A METHOD FOR DESIGN OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES INVOLVING GEOMETRIC NONLINEARITIES

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SUMMARY

Minimum weight design of laminated composite structures involving geometric nonlinearities is considered. A constrained multi-criteria optimization problem is formulated for maximizing failure margin and critical buckling load factor of a composite structure with the minimum number of layers. The constrained optimization problem is tranferred into a sequence of unconstrained problems and solved iteratively using deterministic search with the achievement scalarizing function approach of Wierzbicki. Reissner-Mindlin-Von Kármán type plate model has been implemented to determine the actual nonlinear mechanical response of the structure. Load-displacement behaviour of the optimized structure as well as the failure prediction and the identification of the critical areas of the FE-model are illustrated with a numerical example.

INTRODUCTION

Two types of geometric nonlinearities may arise in the analysis of shell structures, namely large deformation and large rotation nonlinearities. Large deformation nonlinearity is induced by the membrane stress developed due to the midplane stretching when the shell experience large displacements as compared to its dimensions. Large rotation nonlinearity is produced by large change of the shell midsurface slope during the analysis.

Composite materials are desirable in lightweight structures due to their high specific stiffness and strength, and due to their dimensional stability under hygrothermal loads. In laminated composites, layer orientations can be varied to tailor the laminate properties to obtain the optimal response of the structure for the maximum efficiency in weight. Frequently, however, high modulus and strength characteristics of composite materials result in structures with very thin sections that are prone to buckling. For thin laminate structures the buckling load is fairly low and there is a long postbuckling behavior, which illustrates well the importance of being able to design plates in the postbuckling region to take advantage of the load carrying capability [18].

For many practical design problems of laminated composite structures, ply thicknesses are fixed and layer orientations are limited to a small set of angles such as 0, 90, and $\pm\theta$ deg. However, designing with composites leads to high dimesional, multimodal, and non-differentiable optimization problems which are difficult to solve. Furthermore, usually there exists no unique laminate lay-up configuration which would give an optimum for all the structural design criteria simultaneously. Hence the traditional single objective structural optimality concept has to be replaced by a another one, particularly adapted to a multi-criteria problem.

The multi-criteria optimization method of Wierzbicki is based on the reference objectives defining the aspiration levels for the criteria. The reference point is a feasible or infeasible point in the objective space which is reasonable or desirable to the designer. The reference points can be used to derive achievement function having minimal solutions at Pareto optimal points. The method works for nonconvex problems and is hence applicable in structural optimization, particularly applied in discrete optimization of laminated composite structures.

The objective of our project is to develop an optimization system for laminated composite structures involving geometric nonlinearities. Solving a nonlinear problem is always a computationally expensive task. To reduce computational costs, we have considered in this paper the use of linear buckling load factor and failure margin of a critical area of the FE-model as criteria in the vector objective function to be maximized. Instead of only solving the linear eigenvalue problem for obtaining the critical loads, we consider determining the actual load response of selected structures, which requires solving nonlinear, large deflection plate bending equations. We have included as a constraint the failure prediction of the laminate. Critical areas of the structure are identified and illustrated in the postbuckling and prefailure region. Initial results of our technique were introduced in the studies [11, 13]. In this paper, we give an introduction to the design method and summarize the employed model for the structural analysis.

STRUCTURAL LAYUP DESIGN OPTIMIZATION PROBLEM

For thin laminates the buckling load is fairly low and there is a long postbuckling and prefailure region to take into account in the structural design of the plate [3]. Therefore, it is not meaningful to consider the linear buckling load factor as a constraint but rather a criterion to be maximized in the design optimization problem. The actual load response and failure prediction is to be done by nonlinear analysis with load scaling to determine the mechanical behavior of the structure in the postbuckling region.

A structural weight minimization problem is formulated in discrete form

$$\mathrm{S} = \{\vec{y} \mid \vec{y} = \arg\min_{\vec{x}\in\tilde{\mathrm{S}}} \; n(\vec{x})\} \tag{1}$$

where $\vec{x} = (x_1, x_2, \ldots, x_n)$ is the layer orientation index design variable vector defining the laminate lay-up configuration with n layers and $\tilde{\mathrm{S}}$ the feasible set of lay-up configurations defined as

$$\tilde{\mathrm{S}} = \{\vec{x} \mid \tilde{g}(\vec{x}) = 1 - \tilde{RF}(\vec{x}) \leq 0\} \tag{2}$$

Reserve factor $\tilde{RF} > 0$ [10, 14, 19] denotes failure margin of the structure measuring the criticality of the effective load with respect to the failure load in the postbuckling region.

