Rakenteiden Mekaniikka (Journal of Structural Mechanics)
Vol. 41, No. 3, 2008, pp. 137-151

Engineering oriented formulation for laminate lay-up optimization

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Summary. The concept of elementary laminates is used to formulate the design problem for a laminated composite structure. A parameterized laminate is divided into stacks with periodic patterns of elementary laminates. With elementary laminates desired regularity for the laminate lay-ups is achieved, which is practical for multi-layer laminates. Due to the reduced design space solution time can be considerably reduced. Two laminate lay-up formulation concepts are presented and their performance is evaluated. The design problem used as a reference case involved the optimization of the stacking sequences to maximize plate buckling loads using a genetic algorithm.

Key words: laminated composites, design optimization, genetic algorithm

Introduction

The design of composite structures is challenging due to the complex mechanical behavior of the structures and the number of design variables involved. A composite laminate is typically formed from a number of fiber-reinforced layers having directional properties. Basically, for each layer of the laminate, design variables are the choice of material system, thickness of the layer, and orientation of the layer. In practice, laminate design is more constrained. The choice of material systems is almost limitless, but as a result of the conceptual design phase, a few possible candidates are usually left. For solid laminates the use of multiple material systems is beneficial in several applications, though the use of a single material system is more common. Sandwich structures consist of two material systems in minimum. The thickness of reinforced layers is usually determined by the choice of material system and processing. However, in sandwich panels the thickness of the core material can be chosen quite freely. The choice of layer orientations is often constrained to 0, 90, +45, and -45 deg. Other off-axis directions may substitute ±45 deg. To avoid undesirable anisotropy of the structure, various constraints are often set for the lay-up. For instance, symmetry with respect to the laminate mid-plane may be required and an equal number of off-axis layers with minus and plus orientation is usually preferred, i.e., a balanced lay-up. Due to manufacturing, some regularity may be desired in the laminate lay-up. This can be achieved, for instance, by repeating the same elementary laminate a number of times. Some stacking sequences may be preferred against others. For example, off-axis layers on the surface may provide better impact tolerance for the laminate and thick sections of unidirectional layers may be undesirable.

The formulation of a structural optimization problem for a laminated composite structure is not trivial since the aspects described above should be taken into account.

The size of the design space needs to be constrained in a practical manner so that the problem can be solved in a reasonable time. On the other hand, too severe constraining of designs leads to a limited objective space and the possible loss of the optimum design. Different concepts for the formulation of the laminate design optimization can be found in the literature depending on the optimization strategy used. These concepts tend to produce laminate solutions that are impractical to manufacture, however.

For a laminate lay-up design, values of the design variable vector are typically restricted integers. Stochastic population-based search methods have been successfully used to solve complex discrete structural optimization problems. Population-based search methods like genetic algorithms [1, 2, 4, 7] and particle swarm optimization (PSO) techniques [5, 6] maintain and manipulate a population of solutions in the search for better solutions.

Laminate lay-up formulations for optimization problems

Frequently, for a fixed number of layers $k = 1, 2,\ldots, N$ and a fixed set $\Theta = \{\theta_1, \theta_2,\ldots, \theta_l\}$ of l allowed layer orientations, a standard laminate code is encoded with a representing layer orientation identity string. For instance, the standard laminate code $[0/\pm45/90]_{SE}$ can be encoded as (1, 3, 4, 2, 2, 4, 3, 1), where the layer orientation identity design variables 1, 2, 3, and 4 refer to the layer orientations 0, 90, +45, and -45 deg, correspondingly, and SE stands for the symmetric even laminate structure.

For many applications with in-plane loading, the stacking sequence is not as important as the number of layers in each orientation. Fixed layer thicknesses can be assumed, as it is often the case with predetermined ply selection. A laminate with layer orientations $\theta_1, \theta_2,\ldots, \theta_l$ is simply stacked, i.e., all layers of the same orientation are consecutive like in $[(\theta_1)N_1, (\theta_2)N_2,\ldots, (\theta_l)N_l]$ where N_l are multipliers for each layer orientation. For a 12-layer laminate using three layer orientations a representation would be (N_1, N_2, N_3) with the constraint $N_1+N_2+N_3 = 12$. Gürdal et al. [2] remark that for a genetic search, a formulation of a string (N_1, N_2) with $N_3 = 12-N_1-N_2$ and the constraint $N_3 \geq 0$ is much more efficient than the first one. It is obvious since the design space of the second formulation is thirteen times smaller than the first one, while the number of feasible designs remains the same for both variants.

When the laminate is loaded with bending or out-of-plane shear forces, the stacking sequence plays a vital role. One way of considering this aspect is to introduce a stacking sequence vector Σ, which determines the locations of the specific layer orientations in the laminate. For a laminate with l allowed layer orientations, the stacking sequence vector includes $l!$ permutations. For example, for a $[\theta_1/\theta_2/\theta_3]$ laminate $\Sigma = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$, where 1, 2, and 3 refer to the layer orientations θ_1, θ_2, and θ_3, respectively.

Laminate lay-up formulation concepts

In this work the design study involved a 48-layer laminate with symmetric even lay-up. Allowed layer orientations were restricted to 0, 90, and ±45 deg. Furthermore, it was assumed that the laminate is constructed of bunches of two plies with the same orientation. Therefore, allowed layer orientation pairs were 0_2, 90_2, and ±45. As a result, the number of layer pairs $s = 12$, and the number of layer orientations $l = 3$.

The basic ply angle alphabet system was utilized in the benchmark case and therefore, it is introduced here as well. The symmetric even laminate lay-up using the reference concept is formulated in the following:

A:
$s = 12, p = 1, q = 12, r = 1, l = 3$
$[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]_{SE}$

where the design variables $x_i \in \{1, 2, 3\}$, $i = 1, 2, \ldots, 12$, correspond to the layer pairs 0_2, 90_2, and ±45 deg, respectively. The number of regular stacks in a laminate, the number of layer pairs in the elementary laminate, and the number of elementary laminates are denoted by p, q, and r, respectively. The reference concept is herein referred as the concept A.

