{aligned}. \tag{39}$$

It is expected, that the crack begins to grow when σ_f achieves the value of fibre tension strength. Assuming that $\sigma_c = \sigma_f V_f$ and $C = 0$ [6] the following can be obtained

$$G_c = \frac{r}{6 E_f \tau} V_f \sigma_f^3 . \tag{40}$$

From eq. (40) follows, that resistance of crack propagation increases together with fibre strength and volume density of fibres content in the material. However, it does not mean there is a simple linear relationship. At the same time friction forces, which are equal to matrix shear resistance, should not be too high.

The third stage of researched material could include the identification of fracture mechanics. The main idea here might be to introduce the damage parameter as a macroscopic quantity growing with progressive damage in such a way, that it should be possible to relate reducing effective area of cross section to the growth of the damage parameter [7, 10].

Theoretical background of classical analysis of concrete structures

As it is already known, when considering concrete at microscale, the structure of this material is heterogeneous. But when reinforcement bars are added to the tensioned cross sections of concrete in macroscale it becomes an orthotropic material. The presence of fibres in fibre reinforced concrete makes it necessary to consider this material at meso- and microscales. Regarding to this, a variable characterizing the characteristic length of the inner structure of the material and associated with the orientation of fibres has to be added to the governing equations. Determining the arrangement of fibres in the matrix is the most important starting point for further development of design rules for fibre reinforced concrete structures. Due to different structural scales, theory of reinforced concrete structures cannot directly be applied to the fibre reinforced concrete. However, brittle behaviour of concrete matrix is a common feature for both materials influencing on both the cross-sectional level and on the level of material interfaces. These common features may be possible to utilize in the dimensioning methods for load bearing structures of steel fibre reinforced concrete. For this purpose the dimensioning principles of concrete structures with ordinary reinforcement are introduced in this paragraph.

Experimental basis of the theory of resistance of reinforced concrete and calculation methods of concrete structures

The dimensioning principles of reinforced concrete structures are built up on the experimental data and the principles of mechanics proceeding from the stresses and deformations of members in various loading stages [8].

Three stages of stress and deformation state of reinforced concrete members in case of pure bending

Having loaded reinforced concrete member by gradually increasing loading, it is possible to observe three stages of stresses and deformations:

Stage one (I): uncracked tension zone. In this stage tension stresses are taken both by concrete and reinforcement (Figure 12), where f_{ct}, f_y denotes respectively concrete and

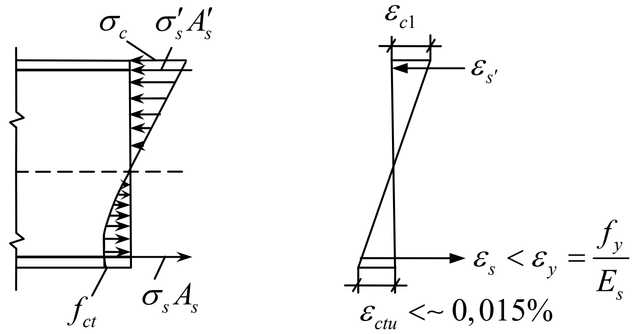

Figure 12. Stage one (I) of stress [σ] deformation [ε] state in reinforced concrete

reinforcement tension strength, σ_c is the concrete stress in compression zone, $\sigma_s A_s$, $\sigma'_s A'_s$ the stress and the cross section area of reinforcement bars respectively in tension and compression zones, E_s is the Young's modulus of the reinforcement, ε_{ctu} is the concrete ultimate strain in tension, ε_{c1} is the concrete maximum strain in compression, ε_s, $\varepsilon_{s'}$ is the reinforcement strain and ε_y is the relative strain of tensioned reinforcement. *Stage two (II)*: cracks in tension zone, but stress in compressed concrete remains under its maximum strength (Figure 13).