The design variable x_l represents an alternative layer orientation, where $1, 2, 3, 4$ stands for the four possible layer orientations $0, 90, +\theta, -\theta$ deg, respectively. A set of allowable layer orientations is defined by $X = (1, 2, 3, 4)$ corresponding to $\Theta = (0, 90, +\theta, -\theta)$. For instance, the symmetric even (SE) laminate structure $[0/\pm\theta/90]_{\mathrm{SE}}$ with $n = 8$ layers is encoded as $\vec{x} = (1, 3, 4, 2)$.

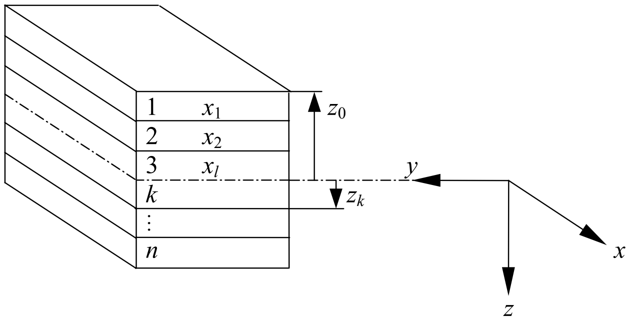

Figure 1. The laminate xyz-coordinate system and the layer numbering convention.

The solution for the structural optimization problem is not unique, i.e., there might be several feasible lay-up configurations with the minimum number of layers. Since we want to find the lay-up configurations that maximize the two design criteria with the minimum number of layers, we introduce a multi-criteria optimization problem as follows.

The multi-criteria optimization problem is formulated as

$$\max_{\vec{x}\in\mathrm{S}} \begin{bmatrix} RF(\vec{x}) \\ \lambda(\vec{x}) \end{bmatrix} = \max_{\vec{x}\in\mathrm{S}} \vec{z}(\vec{x}) \tag{3}$$

where RF and λ are the failure margin and the critical buckling load factor determined according to the Reissner-Mindlin plate model.

Generally, the components $z_i : \mathrm{S} \to \mathbb{R}, i = 1, 2, \ldots, m$ of the vector objective function are called criteria and they represent the design objectives by which the performance of the design point is measured. The image of the feasible set in the criterion space is $\Lambda = \{\vec{z} \in \mathbb{R}^m \mid \vec{z} = \vec{z}(\vec{x}),\ \vec{x} \in \mathrm{S}\}$.

Definition for Pareto optimal solutions of the problem (3) can be found in various references. Thoroughful definitions and illustrations can be found for instance in [17].

Definition 1. A solution $\vec{x}^*$ is Pareto optimal for the problem (3) if and only if there exists no $\vec{x} \in \mathrm{S}$ such that $z_i(\vec{x}) \geq z_i(\vec{x}^*)$ for all $i = 1, 2, \ldots, m$ and $z_i(\vec{x}) > z_i(\vec{x}^*)$ for at least one $i = 1, 2, \ldots, m$. The points $\vec{z}^* = \vec{z}(\vec{x}^*) \in \Lambda$ in the criterion space are called the maximal points.

Definition 2. A solution $\vec{x}^*$ is weakly Pareto optimal for the problem (3) if there does not exist another $\vec{x} \in \mathrm{S}$ such that $z_i(\vec{x}) > z_i(\vec{x}^*)$ for all $i = 1, 2, \ldots, m$. The corresponding points $\vec{z}^* = \vec{z}(\vec{x}^*) \in \Lambda$ in the criterion space are called the weakly maximal points.

ACHIEVEMENT FUNCTION APPROACH USING REFERENCE OBJECTIVES

Let $\bar{z}_i \in \mathbb{R}, i = 1, 2, ..., m$ be arbitrary reference objectives characterizing aspiration levels for the given criterion vector ($\vec{\bar{z}} \in \mathbb{R}^m$ denotes the reference objective vector) and let $s_{\bar{z}} : \Lambda \to \mathbb{R}$ be a continuous achievement function. The achievement problem to be solved is

$$\min_{\vec{z} \in \Lambda} \ s_{\bar{z}}(\vec{z}) \tag{4}$$

The design points are generated iteratively, each point being computed on the basis of the preceding point. At each iteration cycle, the selection of the design point is based on the achievement function value. It has been shown by Wierzbicki [22] that Pareto optimal solutions can be characterized by achievement scalarizing functions if the functions satisfy certain requirements. In this work, we consider an order-representing achievement function [17] as follows.

Definition 3. A function $s_{\bar{z}}$ is strictly decreasing if for $\vec{z}^{(j)}, \vec{z}^{(j+1)} \in \mathbb{R}^m, z_i^{(j)} < z_i^{(j+1)}$ for all $i = 1, 2, \ldots, m$ imply $s_{\bar{z}}(\vec{z}^{(j)}) > s_{\bar{z}}(\vec{z}^{(j+1)})$.

