CRITERIA FOR MATRIX FAILURE IN CONTINUOUS FRP-COMPOSITES - A LITERATURE STUDY. PART I: MATRIX CRACKING

Juha Isometsä and Hannu Lahtinen

Rakenteiden Mekaniikka, Vol. 29
No. 1, 1996, s. 3-28

ABSTRACT

Several criteria for the onset of matrix cracking in brittle FRP-composites have been reviewed and different methods to determine the stress distribution in cracked cross-ply laminate in conjunction with fracture based criteria have been compared with each other. A critical survey of these criteria showed that due to heterogenity of composite material, statistical variability of the mechanical properties and enviromental effects, gross simplifications in the crack growth characterization of these materials must be made. Consequently, the criteria presented here may be applicable only to very limited type of specimen geometry, material configurations, enviromental conditions, and loading types.

INTRODUCTION

The use of composites in load bearing structures is primarily motivated by the low weigth-to-stiffness and weigth-to-strength ratios. In the design phase the stiffness characteristics of a composite can be determined fairly accurately by rigorous micromechanical methods. However, despite the extensive research work on the field of composite materials, a comprehensive failure criterion for the composites has not yet been introduced. There are serious doubts that a general failure criterion could actually be achieved. The anisotropy, heterogeneity, and vulnerability due to environmental effects bring a large number of variables to composite design that are usually related to each other. This makes the detailed analysis of composite structures a formidable task.

The matrix failure has received considerable attention in literature. Several micromechanical models that consider the initiation of matrix failure and/or the stiffness loss due to damage are established. As a contribution to this field, we review some of the most important papers of the initiation of matrix failure in order to give the reader guidelines for further study. We also compare some failure criteria for matrix cracking (intralaminar cracking) to analyse the unidirectional tensile test of cross-ply laminates.

An outline of the study is as follows. First we describe the physical aspects of failure of the composites. Then we review some criteria of the onset of the transverse matrix cracking. Finally we present the comparison between the various matrix cracking criteria.

FAILURE OF FIBER REINFORCED COMPOSITES

Unidirectional composites

The longitudinal tensile behaviour of unidirectional composites is governed by the fibers, whereas the remaining responses are very much dependent on the matrix properties. It follows that the remaining responses show some degree of nonlinearity which is due to damage and yielding of the matrix material. The nonlinearity is usually more pronounced in transverse compression and in-plane shear behaviour than in longitudinal compression and transverse tension response (Chamis (1974)).

In the failure analysis the composite material cannot be considered as a mixture of materials but as a structure. The unidirectional composite consists of strong and stiff fibers embedded in relatively weak matrix, and, consequently, only the fiber, the matrix, or the bonding between these two material phases can fail (Hart-Smith (1992)).

According to Hashin (1980), the primary failure modes in the unidirectional composites are a fiber mode and a matrix mode. In the fiber mode the composite fails because of fiber rupture in tension or due to buckling in compression and in the matrix mode a plane crack parallel to the fibers appears. This suggests that the failure surface can be expressed as a piecewise smooth envelope in the stress or strain space rather than a single smooth function as it is proposed by the quadratic failure criteria (e.g. Tsai-Wu, Tsai-Hill).

Composite laminates

There are four primary failure modes in FRP (Fiber Reinforced Plastics) composites. These modes are matrix cracking, fiber/matrix debonding, fiber breaking, and delamination. The development of these failure modes is shown schematically in figure 1 (Reifsnider (1983)).

Figure 1 shows that the initiation of damage is dominated by the matrix cracking in the off-axis plies. These cracks run parallel to fiber directions and are usually spanning the width of the specimen (Flaggs and Kural (1982), Wang (1984)). The number of cracks grows monotonically with increasing load, and it tends to reach a periodic saturation pattern (CDS) that marks the initiation of a new damage mode. In this new damage mode secondary cracks originating from transverse cracks appear in the interlaminar planes. Subsequent damage consists of delamination initiated from these secondary cracks and leads to highly localized damage with fiber failures and finally to fracture (Talreja (1987)).

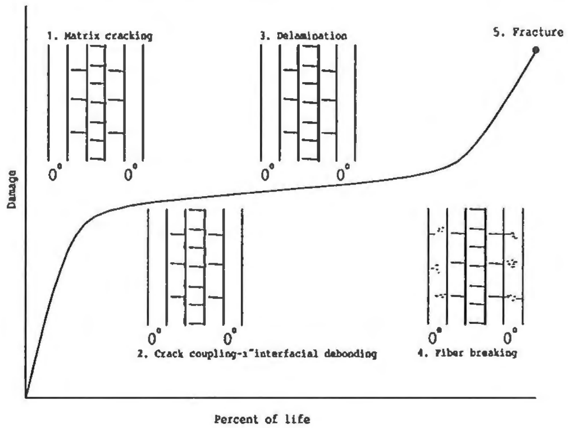

Figure 1. Damage development in [0/90/90/90/0]-laminate.