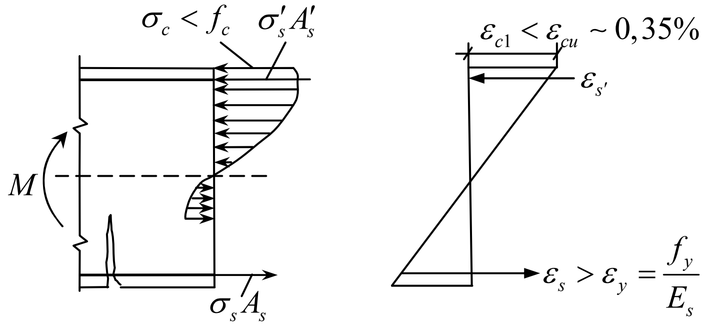

Figure 13. Stage two (II) of stress and deformation state in reinforced concrete, f_c denotes concrete compression strength.

Tension stresses are taken by: a) in cracked sections – by reinforcement and concrete above the crack; b) between cracks – by both concrete and reinforcement.

Stage three (III): the stage of failure. As a result of an increasing external load, the stage two moves to the stage three and, as a consequence, plastic deformations are developing in the compressed concrete zone. The location of the maximum compression stress in concrete moves from the edge of a section to its centre. The reinforcement tension stresses exceed the yield strength. The ultimate strain of concrete ε_{cu} plays a significant role during this stage. Usually it limits plastic deformations that may develop in the reinforcement prior to failure. In design codes the stage three is often divided in to two parts (Figure 14). Case 1. The failure of reinforced concrete begins by yielding of the

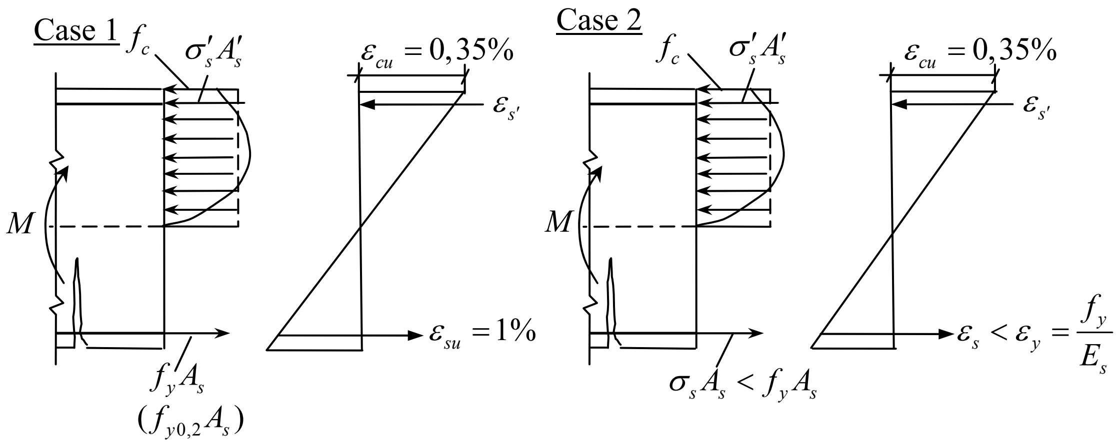

Figure 14. Stage three (III) of stress and deformation state in reinforced concrete

reinforcement in the tension zone and ends by breaking of concrete in the compression zone. This kind of failure has a plastic character. Case 2. The failure of members with excess content of reinforcement or mainly compressed members. The fracture occurs in compression zone before yielding of the steel reinforcement. The failure is brittle.

Stress redistribution in statically indeterminate structures

The essence of the method of stress redistribution in statically indeterminate structures consists of following: under some load value the stresses in tensioned (soft) reinforcement achieve the yield limit. Together with the developing of plastic deformations in bars, the area of large local deformations, called *plastic hinge*, is developing in concrete.

In statically determinate structures, the developing of plastic hinge may cause considerable deflections and decreasing of compressed zone and, as a result, the compression stresses achieve their ultimate value leading to the collapse of the structure. Differently from statically determinate structures, in statically indeterminate structures with the advent of plastic hinge, redundant connections prevent the rotation of parts of structural system in relation of each other and redistribute external loading within the system.

