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Minimum cost steel beam using semi-rigid joints

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Summary. The question to be considered in the paper is: What is the benefit to use semi-rigid joints in structures? A simple example is given to demonstrate the cost effects to a steel beam using semi-rigid joints at both ends. The optimization is done manually step by step for hot rolled beams using one material grade and end plate joints. It is shown, that 14 % savings can be achieved, if semi-rigid joints are used instead of rigid joints. The savings are 30 % compared to the costs when using hinged joints. Similar results can be found for entire frames in the literature. It is believed, that for welded beams the savings can be nearer to the theoretical maximums, 33 % and 100 %, respectively. The use of pre-chamfer to eliminate the deflection constraints and the library for welded beams is almost infinite and these enable to use wider design space for the search.

Key words: steel, semi-rigid joints, costs

Introduction

In Finland the question of the use of semi-rigid joints is often aroused. However, during last 15-20 years it has been demonstrated considerable savings in frame designs when using semi-rigid joints in steel frames [1], [2], [3].

Steel frame design can be done [4] using elastic theory or plastic theory. The most recent European standard [5] includes rules to design semi-rigid joints. The costs of members and joints are calculated typically very approximately.

In this paper a simplified case is solved exactly using the elastic theory. Three joint layouts are considered and cost comparisons are done for all cases using a novel feature based cost analysis [6]. Conclusions and needs for further studies are outlined.

The design space chosen is very simple and rather limited simulating real projects with hot-rolled profiles. The design space in this case available is: steel members, steel grades and end plate joints. The feasibility of the solution means that the requirements of the EN standards are fulfilled. The manufacturing costs of the beams are taken into account.

Case considered

Consider a one span steel beam between stocky columns and supported for shear forces with small steel plates under the beam ends (requirement of the contractor). The beam is supported against lateral torsion. Loads and a span of the beam are known. The ultimate limit state uniform load is q and the serviceability limit state uniform load is q_s. The span of the beam is L. The elastic theory is used for the global analysis.

The design space is as follows:

- Symmetric IPE beams available, steel material grade is S235,
- Grade S235 steel plates available for the joints,
- Bolts of grade 8.8 available for the joints,
- End plate joints should be used.

The constraints are:

- Design requirements following the European standards [4], [5].

The task is to find the minimum cost beam.

Moment resistance

Consider at first the design without using semi-rigid joints, meaning that we use only hinged or rigid joints at both ends of the beam. To begin with the bending moment requirements are considered. The bending moment resistance check is:

$$|M_{\max}| \le M_R \tag{1}$$

where $|M_{max}|$ is the absolute maximum moment along the beam and M_R is the bending moment resistance of the cross-section.

For the hinged beam

$$|M_{\max}| = \frac{q \cdot L^2}{8} \Rightarrow M_R \ge \frac{1}{8} \cdot q \cdot L^2 = 0.125 \cdot q \cdot L^2 \tag{2}$$

and for the rigidly jointed beam:

$$|M_{\max}| = \frac{q \cdot L^2}{12} \Rightarrow M_R \ge \frac{1}{12} \cdot q \cdot L^2 = 0.083 \cdot q \cdot L^2 \tag{3}$$

The rigidly jointed beam requires smaller bending moment resistance, as is known.

Consider next the case where are rotationally elastic springs at both end supports of the beam. The rotation stiffness of the springs are S. Fig. 1. illustrates three bending moment diagrams, hinged beam, rigidly jointed beam and beam with semi-rigid joints at both ends using the elastic theory.

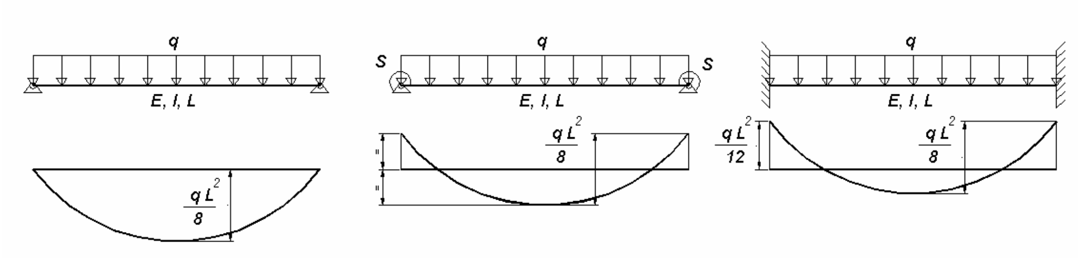

Figure 1. Bending moment diagrams for three cases.

