Rakenteiden Mekaniikka (Journal of Structural Mechanics)
Vol. 42, No 4, 2009, pp. 207-217

Web shear failure in prestressed hollow core slabs

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Summary. Web shear failure is one of the numerous failure modes which have to be taken into account in the design of prestressed hollow core slabs. In the early eighties it came out that the traditional design method which is still in Eurocode 2, is on the unsafe side for numerous slab types. The paper describes the procedure which resulted in the design rules adopted in the European product standard for hollow core slabs.

Key words: prestress, hollow core slab, hollow-core slab, web shear failure, eurocode, transfer of prestress

About prestressed hollow core slabs

Precast, prestressed hollow core slabs or PHC slabs are among the most common load-bearing concrete elements in the world. They are widely used in floors and roofs of office, residential, commercial and industrial buildings. Typical slab cross-sections used in Finland are shown in Fig. 1 and a 3D illustration is given in Fig. 2.

The manufacturing tecnique is simple. Prestressing strands are first tensioned above a long bed, whereafter a casting machine casts and compacts the concrete around and above the strands. After hardening of the concrete the ends of the strands are released and the long slab is saw-cut into units of desired length. Due to the special manufacturing technique no transverse reinforcement is possible.

When subjected to a transverse line load shown in Fig. 3, the heavily prestressed slab units typically fail as shown in Fig. 4. The failure is abrupt and noisy, like a small explosion. Before failure, the failure zone is completely uncracked, and when the first crack appears, the failure takes places immediately. This failure mode is called web shear failure and it is the subject of the following story.

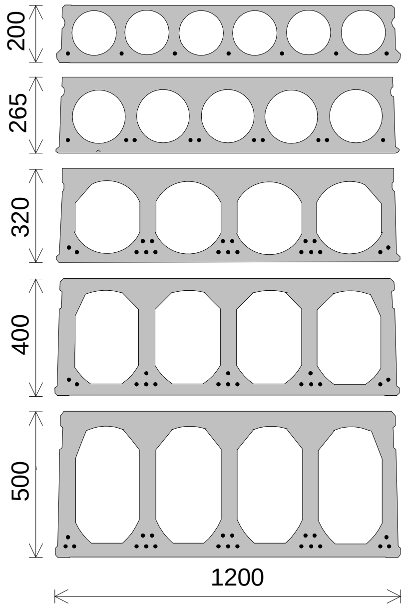

Fig. 1. Slab cross-sections. The black dots indicate possible positions of the longitudinal prestressing strands.

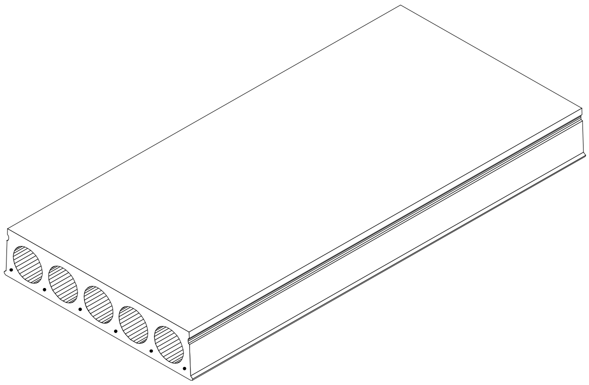

Fig. 2. Illustration of a PHC slab.

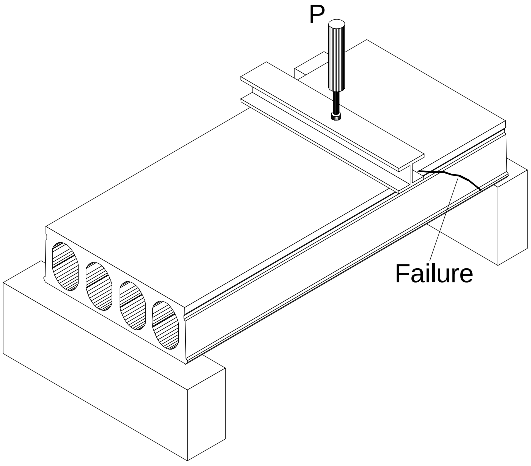

Fig. 3. Shear test.