Stage IIa occurs, when stress in tensioned reinforcement achieves the yield limit, but concrete stress does not achieve its ultimate value (Figure 15). At the same time, deformations in plastic hinge are increasing, but the value of bending moment remains unchanged

$$M = f_y A_s z_c, \tag{41}$$

where z_c is the distance (moment arm) between the pair of internal forces. The fracture of the structural member will occur when plastic hinges will be developed in all redundant connections and redistribution of external forces is not possible anymore.

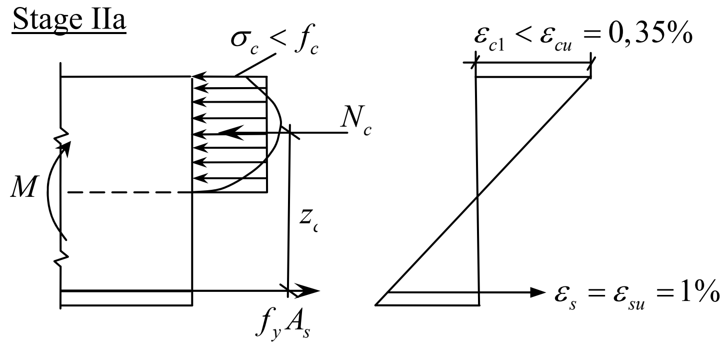

Figure 15. Stage IIa at the section with plastic hinge

Comments

As it was mentioned, concrete with conventional reinforcement behaves like an orthotropic material. The question concerning the orientation of bars is not under consideration, as usually predicted tension in beams or columns coincides with the actual reinforcement. In other words, the principle of determinism can be applied. As a difference from a common reinforced concrete, steel fibre reinforced concrete is anisotropic. The level of anisotropy relies primarily on the degree of fibre orientation. If it would be possible to determine the rule for fibres orientation in the matrix considering reasonably the type of structure, casting method and the rheology of mixture, the calculation of cross-sectional capacities can carried out similarly as done for conventional reinforced concrete cross-sections. However, the issue of anchorage failure must be studied separately. A more precise approach needs to take into account the anchorage length of fibres in the matrix, which depends a lot on the matrix-fibre interface and shape of fibres and, additionally, the work done by the matrix should also be considered. From the latter, the difference between conventional and fibre reinforced concrete rises once again: in common reinforced concrete the tension ability of concrete is simply not taken into account, but reinforcement anchorage is assumed also in tension zone. In general, designing of conventional reinforced concrete structures involves the work of two separate materials: concrete and steel. Each of the materials has its own role for structural capacity: one deals with compression and another with tension. In case of steel fibre reinforced concrete the situation turns to integration of two materials, what finds its explanation in the theory of composites.

Possible solutions

After reviewing the researched material from the point of microstructural material and from the already established theory of analysis of concrete structures, it can be concluded, that a valid, full and objective method, reflecting the behaviour of steel fibre reinforced concrete, does not exist. Fracture mechanics and corresponding calculation methods of steel fibre reinforced concrete are still open questions. The classical theory of reinforced concrete completely ignores the orientation of reinforcement bars. As it was stated above, certain important properties of steel fibre reinforced concrete directly depend on fibres orientation in the matrix. In order to predict the arrangement of fibres in the matrix, certainly both theoretical and practical work must be done. Concerning fibre orientation, a temping approach to this problem could be to employ the theory of probability and implement some initial assumptions about the location of fibres. However, the most valuable and reliable information about the orientation of fibres in the matrix must come from experiments as the orientation is influenced by the manufacturing process of concrete mass with steel fibres.

The diagram, shown in the Figures 16a and 16b may be a conclusion of this article. It shows the formation of composite and the main present ambiguities, which prevent from using SFRC in load-bearing structures safely.