Definition 4. A continuous achievement function $s_{\bar{z}} : \Lambda \to \mathbb{R}$ is order-representing if it is strictly decreasing as a function of $z \in \Lambda$ for any $\bar{z} \in \mathbb{R}^m$ and if $\{\vec{z} \in \mathbb{R}^m \mid s_{\bar{z}}(\vec{z}) < 0\} = \bar{z} + \mathrm{int}\ \mathbb{R}^m_+$. For a continuous order-representing achievement function $s_{\bar{z}} : \Lambda \to \mathbb{R}$ we have $s_{\bar{z}}(\vec{z}) = 0$.

Based on the results represented by Wierzbicki, Miettinen [17] has given the following conditions concerning the the solutions of an order-approximating achievement function to be Pareto optimal.

Sufficient condition for a solution of an achievement function to be Pareto optimal. If the achievement function $s_{\bar{z}} : \Lambda \to \mathbb{R}$ is order-representing, then, for any $\vec{\bar{z}} \in \mathbb{R}^m$, the solution of the problem (4) is weakly Pareto optimal.

Necessary condition for a solution of an achievement function to be Pareto optimal. If the achievement function $s_{\bar{z}} : \Lambda \to \mathbb{R}$ is order-representing and $\vec{z}^* \in \Lambda$ is weakly Pareto optimal or Pareto optimal, then it is a solution of the problem (4) with $\vec{\bar{z}} = \vec{z}^*$ and the value of the achievement function is zero.

We employ an optimization procedure where the problem (4) is solved iteratively by transferring the constrained problem into a sequence of unconstrained problems. Let an order-representing achievement function be defined as

$$s_{\bar{z}}(\vec{z}) = \max_{i=1,2} \{\rho_i(\bar{z}_i - z_i)\} \tag{5}$$

where at the jth cycle $\bar{z}_i = \max z_i^{(j)}$ and $z_i = z_i^{(j)}$ and the criterion values are scaled as $\rho_i = w_i^{(j)}/(\max z_i^{(j)} - \min z_i^{(j)})$ with some fixed weighting vector $\vec{w} > 0$.

Termination condition for the algorithm is defined as

$$\min\{s_{\bar{z}}(\vec{z}), \tilde{g}(\vec{x})\} \leq \delta \tag{6}$$

where $\delta > 0$ is a small predefined termination scalar.

REISSNER-MINDLIN-VON KÁRMÁN MODEL FOR PLATE BENDING

The plate bending problem will be formulated for a thin or moderately thick laminated composite plate which in its undeformed configuration occupies the region $\Omega \times (-t/2, t/2)$, where $\Omega \subset \mathbb{R}^2$ is the midsurface and $t > 0$ is the laminate thickness. The kinematical unknowns in the model are transverse deflection w, in-plane displacement $u = (u_1, u_2)$, rotation of the middle surface $\beta = (\beta_1, \beta_2)$, and drilling rotation ω. The plate is subjected to the in-plane load $f = (f_1, f_2)$ and the transverse pressure g.

We will use standard dyadic notation of tensor calculus. The functions u, w, β, and ω are determined from the condition that they minimize the potential energy of the plate. The energy is defined as

$$\begin{aligned}\Pi(u,w,\beta,\omega) &= \frac{1}{2}\int_\Omega \varepsilon(u) : A : \varepsilon(u)\, d\Omega + \int_\Omega \varepsilon(u) : B : \varepsilon(\beta)\, d\Omega \\ &+ \frac{1}{2}\int_\Omega \varepsilon(\beta) : D : \varepsilon(\beta)\, d\Omega + \frac{1}{2}\int_\Omega \gamma(w,\beta)\cdot A^\star \cdot \gamma(w,\beta)\, d\Omega \\ &+ C\int_\Omega [\omega - rot(u)]^2\, d\Omega + \frac{1}{2}\int_\Omega \varphi(u,w) : A : \varphi(u,w)\, d\Omega \\ &+ \int_\Omega \varepsilon(u) : A : \varphi(u,w)\, d\Omega + \int_\Omega \varepsilon(\beta) : B : \varphi(u,w)\, d\Omega - \int_\Omega f\cdot u\, d\Omega - \int_\Omega gw\, d\Omega\end{aligned} \tag{7}$$

where ε is the linear strain tensor

$$\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T) \tag{8}$$

φ is the nonlinear membrane strain tensor

$$\varphi(u,w) = \frac{1}{2}(\nabla u_1 \otimes \nabla u_1 + \nabla u_2 \otimes \nabla u_2 + \nabla w \otimes \nabla w) \tag{9}$$

