SENSITIVITY ANALYSIS OF THE EFFECT OF INITIAL IMPERFECTIONS ON THE STEEL PLATE GIRDER FATIGUE LIMIT STATE

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SUMMARY

The influence of initial imperfections on the behaviour of a thin-walled girder welded of slender plate elements is analysed in the present paper. The study is divided into two parts: (i) in the former one, the plate girder is considered to be exposed to quasi-constant loading (i.e. to loads which are either constant or repeated in a very small number of cycles), while (ii) in the latter; the girder is assumed to be subjected to loading repeated many times. Then it is understandable that the objective of the first part is to investigate into the influence of initial imperfections on the static ultimate load of the girder related to the formation of a plastic failure mechanism in it, while that of the second part is studying the effects of imperfections on the stress state under considerably lesser loads, i.e., under such ones as to correspond to the development of fatigue cracks in the girder and, consequently, to its fatigue limit state. In doing so, the girder was modelled, using the geometrically and materially non-linear shell finite element method, by the ANSYS program. All input imperfections were considered to be random quantities. The Latin Hypercube Sampling (LHS) numerical simulation method was used. It was studied by sensitivity analysis to what extend the variability of initial imperfections was reflected in the variability of (i) static ultimate load and of (ii) stresses in the crack-prone areas of the girder.

INTRODUCTION

Thin-walled steel structures mostly consisting of slender components are often used in practice. A typical example of a thin-walled structure is the girder welded of slender plates, see Fig. 1.

The design of a thin-walled system is always rather complicated because it is necessary to take into account also the influence of initial imperfections to a greater extent. For thin-walled girders, buckling of thin plates represents the major type of stability loss. Fatigue cracks in a breathing web are caused by a number of times repeated stability loss of slender plate elements of the structure - by the so-called breathing at cyclic loading. So, the slenderness of girder plates predetermines the behaviour of thin-walled structures under repeated loading above all. On behalf of the sensitivity analysis, it is possible to distinguish the extent of influence of individual imperfections on the (i) static ultimate load and (ii) stresses in the crack-prone areas. As all the imperfections are, according to their character, random quantities, the so-called stochastic sensitivity analysis has been applied.