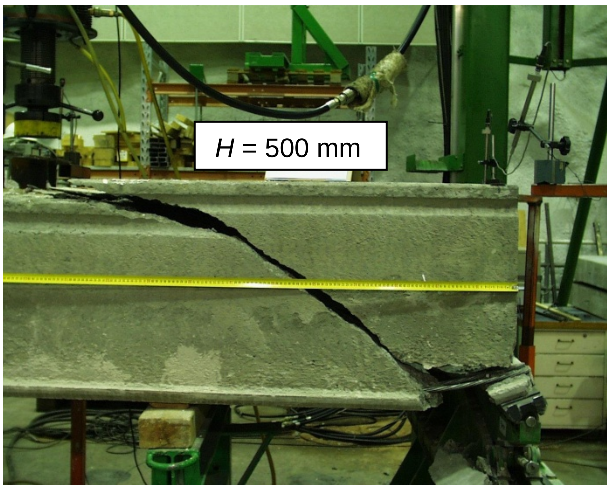

Fig. 4. Web shear failure.

Traditional design method for web shear failure

To simplify the design, it is generally assumed that the PHC slabs behave like simply supported beams. This simplification means that the mechanical model of a PHC floor is a number of parallel I-beams. To get an impression of the behaviour of a large floor subjected to a uniformly distributed load, it is enough to study a longitudinal cut taken from one slab unit as shown in Fig. 5. A more accurate way would be to regard one slab unit as a beam, the width of which at a given depth being equal to the total width minus the sum of core widths.

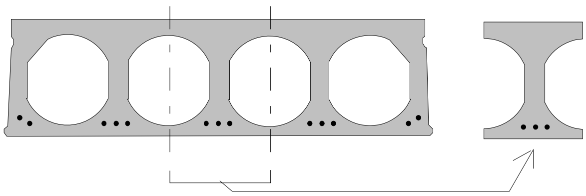

Fig. 5. A longitudinal cut representing the whole slab unit.

A two-dimensional plane stress analysis has been carried out for a cut shown in Fig. 6.a. For this purpose the cut section is modelled as shown in Fig. 6.b. The principal stresses σ_I and σ_{II} ($\sigma_I > \sigma_{II}$) at the end of the cut, calculated in the integration points of the finite elements, are shown in Fig. 7. The stresses are due to prestressing force and a vertical point load at a distance of three times the slab depth.

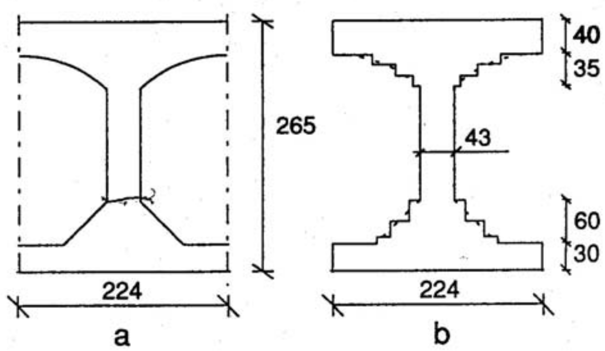

Fig. 6. a) Cross-section of one web and the flanges on both sides. b) Approximate cross-section for two dimensional FEM-model.

The point load on the modelled cut corresponds to a typical experimental failure load. The maximum principal stress $\sigma_{I,max}$ in the web, see Fig. 7, is positive and equals roughly the tensile strength of the concrete. Furthermore, when an inclined crack appears in the web close to the support, its immediate propagation upwards cannot be prevented. The propagation downwards is also possible because the anchorage length of the strands is short. These considerations suggest that setting the maximum principal stress $\sigma_{I,max}$ in the web equal to the tensile strength constitutes a satisfactory failure criterion. It seems that this criterion has been applied since the early ages of prestressed I-beams, not encouraged by FEM results as here but based on simple engineering reasoning. As early as in 1972, maybe even earlier, the British concrete code CP 110 [1] had adopted this criterion.

The actions affecting the stress state in the concrete are the prestressing force, the self-weight of the slab unit and the imposed external load. The relevant stress components in two-dimensional analysis are horizontal normal stress σ, vertical normal stress σ_v and shear stress τ . When a web shear failure takes place, σ_v practically vanishes except next to concentrated loads and in the nearest neighbourhood of the support where σ_v is negative and where the maximum principal stress never occurs. So, it is natural to assume that only the horisontal normal stress σ and shear stress τ need to be taken into account.