The rotation spring can be adjusted so that the same absolute moment appears at the supports and at the mid section of the beam as shown in Fig 1. Then:

$$|M_{\max}| = \frac{q \cdot L^2}{16} \Rightarrow M_R \geq \frac{1}{16} \cdot q \cdot L^2 = 0.0625 \cdot q \cdot L^2 \tag{4}$$

It can be seen that now the required bending moment resistance is clearly the smallest of the three cases. The relative moments are as follows:

- Hinged to semi-rigid: 100*0.125/0.0625 = 200 %,
- Rigid to semi-rigid: 100*0.083/0.0625 = 133 %.

Varying the rotation stiffness S the following equations can be derived:

$$M_{span} = \frac{q \cdot L^2}{8} - M_{\sup p} \tag{5}$$

$$M_{\sup p} = \frac{q \cdot L^2}{12} \cdot \left(\frac{S \cdot L}{S \cdot L + 2 \cdot EI} \right) \tag{6}$$

$$|M_{\max}| = \max\left(|M_{span}|;|M_{\sup p}|\right) \tag{7}$$

where EI is the bending stiffness of the beam.

The required moment resistance versus the joint stiffness is shown in Fig. 2.

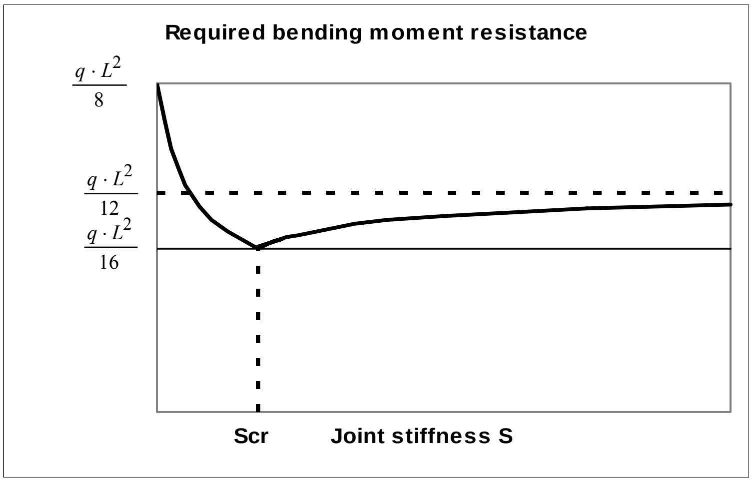

Figure 2. Required moment resistance versus joint stiffness.

The rotational stiffness S_{cr} of the joints for the minimum bending resistance can be derived from Eqs. (5)–(6). This critical stiffness means the stiffness, when the maximum moment changes from the span moment to the support moment. The result is:

$$S_{cr} = \frac{6 \cdot (EI)}{L} \tag{8}$$

Example for moment resistance

Consider the beam with q = 33 kN/m and L = 5.0 m. The minimum required moment resistance is then 0.0625*33*52 = 51.6 kNm. The required static moment W is:

$$M_R = f_y \cdot W \Rightarrow W = \frac{M_R}{f_y} \tag{9}$$

where f_y is the design strength of the steel material using the material factor one.

The required static moments and corresponding minimum weight IPE cross-sections available in the library are given in Table 1. The plastic static moments are used for the cross-sections supposing the cross-section class 2 to be valid applying the standard [4]. In the table are given the minimum weight IPE profiles for hinged and rigid joints using S235 steel grade for all cases.

Table 1. Profiles according to the required bending resistances.

	Semi-Rigid	Hinged	Rigid
Required W (m^3)	2.19×10^{-4}	4.39×10^{-4}	2.93×10^{-4}
Profile	IPE 200	IPE 270	IPE 240
W profile (m^3)	2.21×10^{-4}	4.84×10^{-4}	3.67×10^{-4}
I profile (m^4)	1.94×10^{-5}	5.79×10^{-5}	3.89×10^{-5}
S_{cr} (kNm/mrad)	4.9	$\leq$ 0.9	$\geq$ 6.5
Support moment (kNm)	51.6	0	68.8

It can be seen that the moment resistance requirements do not follow the weights of the beams. The relative weights of the IPE beams are as follows when comparing to the semi-rigid solution:

- Hinged: 100x36.0/22.4 = 161 %,
- Rigid: 100x30.7/22.4 = 137 %.

The relative weights are not the same as the relative moments due to fact that the library for hot rolled sections is not continuous.