γ the transverse shear strain vector

$$\gamma(w,\beta) = \nabla w - \beta \tag{10}$$

and $C > 0$ is a penalty parameter for imposing the condition $\omega = rot(u)$ (see [8]), and

$$rot(u) = \frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1} \tag{11}$$

The tensors A, B, D, and $A^\star$, are defined according to the Classical Lamination Theory (CLT) as

$$A = \sum_k \int_{z_{k-1}}^{z_k} \bar{Q}\, dz = \sum_k (z_k - z_{k-1})\bar{Q}^{(k)} \tag{12}$$

$$B = \sum_k \int_{z_{k-1}}^{z_k} \bar{Q}z\, dz = \frac{1}{2}\sum_k (z_k^2 - z_{k-1}^2)\bar{Q}^{(k)} \tag{13}$$

$$D = \sum_k \int_{z_{k-1}}^{z_k} \bar{Q} z^2 \, dz = \frac{1}{3} \sum_k (z_k^3 - z_{k-1}^3) \bar{Q}^{(k)} \tag{14}$$

$$A^\star_{ij} = \sum_k \int_{z_{k-1}}^{z_k} \bar{Q}_{3i3j} \, dz = \sum_k (z_k - z_{k-1}) \bar{Q}^{(k)}_{3i3j} \tag{15}$$

where $\bar{Q}^{(k)}$ defines the constitutive relation for linear orthotropic materials in plane stress state for layer k in the laminate coordinate system. Detailed expression for computing $\bar{Q}^{(k)}$ as well as deriving the model is considered more thoroughfully in [11, 13].

The differential equilibrium equations of the minimization problem are obtained using standard variational calculus and integration by parts. In the computations we solve these nonlinear equations iteratively by Riks' method with Crisfields's elliptical constraint for arc length [15]. The linearized equations are then discretized by the finite element method. In the postbuckling region the algorithm follows the principal equilibrium path with the minimal stifness.

We will also consider linear stability analysis in the optimization process. In stability analysis we first solve (7) with $\varphi(u,w) = 0$, and compute the normal force $\mathcal{N} = A : \varepsilon(u) + B : \varepsilon(\beta)$. The critical buckling load factor with respect to $\mathcal{N}$ is denoted by λ and determined by minimining the Rayleigh quotient [6]

$$\begin{aligned} R(u,w,\beta,\omega) = \Big\{ \frac{1}{2} \int_\Omega \varepsilon(u) : A : \varepsilon(u) \, d\Omega + \int_\Omega \varepsilon(u) : B : \varepsilon(\beta) \, d\Omega \\ + \frac{1}{2} \int_\Omega \varepsilon(\beta) : D : \varepsilon(\beta) \, d\Omega + \frac{1}{2} \int_\Omega \gamma(w,\beta) \cdot A^\star \cdot \gamma(w,\beta) \, d\Omega \\ + C \int_\Omega [\omega - rot(u)]^2 \, d\Omega \Big\} \Big/ \int_\Omega \mathcal{N} : \varphi(u,w) \, d\Omega \end{aligned} \tag{16}$$

The minimizer (u, w, β, ω) of R is then the buckling mode related to the critical load factor $\lambda = R(u, w, \beta, \omega)$.

Let us finally remark that our plate model contains the following established plate models [1, 21]:

- for $\varphi(u,w) = 0$, the model reduces to the plate model of Reissner and Mindlin
- for $\beta = \nabla w$, the model reduces to the classical Von Kármán plate model
- for $\varphi(u,w) = 0$ and $\beta = \nabla w$, the model reduces to the classical plate model of Kirchhoff.

In the FE-implementation we partition the plate into straight sided triangles and use the linear stabilized MITC plate elements [2, 16]. In the computation, the in-plane forces and the bending moments are obtained consistently from the constitutive relations of the laminate while the shear forces are computed using the reduced shear strains [2, 16].

LAMINATE FAILURE PREDICTION

Exact solutions for reserve factors can be found for various failure criteria of composite materials. However, the failure criteria may consist of different expressions for the assessment of different types of failure. Finding exact solutions for this type of criteria is difficult or even impossible. Therefore, a numerical method independent of the failure criterion formulation to obtain the reserve factors [12, 14] has been employed.

