English summary

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Integration by parts

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Summary. The article deals with integration by parts mainly in one and two dimensions. The purpose is to present practical formulas, which can be used when discontinuity points or lines appear in the domain under study. Special emphasis is given to the jump conditions in connection with the finite element method. Three application examples are considered to illuminate the general formulas. The first example concerns a stretched rod, the second a stretched plate and the third the dynamics of a stretched rod with space-time formulation. Physical basis for discontinuities is discussed. The importance of the principle of virtual work is emphasized. Illustrative notation is strived for and some possible Finnish terminology is suggested. Most of the formulas presented can be used also in other connections than in structural mechanics.

Key words: integration by parts, space-time domain, discontinuous function, jump condition

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Complex modes, bi-orthogonality and general viscous damping

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Summary. The vibrations of a non-proportionally damped discrete system are studied. In the case of non-proportional damping the eigenmodes are complex and the traditional orthogonality conditions are not valid. However, the so called bi-orthogonality conditions can be formed using the eigenvectors of the original problem and its adjoint problem in the first order state space formulation. Utilising this property, the coupling between the eigenvectors in the forced vibration case can be removed. In the state space formulation, the components of the state vector consist of the generalized coordinates and the corresponding velocities so that these components are not independent of each other. According to literature, this may imply problems in the complex eigenvector expansion of a forced vibration problem. It is shown in this work that the eigenvector expansion is internally consistent and that the relations between the components are automatically satisfied through the solution of the equations of motion. The complex eigenmode analysis presented is compared with the traditional non-damped or proportionally damped eigenmode analysis. It is shown, in addition, that the non-proportionality of the damping brings about a new contribution in the frequency response function of the system which has no counterpart in the case of proportional damping. The complex eigenmode analysis can also be applied to gyroscopic and circulatory systems. The method is illustrated by a numerical example.

Key words: non-proportional damping, non-selfadjoint, bi-orthogonality, non-coupled complex modes, eigenmode analysis