Concept B utilizes elementary laminates and it is based on the ply angle alphabet system. The laminate structure consists of two elementary laminates, which may have different lay-ups. The additional information required to build a lay-up is how many times the specific elementary laminate occurs in a row. This is specified by the elementary laminate multipliers M_1 and M_2. The sum of multiplier values indicates the number of regular stacks p in the laminate. When the formulation is applied to the above described design study, and when it is assumed that the number of layers in both elementary laminates is the same, it is reasonable to construct two different sub-concepts B1 and B2:

B1:
$s = 12, p = 3, q = 4, r = 2, l = 3$
$[(x_1, x_2, x_3, x_4)\, x_5, (x_6, x_7, x_8, x_9)\, x_{10}]_{SE}$
$0 \leq x_5 \leq 3, x_{10} = 3 - x_5$

where the design variables $x_i \in \{1, 2, 3\}$, $i = 1, 2, 3, 4, 6, 7, 8, 9$ correspond to the layer pairs 0_2, 90_2, ±45 deg, respectively, and the variables $x_i \in I$, $i = 5, 10$, $I = 1, 2, \ldots, K$, correspond to the elementary laminate multipliers.

B2:
$s = 12, p = 4, q = 3, r = 2, l = 3$
$[(x_1, x_2, x_3)\, x_4, (x_5, x_6, x_7)\, x_8]_{SE}$
$0 \leq x_4 \leq 4, x_8 = 4 - x_4$

where the design variables $x_i \in \{1, 2, 3\}$, $i = 1, 2, 3, 5, 6, 7$ correspond to the layer pairs 0_2, 90_2, ±45 deg, respectively, and the variables $x_i \in I$, $i = 4, 8$, correspond to the elementary laminate multipliers.

All parameters are summed up in Table 1. In the elementary laminate concepts, the M_1 multiplier is the independent variable and the support variable M_2 is solved from the equation $M_1 + M_2 = p$. In case M_1 or M_2 equals to zero, only a single elementary laminate is applied. The number of allowed elementary laminates r must be specified so that $r < p$ is valid. With equality condition the design space is not reduced compared to the concept A.

Table 1. Support parameters used in the laminate parameterization.

Parameter	Comment
s	number of layer pairs in the laminate, s=p x q, s=12
p	number of stacks in the laminate
q	number of layer pairs in the elementary laminate
r	number of allowed elementary laminates, r=2
l	number of layer orientations, l=3

Concept C couples the stacking sequence vectors $S_1 \in \Sigma$ and $S_2 \in \Sigma$ into the elementary laminate concept. The two stacking sequence vectors can independently have the six permutations presented above while 1, 2, and 3 refer to the layer pairs 0_2, 90_2, and ±45 deg, respectively. Layer pairs of the elementary laminate can have different multipliers. The formulation of the concept is presented as:

C:
$s = 12, p = 4, q = 3, r = 2, l = 3$
$[((\zeta_1)\, x_{\zeta 1}, (\eta_1)\, x_{\eta 1}, (\xi_1)\, x_{\xi 1})\, x_1, ((\zeta_2)\, x_{\zeta 2}, (\eta_2)\, x_{\eta 2}, (\xi_2)\, x_{\xi 2})\, x_2\,]_{SE}$
$(\zeta_i, \eta_i, \xi_i) = S_i$, i = 1, 2
$x_{\zeta 1} + x_{\eta 1} \leq 3,\; x_{\xi 1} = 3 - x_{\zeta 1} - x_{\eta 1}$
$x_{\zeta 2} + x_{\eta 2} \leq 3,\; x_{\xi 2} = 3 - x_{\zeta 2} - x_{\eta 2}$
$0 \leq x_1 \leq 4,\; x_2 = 4 - x_1$

where ζ_i, η_i, and ξ_i, $\zeta_i \neq \eta_i \neq \xi_i$ define the locations of the layer pairs 0_2, 90_2, ±45 deg, respectively, in S_1 and S_2. Variables $x_{\zeta 1}$, $x_{\eta 1}$, $x_{\xi 1}$, $x_{\zeta 2}$, $x_{\eta 2}$, and $x_{\xi 2}$ are layer pair multipliers and they correspond to the associated layer pair orientations. Variables $x_i \in I$, $i = 1$, 2, correspond to the elementary laminate multipliers.

The design problem

The design problem is described in detail in [3] and [8] where the stacking sequence of a laminated plate is optimized considering various constraints. Compressive in-plane forces λN_x and λN_y, where λ denotes the load factor, are applied to a simply supported plate of the dimensions $a \times b$, with $a = 20$ in ($a = 508$ mm) and an aspect ratio of $a/b = 4$ (see Figure 1). Material properties are shown in Figure 2.