For PHC slabs with circular or oval hollow cores the position of the critical point, i.e. point (x,z) which gives the highest principal stress, tends to be at the depth where the web width is narrowest. This is typically so close to the centroidal axis that the horizontal stress due to the bending moment may be ignored. The horisontal compression due to the prestressing force equals zero at the end and increases with x. Since the horisontal compression reduces σ_I, the critical point must be as close to the support as possible but not too close to be affected by σ_V. Based on this reasoning the critical point shown in Fig. 8 seems justified.

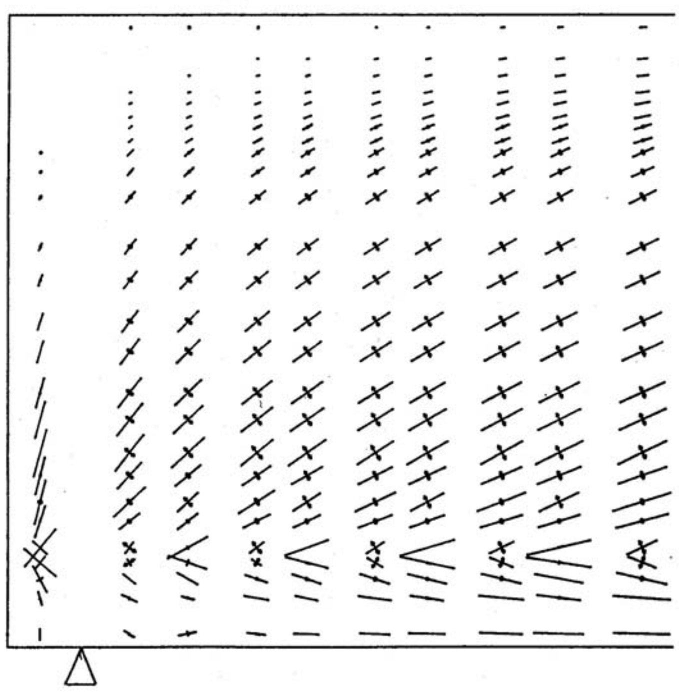

Fig. 7. Principal stresses illustrated as vectors. Tensile stresses are indicated by arrows. The concrete stresses at the depth of the strands and in the nearest near the support are inaccurate due to the concentrated transfer of the prestressing force and support reaction.

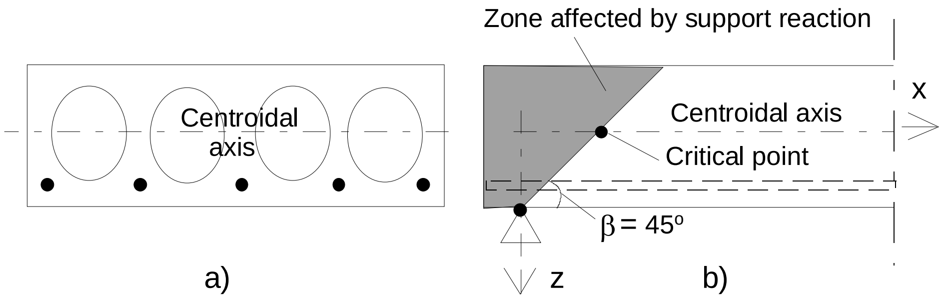

Fig. 8. Critical point for web shear failure in traditional method.

When σ and τ are known, the failure criterion becomes

$$\sigma_{I,max} = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2} = f_{ct} \tag{1}$$

where f_{ct} is the tensile strength of the concrete. As first estimate from elementary beam theory,

$$\sigma = -\frac{P}{A} + \frac{Pe + M}{I} z \quad \text{and} \tag{2}$$

$$\tau = -\frac{VS}{Ib_w} \tag{3}$$

where P is prestressing force, M bending moment due to actions other than P, z vertical coordinate, origin at centroidal axis, positive downwards, e eccentricity of P, positive downwards, A cross-secctional area, I second moment of area of cross-section, S first moment of area above considered horizontal axis, around the centroidal axis, V shear force and b_w width of web at the considered depth.