For determining the layer margin to failure, we formulate an unconstrained minimization problem using the load criticality factor μ as

$$\min_{\mu \in [a,b]} v(\mu) = |1 - \mathcal{F}(\mu)| \tag{17}$$

where $\mathcal{F}$ denotes the generalized failure criterion. One of the common failure criteria for polymer matrix fiber-reinforced composites, i.e., maximum strain, maximum stress, Tsai-Wu, Hoffman, Tsai-Hill, simple Puck, modified Puck or Hashin failure criterion [7, 9, 20] can be used. In stress space, the failure criterion value is obtained from

$$\mathcal{F}(\mu) = \mathcal{F}[\sigma(\mu)] \tag{18}$$

where σ denotes the layer actual stress state. If only external mechanical loads are involved, then $\sigma(\mu) = \mu\sigma$. The objective function is minimized over the closed bounded interval by iteratively reducing the interval of uncertainty $[a, b]$. In the golden section line search method employed in this work, the interval of uncertainty is reduced each time by a factor of the golden section ratio until the final length of the uncertainty $b_{\mathcal{F}} - a_{\mathcal{F}} \leq \delta_{\mathcal{F}}$ for some $\delta_{\mathcal{F}} > 0$ is reached. Hence, the point $\mathcal{F}(\mu) = 1$ where failure occurs is achieved with

$$\mu_{\mathcal{F}} = (a_{\mathcal{F}} + b_{\mathcal{F}})/2 \tag{19}$$

The final length of uncertainty $\delta_{\mathcal{F}}$ reflects the desired degree of accuracy of the results.

Finally, the minimum of the layer reserve factors determined typically on the top and bottom surfaces of each layer of the laminate defines the margin to laminate First Ply Failure (FPF) at the element K as

$$RF_K = (\min \mu_{\mathcal{F}})_{|K} \tag{20}$$

The critical region in the structure is defined as the minimum of the element reserve factors

$$RF = \min_{K \in \mathcal{C}_h} RF_K \tag{21}$$

where $\mathcal{C}_h$ denotes partitioning of the plate. In this paper we use RF to denote the reserve factor computed with $\varphi(u, w) = 0$ in Eq. (7) and $\tilde{RF}$ to denote the reserve factor determined using the stress-strain state achieved with the Reissner-Mindlin-Von Kármán plate model.

NUMERICAL EXAMPLE

To illustrate the design optimization technique, we consider the design optimization problem (3) of a clamped Z-profile elastic beam shown in Figure 2. For solving the design optimization problem, we employ the Elmer software package [4] recently extended to cover analysis and design of laminated composite structures as well.

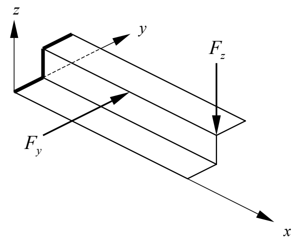

Figure 2. Z-profile beam.

The beam is composed of layers having the mechanical properties of in plane 23 transversely isotropic AS4 carbon/epoxy ply listed in Table 1. The length of the beam is 1.0 m, the height of the web and the width of the flanges are 0.1 m. The beam is subjected to its own weight and to the design loads $F_y = 1290$ N at the midlength of the beam and $F_z = -860$ N at the free end of the beam as shown in Figure 2. The web and the flanges are assumed to have identical lay-up configuration. A finite element mesh with 6000 linear triangular elements is used in the computation. The material coordinate system is given parallel to the global x-axis. In the design optimization, Tsai-Hill failure criterion is used to predict the failure with $\delta = \delta_{\mathcal{F}} = 0.5 \cdot 10^{-3}$. In our numerical example we let the stabilation parmeters $\alpha = 0.2$ and $C = t(A^{\star}_{11} + A^{\star}_{22})$.

AS4/3501-6	$t_{\text{ply}} = 0.134$ mm
$E_1 = 139.3$ GPa	$E_2 = 11.1$ GPa
$G_{12} = 6.0$ GPa	$\nu_{12} = 0.3$
$G_{23} = 3.964$ GPa	$\nu_{23} = 0.4$
$X_t = 1950$ MPa	$Y_t = 48$ MPa
$X_c = 1480$ MPa	$Y_c = 200$ MPa
$S_{12} = 79$ MPa	$\rho = 1580\ \text{kg/m}^3$

Table 1. Mechanical properties of the AS4/3501-6 carbon/epoxy ply.

The optimization algorithm begins with the generation of permutations of symmetric even (SE) lay-ups $\Theta = (0, 90, +\theta, -\theta)$, $\theta \in \{0, 5, 10, \ldots, 90\}$ deg with $n = 8$. The design alternatives are mapped into the criterion space and the achievement function values are computed for each alternative.

Choosing interactively the weighting factors $w_1 \in \{0.15, 0.19, 0.24, 0.3\}$, for instance, and $w_2 = 1 - w_1$ produces the lay-up configurations $[90/\mp\theta/0]_{\text{SE}}$ with $\theta \in \{50, 45, 40, 35\}$ deg, respectively, as parents at the first iteration cycle. The initial and the following generations as well as the selected parents are shown in Figure 3.