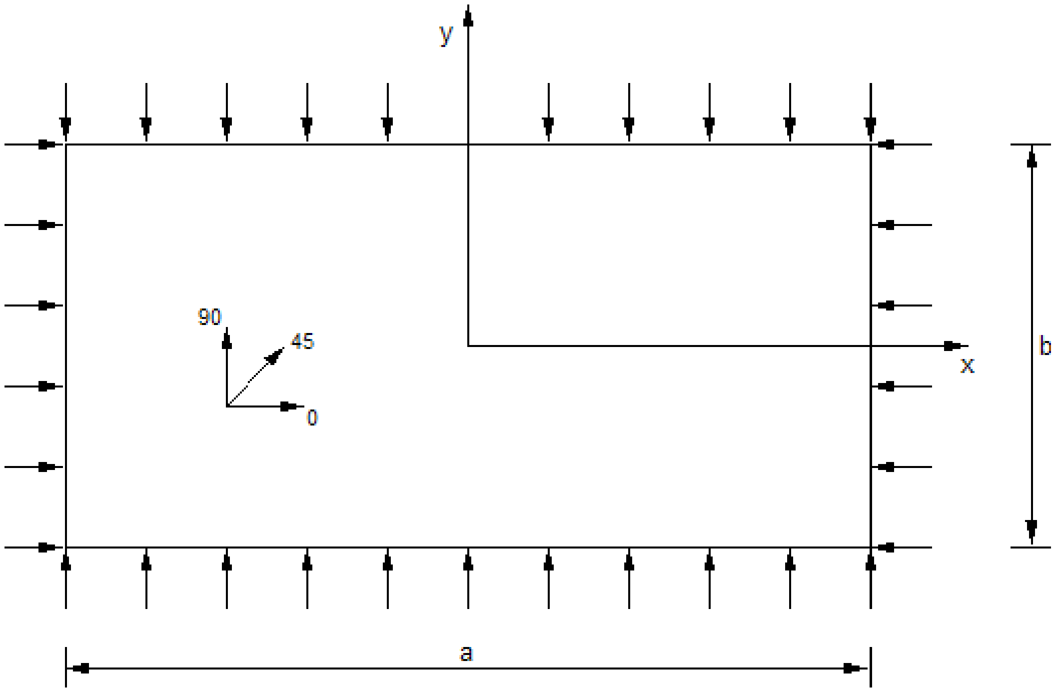

Figure 1. Plate geometry and load components for the design problem.

The critical buckling load factor is calculated using the analytical formula

$$\lambda_{cr}(m,n) = \pi^2 \frac{[D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4]}{(m/a)^2 N_x + (n/b)^2 N_y} \tag{1}$$

where the elements of the bending stiffness matrix D were solved using classical lamination theory CLT [2]. In the minimization of λ_{cr}, all values of m and n between 1 and 3 were checked. It was expected that these numbers cover the number of half-waves for the lowest buckling mode. The loading ratios N_y/N_x = 0.125, 0.25, 0.5, were applied to the plate.

