SOLUTION STRATEGIES FOR FPK-EQUATION USING STANDARD FEM SOFTWARE FOR DIFFUSION PROBLEMS

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ABSTRACT

The paper studies the response characteristics of stochastic mass damper. The excitation is assumed to be the stochastic process causing random vibration in the structure. The primary structure is appended with mass damper exhibiting nonlinear stiffness characteristics and viscous or internal friction type damping characteristics. The response of the primary structure is analyzed and using FEM solutions for FPK-equation (Fokker – Planck – Kolmogorov) with the aid of standard software for transient diffusion problems. The effectiveness of the damper is assessed for various alternative stiffness and damping configurations for the mass damper. The solution will be presented in the form of marginal distribution for displacement, velocities and accelerations of the primary structure.

Keywords: Fokker-Planck-Kolmogorov-equation, random process, stochastic mass damper, Markov process.

INTRODUCTION

The work of physicists on Brownian motion enabled the modeling of the response of dynamic systems to wide-band random excitation in terms of multi-dimensional Markov processes [1]. The state transition probability function for Markov a process is governed by a partial differential equation, known as the Fokker Planck Kolmogorov (FPK) equation, or forward diffusion equation [2].

The Fokker Planck Kolmogorov equation, governing the diffusion of probability mass in state space, is analogical to the diffusion equations, which govern the diffusion of heat, or mass, in thermo-hydraulic problems. The drift and diffusion coefficients in the FPK equation can be related to the parameters in the dynamic equations of motion.

The theoretical framework of Markov process theory offers, an approach to the treatment of non-linear random vibration problems. For cases where The excitation must be approximated as white noise process or the excitation pre-filtered in order to generate excitation processes for the system with the required power spectra.

Then Markov process theory is also applicable to a high dimensional combined system consisting of the oscillators in series with the original system. These systems lead to multi-dimensional problems governed by FPK – equation.

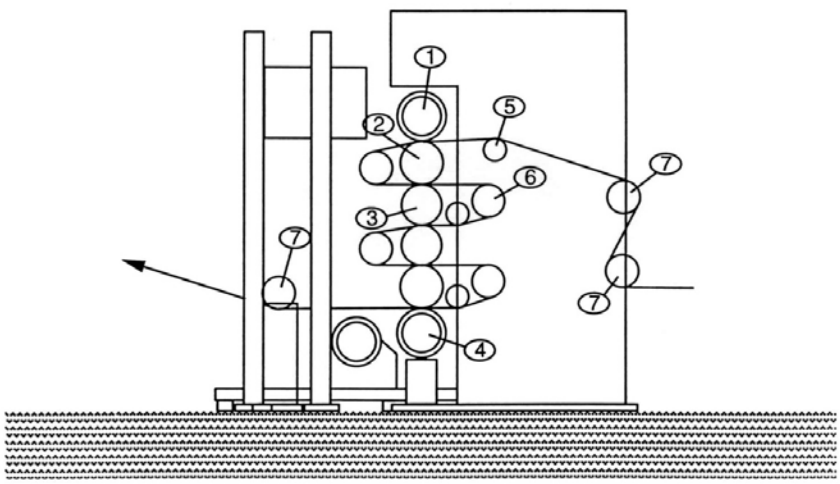

Figure 1. Schematic figure of calender frame in paper machine.

The class of non-linear random vibration problems for which the Fokker Planck Kolmogorov equation can be solved exactly is limited. The broadest class of single degree of freedom structural systems, for which the stationary solution of the associated FPK equation can be determined, requires that the mass, damping and stiffness of the oscillator are functions of displacement and velocity of a particular form. This class does not include such structural systems that are common in practical applications.
This paper presents the two solution methods for the FPK with the application to the dynamic vibration absorber.
The numerical examples presented are solved by statistical equivalent linearization or by finite elements in transient problem formulation.
The statistical linearization used here follows the approach of reference [1] and the finite element solution approach follows the references [3], [4] and [5].

PROBLEM FORMULATION

The application of dynamic vibration absorbers is especially effective for high slender structures like, for instance, supercalender frames in paper machine construction. The material of these calender frames is steel and they are loaded by stochastic excitation of calender rolls with various diameters. The excitation frequencies of various calender are different because the web speed in the paper machine is same for all rolls and the roll sizes vary. Dynamic vibration absorbers are used for decreasing the vibrations in calender frames in cases when the dominant vibration mode of the frame happens to coincide with the running speed of the paper machine. The allowable vibration amplitudes are specified in root mean square (rms) velocity values for the top of the calender frame. The typical allowable rms velocity value is 4.5 mm/s. The vibration absorber design for deterministic harmonic excitation loading and linear vibration absorber characteristics has been discussed in references [6] and [7]. The typical calender frame for paper machine is shown in Figure 1.

