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# Reliability of ice-strengthened shell structures of ships navigating in the Baltic Sea

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**Abstract**. The aim of this paper is to determine the probability of ice damage on a hull for a ship operating in the Baltic ice conditions. Limit state equations for the permanent deflection of the plating and for the development of the three plastic hinge mechanism in the frame are presented. These equations are used for the safety index analysis, which is applied for transversely framed plating and transverse frames at the bow part of a typical ice-strengthened vessel. Long term ice load distributions are based on the full scale measurements.

*Key words:* Reliability of ship structures, ice loads, ice damages

# Introduction

Extreme consequences, such as loss of life, environmental damage and ship losses, that can be the result of severe structural damages in ice, are almost non-existent in the Baltic Sea. These problems have been practically overcome e.g. by the development of the winter navigation system, composed of reasonable ice class rules (regarding the strengthening of the ship hull structure against ice loads), traffic restrictions and efficient icebreaker assistance.

The typical damage case in ice is a permanent dent of the plating or frames on the ice-strengthened region. The main challenge for reliable risk based analysis is the determination of long term ice loads. The ice loads have a strong statistical natures with a lot of affecting parameters such as: time in ice, variation in ice conditions and variations in the operations principles of the ship in ice. In this paper ice loads are based on the long term full scale measurements onboard a typical ice-strengthened vessel navigating in the Baltic Sea.

The character and magnitude of the deformation of a structure depends on the load level. When the load level is low enough the structural deformation will remain elastic, i.e. with no permanent changes. However, when the load level is increasing, exceeding a certain limit, some element of the structure starts to yield and so the deformation becomes permanent causing ice induced damage. The higher the load the deeper the permanent deflection will be. Therefore the depth of the permanent deflection is used here as the studied limit state on the reliability analysis.

#### The basic principle of level 2 reliability analyses

As also described in [1], there are three various levels of reliability analysis. Level 2 is the so called safety index approach and it is used in the following. The principle of reliability analysis is to find a proper level for the failure probability  $P_{f}$ :

$$P(R-S \le 0) \le P_f = 1-L$$

$$P(g(X) \le 0) \le P_f = 1-L$$
(1)

where  $P_f$  is the failure probability, L=1- $P_f$  is the reliability, R is the capacity of structures, S is the load on the structure, g (X) is the limit state function, X is a vector including all the statistical variables affecting the limit state such as: load, material properties, dimensions of the structure etc.

Assuming that the load  $f_S(s)$  and strength distributions  $f_R(r)$  are statistically independent, the probability of the event R-S<=0 can be formulated as:

$$P_{f} = P(R - S \le 0) = \iint_{\{r,s|r-s\le 0\}} f_{R}(r) f_{S}(s) dr ds$$

$$= \int_{r=-\infty}^{\infty} \int_{s=r}^{\infty} f_{R}(r) f_{S}(s) dr ds$$

$$= \int_{-\infty}^{\infty} f_{R}(r) [1 - F_{S}(r)] dr = 1 - \int_{-\infty}^{\infty} f_{R}(r) F_{S}(r) dr$$
(2)

Equation (2) is graphically illustrated in Fig. 1. The failure probability increases when the overlap between the distributions increases. The failure probability approaches zero, when there is no overlap between the distributions.



Figure 1. Illustration of the load, strength and failure probability distributions

In a special case of assuming that both R and S are normally distributed, the equation (2) can be somewhat simplified. We assume that the mean value of the strength and load are known:  $\mu_R$ ,  $\mu_S$ , as well the standard deviations:  $\sigma_R$ ,  $\sigma_S$ . In this case also the safety margin Z=R-S is normally distributed with the following mean value and standard deviation:

$$\mu_{Z} = \mu_{R} - \mu_{S}$$

$$\sigma_{Z} = \sqrt{\sigma_{R}^{2} + \sigma_{S}^{2}}$$
(3)