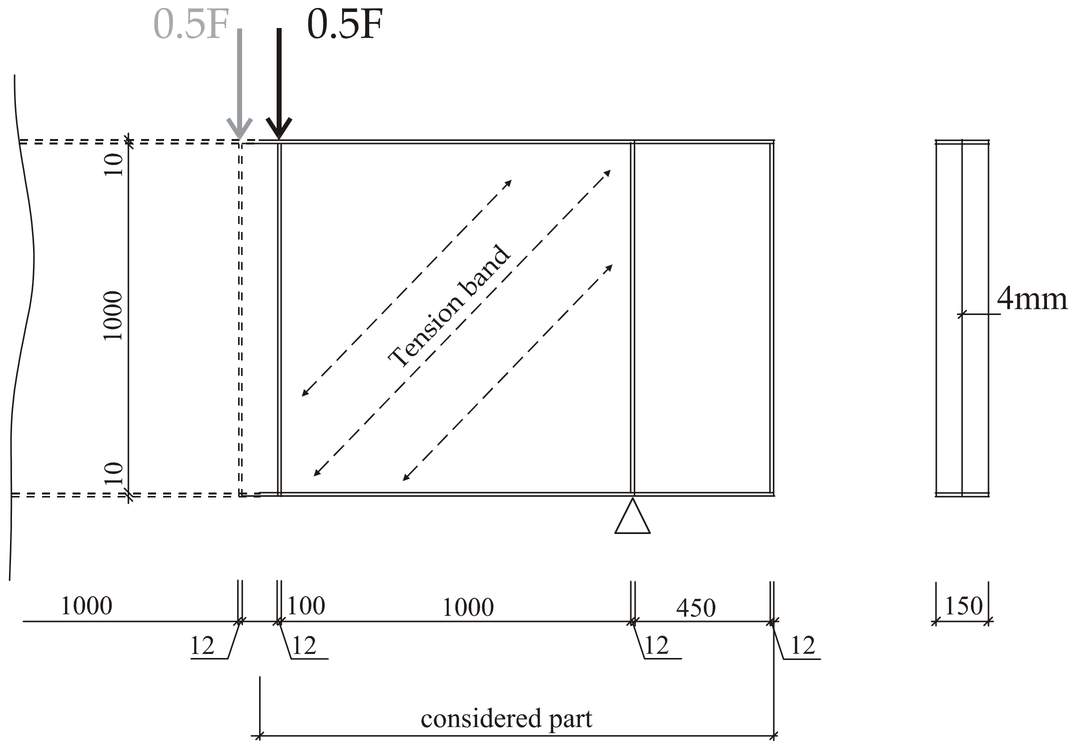

Fig. 1: Geometry of a thin-walled girder

STOCHASTIC SENSITIVITY ANALYSIS

The sensitivity analysis enabled us to assess the relative sensitivity of the random variability of a phenomenon studied (load-carrying capacity, stress state, etc.) to the random variability of individual input quantities (yield strength, initial web curvature, etc.). An evaluation of the sensitivity analysis by applying experimental results would be one of the possibilities to be chosen. For the evaluation of the sensitivity analysis mentioned, all the girder characteristics obtained experimentally by non-destructive methods would be needed, which is practically impossible.

Considering the large heterogeneity of initial imperfections and the possibility of their complex interaction, it is more advantageous to apply the numerical sensitivity analysis. In the first studies, the methods of *deterministic sensitivity analysis* were applied [6]. A similar problem applying the deterministic sensitivity analysis was studied also by Rangelov [14]. The deterministic sensitivity analysis is a method sometimes applied in the computer design of structures. Such an analysis thus consists of a sequence of calculations with a gradually changing value of the input parameters X_i studied, specifically in each calculation run j ($j = 1, 2,\dots,k$) taking place within a certain real extent. By comparison of calculation result of Y, the influence of parameter X_i on the response of Y can be evaluated. The calculation result of Y may mean, e.g., load-carrying capacity or stress state. So, if applying the deterministic sensitivity analysis, we deliberately disregard valuable information about the variability of the input data.

The objective of this paper is therefore to carry out a *stochastic sensitivity* analysis which provides more extended information about the problem studied. The random input quantities are considered as if they have been obtained by measurements, this enabling us to get quantified information about the influence of the scatter of individual parameters involved. This procedure can be employed with advantage in connection with the numerical simulation method Latin Hypercube Sampling Method (LHS), as suggested in the review paper [13], where also other variants of the stochastic sensitivity analysis are described. The LHS method is the Monte Carlo type method, which makes it possible to simulate the realizations of input random quantities as if they have been obtained by measurements. In the paper, the sensitivity analysis is evaluated in the form of Spearman rank-order correlation coefficient.

$$r_i = 1 - \frac{6\sum_{j=1}^{N}(q_{j,i} - p_j)^2}{N^3 - N}, \qquad r_i \in [-1,1]$$

where r_i is the order representing the value of random variable X_i in an ordered sample among N simulated values applied in the j_{th} simulation (the order j of $q_{j,i}$ equals the permutation at LHS), p_j is the order of an ordered sample of the resulting variable for the j_{th} run of the simulation process ($q_{j,i} - p_j$ is the difference between the ranks of two samples, and N is sample size).

The method is based on the assumption that the random quantity influence (both positive and negative) on the output quantity change. A significant quantity change will have a higher correlation coefficient. Opposite to this, the coefficient with its value near zero will signalise a low influence. Let us note that the sensitivity coefficients provide information on the relative influence of the input random quantities change on the output quantity change (e.g., on the stress state). Other sensitivity analysis methods applicable for a stability problems solution of thin-walled structures have been described in [4, 5].