In the standard [5] are given the following requirements for the joint classifications:

Hinged joint: $$S \leq \frac{0.5 \cdot EI}{2 \cdot L} \tag{10}$$

Rigid joint: $$S \geq \frac{8 \cdot EI}{2 \cdot L} \tag{11}$$

The values of Table 1 for hinged and rigid joints are calculated using these equations (E = 210000 MPa). In the standard [5] are given the initial rotational stiffnesses which are in this case (η = 2) two times the values given in Table 1 for hinged and rigid joints.

Deflections

The maximum deflection of the beam with semi-rigid joints at both ends is:

$$v = \frac{5 \cdot q_s \cdot L^4}{384 \cdot (EI)} - \frac{M_{sup\,p} \cdot L^2}{8 \cdot (EI)} \tag{12}$$

where the support moment is calculated using Eq. (6) and the serviceability limit state load q_s. The deflection is decreasing when the rotational stiffness of the joint is increasing as shown in Fig. 3.

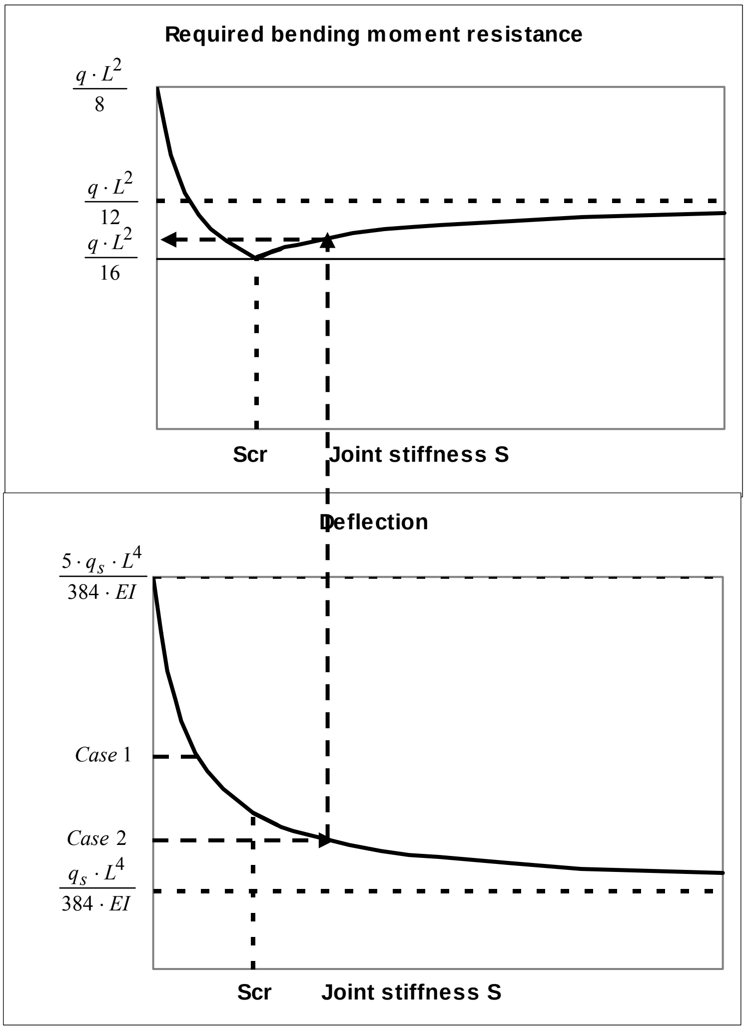

Figure 3. Deflection versus joint stiffness and Cases 1-2.

Referring to Fig 3. there arise two cases:

- Case 1: If the allowed deflection (e.g. $L/400$) is greater than the deflection calculated based on Eq. (12) using S_{cr}, then the minimum resistance of Fig. 2 can be used.
- Case 2: If the allowed deflection is smaller than the deflection calculated based on Eq. (12) using S_{cr}, then the allowed deflection defines the required stiffness and the moment resistance as shown in the path of Fig. 3.

The limit deflections for hinged and rigid joints can be seen in Fig. 3.

Example continues

Consider the same example as above for the moment resistance. The load in the serviceability limit state is q_s = 22 kN/m. The deflections for three cases are shown in Table 2.

Table 2. Deflections.