A consecutive ply constraint was introduced because laminates with a large number of consecutive layers with the same orientation tend to failure due to matrix cracking. Here, the number of consecutive plies was limited to $\bar{o} = 4$ for certain loading cases. Applied loads were used in the failure analysis and a reserve factor $RF \geq 1.5$ was required. Maximum strain failure criterion was used in the failure analysis, i.e., layer principal strain components are compared to the material allowables when determining the RF.

```
Ply: CFRP

Physical nature : reinf.ply       Mech. behavior : transv.is.23
  Form of reinf. : unidirectional

 t =  0.127 mm


Engineering constants (transv.is.23)

 E_1  =  127.553 GPa       G_12  =  6.41212 GPa       nu_12  =  0.3
 E_2  =  13.0311 GPa       G_31  =  6.41212 GPa       nu_13  =  0.3
 E_3  =  13.0311 GPa       G_23  =  5.01196 GPa       nu_23  =  0.3


First failure stresses and strains (transv.is.23)

 X_t / X_eps,t =  1020.42 MPa / 0.8 %      X_c / X_eps,c =  1020.42 MPa / 0.8 %
 Y_t / Y_eps,t =  377.902 MPa / 2.9 %      Y_c / Y_eps,c =  377.902 MPa / 2.9 %
 Z_t / Z_eps,t =  377.902 MPa / 2.9 %      Z_c / Z_eps,c =  377.902 MPa / 2.9 %

 S   / S_eps   =  96.1819 MPa / 1.5 %      (12)
 R   / R_eps   =  96.1819 MPa / 1.5 %      (31)
 Q   / Q_eps   =        - MPa /   - %      (23)
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Figure 2. Material properties used in the design study.

The basic optimization formulation was the same for all concepts. However, each concept had its own formulations for the consecutive ply constraint and a different range of design variables $x = (x_1, x_2, \ldots, x_I)$. Depending on the case, the consecutive ply constraint and the strain constraint were active or not. The optimization problem is described as follows.

$$
\begin{aligned}
&\max \lambda(x) \\
&\text{subject to} \\
&\lambda \le \lambda_{\mathrm{cr}}(m,n) \\
&\mathrm{o} \le \bar{\mathrm{o}} \\
&RF \ge 1.5 \\
&x \in \Omega \\
&\mathrm{S}_1, \mathrm{S}_2 \in \Sigma
\end{aligned}
\qquad (2)
$$

where Ω refers to a set of the feasible design variable vectors of each concept.

Software implementation

The project has been realized using ESAComp [9] and modeFRONTIER [10] simulation tools. ESAComp is software for analysis and design of composite structures. Respectively, modeFRONTIER is a design optimization and process integration software package. The optimization process workflow is defined in modeFRONTIER. Basically it is divided into two parts where the laminate creation forms the first part. Parameterized elementary laminates are defined and constraints such as consecutive plies and symmetry conditions are implemented. For each laminate design modeFRONTIER creates a layer list, which is directed to ESAComp. Material data is stored in ESAComp. Based on the imported layer list, a laminate is created. The second part consists of the calculation of the bending matrix, the buckling load factor, and *RF* for the strain constraint. ESAComp stiffness analysis determines the elements of the bending matrix, which are then used to calculate the buckling load factor with different values of m and n. ESAComp failure analysis determines if the laminate is feasible with respect to the strain constraint when the load condition is determined by the critical buckling load. All designs and their results are sent to the modeFRONTIER optimizer, where their fitness is evaluated and all genetic operators are applied to achieve improved designs.