INPUT RANDOM VARIABLES AND EXPERIMENTAL RESEARCH

The experimental research with statistical evaluation of results can serve as the basic information source of statistical distribution of input random quantities. In the problem solved, we issued from the large amount of experimental material and geometrical characteristics of steel products made by a dominant Czech producer, see [12].

For the not measured quantities (e.g., Young's modulus), the study was based on the data given in the technical literature; statistical characteristics of Young's modulus, e.g., are given in [2, 15].

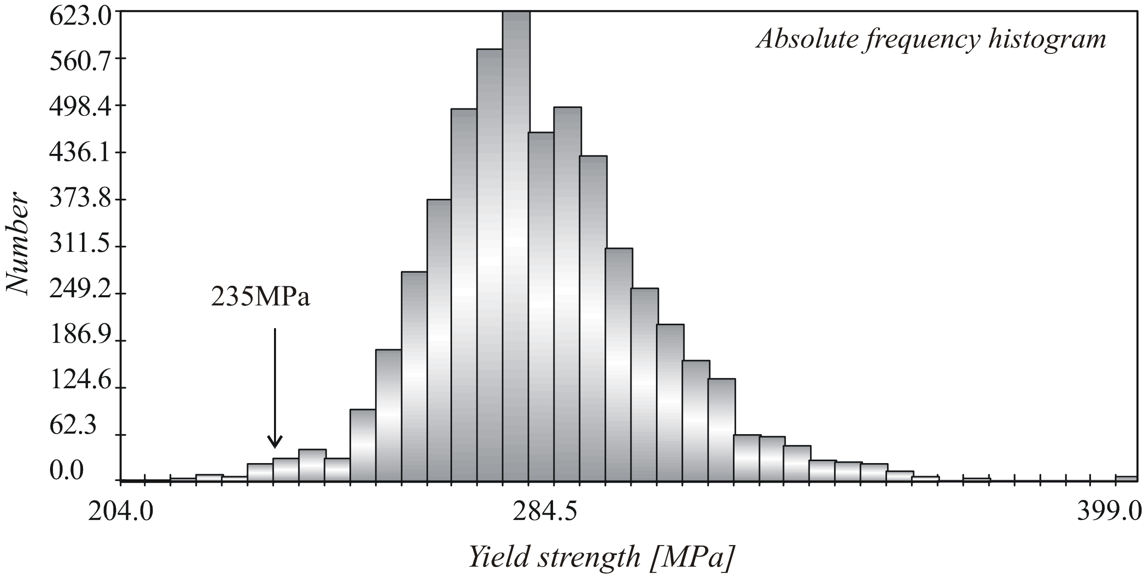

Fig. 2: The histogram of yield strength of tensile coupons – steels S235, 5493 samples

NO.	*Name of random quantity*	*Type of distribution*	**DIMENSION**	**MEAN**	*Standard deviation*
1.	**Amplitude of sine initial web curvature**	Lognormal	mm	3.574	3.335
2.	*Web thickness*	Gauss	mm	4	0.2
3.	*Web yield strength*	Histogram	MPa	284.5	21.5
4.	**Web Young's modulus**	Gauss	GPa	210	12.6
5.	*Thickness of upper flange*	Gauss	mm	10	0.7
6.	*Yield strength of upper flange*	Histogram	MPa	284.5	21.5
7.	*Young's modulus of upper flange*	Gauss	GPa	210	12.6
8.	*Thickness of lower flange*	Gauss	mm	10	0.7
9.	*Yield strength of lower flange*	Histogram	MPa	284.5	21.5
10.	*Young's modulus of lower flange*	Gauss	GPa	210	12.6
11.	*Thickness of left stiffener*	Gauss	mm	12	0.84
12.	*Yield strength of left stiffener*	Histogram	MPa	284.5	21.5
13.	*Young's modulus of left stiffener*	Gauss	GPa	210	12.6
14.	*Thickness of middle stiffener*	Gauss	mm	12	0.84