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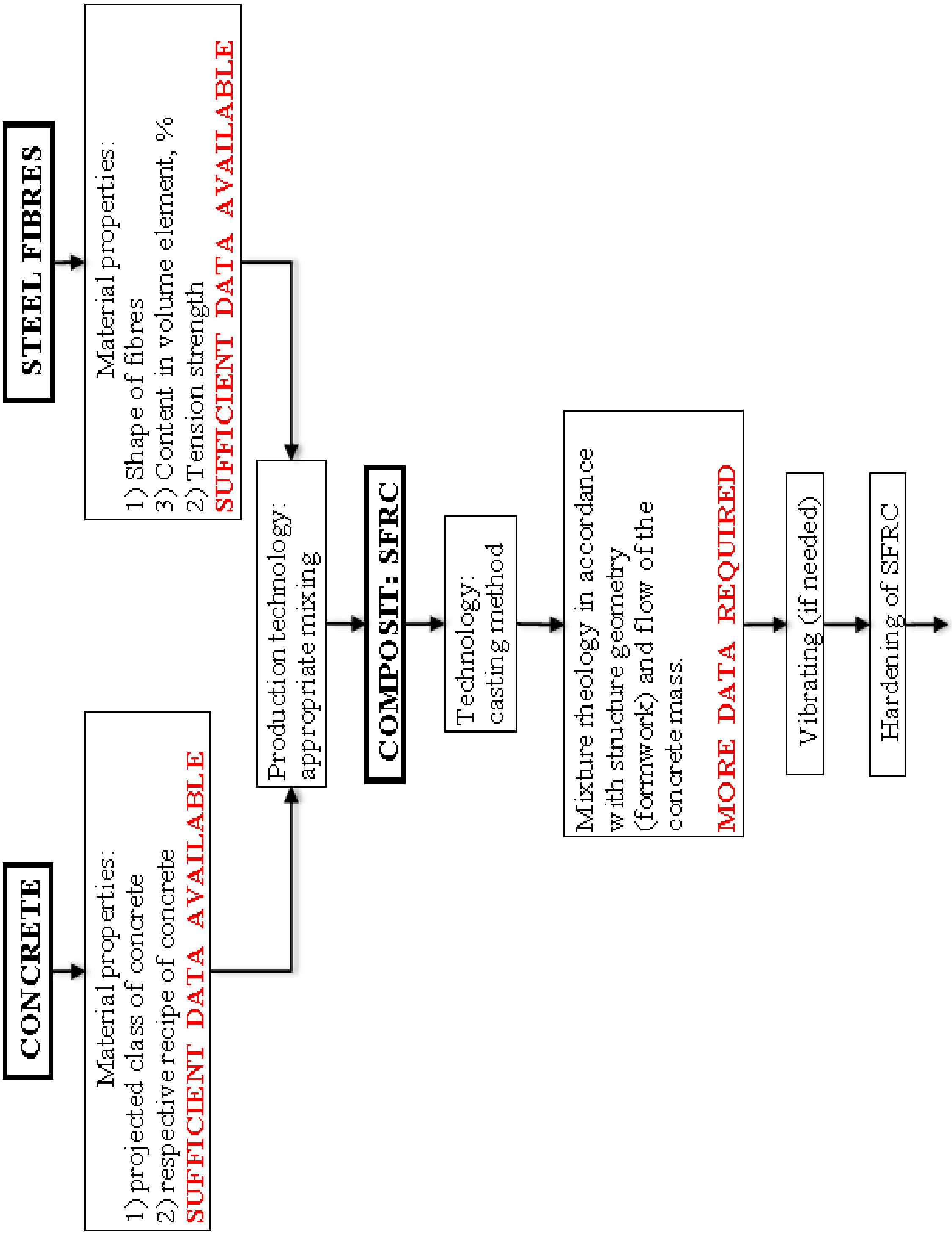

Figure 16. The formation and the main present ambiguities of steel fibre reinforced concrete. (The figure continues on the next page.)