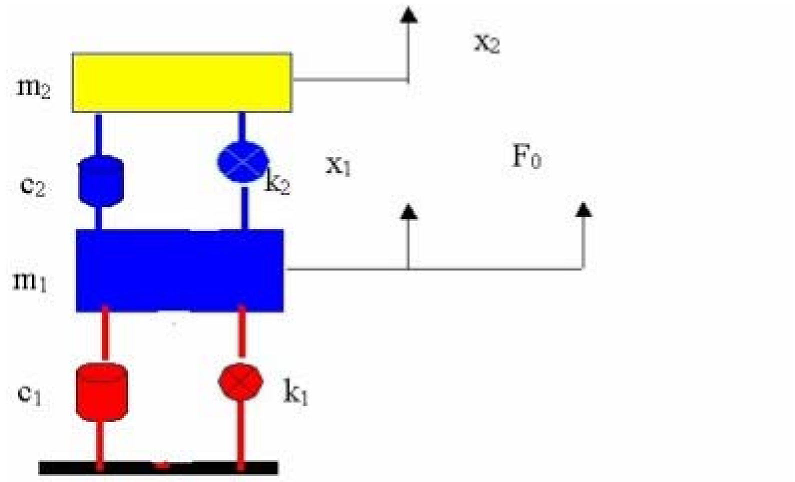

Figure 2 Dynamic vibration absorber

FOKKER-PLANCK-KOLMOGOROV EQUATION FOR VIBRATION ABSORBER

The state transition probability function for such a Markov process [1] is governed by a linear partial differential equation, known as the Fokker-Planck-Kolmogorov (FPK) equation. This equation, governing the diffusion of probability mass in state space, is analogous to the diffusion equations which govern the diffusion of heat, or mass, in seepage problems. It is possible to relate the 'drift' and 'diffusion' coefficients in the FPK equation directly to the parameters in the dynamic equations of motion of the system under consideration. For the white noise excitation process, the Markov process theory offers a direct approach to the exact treatment of non-linear random vibration problems. For the secondary system of Figure 2 the equation of motion can written be [2]

$$X''(t) + 2X'(t) + kX(t) = W(t) \tag{1}$$

With $X(t) = X_1(t)$, $X'(t) = X_2(t)$ and $X(t) = \left[X_1(t) \quad X_2(t)\right]^{\mathrm{T}}$, equation (1) can be presented

$$\mathrm{d}X(t) = \left[X_2(t) \quad -k\left(X_1(t) - 2\beta X_2(t)\right)\right]^{\mathrm{T}} + \left[0 \quad \mathrm{d}\beta(t)\right]^{\mathrm{T}} \tag{2}$$

where

$$\begin{aligned} &E\left\{\mathrm{d}\beta(t)\right\} = 0 \\ &E\left\{\left[\mathrm{d}\beta(t)\right]^2\right\} = 2D\mathrm{d}t \end{aligned} \tag{3}$$

The Fokker-Planck-Kolmogorov equation can then be written

$$\left.\frac{\partial f(x,t)}{\partial t}\right|_{x_0,t_0} = -\frac{\partial(x_2 f)}{\partial x_1} + \frac{\partial([k(x_1) - 2\beta x_2]f)}{\partial x_2} + \frac{\partial^2(Df)}{\partial x_2^2} \tag{4}$$

The solution for the stationary density function $f_s(x_1, x_2)$ can then be written [5]

$$f_s(x_1, x_2) = C\exp\left[\frac{-2\beta}{D}\int_0^{x_1} k(x)\mathrm{d}x + \frac{x_2^2}{2}\right] \tag{5}$$