Matrix cracking

The formation of matrix cracks does not necessarily lead to a catastrophic failure but it can affect the global behaviour of the composite (Bailey et al. (1979), Nairn (1989), Talreja (1994)). It decreases the stiffness of a laminate and helps the formation of other failure modes. Matrix cracks change the behaviour of laminates under thermal loading, which is undesirable for structures that are designed to be dimensionally stable. They can also provide pathways for fluids to penetrate into the composite, and this can cause leaks in vessels, pipes etc. and accelerated moisture absorption in wet environments.

Garrett and Bailey (1977) reported that in GRFP-laminates transverse cracks are associated with voids and areas of high fiber content, and they may also intersect resin-rich areas. A common hypothesis is that microcracks coalesce from local stress raisers such as voids, fiber/matrix debonds etc. and when they achieve a critical size, a transverse matrix crack will form. In the paper of Garrett and Bailey (1977) it is mentioned that the strain magnification factor in the matrix between two closely spaced fibers can be as high as 20. The location and density distribution of the observed microflaws are probabilistic in nature, and their size is usually the order of fiber diameter. Crossman et al. (1980) underlined that in $[\pm 25/90_n]_S$ laminates matrix cracks seem to form first near the free edges of tensile test specimens.

One important phenomenon associated to matrix cracking is that the crack initiation strain depends on ply thickness (Parvizi et al. (1978), Flaggs and Kural (1982)). This phenomenon is known as constrained cracking. Flaggs (1985) found that the crack initiation strain for $[0_2/90]_s$ graphite/epoxy laminate was 2.48 times that would have been expected from a unidirectional transverse tensile test. This led the researchers to conclude that the laminate strength is not a lamina property, and that the matrix cracking is governed by fracture mechanics criteria. Flaggs and Kural (1982) used the term in situ transverse lamina strength for the phenomenon.

In figure 2 the in situ transverse lamina strengths of graphite/epoxy (T300/934) laminates are presented (Flaggs and Kural (1982)). The effect of residual stresses is found by using the laminate theory with the temperature difference $\Delta T = -130\,^{\circ}C$. Here, we see that the transverse lamina strength is predominantly influenced by the thickness of the transverse ply group and by the orientation of adjacent lamina. Furthermore, transverse lamina strength of the measured laminates approached asymptotically the strength of an unidirectional laminate as the thickness of the 90°-ply group is increased.

The constrained cracking behaviour have often been explained by fracture mechanics. Dvorak and Laws (1987) described flaw growth mechanisms in composite laminates. They assumed that cracks initiate from a nucleus created by localized fiber debonding and matrix cracking. These crack nucleuses may propagate on two planes which are parallel to the fiber axis and to the midplane of the ply. In thin plies the crack propagates primarily in the fiber direction, while in thick plies crack nucleus first grows in direction perpendicular to fibers until crack tips reach the adjacent plies, and then propagates in the fiber direction. It follows that the crack length may be regarded as relative to the thickness of the plies, and, therefore, a laminate with a thin off-axis ply group is stronger.

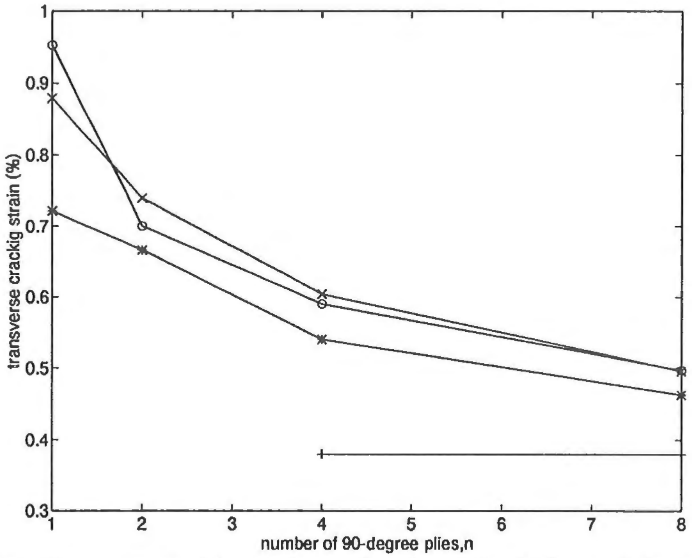

Figure 2. Transverse cracking strain as a function of the thickness of the transverse ply-group. and the orientation of adjacent $\pm\theta$ laminae from Flaggs and Kural (1982). o shows results for $[0_2/90_n]_S$ -laminate, x shows results for $[\pm 30/90_n]_S$ -laminate, $*$ shows results for $[\pm 60/90_n]_S$ -laminate and + the failure strain value for $[90]_{16}$ -laminate.