	Semi-Rigid IPE 200	Hinged IPE 270	Rigid IPE 240
Deflection (mm)	17.6	14.7	4.4
S used in calculations (kNm/mrad)	4.9	0	∞

Use the limit deflection L/400 = 5000/400 = 12.5 mm. Only rigid solution is feasible. Using profile IPE 300 for hinged solution the deflection is 10.2 mm which is feasible solution. For the semi-rigid solution we ended up to the Case 2 in Fig. 3. The proper rotational stiffness for the profile IPE 200 is 13.8 kNm/mrad to get the deflection 12.5 mm. Then the maximum moment is 61.5 kNm which is too much to this profile, so we must take bigger profile for the semi-rigid solution, such as IPE 220.

For IPE 220 the critical rotational stiffness is 7.0 kNm/mrad and the corresponding deflection is 12.3 mm so we end up to the Case 1 in Fig.3. using this profile. In Table 3 are given the final solutions.

Table 3. Final solution.

	Semi-Rigid IPE 220	Hinged IPE 300	Rigid IPE 240
Deflection (mm)	12.3	10.2	4.4
S_{cr} (kNm/mrad)	7.0	≤ 0.9	≥ 6.5
Support moment (kNm)	51.6	0	68.8

The relative weights of the IPE beams are as follows when comparing to the semi-rigid solution:

- Hinged: 100x42.2/26.2 = 161 %,
- Rigid: 100x30.7/26.2 = 117 %.

All the profiles belong to the cross-section class 2 following [4] so the plastic moment resistances could be used.

The shear force at the ultimate limit state is 2.5*33 = 82.5 kN. All cross-sections can resist this shear load.

The cross-sections fulfill all requirements of the standard [4], so they are feasible solutions. Consider next the joints.

Joints

The requirements (support moment and stiffness) for the joints are given in Table 3. End plate joints are used. The joint design was done using the program CoP [7]. The joint layouts are shown in Fig. 4. The stocky columns used in the calculations were HEM 220 (S355).

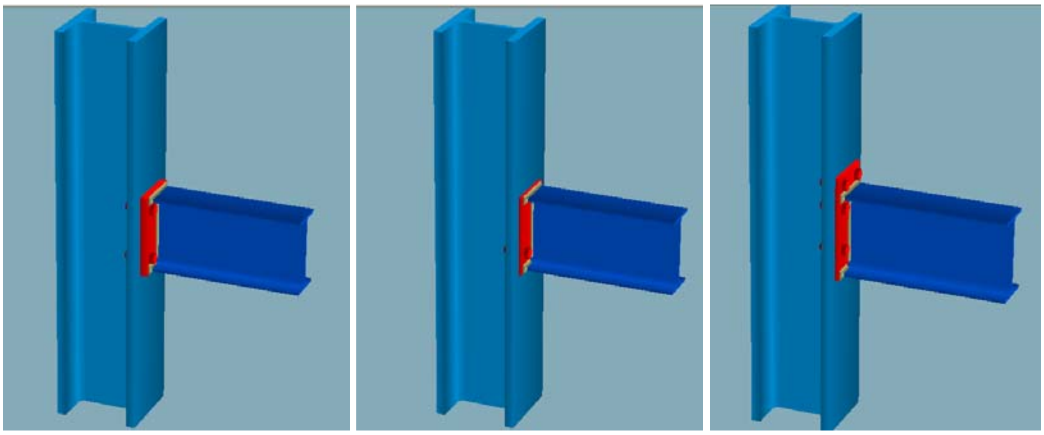

Figure 4. Semi-rigid (a), hinged (b) and rigid (c) joint (shear plates not shown).

The results are:

- Semi-rigid joint:
  - Moment resistance 51.6 kNm,
  - Rotational stiffness: 7.1 kNm/mrad,
  - End plate: S235, 240x150x20 mm^3,
  - Bolts: 4M20, 8.8 (holes 22 mm),
  - Welds: flange 6.4 mm, web 4.1 mm.
- Hinged joint:
  - End plate: S235, 320x150x8 mm^3,
  - Bolts: 2M20, 8.8 (holes 22 mm),
  - Welds: flange 3.0 mm, web 3.0 mm.
- Rigid joint:
  - Moment resistance 70.0 kNm,
  - Rotational stiffness: 14.4 kNm/mrad,
  - End plate: S235, 310x150x12 mm^3,
  - Bolts: 6M20, 8.8 (holes 22 mm),
  - Welds: flange 6.9 mm, web 4.3 mm.

Comparing the semi-rigid solution to two other:

- Semi-rigid joints means the lighter beam and the joints than the rigid joints and the moment to the column is smaller,
- Semi-rigid joints mean much lighter beam than the hinged solution.