15.	*Yield strength of middle stiffener*	Histogram	MPa	284.5	21.5
16.	*Young's modulus of middle stiffener*	Gauss	GPa	210	12.6
17.	*Thickness of right stiffener*	Gauss	mm	12	0.84
18.	*Yield strength of right stiffener*	Histogram	MPa	284.5	21.5
19.	*Young's modulus of right stiffener*	Gauss	GPa	210	12.6

Tab. 1: Model of input random quantities

The initial web curvature was introduced by one half-wave of the sine function both in horizontal and vertical directions. The results of the measurements carried out on the webs of 16 large-size test girders, the web of which was 1000 mm in depth (and consequently similar in character to real steel plate girders), will be used as a data set for the following sensitivity analysis [10]. In general, the initial curvature can be selected in form of a random field. In the future, the study will be amplified in this sense. The one-dimensional random field has been applied, within our research project, only for the problem of a member under axial compression, see [3].

FINITE ELEMENT ANALYSIS OF THE STEEL GIRDER

The girder load-carrying capacity and state of stress analysis is the major subject of the study presented. As the girder presented in Fig. 1 consists of slender plates, its behaviour is significantly influenced by stability phenomena. In the theoretical analysis, it is therefore necessary to apply the geometrically and materially nonlinear FEM solution. The girder was modelled by applying the ANSYS program in a very minute manner, by means of a mesh of shell four-node elements SHELL 181 [1]; see Fig. 3.

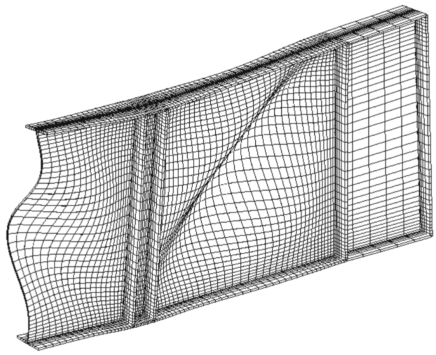

Fig. 3: The section of the web deformation in the limit state

The girder symmetry and that of loading were made use of. For steel grade S235, bilinear kinematic material hardening was supposed (the course of the second part of the diagram line is horizontal). Further on, it was assumed that the onset of plastification occurred when the Equivalent Mises stress exceeded the yield stress.

The influence of initial imperfections on the random load-carrying capacity was analysed in the first part of the study. Within the framework of each run of the LHS method, the load-carrying capacity was found out by the geometrically and materially nonlinear FEM solution**.** The Euler method based on proportional loading in combination with the Newton-Raphson method was used. The loading test was simulated by the incrementation of a loading step in the Euler method. The load-carrying capacity was determined as the loading rate at which the determinant of tangential stiffness matrix K_t of the structure approached zero with accurateness of 0.1 %. The state of stress along the slender web (see Fig. 1) for the load corresponding to the current working load was analysed in the second part of the study.

STATISTICAL ANALYSIS OF THE LOAD-CARRYING CAPACITY

The histogram of random load-carrying capacity calculated by applying the nonlinear calculation by the ANSYS program is presented in Fig. 4.