Since $z \approx 0$ at the critical point, Eq. (2) reduces to

$$\sigma = -\frac{P}{A} \tag{4}$$

Note that here P is not the fully transferred prestressing force but varies within the transfer length as shown in Fig. 9.

In this way a very simple method is obtained. The method, later called traditional method, is relatively accurate for some slab types, e.g. for 265 mm slabs with circular hollow cores. The only problem is that e.g. for 320 mm, 400 mm and 500 mm slabs shown in Fig. 1 this method overestimates the shear resistance in worst cases by tens of percent. This is no wonder because e.g. in Fig. 7 the maximum principal stress is not at the mid-depth where it should be according to the method and because the maximum value of the principal stress is considerably higher than that that predicted by the method. The primary reason for the poor fit to test results is the fact that the method does not take into account shear stresses due to the transfer of the prestressing force.

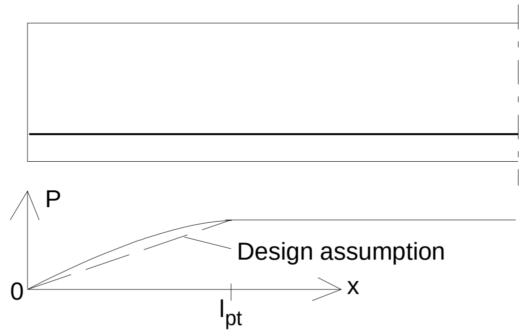

Fig. 9. Prestressing force P within transfer length l_{pt} and beyond it.

To allow for the discrepancy between the test results and theory, Walraven & Mercx [2] recommended in 1983 that the theoretical shear resistance be reduced by 25% in the design. This was no final solution because the need for reduction is different for different cross-sections. At least in Germany attempts were made to modify the method to allow for the addditional shear stress but apparently the resulting approximate method was not published.

Despite the nonconservatism of the traditional method it was taken to Eurocode 2 [3] and to the first version of EN 1168 [4] which is the European product standard for PHC slabs.

The resistance against web shear failure could numerically be solved, but how to develop a simple and reliable method to be used in everyday design? In 1991 the author presented this problem to postgraduate student Lin Yang at the Royal Institute of Technology, Sweden. Yang soon came with a simple solution which he later verified by test results and published as a part of his doctoral thesis [5]. He remembered that the prestressing force is not constant at the end of a PHC slab and followed the way how the shear stress expression is deduced from the normal stress distribution in all basic textbook of structural mechanics.

Yang deduced his shear formula for a case with only one strand layer in the bottom flange. In the following, his approach is extended to a case with n layers of strands some of which may be above the critical web point.

The equilibrium of horizontal forces acting on the free body shown in Figure 10 gives

$$-\tau b_w \Delta x \approx \int_{A_{cp}} (\Delta\sigma) dA + \sum_j \Delta P_j \quad \text{or} \quad \tau = -\frac{1}{b_w}\left(\int_{A_{cp}} \frac{d\sigma}{dx} dA + \sum_j \frac{dP_j}{dx} \right) \tag{5}$$

where σ denotes the axial stress in the concrete, b_w the width of the web at the considered horizontal plane where τ_w is calculated and A_{cp} the cross-sectional area above the considered plane. $\mathbf{\Sigma} P_j$ is the sum of all prestressing forces in strand layers above the considered horisontal plane.

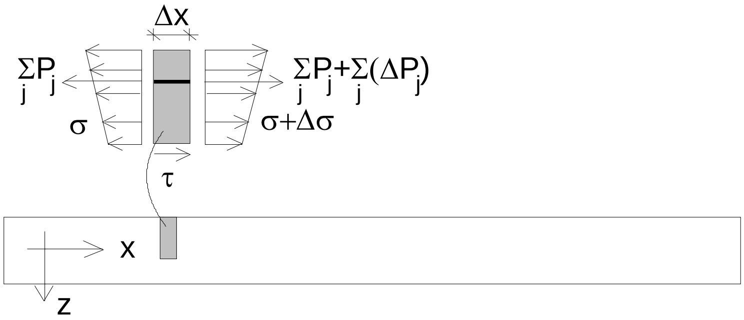

Fig. 10. Free body diagram for a cut of a slab with upper tendons.