Manufacturing costs

Manufacturing costs were calculated using a feature based estimation model. In this model manufacturing is divided to fabrication processes, and each process is evaluated based on time consumed for the process. Features of the fabricated parts determine the time used. On the other hand fixed costs for workshop are estimated. These include labour, equipment, maintenance, real estate and service (heating, cleanup, real estate maintenance) cost for each cost center. In addition costs of raw material, consumables and energy used for processing are considered. Overheads and effect of utilisation rate of cost center are excluded. [6]

Using this type of approach it is possible to estimate cost variations of different structural solutions, including main part (beam) and joints.

In this case following processes are included:

- blasting of beam and end plates,
- sawing of main part to proper length,
- fabricating of assemblies, i.e. end plates, including cutting of plates and bolt holes,
- assembling of end plates, including tack welding and fillet welding,
- post treatment, including edge grinding,
- coating.

In addition the material costs for the main part, assemblies and bolts, nuts and washers are included.

Blasting is executed with steel grains in automatic blasting line, sawing is executed with band saw, cutting of plates and holes are done with plasma cutting machine, welding is executed with MAG-method in flat position, and coating is carried out with alkyd painting system AK160/3-FeSa2½.

In Table 4 are shown cost distributions for each beam-joint combinations and cost components. In Fig 5. are shown the cost component distributions in Euros for three solutions.

Table 4. Manufacturing cost distribution.

		Semi-rigid IPE 220	Hinged IPE 300	Rigid IPE 240
Material unit cost	[€]/kg	1.5	1.5	1.5
Beam material	[€]	196.50	316.50	230.25
Blasting	[€]	1.82	1.82	1.82
Sawing	[€]	8.80	10.12	9.14
Assembly fabrication	[€]	19.73	8.92	22.66
Assembling	[€]	14.18	6.92	17.33
Post treatment	[€]	0.22	0.31	0.24
Coating	[€]	18.38	25.00	20.19
Total cost	**[€]**	**259.63**	**369.59**	**301.63**
Cost difference		100 %	142 %	116 %
Total unit cost	[€]/kg	1.98	1.75	1.97

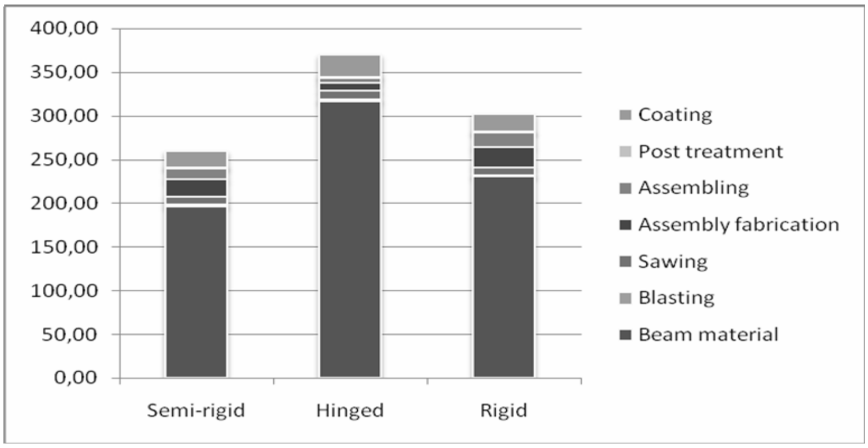

Figure 5. Cost component distributions in Euros of assembly.

Considerable cost savings may be achieved using semi-rigid joints. 80 % of the total savings (-14 %) compared with rigid joint structure is due to the decreasing of cross section resulted from smaller moment, while rest of the savings (20 %) is due to the decreased manufacturing costs. Compared with beam having hinged joints a saving of 30 % is achieved with semi-rigid joint beam.

Although erection costs and fabrication costs of the column (hole drilling) were not included in this study, it is rather obvious that these costs of semi-rigid joint are smaller than with rigid joint due to the less amount of bolts, thus increasing the cost efficiency of semi-rigid joint compared with rigid joint. Accordingly hinged joint will reduce the cost difference with other solutions due to the smaller amount of bolts.

Frame optimisation

Only one beam was considered above. Similar results can be found in the literature for frames. Two examples are given in Fig. 6 [1]. The costs are calculated by taking into account profile costs and joint costs approximately. The designs that account for semi-rigid joints weights 14 % (upper case) and 12 % (lower case) less than the designs found when only fully rigid joints are considered.