A new subset of design points is created at each cycle from each selected parent at that cycle. Here $\vec{x}^{(j)}$ denotes half of the selected laminate structure at the jth iteration cycle, e.g., $[0/90/\pm\theta]_{\text{SE}}$ laminate is denoted by $\vec{x}^{(j)} = (1, 2, 3, 4)$. Accordingly, $\vec{x}^{(j+1)} = (\vec{x}^{(j)}, 1, 1)$ in this case denotes $[0/90/\pm\theta/2(0)]_{\text{SE}}$ lay-up.

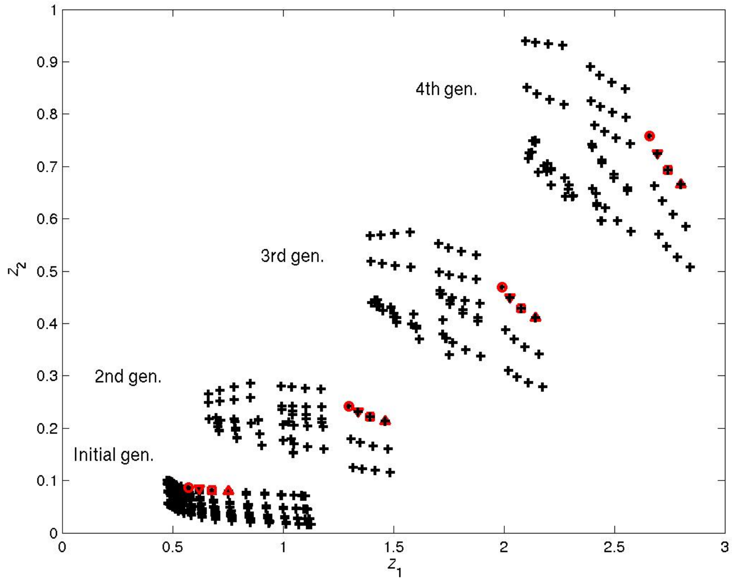

Figure 3. The four generations of design points in the criterion space. At each iteration cycle, symbols denote the selected parents for the next generation. At the last cycle achieved subset of Pareto optimal solutions are listed in Table 2.

The algorithm generates from one selected parent the following design alternatives.

$$\vec{x}^{(j+1)} \in X^{(j+1)} = \left\{ \begin{array}{llll} (\vec{x}^{(j)}, 3, 4), & (\vec{x}^{(j)}, 4, 3), (3, 4, \vec{x}^{(j)}), & (4, 3, \vec{x}^{(j)}), \\ (\vec{x}^{(j)}, 1, 1), & (1, \vec{x}^{(j)}, 1), (1, 1, \vec{x}^{(j)}), & (\vec{x}^{(j)}, 1, 2), \\ (1, \vec{x}^{(j)}, 2), & (1, 2, \vec{x}^{(j)}), (\vec{x}^{(j)}, 2, 1), & (2, \vec{x}^{(j)}, 1), \\ (2, 1, \vec{x}^{(j)}), & (\vec{x}^{(j)}, 2, 2), (2, \vec{x}^{(j)}, 2), & (2, 2, \vec{x}^{(j)}) \end{array} \right\} \qquad (22)$$

To avoid the generation of undesirable thick $n(+\theta)$ or $n(-\theta)$ sublaminates, permutations $(3, \vec{x}^{(j)}, 4)$ and $(4, \vec{x}^{(j)}, 3)$ are neglected in the set. Furthermore, the set of allowable permutations can be easily extended to cover also additional layer orientations and laminate structures when necessary, e.g. $(\pm\theta_1)$ and $(\pm\theta_2)$ layer orientations.

Finally, a subset of Pareto optimal lay-up configurations with $n = 20$ has been found as a solution of the design problem. The lay-up configurations as well as the corresponding criterion and constraint function values are represented in Table 2.

In this example, we selected four parents from the initial generation to the next iteration cycle. The initial phase requires some 460 stability or nonlinear analyses including failure prediction of the laminate to be performed. At each iteration cycle 76 corresponding analyses are performed resulting the total price of the search less than 700 analyses. The final laminate structure is 2.68 mm thin for which there is a long postbuckling behavior to be taken into account prior to failure. For thin laminated rectangular plates with various boundary conditions analogous results have been shown in [18] when comparing computational results with experimental data.