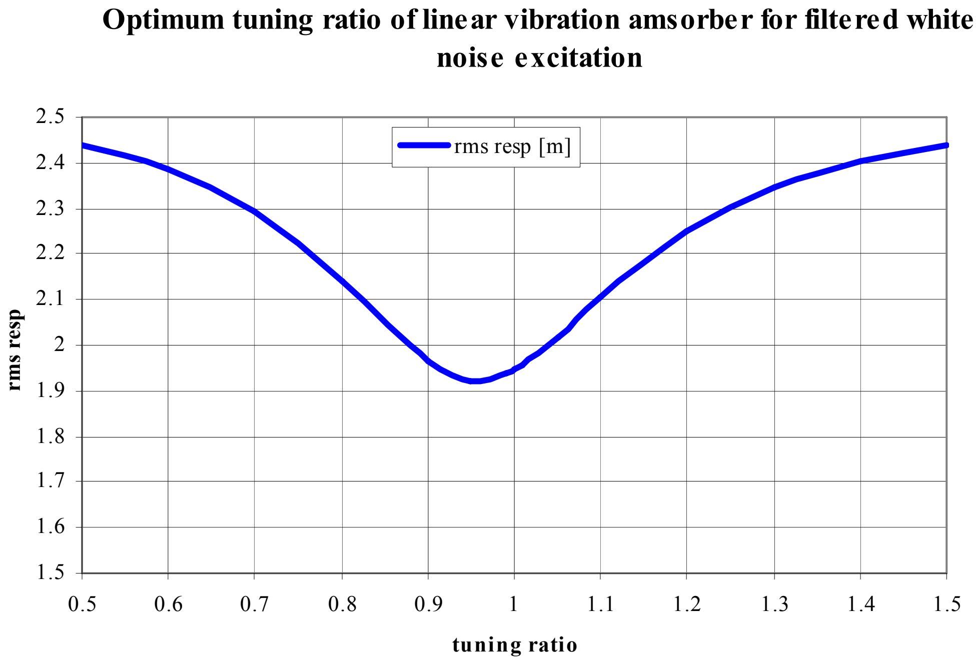

Figure 3 The optimum tuning for stochastically excited linear vibration absorber.

THE RMS RESPONSE OF THE SYSTEM IN FIGURE 2 FOR FILTERED WHITE NOISE

It will be assumed that an external force, $F_0(t)$, acts on the mass m$_1$, The absolute displacements of m_1, and m_2, measured from the static equilibrium position, are denoted by x_1, and x_2, respectively. A linear damper, with coefficient c_1, connects m_1, to the foundation, whereas a linear spring, of stiffness k_2, connects m_1 and m_2. It is assumed that Duffing type linear-plus-cubic spring connects m_1 to the foundation. m_1 and m_2 are connected by linear-plus-quadratic type damper. The force in nonlinear spring is given by $k_1 x_1(1+\varepsilon_1 x_1^2)$ and the force in non-linear damper by $c_2(x_2' - x_1')(1+\varepsilon_2 |x_2' - x_1'|)$. The power spectrum shape of the excitation $p(t) = F_0(t)/m_1$ is assumed to be of the

form $S_p(\omega) = S_0 / (\alpha^2 + \omega^2)$, where ω is the frequency. The porblem is solved by equivalent linearization technique according to reference [4].
For application example following numerical values are taken to the parameters of system in Figure 2:

$m_2 / m_1 = 0.05,$

$\omega_1 = \sqrt{k_1 / m_1} = \omega_2 = \sqrt{k_2 / m_2} = 1 \text{ rad s}^{-1},$

$\zeta_1 = c_1 / 2\sqrt{k_1 / m_1} = \zeta_2 = c_2 / 2\sqrt{k_2 / m_2} = 0.05,$

$\varepsilon_1 k_1 = 0.05 \,\text{Nm}^{-3},$

$\varepsilon_2 c_2 = 0.05 \,\text{Nm}^{-2}.$

The excitation is defined by $S_0 = 1 \,\text{Ns}^2\text{kg}^{-2}$; $\alpha = 2 \,\text{rad s}^{-1}$.

Optimum tuning ω_2 / ω_1 for linear system

The curve showing the optimum tuning for linear system when ε_1 and ε_2 are zero is shown in Figure 3. It can be seen from Figure 3 that the optimum tuning ratio is 0.95.