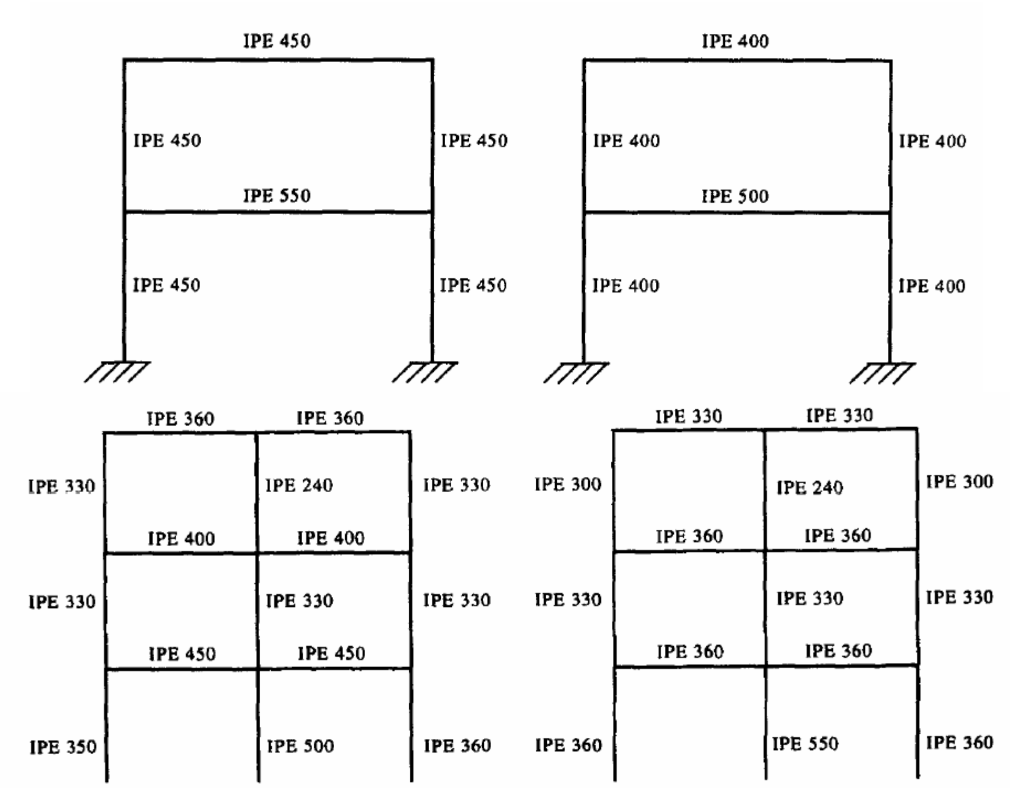

Figure 6. Optimum solutions for fully rigid joints (left) and for semi-rigid joints (right) [1].

Conclusions

One span beam with semi-rigid joints at both ends is considered in the paper. The design space to search the cost optimum was hot-rolled IPE beams made of steel material S235. Comparing the manufacturing costs of the beam with semi-rigid joints to the beams with hinged and rigid joints the following result was got, including costs of end plate joints to beam ends:

- Cost savings from semi-rigid to hinged solution: 30 %,
- Cost saving from semi-rigid to rigid solution: 14 %.

It was shown that theoretically the requirement for the bending moment resistance means the following:

- From semi-rigid to hinged solution: 100 % greater moment resistance requirement,
- From semi-rigid to rigid solution: 33 % greater moment resistance requirement.

When searching the feasible solutions in the hot-rolled section library this theoretical difference is changed because the library is discrete. If the deflection requirement is active then the differences between the solutions are getting smaller. This means that for welded beams it may be possible to come close to the theoretical solution in savings, because the "library of welded profiles" is practically infinite. The pre-chamfer of welded beams is also frequently used so that deflection requirements will not be typically determinate for welded beams.

Only the beam was considered in the paper and one steel grade (S235) for hot-rolled profiles in one loading case. In the literature can be found similar results for frames with semi-rigid joints. The use of semi-rigid joints seems to have potential to get savings in steel structures. To achieve these savings, the designer should have:

- Proper tools to put the semi-rigid joint models into the frame analysis model,
- Proper tools to check the resistances of joints including semi-rigid joints,
- Proper tools to check the cost effects of different joint layouts during the design.

The goals of further researches are to get those tools for designers. The tool integration to the product models widely used in the structural steel design seems to be the most proper way to obtain the goal.

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