Damping methods to mitigate wind-induced vibrations

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Abstract. This paper describes the design of passive dampers for guyed masts and steel chimneys in conditions where the natural frequencies of the structure can not be determined accurately. In such conditions, an account should be taken of an offset in the frequency-tuning of the damping facilities. The paper shows that a traditional tuned mass damper (TMD) or a tuned liquid damper having narrow band control characteristic can be used as a broad damper by adopting a sufficiently high mass in the device. Practical guidelines for the design of damping facilities are provided.

Key words: Wind, vortex-shedding, masts, chimneys, vibrations, damping, mitigation, tuned mass dampers, tuned liquid dampers, design recommendations.

Introduction and background

This paper considers two passive methods to control wind-induced vibrations on guyed masts and industrial steel chimneys. Den Hartog [1] derived design formulas for a tuned mass damper (TMD), which has subsequently found a lot of technical applications on masts and other slender structures. Fisher [2] reports on experiences on a pendulum vibration absorber. This kind of device (Fig. 1) has frequently been used in many European countries. The technical details of the pendulum type TMD vary mainly in the method of providing the damping that is needed within the device [3, 4].

The popularity of the TMD damper is based partly on the simplicity of the formulas that are used in the design of these devices. However, a TMD may occasionally need some maintenance. Therefore, the users of the structures sometimes prefer more simple devices. Impact dampers [5, 6] as well as the chain dampers [7] offer such simplicity in the construction of the device. A further step towards low cost and maintenance-free dampers is the development of tuned liquid dampers (TLD) and tuned liquid column dampers (TLCD) [8, 9, 10].

The devices described in this paper are traditionally called as “dampers”, which can be considered as a misleading term. The real operational principle of these devices is based on the idea of changing the dynamic system such that an auxiliary mass will counteract the vibrations excited by external dynamic actions. The reduction in the response amplitude can then be interpreted in terms of an increase in the effective damping.
Susceptibility to vortex-induced vibrations

The susceptibility of vibrations depends on the structural damping and the ratio of structural mass to fluid mass. This is expressed by the Scruton number \( Sc \), which is given by [11]

\[
Sc = \frac{2 \cdot \delta_s \cdot m_{ie}}{\rho \cdot D^2}
\]

where \( \delta_s \) is the structural damping expressed by the logarithmic decrement, \( \rho \) is the air density under vortex shedding conditions, \( D \) is the reference width of the cross-section at which resonant vortex shedding occurs and \( m_{ie} \) is the equivalent mass per unit length defined as

\[
m_{ie} = \frac{\int_0^1 m(z) \cdot \phi_i^2(z) \, dz}{\int_0^1 \phi_i^2(z) \, dz}
\]
The Scruton number is a dimensionless parameter, which describes how sensitive a structure is for vibrations caused by vortex shedding. Figure 2 shows how the tip displacement $u_{\text{max}}$ of a typical steel chimney of a power plant depends on the Scruton number. This response calculation was done by a method described in Sect. E.1.5.3 of the Eurocode EN-1-1-4, [11]. This method is based on extensive tests and shows that the response amplitude will significantly increase at low values of the Scruton number.

![Figure 2. The maximum amplitude of the tip of the chimney as a function of the Scruton number. Calculated for a steel chimney with $H = 70$ m, $D = 2.6$ m, $m_{1,e} = 570$ kg.](image)

Calculations show that $Sc = 3.3$ in the example concerned if no damping facilities are used. Therefore, this chimney is very susceptible to vortex induced vibrations. The line “Displacement limit” shown in Fig. 2 is a serviceability criterion [12] for the maximum amplitudes of vibration. For the structure concerned, the serviceability criterion is met if $Sc > 15$. This serviceability criterion provides a convenient target for the performance of the damper. Excessive vibrations will not occur if the total damping and the Scruton number can be increased to a sufficiently high level.

**Tuning of TMD- and TLD dampers**

Tuned mass dampers and tuned liquid dampers can be used to suppress vibrations at one natural frequency $f_i$ of the structure. The fundamental mode $\phi_1(z)$ is often the main problem in industrial chimneys that are higher than about 50 m. In very high chimneys the second mode $\phi_2(z)$ (Fig. 3a) may also be susceptible to vortex-induced vibrations.

For the design of the damper, the dynamic behaviour of the structure is substituted at the position $z_D$ of the damper by an equivalent SDOF model depicted in Fig. 3b.
Figure 3. Dynamic characteristics of the steel chimney concerned in Fig. 2. (a) Two eigen-modes, $f_1 = 0.55$ Hz; $f_2 = 2.75$ Hz. (b) A simplified model of the structure-damper system.