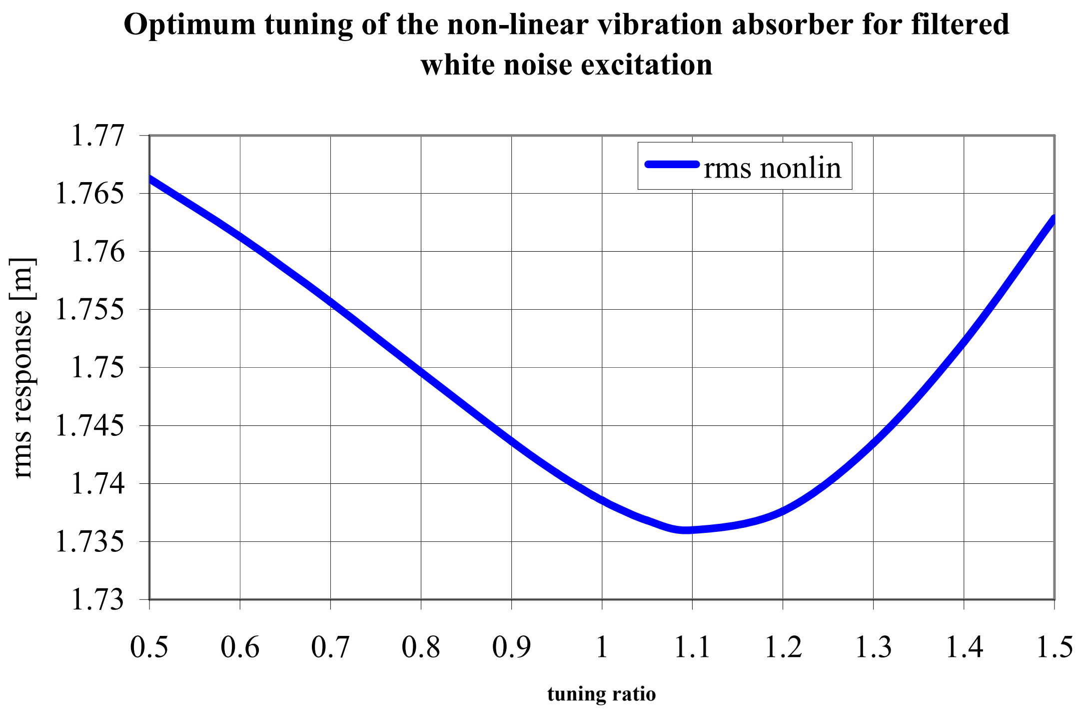

Figure 4 The optimum tuning for stochastically excited non-linear vibration absorber.

Optimum tuning ω_2 / ω_1 for non-linear system

The curve showing the optimum tuning for linear system when $\varepsilon_1 k_1 = 0.05 \,\text{Nm}^{-3}$ and $\varepsilon_2 c_2 = 0.05 \,\text{Nm}^{-2}$ is shown in Figure 4. It can be seen in Figure 4 that for even slightly non-linear system the optimum tuning is high tuned when it for linear system is low tuned. The optimum tuning in Figure 4 is 1.1.

Comparison between linear and non-linear tuning curves

If the tuning curves of Figure 3 and Figure 4 are plotted in the same frame we obtain the Figure 5, which shows the comparison between structural responses for linear and non-linear vibration absorbers for different tuning ratios for filtered white noise excitation

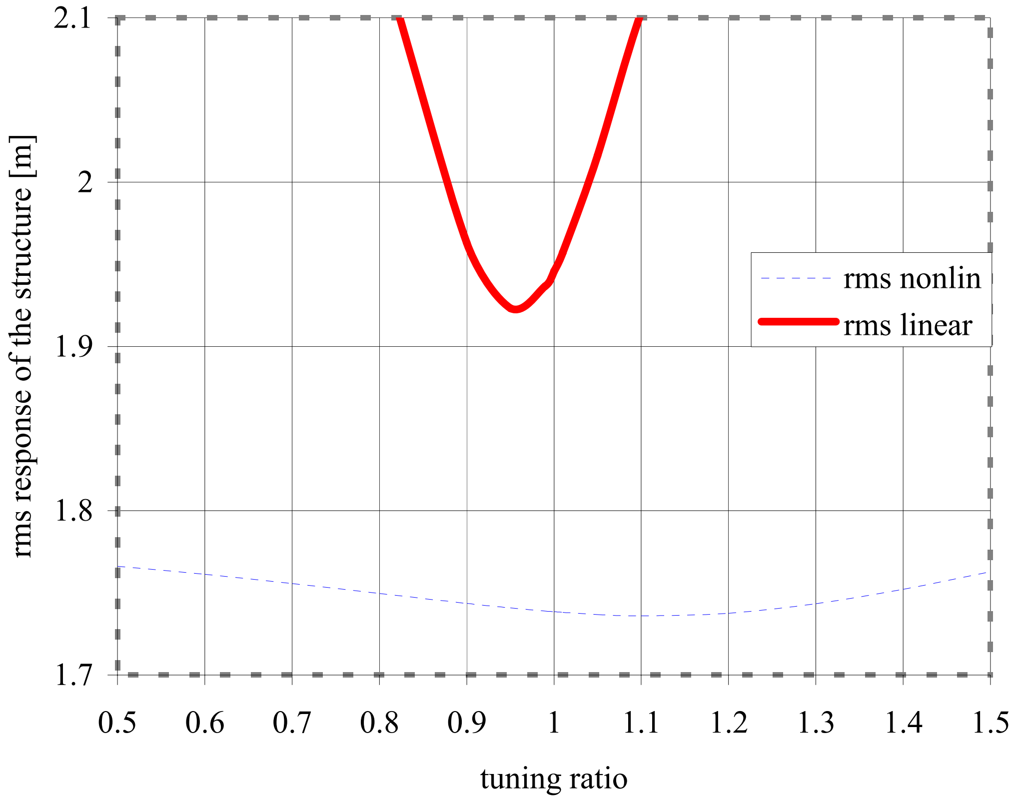

Figure 5 Comparison of tuning curves of linear (solid line) and non-linear (dotted line) vibration absorbers.

CONCLUSION

The response characteristics of linear and non-linear stochastically excited vibration absorber were evaluated. The optimum tuning ratio for linear as well as non-linear absorber were determined. It should be noted that even the slight non-linearity in the system changes the shape of the tuning curve essentially.

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