The horisontal stress in the concrete is obtained from the well-known expression

$$\sigma = \frac{-\sum_{i=1}^{n} P_i}{A} + \frac{-\sum_{i=1}^{n} P_i e_i + M}{I} z \tag{6}$$

where P_i is the prestressing force in tendon layer i (positive), e_i its eccentricity (positive below centroidal axis, negative above it) and A, I, M and z mean the same as in Eq. (2). A straight-forward differentiation of Eq. (6) gives

$$\begin{aligned}\frac{d\sigma}{dx} &= -\frac{1}{A}\sum_i \frac{dP_i}{dx} + \frac{-\sum_i e_i \frac{dP_i}{dx} + \frac{dM}{dx}}{I} z \\ &= -\frac{1}{A}\sum_i \frac{dP_i}{dx} - \frac{z}{I}\sum_i e_i \frac{dP_i}{dx} + \frac{z}{I} V\end{aligned} \tag{7}$$

Substituting this into Eq. (5) and writing

$$S_{cp} = -\int_{A_{cp}} z dA \tag{8}$$

gives

$$\tau = \frac{1}{b_w}\left[\frac{A_{cp}}{A}\sum_{i=1}^{n}\frac{dP_i}{dx} - \frac{S_{cp}}{I}\sum_{i=1}^{n} e_i \frac{dP_i}{dx} + \frac{S_{cp}}{I} V - \sum_j \frac{dP_j}{dx}\right] \tag{9}$$

where $\mathbf{\Sigma} dP_j/dx$ represents the sum of all tendon force gradients above the considered axis at which τ is calculated. If there is only one tendon layer at the bottom of the cross-section, Eq. (9) reduces to

$$\tau = \frac{1}{b_w}\left[\left(\frac{A_{cp}}{A} - \frac{S_{cp} e}{I}\right)\frac{dP}{dx} + \frac{S_{cp}}{I} V\right] \tag{10}$$

which is the expression presented by Yang [5]. The total shear stress can now be expressed as a sum

$$\tau = \tau_P + \tau_V \tag{11}$$

$$\tau_P = \frac{1}{b_w}\left[\frac{A_{cp}}{A}\sum_{i=1}^{n}\frac{dP_i}{dx} - \frac{S_{cp}}{I}\sum_{i=1}^{n}e_i\frac{dP_i}{dx} - \sum_j \frac{dP_j}{dx}\right] \tag{12}$$

$$\tau_V = \frac{S_{cp}}{b_w I}V \tag{13}$$

For a linear transfer of prestressing force dP_i/dx is constant = $\boldsymbol{P}_i/l_{pt,i}$ where $\boldsymbol{P}_i$ is the fully transferred prestressing force and $l_{pt,i}$ is the transfer length. In design calculations the only extra effort is the calculation of A_{cp}; other parameters must already be known for calculation of σ and τ_V.

In practical cases, the layers of lower tendons can be considered as one layer and the layers of upper tendons as another layer. In Finland, the upper tendons are usually missing. In such a case Eq. (12) reduces to

$$\tau_P = \frac{1}{b_w}\left[\frac{A_{cp}}{A} - \frac{S_{cp}}{I}e\right]\frac{dP}{dx} \tag{14}$$

In Yang's method the same failure criterion, i.e. Eq. (1), is applied as in the traditional method, but stress component τ is different. σ may also be different because the critical point is different.

Based on FEM analyses for different slab cross-sections Yang concluded that a web shear failure can only take place outside the zone affected by the support reaction, which is the grey-shaded zone in Fig. 11. In practical cases the maximum principal stress $\sigma_{I,max}$ tends to be on the inclined line A´B´ which connects the highest and lowest point of the web on the inclined line, see Fig. 12. On the right hand side of this line the maximum principal stress decreases very slowly with increasing x. This explains the fact that in shear tests the distance of the failure crack to the support varies. For practical design it is enough to take discrete points under consideration. When calculating σ, Eq. (6) has to be used because the critical point is not necessarily at the centroidal axis.

Yang compared the resistances predicted by his method with results observed in VTT's tests in 1978 - 1987 [5,6]. He found the fitting good. A similar comparison in 2005 [7] with material parameters calculated according to Eurocode 2 and with test results from 1990 – 2003 showed that the fit was much better and safer than with the traditional method but there were still some test results which were lower than those predicted by Yang's method.