The mass of the equivalent SDOF model is given by

$$m_0 = \frac{\int_0^H m(z) \phi^2(z) \, dz}{\phi^2(z_D)}$$  \hspace{1cm} (3)

where $m(z)$ is the distributed mass and $\phi(z)$ the eigenmode concerned. The design parameters of a pendulum damper (Fig. 1) include the apparent natural frequency of the damper, given by

$$f_D = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$  \hspace{1cm} (4)

where $g$ is the acceleration due to gravity and $L$ is the length of the pendulum. Further design parameters are the frequency ratio

$$\nu = \frac{f_D}{f_0}$$  \hspace{1cm} (5)

and the mass ratio

$$\mu = \frac{m_D}{m_0}$$  \hspace{1cm} (6)

According to Den Hartog’s tuning formulas, the optimum frequency ratio is given by
\[
\frac{\nu_{opt}}{1+\mu} = 1 \quad (7)
\]

Correspondingly, the damping coefficient

\[
c_D = 2m_D \left(2\pi f_D \right) \xi_D \quad (8)
\]

is selected such that the damping ratio \( \xi_D \) attains an optimum value given by

\[
\xi_{Dopt} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \quad (9)
\]

Figure 4 shows the optimal frequency- and damping ratios given by Eq. 7 and Eq. 9 as a function of the mass ratio.

The expressions (3) to (9) can be used with minor modifications also in the design of tuned liquid dampers (TLD). First, it is noticed that a liquid damper consists of several steel containers that are partially filled with liquid. While the structure vibrates, the liquid moves inside the containers and creates a counterforce that reduces vibration amplitudes. However, the total mass of the liquid is not effective in the action of the damper. According to [13], the equivalent mass of the TLD can be taken as

\[
m_{D\text{eq}} = 0.8 m_{\text{liquid}} \quad (10)
\]

The natural frequency of a TLD is given by

\[
f_D = \frac{1}{2\pi} \sqrt{\frac{g}{a} \tanh(\eta \frac{h_w}{a})} \quad (11)
\]

where \( a \) is the outer diameter of the container, \( h_w \) is the depth of the liquid and \( \eta \) is a parameter that depends on the geometry of the container [15]. This parameter has the
value of $\eta = \pi$ for rectangular liquid containers. This geometry is preferred in wind engineering applications where the structure vibrates laterally along an elliptical path.

**Tuning offsets**

*Practical considerations*

According to a traditional design practice, a mass ratio in the range of 1% is considered as satisfactory in conditions where the tuning can be done accurately to the desired natural frequency of the structure. This is usually not the case for guyed masts and industrial steel chimneys. Tuning offsets may easily occur due to various kinds of practical problems. Such problems can be discussed by considering the error in identifying the natural frequencies.

![Figure 5](image)

Figure 5. Influence of guy cable models on the natural frequencies.

Figure 5 shows the frequency response at the top of the guyed mast shown in Fig. 1 assuming first a full dynamic model for the guy cables (solid line) and then a spring-mass-spring model (dotted line). The details of these guy models are given in [14]. Figure 5 also indicates by arrows the natural frequencies $f_{n}$ that were received in an eigenmode analysis where the guys were modelled by simple springs. In this case, the lowest natural frequency is 0.36 Hz. The more accurate models yield lower eigenmodes in the frequency range around 0.2 Hz. It was also found that the spring-mass-spring model for the guys yields the second natural frequency as 0.43 Hz. This is 19% higher than the corresponding value due to the simple model. In the case of the full dynamic model for the guy cables, the eigenmode in the frequency range of 0.4 Hz splits into two different modes with frequencies 0.40 Hz and 0.45 Hz.

Corresponding inaccuracies are common also in the analysis of industrial steel chimneys. Three experienced analysers obtained for the chimney described in Fig. 3 the fundamental frequencies as 0.54 Hz, 0.55 Hz and 0.59 Hz. A subsequent measurement showed that the true value was 0.53 Hz.
These practical experiences show that a certain level of tuning offset should be assumed in the design of TMD-and TLD dampers.

**Numerical examples**

Parametric studies were made to clarify how an offset in tuning of the frequency influences the performance of a TLD. The equations of equilibrium for the simplified model shown in Fig. 3(b) are written as

\[
\begin{align*}
    m_0 \ddot{x}_D - c_D (\dot{x}_D - \dot{x}_0) + k_D (x_D - x_0) + k_0 x_0 &= F(t) \\
    m_D \ddot{x}_D + c_D (\dot{x}_D - \dot{x}_0) + k_D (x_D - x_0) &= 0
\end{align*}
\]

where the parameters are defined in Fig. 3(b). The force acting on the primary mass \( m_0 \) is notated as \( F(t) \). The solution of this equation is written in the frequency domain as

\[
X(\omega) = H(\omega) F(\omega)
\]

where \( F(\omega) \) is the Fourier transformation of the excitation \( F(t) \), \( H(\omega) \) is the frequency response function of the main mass and \( \omega \) is the angular frequency. If the excitation occurs at a single frequency, it can be represented as \( F = F_0 e^{i\omega t} \), where \( F_0 \) is the amplitude of the force.