FAILURE CRITERIA FOR MATRIX CRACKING

The failure criteria for the matrix cracking can be characterized by how the crack growth criterion is formulated. The two major ways to predict matrix cracking are the methods based on the voluminal strength and the approaches based on the fracture mechanics. Further, these approaches can be distinguished by the degree of the stress approximation, i.e. 1-d, 2-d, FEM etc. In the following, we shall use this characterization as a guideline.

Failure criteria based on linear fracture mechanics

In the approaches based on fracture mechanics it is postulated that the failure will occur when it is mechanically possible (stress is the same as the failure stress) and energetically favorable (in the cracking process "supply of energy" > "consumption of energy"). These models are usually studying the self-similar 'growth' of a transverse crack. As discussed earlier, it seems that the transverse cracks coalesce from many material flaws. However, in the failure criteria developed it is assumed that the size of the initial flaw can be regarded as an effective material property and the composite as a homogeneous continuum.

Shear lag models

In their landmark paper Aveston et al. (1971) used the shear lag method to study the failure process of fibrous composites, where the failure strain of the (brittle) matrix is lower than that of the fibers (e.g., steel, glass, and asbestos in cement and carbon fibres in glass). They examined the cracking of the matrix phase between stiff fibers. In the work they assumed that as the fibers remain intact and the matrix cracks span the entire cross-section, the fibers debond completely between adjacent cracks (fig. 3). Now, the main idea is that the increased load in the stronger phase due to matrix cracking is transferred back to the weaker phase by a constant shear stress. This implies that there exists a characteristic length x', which shows the point where the normal stress is fully recovered. The constant shear stress is due to frictional sliding and yielding at the fiber/matrix interface. From the load equilibrium (fig. 3) we can conclude that the total shear force P_m at the interface between fiber and matrix is

$$\sum P_m = \sigma_{mu} A_m = 2\pi r \tau x' N \quad . \tag{1}$$

Hence,

$$x' = \frac{\sigma_{mu} A_m}{2\pi r N \tau} \quad , \tag{2}$$

where σ_{mu} is matrix cracking stress, A_m is the area of the cross-section of the matrix, r is the radius of the fiber, and N is the number of fibers. When a value for τ has been determined, the shear lag distance x' can be defined from (2).

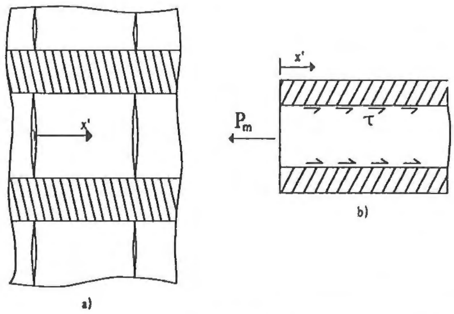

Figure 3. a) Cracked composite cross-section and b) element of load equilibrium.

Aveston et. al. assessed the conditions for multiple matrix cracking to be as follows:

1. Stress in matrix is equal to or greater than the matrix failure stress, and
2. The "consumption" of energy is equal to or less than "supply" of energy between the uncracked state and the state where a single crack appears in the matrix running completely across the specimen.

Hence,

$$2\gamma_m V_m + \gamma_{db} + U_s + \Delta U_f \leq \Delta W + \Delta U_m \quad , \tag{3}$$

where γ_m is the surface energy per unit area of the crack surface, V_m is the matrix volume fraction, γ_{db} is the energy of fiber/matrix interfacial debonding per cross-section of composite, U_s is the work done by the matrix sliding over a distance 2x' on the fiber surface per unit cross-sectional area A, ΔU_f is the increase of elastic energy of fibers per A, ΔW is the work done by external (fixed) load per A, and ΔU_m is the reduction of elastic strain energy of matrix per A. Aveston et al. assumed that the contribution of γ_{db} is neglible and derived an expression to the strain ϵ_{muc} for multiple matrix cracking to be

$$\varepsilon_{muc} = \left(\frac{12\tau\gamma_m E_f V_f^2}{E_c E_m^2 r V_m} \right)^{1/3} \quad , \tag{4}$$

where E_f and E_m are the Young's modulus of fiber and matrix, respectively.