Results

Before optimization, the Design of Experiment (DOE) is run. In this part, initial design configurations are created and analyzed. DOE results are used by the optimization algorithm. In this study, the DOE design space was created with a random sequence method. The genetic algorithm MOGA II was used in the optimization. Parameters that were used in the optimization cycle are presented in Table 2.

Table 2. Basic parameters used in the modeFRONTIER optimization cycle.

Parameter	
Number of Generations	30
Probability of Directional Cross-Over	0.5
Probability of Selection	0.05
Probability of Mutation	0.1
DNA String Mutation Ratio	0.05
Elitism	Enabled
Treat Constraints	Penalising Objectives
Algorithm Type	MOGA - Generational Evolution
Random Generator Seed	1

A typical distribution of designs showing the critical buckling load factor and *RF* for the strain constraint is presented in Figure 3. Also, the design variable vector for the optimal solution is shown in the figure. During the process a constraint does not need to be treated as absolute limit. Instead, a tolerance value can be given. The more the design violates a constraint, the more its fitness value is penalized. In this study, a tolerance value of 0.2 was used, which explains the big number of designs in the region $1.3 \leq RF \leq 1.5$. Fitness is considered when designs are selected for the next generation. Designs with almost satisfactory *RF* are more likely to stay in the population for reproduction.

The optimum laminate lay-ups for the different loading conditions were determined using the concept A. Basically the same lay-ups that were derived in the reference [3] were found. The solutions derived with the concept A are denoted as the global optimum.

For some loading cases, many near optimal solutions occur. In such cases the genetic algorithm may converge prematurely. The loading cases for which the global optimum was not reached were partly rerun with other parameters. For example, the genetic search process could be replaced by the full factorial method in which all possible solutions are surveyed. With this approach, the best attainable solution for the specific elementary laminate concept and the loading case combination could be determined. These solutions are denoted as local optima.

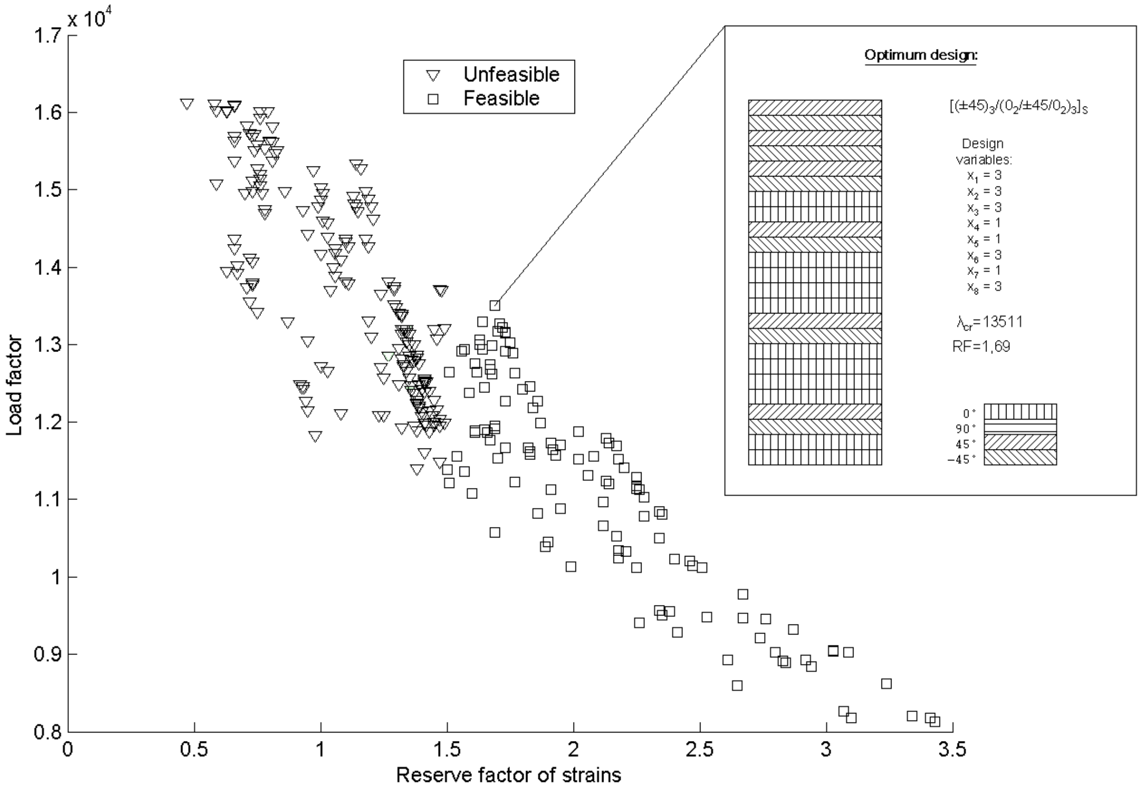

Figure 3. Distribution of designs with respect to the critical buckling load factor and *RF* for strains. The design corresponds to the concept B2 with the constraints B, C, and S and the load ratio of 0.125.

Local optima derived using the different elementary laminate concepts are compared with respect to the global optimum in Figure 4 and Table 3. The applied constraints are denoted by B, C, and S referring to the buckling, consecutive ply, and strain constraints, respectively. In the majority of optimization runs the initial population had 100 designs and 30 generations were calculated altogether. The elementary laminate concepts do constrain the design space. However, for more than two thirds of the loading cases, the laminate lay-ups derived represent the optimal solution and for all loading cases a performance of 96 % minimum is achieved. In the reference [3] laminates with less than 10 % loss of performance are considered practical and with this respect the developed concepts work well.