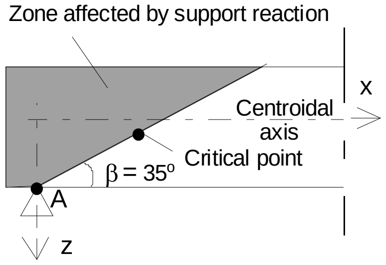

Fig. 11. Zone affected by support reaction.

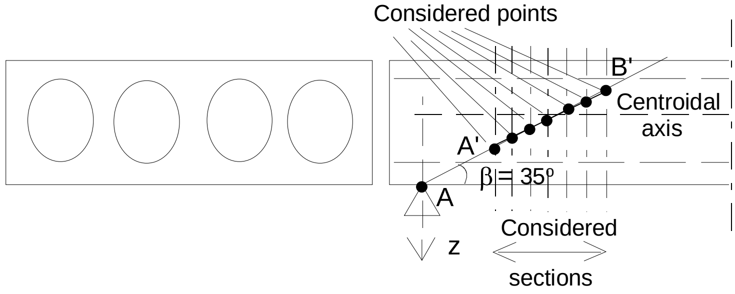

Fig. 12. Points to be considered = possible critical points.

Contribution to standardisation

One might expect that everybody regarding expression $\tau = VS/(Ib_w)$ as correct for members with constant axial force would immediately accept Yang's formula for τ because both are deduced exactly in the same way and because there were many who before Yang knew which role the transfer of the prestressing force played. This has not been the case. After publication of Yang's method it took more than ten years to include it to product standard EN 1168.

The traditional design method for web shear failure was included in Eurocode 2 and it is still there. Bertagnoli and Mancini [8] have recently shown that Eurocode 2 gives satisfactory results when compared with a great number of shear test results. This may seem odd, but there is a natural explanation. In Eurocode 2, the design model for shear compression failure, which is completely different from web shear failure, is over-conservative. This model often predicts a lower resistance than the model for web shear failure. When this lower value is applied to cases in which the actual failure mode is web shear failure, a safe design is obtained.

The result of Bertagnoli and Mancini is reassuring information for those who have used Eurocode 2 for the shear design of PHC slabs, but a poor starting point for the future. A failure model always simulates a certain failure mode, and these two must not be separated from each other.

There is also another point. The shear compression failure seldom occurs in a short shear span, but according to some unpublished test results it always takes place in a heavily prestressed PHC slab when the shear span is so long that a web shear failure is excluded. In such cases the shear compression model of Eurocode 2 seems to be nonconservative.

To put it briefly, in Eurocode 2 the design model for shear compression failure is overconservative near to the support and nonconservative in the span while the model for web shear failure is nonconservative near to the support where it is supposed to be used. It is obvious that the shear design method for members without shear reinforcement needs to be reconsidered.

In 2006, Hawkins and Ghosh [9] have realised that the shear resistance of PHC slabs can be less than that predicted by the American concrete code ACI 318-05 and this conclusion is true both for European and American test data. This is no wonder because the design criterion in ACI 318-05 is based on the same ideas as the traditional model for web shear failure in Eurocode 2.

In Finland noncontradictory complementary information for PHC slabs has been published as a SFS standard. In this way e.g. the problem with shear compression model of Eurocode 2 has been solved.

General viewpoints

When a formula becomes too familiar, there is a risk that we forget where it comes from and apply it to cases where it should not be applied. This risk was realised when $\tau = VS/(Ib_w)$ was applied to the ends of PHC slabs. As in all science, to know it is not enough. To correctly apply the knowledge, we must know, how the knowledge has been acquired. This principle can be demonstrated to the students in the light of the present case.

It is relatively easy to modify and amend the existing design rules when the changes are supported by indisputable research results provided that the changes improve the competitiveness of the product to be designed. In the opposite case it is not so easy. It is a big step for all involved to admit that the design rules used for years or for decades have not met the safety requirements. The evidence must be convincing, preferably experimental. Furthermore, to show the lack of safety is a minor effort compared with the effort of developing simple and safe but not oversafe design rules.

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