As an extension to their work, Aveston and Kelly (1973) considered the case where the assumption of complete debonding between the constituents was removed. They defined the additional load $\Delta\sigma$ carried by fibers due to cracking to be

$$\Delta\sigma(y) = \sigma_f(y) - \sigma_f(y \to \infty) = \sigma_f - E_f\varepsilon_{mu} \quad , \tag{5}$$

where σ_f is the stress in the fiber. They assumed that

$$\frac{d(\Delta\sigma)}{dy} = H(v_f - v_m) \quad , \tag{6}$$

where H is a constant, while v_f and v_m are the elastic displacements in the y-direction of fiber and matrix, respectively (fig. 4). According to Aveston and Kelly the differential equation for load transfer is

$$\frac{d^2(\Delta\sigma)}{dy^2} = \phi\Delta\sigma \quad , \tag{7}$$

where

$$\phi = \frac{HE_c}{E_f E_m V_m} \quad , \tag{8}$$

and where E_c is the Young's modulus of the composite. The general solution for (7) is

$$\Delta\sigma = \Delta\sigma_0 e^{-\sqrt{\phi}y} \quad , \tag{9}$$

where

$$\Delta\sigma_0 = \Delta\sigma \quad . \tag{10}$$

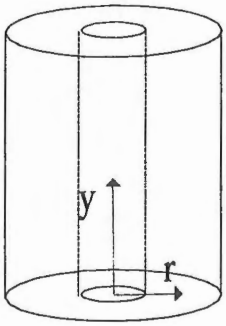

Figure 4. Composite sylinder of fiber and matrix.

The key point is now to define the unknown constant H. To solve H, Aveston and Kelly assumed unrealistically that σ_y is constant, which violates previous assumption that it depends on y (Talreja (1993)). However, they came to an expression for the constant H.

Later, Garret and Bailey (1977) applied the results of Aveston and Kelly to the transverse cracking phenomenon. They concluded that the analysis for the cracking of the matrix phase between stiff fibers could be also used for cross-ply laminates, where a transverse-ply with relatively low stiffness in the loading direction is sandwiched between two 0°-plies. Garrett and Bailey derived the expression for the transverse-crack spacing versus the applied tensile load, but as they applied the results obtained by Aveston and Kelly, they also made the same mistakes.

In their paper Parvizi, Garrett, and Bailey (1978) used the same analysis to study the constrained cracking behaviour and derived an expression for the transverse failure strain to be

$$\varepsilon_2^{cr} = \sqrt{\frac{bE_1G_c\sqrt{\phi}}{(b+d)E_2E_c}} \quad , \qquad (11)$$

where

$$\phi = \frac{E_cG_{23}(b+d)}{E_1E_2bd^2} \quad , \qquad (12)$$

and where E_1 is the longitudinal Young's modulus, E_2 the transverse Young's modulus, G_{23} the transverse shear modulus of a lamina, E_c the effective modulus of the laminate, and G_c the critical strain energy release rate of the composite. In addition, the thicknesses of the 0°- and 90°-plies are b and 2d, respectively (fig. 5). From equations (11) and (12) we see that the transverse cracking strain decreases as the thickness of the 90°-ply group is increased and the others are kept constant. This is a correct trend. In their paper Parvizi et al. compared experimental data for glass/epoxy cross-ply laminates and the results obtained by expression (11), and it seems that the theoretical values agree quite well with the experiments when the ratio 2d/b is lower than 1.

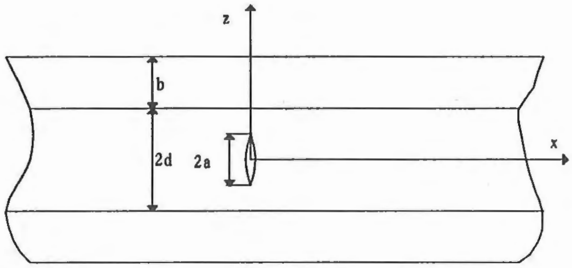

Figure 5. A laminate with a microcrack.

The value of transverse cracking strain is also affected by thermal strains induced in the curing process. Bailey, Curtis, and Parvizi (1979) determined the thermal strains by an elementary one-dimensional thermal analysis and summed them up to the equation (11) in order to have a more realistic value for the transverse cracking strain. They also studied the longitudinal splitting of the 0°-plies, which were observed in the experiments of tensile loaded glass/epoxy cross-ply laminates, and compared the results of their crack constraint theory with the experiments made for [0/90/0]- and [90/0/90]-laminates, which were both glass and carbon reinforced. The results obtained by their theory for carbon/epoxy laminates do not correspond very well with the tests. However, the paper of Bailey et al. is a good summation of the theoretical and experimental results and observations of the matrix cracking phenomenon obtained by Bailey and his co-workers.

