# STATIC ANALYSIS OF SUSPENSION BRIDGES LOADED BY CONCENTRATED FORCES

Valdek Kulbach

Siim Idnurm

Juhan Idnurm

Rakenteiden Mekaniikka, Vol. 34
Nro 2, 2000, s. 3-14

## ABSTRACT

The paper introduces a discrete method for static analysis of suspension bridges. The simplest suspension bridge consists of geometrically non-linear cables, connected by hangers with an elastic linear stiffening girder. In the calculation of suspension bridges the geometrically non-linear behaviour of the parabolic cable is the main problem. A geometrically non-linear continual model is especially useful for classical loading cases - a uniformly distributed load on the whole or on half span. But the modern traffic models consist of uniformly distributed and concentrated loads. The discrete model of a suspension bridge allows us to apply all kinds of loads, such as distributed or concentrated ones. Our assumptions of the discrete method are: linear elastic strain-stress dependence of the material and absence of horizontal displacements of hangers. We present some comparative numerical examples.

## ELASTIC CABLE LOADED BY CONCENTRATED FORCES

Let us consider the elastic cable without the stiffening girder (Fig.1). The cable is prestressed by initial vertical forces $F_{0i}$. If the cable is loaded by a uniformly distributed load, then it takes the parabolic form. Here the cable is loaded by

Equations (2) and (3) give us the coordinates of the catenary's curve, if we know the thrust $H_0$ . If the supporting points of the cable are on the same level ($z_0 = z_{n+1}$), then from the first node equilibrium condition, we get

$$H_0 = \frac{a_0 \sum_{i=1}^{n} F_{0i}(l - x_i)}{l(z_1 - z_0)}. \tag{4}$$

where $l$ is the span of the cable.

The initial form of the catenary's curve is described by three coordinates $z_0$, $z_1$, $z_{n+1}$. Instead of these coordinates you may use known values of the other three arbitrary coordinates $z_{i-1}$, $z_i$ and $z_{i+1}$. By the action of the temporary loads $\Delta F_i$ the equilibrium equation for the node $i$ is expressed as

$$H\left(\frac{z_{i-1} - z_i}{a_{i-1}} + \frac{z_{i+1} - z_i}{a_i} + \frac{w_{i-1} - w_i}{a_{i-1}} + \frac{w_{i+1} - w_i}{a_i}\right) + F_i = 0, \tag{5}$$

where $w_{i-1}, w_i, w_{i+1}$ are vertical displacements, $H$ is the total (summary) thrust under action of temporary and initial load and $F_i = F_{0i} + \Delta F_i$.

From Eq. (5) we get an expression for the vertical displacement

$$w_i = \frac{1}{1 + \frac{a_i}{a_{i-1}}}\left[w_{i-1}\frac{a_i}{a_{i-1}} + w_{i+1} + \frac{(z_{i-1} - z_i)a_i}{a_{i-1}} + (z_{i+1} - z_i) + \frac{F_i a_i}{H}\right]. \tag{6}$$

So, as there are two unknown parameters in Eq. (6): $w_i$ and $H$, we need another equation for calculating $w_i$ and $H$.

$$\sum_{i=0}^{n}\left(u_{i+1}-u_i\right)=u_{n+1}-u_0 \tag{11}$$

we may derive from Eq. (10) the form

$$\frac{H-H_0}{EA}\left\{\sum_{i=0}^{n}a_i\left[1+\left(\frac{z_{i+1}-z_i}{a_i}\right)^2\right]^{\frac{3}{2}}-\frac{EA}{H-H_0}\left(u_{n+1}-u_0\right)\right\}=\sum_{i=1}^{n}\left(w_{i+1}-w_i\right)\left(\frac{z_{i+1}-z_i}{a_i}+\frac{w_{i+1}-w_i}{2a_i}\right) \tag{12}$$

Solution of the system of non-linear equations (5) and (12) enables us to calculate all the displacements $w_i$ and $H$ by the given initial cable form and boundary conditions for $u_0$ and $u_{n+1}$.

## ELASTIC CABLE WITH THE STIFFENING GIRDER

The simplest suspension bridge consists of a geometrically non-linear cable, connected by hangers with an elastic linear stiffening girder ( Fig. 2.).

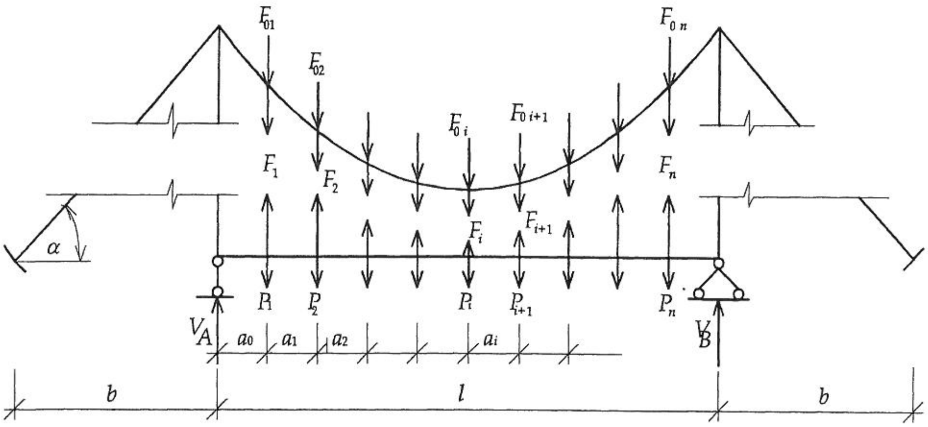


**Figure 2.** The discrete model of the suspension bridge with vertical pylons.

We obtain $n$+1 linear equations for calculating $F_i$, but there are $n$+2 unknown parameters: $F_1$, $F_2$ ... $F_n$ , $V_A$ and $\varphi_0$. An extra equation can be written from the moment equilibrium condition upon support $B$, as follows:

$$\sum_{i=1}^{n} F_i(l - x_i) + V_A l + M_P = 0, \qquad (16)$$

where $M_P$ is the moment of the external forces upon support $B$ and $l$ is the main span of the suspension bridge.