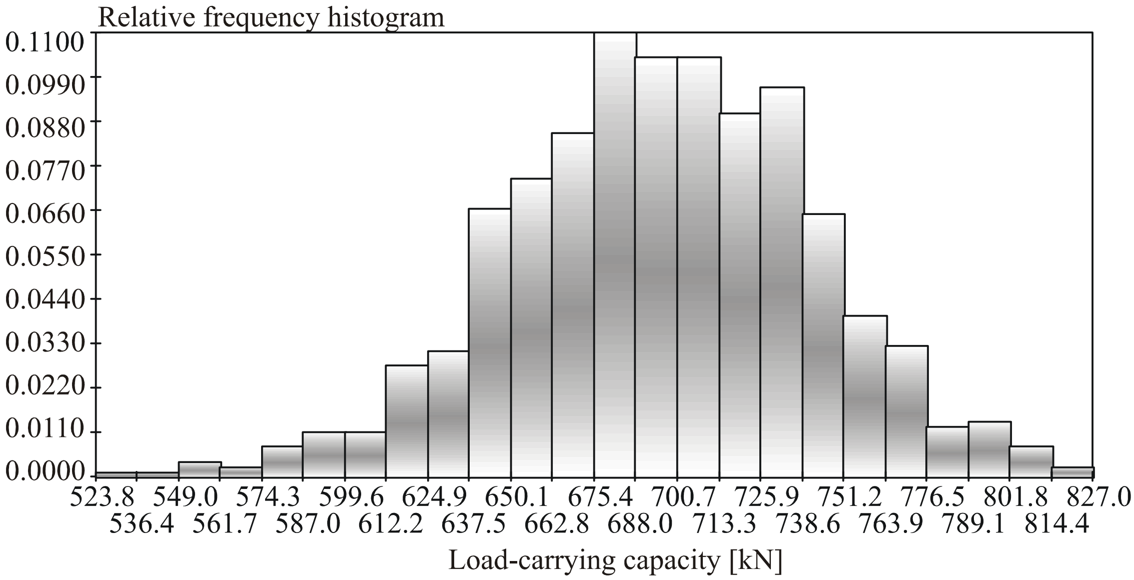

Fig. 4: Histogram of load-carrying capacity

Random realizations of initial imperfections were simulated by applying the LHS method for 800 runs. The random load-carrying capacity mean value is 695.62 kN; its standard deviation is 46.79 kN; standard skewness value is -0.18; that of standard kurtosis is 3.2.

SENSITIVITY ANALYSIS OF THE LOAD-CARRYING CAPACITY

Sensitivity coefficients (1) in Fig. 5 suggest an idea of relative load-carrying capacity sensitivity to initial imperfections. As long as the value of the correlation coefficient increases, the load-carrying capacity increases when also the value of the input quantity grows. A negative value of the coefficient heralds a negative effect of the quantity on the load-carrying capacity (e.g. quantity No. 1). With regard to numerical methods applied, the sensitivity coefficients are burdened by a certain error. The error can be assessed within the interval of approximately ±0.05 and it should decrease with increasing number of simulation runs and finer meshing by finite elements. For our study, the dominant quantities with a high sensitivity coefficient (e.g., No. 2, 5, 6) for which the given solution error is relatively low, are most interesting.