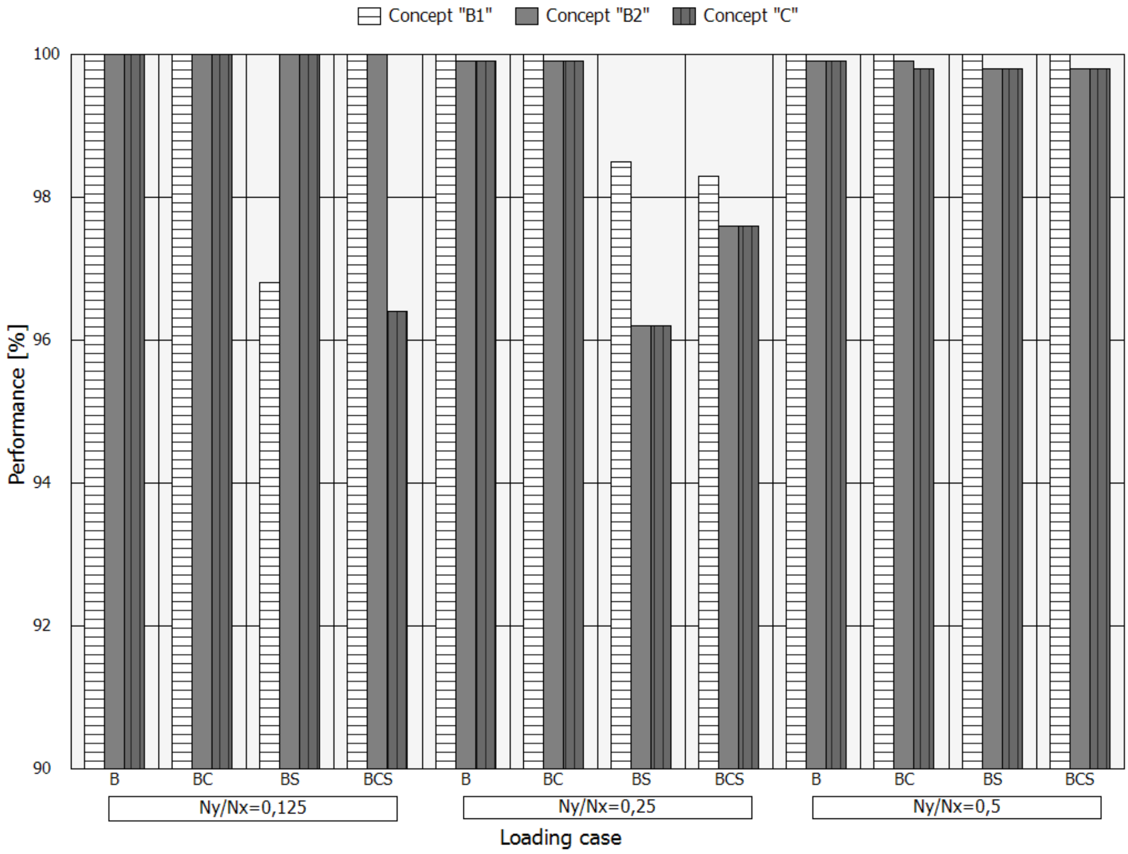

Figure 4. Global performance of the different elementary laminate concepts.

Another performance indicator is the number of designs required to reach the local optimum. Taking into account the 12 loading cases presented in Figure 4 and comparing the average number of designs per concept required to obtain the converged solution, the following can be concluded. Due to the large design space of concept A, the number of required designs, which is approximately 800, is considerably higher than for the other concepts. Concepts B1 and B2 required about 530 and 350 designs, whereas for the concept C the number of required designs was smallest, around 260. In practice these figures are smaller. For a new generation, genetic operations might create a design variable vector already used in previous generations. As a result, duplicate designs are produced. The optimization process is able to identify these designs and use the already computed information. Without duplicated designs, the average number of required designs were 600 (Concept A), 400 (Concept B1), 230 (Concept B2), and 220 (Concept C). With this respect, concepts B2 and C perform equally well.

The third remarkable performance indicator is how reliably the local optimum is obtained. This is presented in Figure 5. Here the performance is determined by the first performed optimization run. To increase the confidence level, more analyses would be required. The number of studied loading cases is 12 and the number of elementary laminate concepts is 3. Thus, the total number of cases examined is 36, which already provides some statistical confidence. The results are very satisfactory since only for a single combination of the elementary laminate and loading condition near optimal solution is not reached. Still, this specific laminate meets the limit of the practical solution.

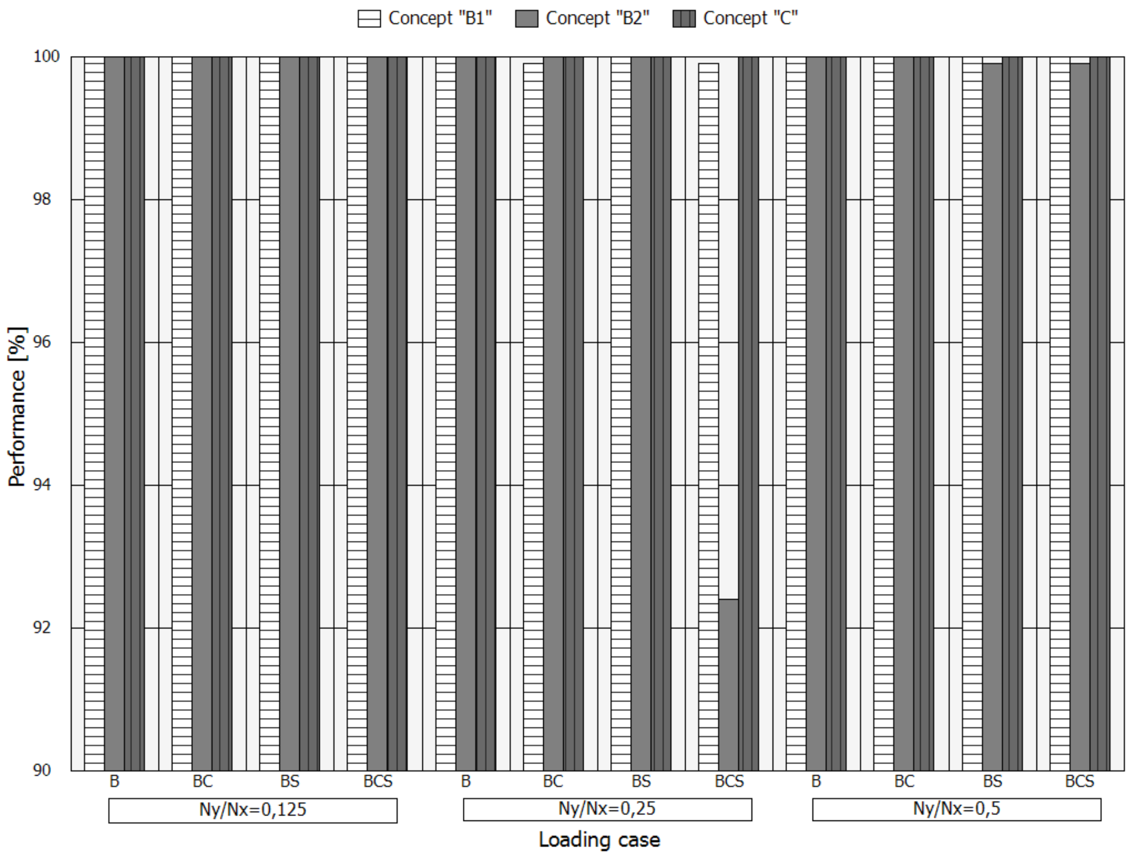

Figure 5. Local performance determined by the first optimization run.

Table 3(a). Global performance of the elementary laminate concepts A and B1.

Applied constraints	Concept "A" lay-up	Failure load factor	RF	Reference lay-up	Failure load factor	Performance
Load case: Nx=1.0; Ny=0.125						
B	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.46	1.0000
B,C	$([\pm45]_{12})_s$	16120.55				
B,S	$([\pm45]_6,0_{12})_s$	14977.99	1.52	$([\pm45]_6,0_{12})_s$	14977.92	1.0000
B,C,S	$([\pm45]_3,0_2,[\pm45,0_4]_2,90_2,\pm45)_s$	13517.03	1.50	$(90_2,[\pm45]_2,0_2,[\pm45,0_2]_4)_s$	13514.13	1.0002
Load case: Nx=1.0; Ny=0.25						
B	$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.85	1.0000
B,C	$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90				
B,S	$(\pm45_2,90_2,\pm45_4,0_{10})_s$	12859.40	1.68	$([\pm45]_2,90_2,[\pm45]_4,0_{10})_s$	12859.37	1.0000
B,C,S	$(90_2,[\pm45]_5,0_2,\pm45,0_4,\pm45,0_2)_s$	12674.85	1.50	$(90_2,[\pm45]_2,0_2,\pm45,0_4,\pm45,0_2)_s$	12674.84	1.0000
Load case: Nx=1.0; Ny=0.5						
B	$(90_2,[\pm45]_2,90_2,[\pm45]_3,90_{10})_s$	9999.37		$(90_2,[\pm45]_4,90_{12},\pm45)_s$	9999.34	1.0000
B,C	$(\pm45,90_2,\pm45,90_4,[\pm45]_4,90_4,\pm45)_s$	9997.03		$(90_2,[\pm45]_3,90_2,\pm45,90_4,[\pm45,90_2]_2)_s$	9999.13	0.9998
B,S	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.18	1.0000
B,C,S	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.18	1.0000

Applied constraints	Concept "B1" lay-up	Failure load factor	RF	Concept "A" lay-up	Failure load factor	Performance