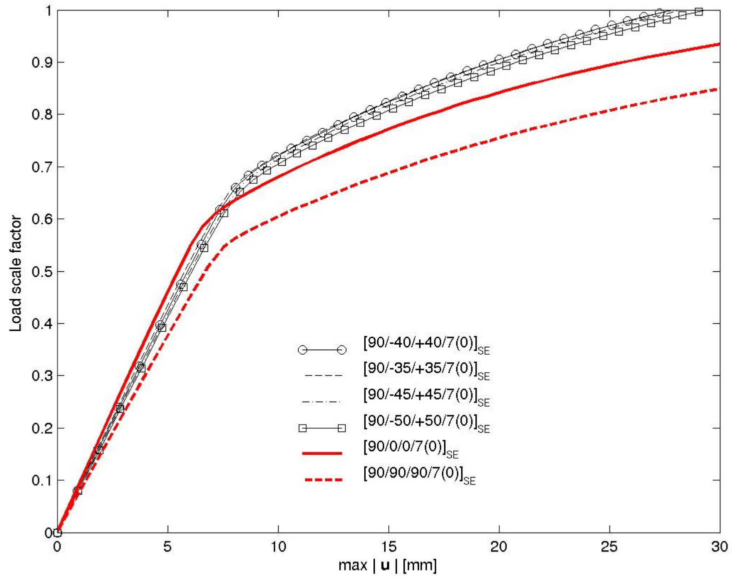

Figure 4. Load-displacement behavior of the Z-beam with the Pareto optimal lay-up configurations listed in Table 2 and with the comparative lay-ups with $\theta = 0, 90$. Load scale factor 1.0 corresponds to the design load.

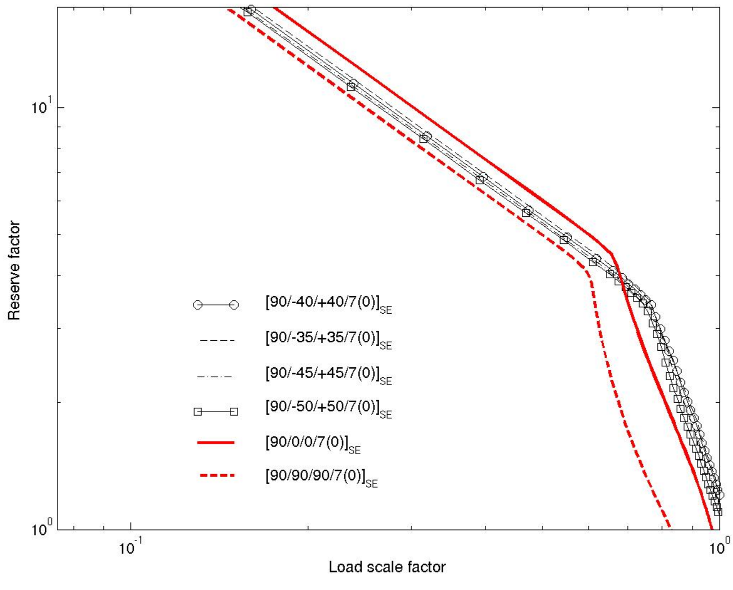

Figure 5. The FPF reserve factor value as a function of the load scale factor with logarithmic scales for the Z-beam with the Pareto optimal lay-up configurations listed in Table 2 and with the comparative layups with $\theta = 0, 90$. Load scale factor 1.0 corresponds to the design load.

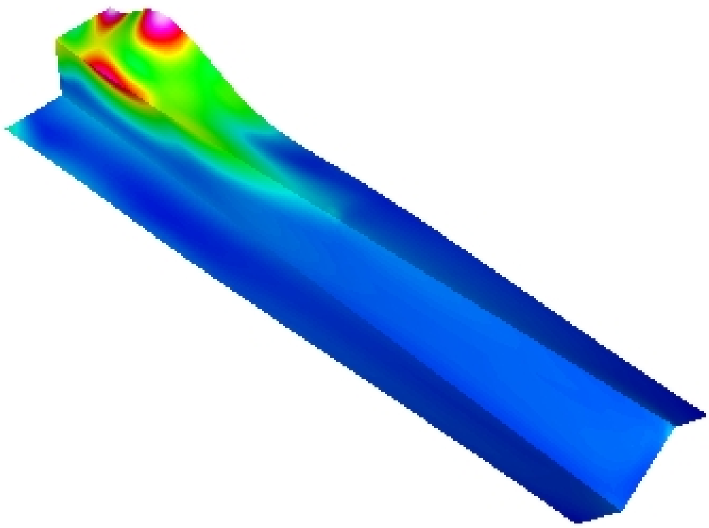

Figure 6. An example of the deformed structure composed of the $[90/\mp50/7(0)]_{\mathrm{SE}}$ lay-up subjected to the design load using the Reissner-Mindlin-Von Kármán model. Colors represent the value of inverse FPF reserve factor computed with the Tsai-Hill failure criterion.