The probability function for the safety margin Z can now be presented as normalised (0,1):

$$F_Z(z) = \Phi(\frac{z - \mu_z}{\sigma_z}) \tag{4}$$

where  $\Phi$  is the cumulative normal distribution. Failure probability  $P_f$  can now be determined from:

$$P_{f} = F_{z}(0) = \Phi(\frac{0 - \mu_{z}}{\sigma_{z}}) = \Phi(-\frac{\mu_{R} - \mu_{s}}{\sqrt{\sigma_{R}^{2} + \sigma_{s}^{2}}})$$
(5)

This means that the failure probabilities can be determined using the standard tables of the cumulative normal distributions,  $\Phi$ . Equation (5) can also be formulated:

$$P_{f} = \Phi(-\beta) = 1 - \Phi(\beta)$$

$$\beta = \frac{\mu_{Z}}{\sigma_{Z}} = \frac{\mu_{R} - \mu_{S}}{\sqrt{\sigma_{R}^{2} + \sigma_{S}^{2}}}$$
(6)

where  $\beta$  is the so called safety index. Fig. 2 shows the meaning of the equation (6) graphically. The safety index  $\beta$  describes the distance of the mean value of the safety margin from the origin and this distance is presented relative to the standard deviation. Typical values for safety index  $\beta$  vary between 2 to 3. This means that the failure probability varies roughly between  $10^{-2}$  to  $10^{-3}$ .

#### Definition of the limit state function for ice damage

Figure 3 illustrates a typical ice damage observed frequently on ships navigating in the Baltic ice conditions. As can be seen from Fig.3, ice induced loads typically cause some permanent deflections on the ice-strengthened plating and frames, the depth of the dents are typically 30-100 mm [3].





Figure 2. Illustration of the safety index approach [2]

Figure 3. A typical ice damage [3]

In estimation of the load causing permanent deflections in the side plating, an approach developed by Hayward [4] is used. The approach is based on extensive finite element calculations to find out a correction  $f_{DT}$ , which takes into account the effect of the load height on the permanent deflection. The starting point for the analytical expressions is the formulations developed by Jones [5] considering yield line theory for uniform pressure on plating. So the Hayward [4] approach for the required line load, q, has the following form when wp/t  $\leq 1$ :

$$q = \frac{p_c h_c}{f_{DT}} \left[ 1 + \frac{w_p^2}{3t^2} \left( \frac{\zeta_0 + (3 - 2\zeta_0)^2}{3 - \zeta_0} \right) \right]$$
(7)

and when  $w_p/t > 1$ :

$$q = \frac{2p_c h_c w_p}{t f_{DT}} \left[ 1 + \frac{\zeta_0 (2 - \zeta_0)}{3 - \zeta_0} (\frac{t^2}{3 w_p^2} - 1) \right]$$
(8)

where t is the plate thickness,  $h_c$  is the load height,  $w_p$  is the permanent deflection in the plating,  $\sigma_y$  is the yield strength of the plate material. The threshold pressure  $p_c$  causing double Y-shaped yield line is [5]:

$$p_{c} = \frac{48M_{p}}{s^{2} \left(\sqrt{3 + \left(\frac{s}{l}\right)^{2} - \frac{s}{l}}\right)^{2}}$$
(9)

where s is frame spacing, l is the frame span, M<sub>p</sub> is the plastic moment of the plating:

$$M_p = \sigma_y \frac{t^2}{4} \tag{10}$$

the shape parameter,  $\zeta_0$ , has the following form:

$$\zeta_0 = \frac{s}{l} \left( \sqrt{3 + \frac{s^2}{l^2}} - \frac{s}{l} \right) \tag{11}$$

The correction factor  $f_{DT}$  is equations (7) and (8) has the following form:

$$f_{DT} = -0.1330 x_T^2 + 0.6701 x_T \tag{12}$$

where

$$x_T = \frac{h_c}{s} \left(\frac{s}{t}\right)^{0.2} \tag{13}$$

For the frames, Varsta et al. [6] has presented an equation to estimate line load on frame, q, causing the three plastic hinge mechanism in the frame:

$$q = 4 \frac{M_p + M_{ps}}{sl} \tag{14}$$

where M<sub>p</sub> is the moment required to cause the plastic hinge at the midspan (no shear)

$$M_{p} = \sigma_{y} W_{p} \tag{15}$$

where  $W_p$  is the plastic section modulus of the frame. The moment required to cause the plastic hinge at the end of the frames (at support) should also include the effect of shear. This means that the line load should be solved by an iterative approach complicating remarkably the reliability analysis and therefore to simplify the calculations it is assumed that the plastic section modulus at the end is 50 % of the plastic section at midspan  $W_{ps} = 0.5 W_p$ .