Thus, the system of linear equations (15) and (16) allows us to calculate the internal forces $F_i$ in the hangers, if the displacements $w_i$ are known.

Now we can construct an iteration process for calculating the displacements $w_i$ , the thrust $H$ and the internal forces in hangers $F_i$ as follows:

1. Prestress of the cable (initial forces in the cable) may be calculated by Eq. (4).
2. Let us assume that the initial displacements of the stiffening girder are $w_{g,i}^{0}$.
3. From the system of linear Eqs. (15) and (16), the internal forces $F_i$ are calculated from the additional load $P_i$.
4. From the system of non-linear Eqs. (6), (13) and (14), the vertical displacements of the cable are calculated.
5. Comparison between vertical displacements of the stiffening girder and the cable. The difference for every hanger node $i$ is

$$\Delta w_i^{k} = w_{c,,i}^{k} - w_{g,i}^{k-1}, \qquad (k = 1,2,\ldots.) \qquad (17)$$

where $w_{c,,i}^{k}$ , $w_{g,i}^{k-1}$ are the vertical displacements of the cable and the girder.

6. The new stiffening girder displacements will be calculated

$$w_{g,i}^{k-1} = w_{g,i}^{k} + K \times \Delta w_i^{k}, \qquad (18)$$

In this table, the load is calculated for one stiffening girder using the lever rule. The traffic load is presented here by its characteristic value. The whole or half span uniformly distributed traffic load (effective traffic load) from the double axle concentrated load for the continual model is calculated as follows:

$$p_2' = 2\times(2P_2')/l, \quad (19)$$

where $P_2'$ is the axle load of the double-axle traffic load.

A comparison of maximum deflections from the whole span traffic load is shown in Fig. 3. Since the linear principle of superposition of the load is not valid for suspension bridges, the deflection caused by the traffic load is calculated by the following expression:

$$w(p_2) = w(p_0 + p_1 + p_2) - w(p_0 + p_1). \quad (20)$$

The continual method used here is described in [4]. The linear method is the usual finite elements method (FEM).

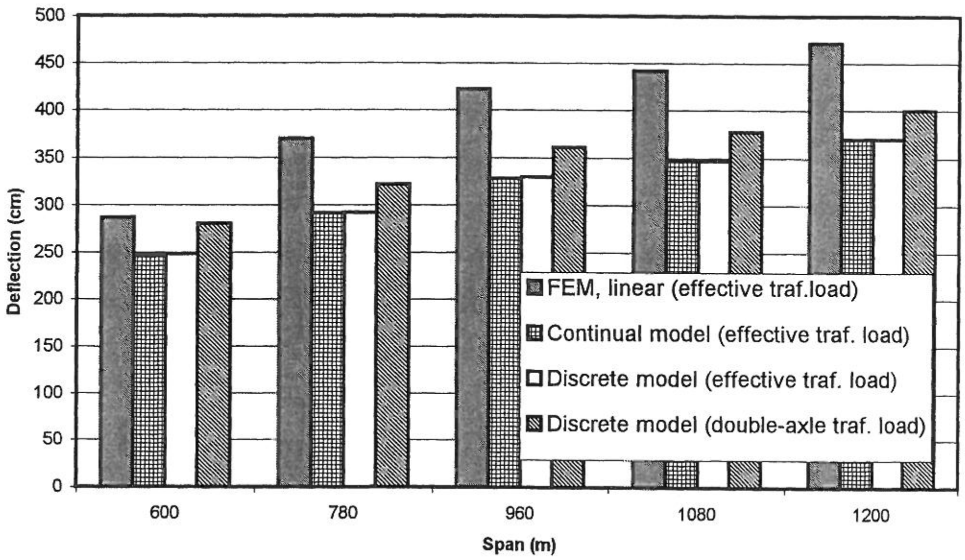


**Figure 3.** Deflections from the whole span traffic load.

Figure 4 shows the bending moments from the whole span traffic load and self-weight load.

load. However, the linear FEM method is not applicable in the case of double axle concentrated forces.

## REFERENCES


1. Kulbach, V. and Õiger, K. *Static Analysis of Suspended Systems.* Tallinn, 1986 (in Russian).
2. Kulbach, V. Statical Analysis of Girder- or Cable–Stiffened Suspended Structures. *Proc. Estonian Acad. Sci. Eng.,* 1995, **1** (1), 2-19.
3. Smirnov, V.A. *Long Span Suspension Bridges.* Moscow, 1975 (in Russian).
4. Aare, J. and Kulbach, V. Accurate and Approximate Analysis of Statical Behaviour of Suspension Bridges. *Journal of Struct. Mech.*, 1984, **17,** 3, 1-12.
5. Kulbach, V. and Talvik, I. Efficiency of Steel in Suspended Structures. *Journal of Constructional Steel Research,* 1998, **46**, 1-3, Paper No. 185.



Valdek Kulbach, Dr. Tech. Sc.
Department of Civil Engineering
Tallinn Technical University
Ehitajate tee 5
19086 Tallinn, Estonia
email: vkulbach@edu.ttu.ee

Siim Idnurm, PhD.
Department of Civil Engineering
Tallinn Technical University
Ehitajate tee 5
19086 Tallinn, Estonia
email: sidnurm@edu.ttu.ee