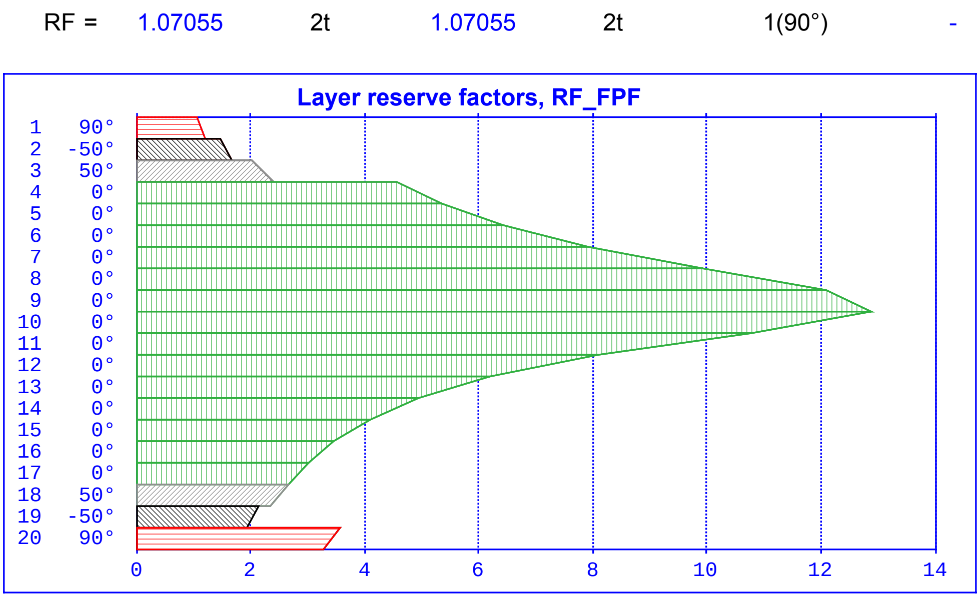

Figure 7. An example of postprocessing the results with the ESAComp software [5]. The layer reserve factor value is shown for the Pareto optimal $[90/\mp50/7(0)]_{\mathrm{SE}}$ laminate at the critical point of the Elmer FE-model using the Tsai-Hill failure criterion.

Lay-up	RF	λ	$\tilde{RF}$	$\max \lvert \vec{u} \rvert$ [mm]
$[90/\mp 50/7(0)]_{\mathrm{SE}}$	2.66117	0.75764	1.07044	29.7481
$[90/\mp 45/7(0)]_{\mathrm{SE}}$	2.69606	0.72423	1.13430	28.8957
$[90/\mp 40/7(0)]_{\mathrm{SE}}$	2.74338	0.69313	1.20555	27.9818
$[90/\mp 35/7(0)]_{\mathrm{SE}}$	2.80165	0.66563	1.20866	28.4513

Table 2. A subset of Pareto optimal lay-up configurations with $n = 20$ for the problem (3).

In Figure 4, load-displacement behavior in terms of $\max \lvert \vec{u} \rvert, \vec{u} = (u_1, u_2, w)$ is shown for the Pareto optimal solutions listed in Table 2 and for the two comparative lay-up configurations with analogous stacking sequence but with $\theta = 0, 90$. In Figure 5, the FPF reserve factor computed with the Tsai-Hill failure criterion as a function of the load scale factor is shown for the corresponding lay-up configurations. For the Pareto optimal configurations, the buckling load is clearly higher and the maximum displacement lower than for the comparative configurations in which also the first ply failure occurs prior to the design load.

Examples of postprocessing the results are shown in Figures 6 and 7. The critical values of reserve factors are less than one whereas the non-critical values range from one up to infinity. For that reason in color charts, for instance, margin to failure is more practical to view as inverse reserve factors. In Figure 6, an example of the deformed structure subjected to the design load is represented. Colors show the value of the inverse reserve factor computed with Tsai-Hill failure criterion. In Figure 7, the ESAComp software [5] has been used to plot the chart of the layer reserve factors at the critical point of the Elmer FE-model.

CONCLUSIONS

An efficient technique for solving analysis and design problems of laminated composite structures involving geometric nonlinearities is considered. For solving the design problem, a constrained multi-criteria optimization problem for maximizing failure margin of a critical point in the FE-model and critical buckling load factor has been formulated. The constrained problem has been transferred into a sequence of unconstrained problems and solved using deterministic search and the achievement function approach of Wierzbicki. The actual mechanical response of the structure is determined solving large deflection plate bending equations within the Elmer PDE solver in which the Reissner-Mindlin-Von Kármán type plate model has been implemented. Some of the advantages of the design method are that it is easy to implement and the design space is facile to expand, and it is also employable at distributed computing environment. The method is also relatively fast, which could make it a versatile tool in a preliminary design stage.

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