## Definition of the long term ice load

The long term loads used in the analysis is based on the long term measurements onboard MT Kemira [7]. It is assumed that the line load follows the Gumbel I distribution. Cumulative distribution function, CDF, of the Gumbel I distribution  $G(y_n)$  is [11]:

$$G(y_n) = e^{-e^{-c(y_n - u)}}$$
(16)

where c is the inverse measure of dispersion of the measured maxima, u is the characteristic largest value and  $y_n$  is the extreme value at the measured 12 hour intervals. The parameters c and u and can be determined once the mean,  $\mu_{yn}$ , and standard deviation,  $\sigma_{yn}$ , of the measured 12 hour maximum values are known:

$$c = \frac{\pi / \sqrt{6}}{\sigma_{y_n}}$$

$$u = \mu_{y_n} - \frac{\sqrt{6}}{\pi} \gamma \sigma_{y_n}$$
(17)

where  $\gamma$  is the so called Euler constant and has the numerical value of 0,577.

The long term extreme value distributions after N events can now be determined from the relationship:

$$G(y_n) = \left(e^{-e^{-c(y_n-u)}}\right)^N \tag{18}$$

Usually the long term loads are presented as a function of the so called return period, T  $(y_n)$ , which defines the required time in ice [days] to achieve the estimated long term load level and it can be solved from the relationship:

$$T(y_n) = 0.5 \frac{1}{1 - G(y_n)}$$
(19)

when the 12 hour maximum values form the basic measured data base. Figure 4 shows a typical long ice load distribution, which is based on the long term measurements onboard MT Kemira during winters1985-1991. Figure 4 gives the measured load at the bow frame of the ship and the load is divided by the frame spacing of the ship, 0,35 m, to get the line load/m. The Gumbel parameters are in this case u=97,87 kN/m and c=0.01256 [9]. As can be sent he Gumbel fits very well on the measured data up to the return period level of 20-30 days. There is higher scatter with the high load values and longer return periods and this due to the fact the highest load typically takes place in an extreme ice condition e.g. when we have heavy ridges or moving thick ice. These occur seldom and therefore the distribution fitted on the long term data can underestimate these rare events. It is believed, however, that the fitted distribution estimate well the basic load events during ship's lifetime and also the measured distribution gets closer the fitted distribution if there have been longer measuring period.

# Calculation of the safety index

Based on the approach described above, the safety index is determined for MT Kemira operating frequently in ice. The ship life time of 25 years is used to determine the long term loads distributions. The ship is sailing between Kokkola and some European ports. In this period the ship will be about 980 days in ice from which about 200 days on the Gulf of Finland. The basic equations (4-6) to determine the safety index require that the distributions are normally distributed. In the case of non-normal distributions such as Gumbel 1, the distributions are presented as the so called equivalent normal distributions at the failure point., see e.g. [10,11,12] for further details.

When looking at the used equations for the strength part (eq. 7-15), we can see that the equations include load height, structural dimensions and material yield strength as variables. As analysed by Kujala [10], the statistical variation for the structural dimensions are small compared to the loads and material yield strength, therefore these are taken here as deterministic variables. The actual height of the ice load during the ice-breaking is naturally varying a lot, unfortunately there are not many measured observations of the load height to determine the statistical characteristics of it. However, it is known that the load height is typically small and the assumption to take the ice load as a line like loading is widely accepted [12].