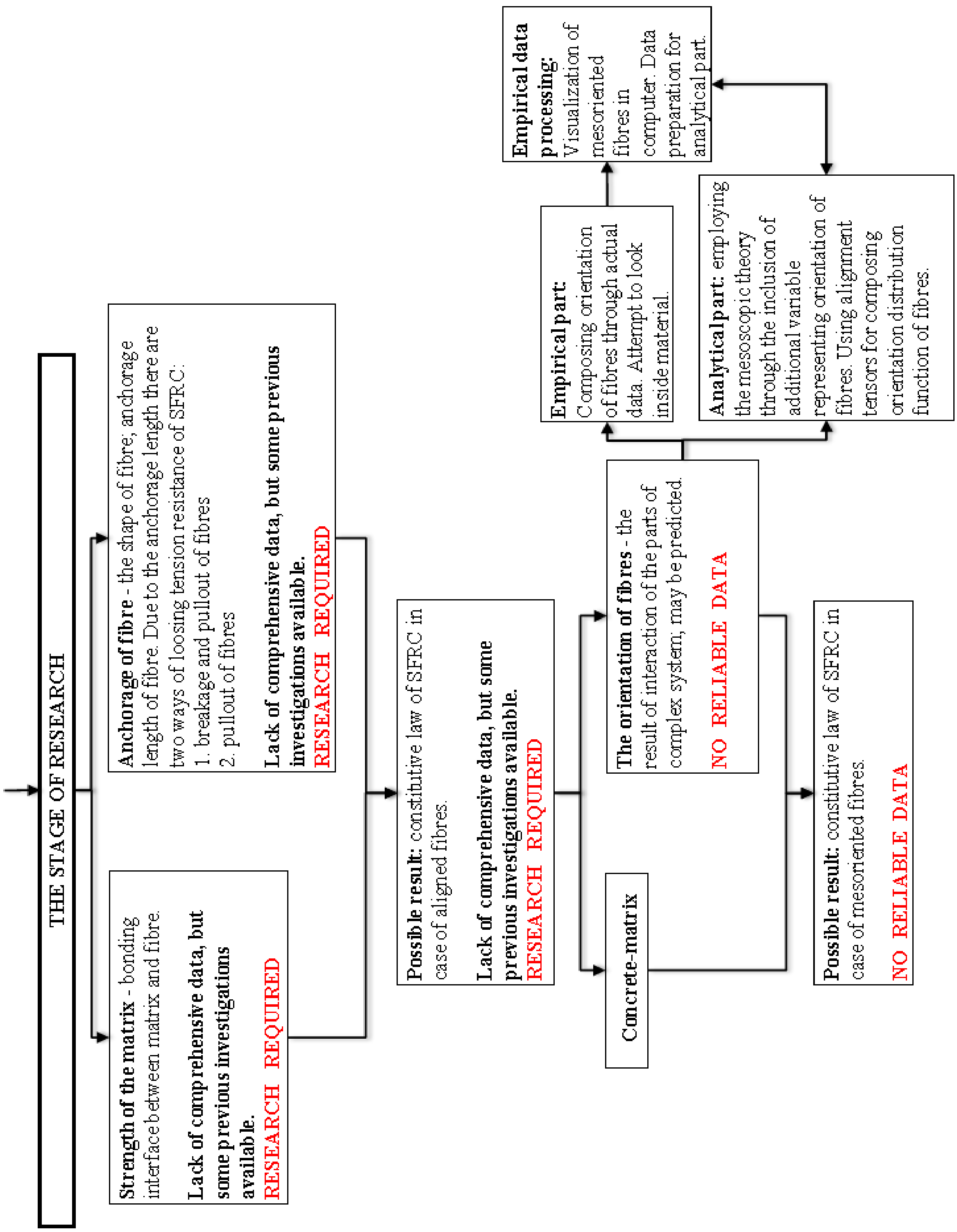

Figure 16 (continued). The formation and the main present ambiguities of steel fibre reinforced concrete.

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