The magnitude of the frequency response is scaled as

\[
H_d = \frac{|H(\omega)|}{x_s}
\]

where \( x_s = F_0 / k_0 \) is the static displacement corresponding to the force amplitude \( F_0 \). The parameter \( H_d \) is the non-dimensional amplitude of the main mass [1]. It is also known as the dynamic magnification factor [16].

Figure 6 shows dynamic magnification functions for a typical steel chimney with \( H = 70 \text{ m} \), \( D = 2.6 \text{ m} \), \( f_1 = 0.53 \text{ Hz} \) and \( m_1,e = 570 \text{ kg} \). The structural damping of the main structure was assumed as 0.0040 as a fraction of critical, which is equal to 0.025 as logarithmic decrement. Three cases were considered as follows:

- Case 1: The chimney was modelled as a single-degree-of-freedom (SDOF) model, see Fig. 3b,
- Case 2: Two-degree-of-freedom model for the chimney having a TMD or a TLD at the chimney tip in optimal frequency tuning. The effective mass of the damper and the mass ratio were taken correspondingly as \( m_D = 267 \text{ kg} \) and \( \mu = 0.0283 \),
- Case 3: The influence of an offset in frequency tuning was studied by assuming that the fundamental frequency is 0.50 Hz instead of 0.53 Hz.
The performance of the two-degree-of freedom system is described by identifying first the maximum value $H_{d, \text{max}}$ of the dynamic magnification (Fig. 6). The total damping of the system is then estimated in terms of a corresponding SDOF model [16] as

$$\delta_{s, \text{tot}} = 2\pi \frac{1}{2 H_{d, \text{max}}}$$

where $\delta_{s, \text{tot}}$ is an equivalent logarithmic decrement describing the performance characteristics of the two-degree-of-system. This parameter is used in Eq. (1) to estimate the Scruton number for the chimney having a TMD or TLD.

Figure 6. Dynamic magnification of the main mass.

Figure 7. Influence of frequency offset on the performance of the damping system.
A parametric study on the frequency tuning problems was performed by assuming that the natural frequency $f_1$ of the main mass in the Case 2 deviates from the basic value of 0.53 Hz. The results of this study are shown in Fig. 7. It can be seen that the Scruton number assumes values that are higher than the requested minimum of $\text{Sc} = 15$ if the frequency offset is not excessive.

**Design principles used for TMD and TLD facilities**

The basic theory of passive dampers that was described by the expressions (3) to (11) is based on a condition where the main mass $m_0$ and the auxiliary mass $m_D$ execute unidirectional motions. Full-scale measurements [17] show, however, that vortex-excited vibrations of slender structures create biaxial vibrations in cross-section of the structure. Accordingly, the tip of a chimney or a mast almost always follows an elliptic path during the vibrations. Due to this, the liquid inside the TLD containers lose some of its effectiveness as compared to uniaxial excitation. The TMD devices have the same problem.

This loss of the efficiency shall be compensated in the design by selecting for the effective Scruton number a higher target value than the minimum of $\text{Sc},\text{min} = 15$ suggested by Fig. 2. Another issue is the offset in the frequency tuning, which will always be present due to practical reasons.

Due to these reasons, it is recommended that the target value for the effective Scruton number should be in the range of $\text{Sc} > 35$. This recommendation pertains to a situation where simple engineering methods are used to determine the dynamic characteristics of the main structure and the damping facility is designed by expressions (1) to (11). A substantial reduction in the mass of the TLD or TMD device would be achieved if a semi-active control method is applied such the tuning offset is automatically compensated.

**Conclusions**

High-rise structures such as guyed masts and industrial steel chimneys are susceptible to vibrations induced by a wind action known as vortex shedding. Passive dampers can be used to mitigate these vibrations. The design formulas and principles of tuned mass dampers and tuned liquid dampers are described in this paper. These devices are very simple and suitable for masts and towers where only robust and cost-effective damping methods can be used. The method that is described in this paper is based on the idea that a dimensionless parameter, the Scruton number, is increased to a sufficiently high level by installing a damping facility on the structure. This design method has been used by the author in 1985-2008 to design a total of fifty dampers for masts and chimneys in Finland and other countries.
References


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