Load case: Nx=1.0; Ny=0.125						
B	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,C	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,S	$([\pm45]_4,[\pm45,0_6]_2)_s$	14501.92	1.57	$([\pm45]_6,0_{12})_s$	14977.99	0.9682
B,C,S	$(90_2,[\pm45]_2,0_2,[\pm45,0_2]_4)_s$	13514.15	1.50	$([\pm45]_3,0_2,[\pm45,0_4]_2,90_2,\pm45)_s$	13517.03	0.9998
Load case: Nx=1.0; Ny=0.25						
B	$([\pm45]_4,[90_2,[\pm45]_2,90_2]_2)_s$	13435.79		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9995
B,C	$([\pm45]_4,[90_2,[\pm45]_2,90_2]_2)_s$	13435.79		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9995
B,S	$(\pm45,90_2,[\pm45]_2,[[\pm45]_2,0_4]_2)_s$	12671.55	1.50	$(\pm45_2,90_2,\pm45_4,0_{10})_s$	12859.40	0.9854
B,C,S	$(\pm45,90_2,[\pm45]_2,[\pm45,0_2]_4)_s$	12456.71	1.53	$(90_2,[\pm45]_5,0_2,\pm45,0_4,\pm45,0_2)_s$	12674.85	0.9828
Load case: Nx=1.0; Ny=0.5						
B	$([\pm45,90_2]_4,[\pm45]_2,90_2,\pm45)_s$	9997.03		$(90_2,[\pm45]_2,90_2,[\pm45]_3,90_{10})_s$	9999.37	0.9998
B,C	$([\pm45,90_2]_4,[\pm45]_2,90_2,\pm45)_s$	9997.03		$(\pm45,90_2,\pm45,90_4,[\pm45]_4,90_4,\pm45)_s$	9997.03	1.0000
B,S	$([\pm45,90_2]_4,[\pm45]_4)_s$	9994.84	1.53	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9997
B,C,S	$([\pm45,90_2]_4,[\pm45]_4)_s$	9994.84	1.53	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9997

Table 3(b). Global performance of the elementary laminate concepts B2 and C.

Applied constraints	Concept "B2" lay-up	Failure load factor	RF	Concept "A" lay-up	Failure load factor	Performance
Load case: Nx=1.0; Ny=0.125						
B	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,C	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,S	$([\pm45]_6,0_{12})_s$	14977.99	1.52	$([\pm45]_6,0_{12})_s$	14977.99	1.0000
B,C,S	$([\pm45]_3,[0_2,\pm45,0_2]_3)_s$	13511.37	1.69	$([\pm45]_3,0_2,[\pm45,0_4]_2,90_2,\pm45)_s$	13517.03	0.9996
Load case: Nx=1.0; Ny=0.25						
B	$([\pm45]_2,90_2,[\pm45]_9)_s$	13433.50		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9994
B,C	$([\pm45]_2,90_2,[\pm45]_9)_s$	13433.50		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9994
B,S	$([90_2,[\pm45]_2]_2,[0_4,\pm45]_2)_s$	12374.86	1.50	$(\pm45_2,90_2,\pm45_4,0_{10})_s$	12859.40	0.9623
B,C,S	$([90_2,[\pm45]_2]_2,[0_4,\pm45]_2)_s$	12374.86	1.50	$(90_2,[\pm45]_5,0_2,\pm45,0_4,\pm45,0_2)_s$	12674.85	0.9763
Load case: Nx=1.0; Ny=0.5						
B	$(\pm45,90_2,\pm45,[\pm45,90_4]_3)_s$	9988.37		$(90_2,[\pm45]_2,90_2,[\pm45]_3,90_{10})_s$	9999.37	0.9989
B,C	$([90_2,[\pm45]_2]_2,[90_2,\pm45,90_2]_2)_s$	9985.93		$(\pm45,90_2,\pm45,90_4,[\pm45]_4,90_4,\pm45)_s$	9997.03	0.9989
B,S	$(90_4,\pm45,[\pm45]_9)$s	9977.60	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9979
B,C,S	$(90_4,\pm45,[\pm45]_9)$s	9977.60	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9979

Applied constraints	Concept "C" lay-up	Failure load factor	RF	Concept "A" lay-up	Failure load factor	Performance
Load case: Nx=1.0; Ny=0.125						
B	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,C	$([\pm45]_{12})_s$	16120.55		$([\pm45]_{12})_s$	16120.55	1.0000
B,S	$([\pm45]_6,0_{12})_s$	14977.99	1.52	$([\pm45]_6,0_{12})_s$	14977.99	1.0000
B,C,S	$([\pm45]_3,[0_4,\pm45]_3)_s$	13027.42	1.75	$([\pm45]_3,0_2,[\pm45,0_4]_2,90_2,\pm45)_s$	13517.03	0.9638
Load case: Nx=1.0; Ny=0.25						
B	$([\pm45]_2,90_2,[\pm45]_9)_s$	13433.50		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9994