Consequently, the main statistical variables in this study are the yield strength and the ice load. The used ice load distribution is shown in Figure 4 for the Bay of Bothnia. The mean value of the yield strength is assumed to be 290 MPa and the standard deviation is 22,40 MPa [10]. These values are based on the measured data by the steel manufacturer Ruukki. The strength is assumed to be normally distributed as the normal distribution gave the best statistical fit on the measured yield strength database [10]. The plate thickness used for the bow of MT Kemira has been 20 mm and the transverse frames at the bow are assumed to be HP 260\*10 profiles with a frame spacing of 350 mm. The span of the frames is assumed to be 3.2 m. These are the minimum requirements for 1A Super class ship. The ice load is assumed to be line like with the load height of 0.075 m [12]. With these parameters, the limit state equations (7-8) get the following forms as a function of the relative permanent deflection  $w_p/t$ :

a) plating with  $w_p/t \le 1$ :

$$0.00472 \sigma_y \left(1 + \frac{w_p^2}{3t^2} 2.5407\right) - q_{ice} = 0$$
<sup>(20)</sup>

b) plating  $w_p/t > 1$ 

$$0.00944 \sigma_y \frac{w_p}{t} (1 + 0.11483 \left(\frac{t^2}{3w_p^2} - 1\right) - q_{ice} = 0$$
(21)



Figure 4. Measured long term ice loads onboard MT Kemira and Gumbel 1 distribution fitted on the data [8].



Figure 5. Obtained distributions for the strength and ice induced load.

where  $q_{ice}$  is the ice-induced line load (see e.g. Figure 4). For the frames (eq. 14-15) we obtain as the limit state function:

$$0.00646\,\sigma_{v} - q_{ice} = 0 \tag{22}$$

An example of the obtained distributions for the strength and load is shown in Figure 5 for the plating.

In the case of Figure 5, the permanent deflection of 30 mm is used as the limit state of the plating. Once the permanent deflection is used as a parameter, the obtained safety indexes are shown in Figure 6. The safety index are calculated using the whole measured data up to the Bay of Bothnia as one case and only the measured data on the Gulf of Finland as the second case. In the Gulf of Finland case, Gumbel parameters are u=92,8 kN/m and c=0.0187 for the bow frame [9]. For the bow frames the safety index for the Bay of Bothnia is -0,98 and for the Gulf of Finland it is 2,56.



Figure 6. Obtained safety index as a function of the permanent deflection of the plating [12].

Figure 6 clearly indicates that small dents on the plating take place frequently when ships navigate in ice, especially when navigating in the northern Baltic Sea. If safety index 2 is considered as a good level, this is achieved by using permanent deflection of 25 mm as the limit state. This means  $w_p/t$  relation getting the value of 1.25 which seems to be a fairly reasonable level.

For frames, the obtained value of -0.98 for the safety index when navigating in the northern Baltic Sea seems to be fairly low. This means that the ultimate strength of the transverse frames at the bow is frequently reached when navigating in ice. However, the gathered damage statistics do not support this finding [3,13]. One reason for the some-

what too low value can be the fact that the brackets usually fitted at the upper and lower end of the frame are not included in the theoretical model.

# Conclusions

Level 2 reliability analysis is conducted to study the probability of ice induced damages on a typical ice-strengthened vessel navigating in ice in the Baltic Sea. Dents on the plating with varying depth are used as limit states for the plating and formation of the three plastic hinge mechanism is used as the limit state for the frames. The measured long term ice loads onboard MT Kemira is used to represent the statistical distribution of ice induced loads.

The results show that small dents take frequently place on the plating when navigating in ice, especially in the northern part of the Baltic Sea. However, the level of safety seems to be adequate as the level 2 of the safety index  $\beta$  is achieved with permanent deflection  $w_p/t$  getting the value of 1.25. The strength model for the frames seems to under estimate the real strength as the obtained low values for the safety index  $\beta$  are not supported by the gathered damage statistics.

The level 2 approach gives a good basis to study the proper level of icestrengthening for ships on the various sea areas of the Baltic Sea. The design rules for ice strengthening is today based mainly on a deterministic approach. The statistical approach gives much more reliable basis to determine proper scantlings for the shell structures of ice-strengthened vessels so that the adequate safety is achieved.

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