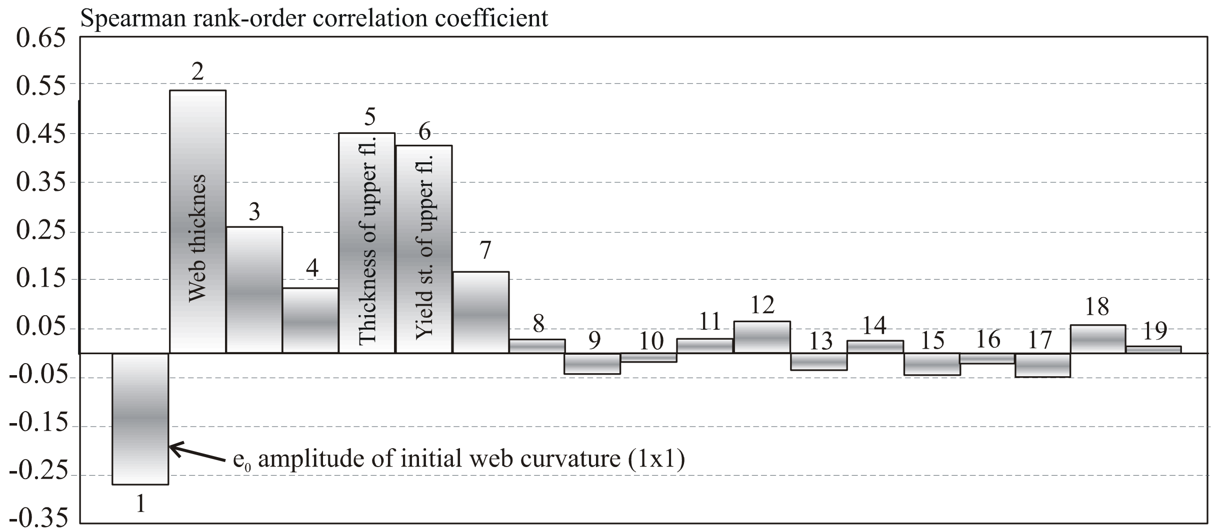

Fig. 5: Sensitivity coefficients of the load-carrying capacity

SENSITIVITY ANALYSIS OF EQUIVALENT MISES STRESS

The main objective of the sensitivity analysis described in the paper is to study the behaviour of girders, the web of which is under the action of combined shear and bending stress. For the fatigue behaviour, it is relevant to study stress state near the web edges, as it follows from experimental results [16, 8]. In the following part of the study, it was examined how the variability of initial imperfections took part in the state of stress variability. Equivalent Mises stress was obtained by ANSYS program and evaluated in 152 nodes of finite elements along the web edges. Two loading levels were considered; (i) equal to 10 % and (ii) equal to 60 % of the mean static load-carrying capacity 695.62 kN; they represent the interval of working loads to which a typical bridge structure is usually exposed during its lifetime. The resulting distribution of the sensitivity coefficients along the web edges is presented in Figs. 6 and 7, but only for those quantities where the Spearman rank–order correlation coefficients are superior to 0.15.

CONCLUSION

The results of the sensitivity analysis according to Figs. 6 and 7 are relevant for fatigue phenomena. According to the results of experimental studies [16, 11], the fatigue crack initiation and propagation occur near the left bottom and right upper web edges most frequently. In those points, the maximum sensitivity coefficients values were obtained for the initial web curvature, for the working load value equalling to 10 % of mean load-carrying capacity, see Fig. 6.