B,C	$([\pm45]_2,90_2,[\pm45]_9)_s$	13433.50		$([\pm45]_4,90_2,\pm45,90_2,[\pm45]_5)_s$	13441.90	0.9994
B,S	$([90_2,[\pm45]_2]_2,[0_4,\pm45]_2)_s$	12374.86	1.50	$(\pm45_2,90_2,\pm45_4,0_{10})_s$	12859.40	0.9623
B,C,S	$([90_2,[\pm45]_2]_2,[0_4,\pm45]_2)_s$	12374.86	1.50	$(90_2,[\pm45]_5,0_2,\pm45,0_4,\pm45,0_2)_s$	12674.85	0.9763
Load case: Nx=1.0; Ny=0.5						
B	$([90_2,[\pm45]_2]_3,90_6)_s$	9986.22		$(90_2,[\pm45]_2,90_2,[\pm45]_3,90_{10})_s$	9999.37	0.9987
B,C	$([90_2,[\pm45]_2]_3,[90_4,\pm45])_s$	9984.06		$(\pm45,90_2,\pm45,90_4,[\pm45]_4,90_4,\pm45)_s$	9997.03	0.9987
B,S	$(90_4,\pm45,[\pm45]_9)_s$	9977.60	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9979
B,C,S	$(90_4,\pm45,[\pm45]_9)_s$	9977.60	1.56	$(90_2,[\pm45]_2,90_2,\pm45,90_2,[\pm45]_6)_s$	9998.20	0.9979

Results for a specific loading case are shown in Figure 6 including the critical buckling load factor λ_{cr} and the reserve factor *RF* for each elementary laminate concept. Optimal lay-ups with the indication of the repeated stacks are illustrated in the figure as well. Performance for the concepts A, B1, and B2 can be regarded as optimal since the worst solution deviates less than 0.05 % from the global optimum. In fact, the laminate found with the concept A is even slightly better than the optimal solution presented in reference papers. The laminate obtained with the concept C with its 4 % lack of performance is still near optimal and thus practical. On the other hand, *RF* for the strain constraint is 16 % higher for this laminate compared to the limit value.

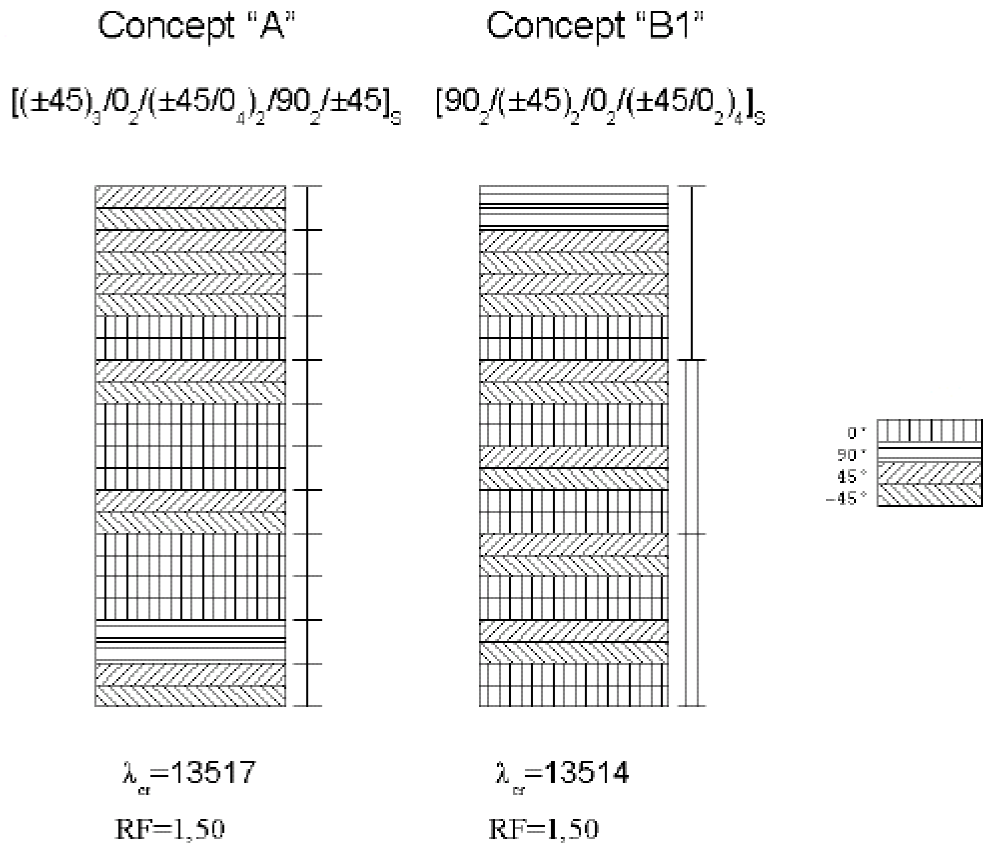

Figure 6(a). Optimal lay-ups with the indication of the repeated stacks for the loading case with the constrains B, C, and S and the load ratio of 0.125 for the elementary laminate concepts A and B1.

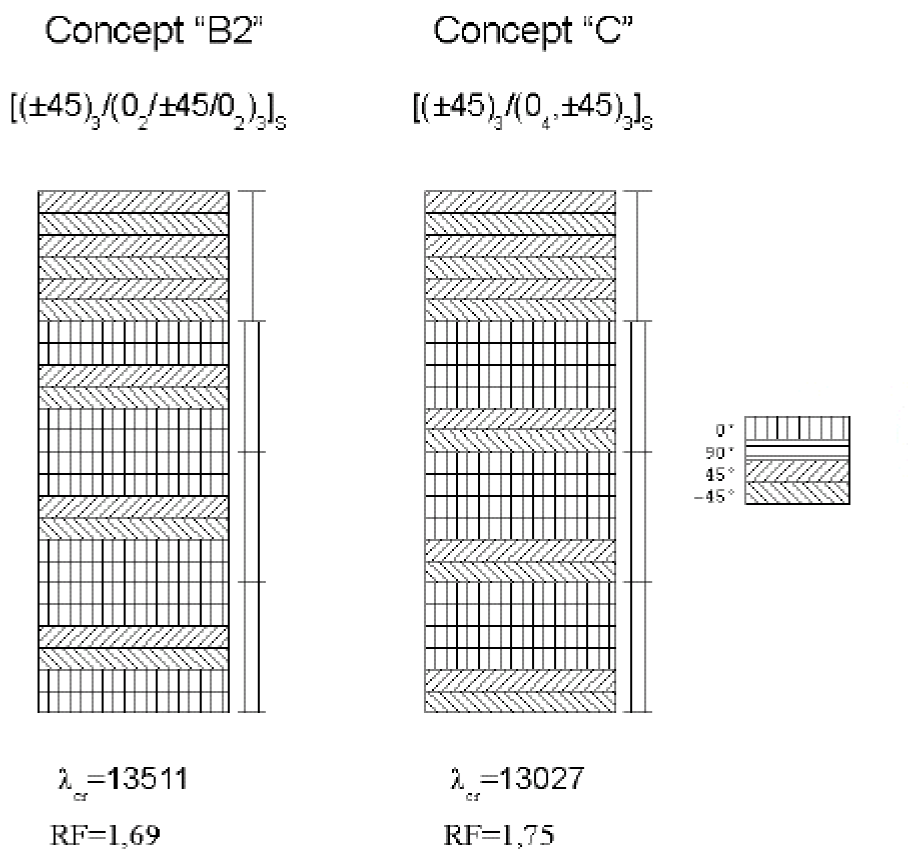

Figure 6(b). Optimal lay-ups with the indication of the repeated stacks for the loading case with the constrains B, C, and S and the load ratio of 0.125 for the elementary laminate concepts B2 and C.

Conclusions

Two elementary laminate concepts have been introduced and reviewed with respect to the reference case. In these concepts, the laminate lay-up is divided into two elementary laminates, which are then repeated a number of times. In this work, the approach has been applied to a design study with fixed number of layers. Nevertheless, the approach can be adapted to problems where the mass of the structure is minimized. Consequently, optimal number of layers is determined. In such cases, the first elementary laminate can represent a variable part in which the number of layers is varying. The second part would form the regular elementary laminate.

The aim of the concept is to provide regular laminates, which are practical in the manufacturing point of view. Still, the concepts provide flexibility and, for example, they are able to take into account multi-axial load state by means of the optimal stacking sequence.

Using different performance criteria, quality of the concepts has been demonstrated. Both solution time and the ability to reach global optimum are very competitive with respect to traditional concepts. Performance can further be enhanced. For parameterized laminates it is typical that different design variable vectors generate identical laminate lay-ups. Overlapping designs can be filtered out without disturbing the optimization process.

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