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A note on traction and displacement boundary conditions

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Summary. The note is of pedagogical nature and gives a simple physically based explanation on the fact that the traction and the displacement cannot be prescribed at the same point on a boundary in a solid mechanics problem.

Key words: boundary conditions, solid mechanics

Introduction

We borrow from the text-book of Belytschko et al. [1, p. 26]:

> "The traction and displacement cannot be prescribed at the same point, but one of these must be prescribed at each boundary point; this is indicated by
> $$\Gamma_u \cap \Gamma_t = 0 \quad \Gamma_u \cup \Gamma_t = \Gamma \qquad (2.2.28)"$$

There might arise a simple question: why? We will present below an easily understandable physically based explanation.

Explanation

Using pure mathematics to give an explanation for the content of the statement referred to above seems difficult especially if the setting is an introductory continuum mechanics course. However, a visual type of demonstration may help the student to assimilate and also to remember the point under study.

The sensibility of possible boundary conditions can sometimes be inquired using suitable thought experiments. An imaginary laboratory arrangement is set up, in which certain boundary conditions are attempted to be realized. If this succeeds physically, one can probably be sure that the corresponding boundary condition system is also mathematically sound.

Let us imagine that we have in our laboratory available a huge number of "boundary slaves" situated along the boundary of the body. Each slave A possesses a force measuring device B consisting of a spring with a given stiffness and a force reading meter by which he can interact with the body surface at point P by pushing or pulling (Figure 1). To retain some clarity, at a point only one slave acting along one coordinate

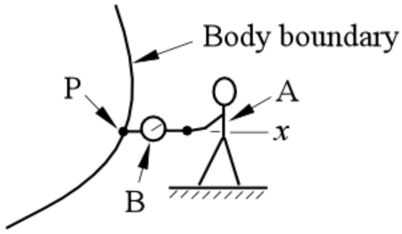

Figure 1. A boundary slave

direction, say *x*, has been drawn in the figure. The slave can also read the displacement *u* of point P in the direction of ***x*.**

Let us give the slave a displacement command: the value of the displacement at point P must be $\bar{u}$. The slave tries to satisfy this command by moving the end of the device in his hands until he can see the correct displacement value. Clearly, however, he cannot give simultaneously a possibly predetermined value for the elongation of the spring (and thus for the force in the spring); these values are consequences of the satisfaction of the displacement command.

Let us then consider another command; a traction command: the value of the "traction" (the spring force in the spring) must be $\bar{t}$. Now the slave moves again the end of the device and follows the reading of the elongation gauge of the spring, —which with a suitable calibration — gives the value of the spring force, and he is contended when the reading is right. Clearly again, he cannot simultaneously control the displacement of point P; it is a consequence of the solution.

The line of thought described above is actually not so far fetched. In [2, p. 351], a boundary value problem is called a "jury problem" where thus a certain set of people watches the satisfaction of the correct boundary conditions.

As is well known, a pure traction or displacement boundary condition is not the only possibility. For instance, the traction can depend on the boundary displacement as for example in the Winkler foundation model. Further, more complicated boundary conditions can be devised. However, these are not included in the theme of the present note.

References

[1] T. Belytschko, W. K. Liu and B. Moran, *Nonlinear Finite Elements for Continua and Structures*, Wiley, 2000.

[2] S. H. Crandall, *Engineering Analysis, A Survey of Numerical Procedures*, McGraw-Hill, 1956.

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