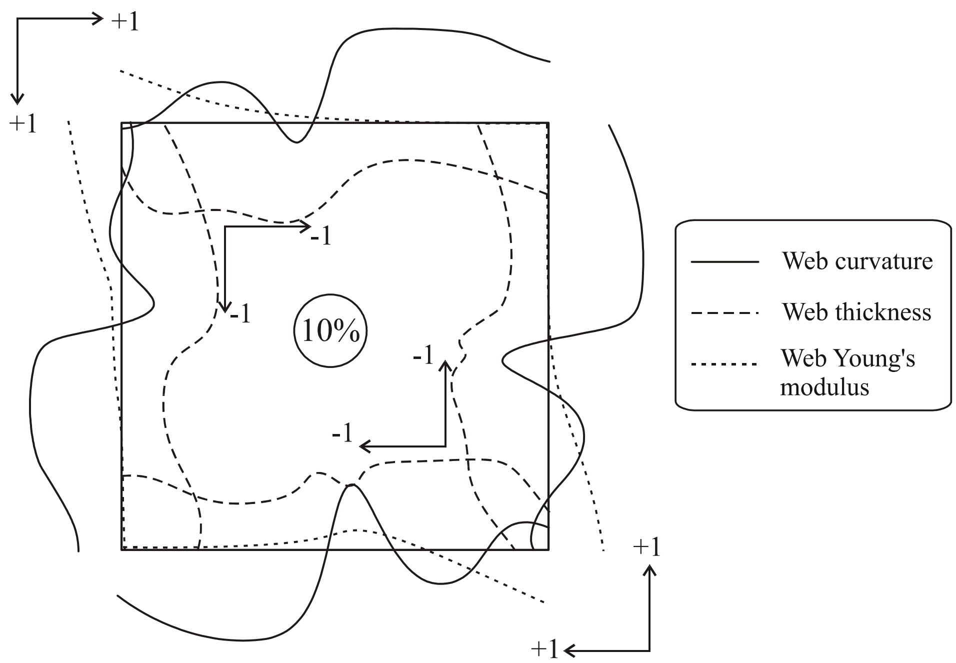

Fig. 6: Sensitivity coefficients along the web edges – working load 10%

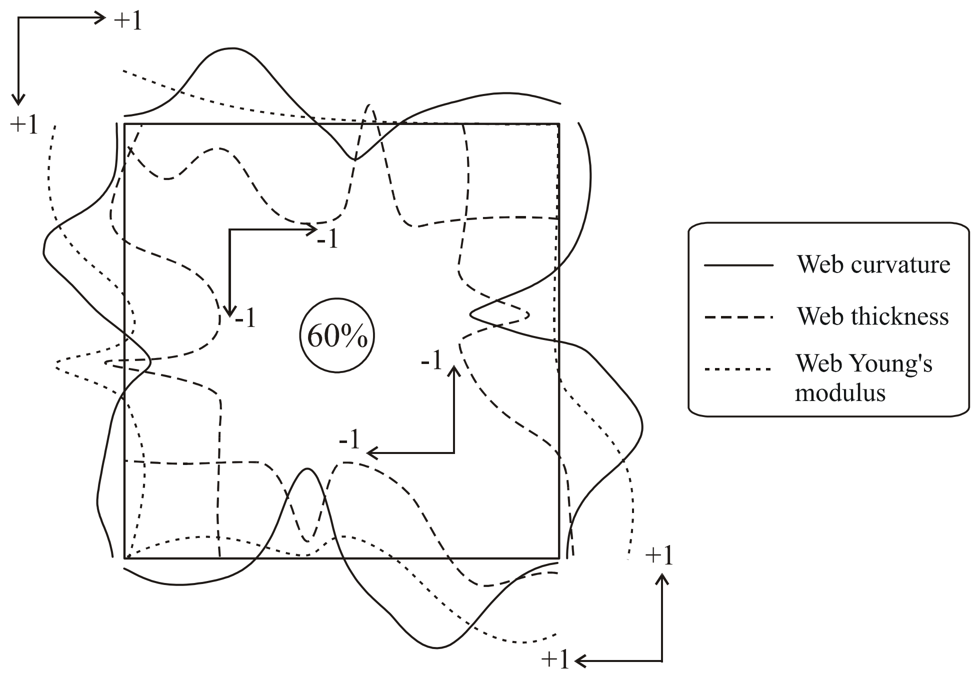

Fig. 7: Sensitivity coefficients along the web edges – working load 60%

On the contrary, the web thickness has the maximum influence if the value of working load equals to 60 %, see Fig. 7. The results of the authors' sensitivity analysis confirmed a great influence of the initial curvature of a slender web on the variability of bending stresses [7] and principal stresses [8], i.e., in the crack-prone areas of the web if the girder was subjected to repeated loading.

The results of the load-carrying capacity sensitivity analysis are presented in Fig. 5. The dominant quantities influencing the load-carrying capacity positively are: web thickness, upper flange thickness and upper flange yield point. The web initial curvature influences the load-carrying capacity negatively, but its influence is not dominant. A higher effect of initial curvature amplitude on the load-carrying capacity was observed for the same girder but with the web plate thickness of 6 mm, see [9].

The random variability (caused by manufacturing, assembling, etc.) of the random imperfections for which relatively high values of sensitivity coefficients were obtained should be checked with increased accuracy [4]. The input random imperfections can be divided into two basic groups [4]. The quantities, the statistical characteristics of which can be favourably influenced by manufacturing (yield strength, geometrical characteristics) can be included into the former group, and those not satisfactorily sensitive to manufacturing technology changes (e.g., Young's modulus E variability), into the latter one. The first group of quantities can be subdivided into two subgroups: (i) quantities for which both mean value and standard deviation can be changed by improvement in quality of manufacturing - such a quantity is, e.g., yield strength; (ii) quantities the mean value of which cannot be changed substantially as the mean value should equal the nominal value, e.g., geometrical characteristics of cross-section dimensions.

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