The shear lag approach can be modified by removing the assumption that the whole 90°-ply group is the shear transfer region as discussed previously. This is done by assuming that there exists a thin interlaminar shear layer between the cracked and constraining ply group which transfers the load. Lim and Hong (1989) applied the modified shear lag approach to cross-ply laminates, and they incorporate the effect of the interlaminar shear layer, thermal strains, and Poisson's effect to the matrix cracking problem. They used the Griffith energy balance criterion similar to that described before (eq. (3)) in order to predict the onset of transverse cracking. However, in this type of analysis some value for the thickness (d_0) and the shear modulus (G^i) of the shear layer must be assumed.

All the analyses so far have dealt with cross-ply laminates. Flaggs and Kural (1982) showed that the transverse cracking failure strain determined by eq. (11) for $[\pm 30/90_n]_s$- and $[\pm 60/90_n]_s$-laminates did not agree well with the experiments. In his article Flaggs (1985) studied the tensile matrix failure in the 90°-ply of $[\pm \theta/90_n]_s$-laminate and mixed mode failure in θ°-ply of $[0_2/\theta]_s$-laminates by an approximate two-dimensional shear-lag model with the fracture mechanics criteria. He used the shear lag approach to analyse the load transfer mechanism for inplane shear and normal stress between cracked inner ply-group and outer ply-group (fig. 6). The governing equations are equivalent to expression (7) with the exception that the contribution of the inplane shear stress is added to the expression, which makes the analysis two dimensional.

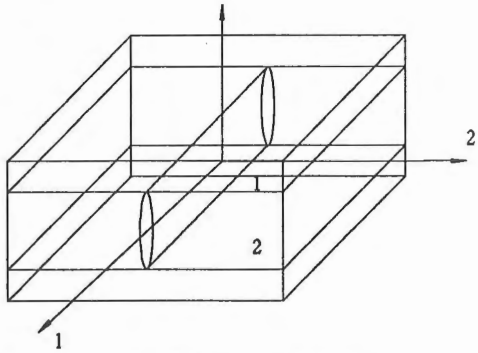

Figure 6. Laminate geometry for 2-d shear-lag model.

Load redistribution is now governed by equation

$$\begin{bmatrix} \dfrac{d^2}{dx_1^2} - L_{11} & -L_{16} \\ -L_{61} & \dfrac{d^2}{dx_1^2} - L_{66} \end{bmatrix} \begin{Bmatrix} \Delta n_1 \\ \Delta n_6 \end{Bmatrix}^{(1)} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad , \tag{13}$$

where L_{ij} are constants, while Δn_1 and Δn_6 are additional load increments (stress resultants of the lamina) of the normal and shear stress, respectively. The unknown constants L_{ij} are obtained from the equilibrium of the interlaminar shear components in consistence with the assumption that each ply-group behaves like a separate Mindlin plate. The transverse cracking criterion is similar to used by Aveston et al., namely (in sligthly different form than eq. (3))

$$\Delta W - \Delta U \geq 2a_0 G_c \quad , \tag{14}$$

where a_0 is the change in crack length between the initial crack-free state and the current state, and G_c is the critical strain energy release rate in the transverse direction of the unidirectional lamina. In the previously mentioned shear-lag analyses (Garrett and Bailey (1977)) and related papers (Lim and Hong (1989)), it was assumed that $2a_0 = 2d$, which is the thickness of the whole transverse ply-group. Flaggs assumed (motivated by the work of A.S.D Wang and his co-workers, which will be discussed later) that

$$a_0 = \begin{cases} d \ \mathit{if} \ d \le a_c \\ a_c \ \mathit{if} \ a_c > d \end{cases} , \tag{15}$$

where a_c is the critical flaw size. He assumed that a_c is 2.5 times the thickness of a single ply. This model gives quite good results compared with experimental data for $[0/90_n]_{\bar{s}}$, $[\pm30/90_n]_s$- and $[\pm60/90_n]_s$-laminates. The 2-d shear-lag model agrees also with the measured results better than the shear-lag model applied by Bailey et al. (1979) with the assumption $2a_0 = 2d$. However, the choice the value for a_c, as it will be discussed later, affects the difference between the data and the theoretical results.

Flaggs considered also the mixed mode matrix cracking of $[0_2/70]_s$- and $[0_2/80]_s$-laminates, where the inplane shear stress contributes the failure. In the mixed mode fracture G_c is not necessarily a constant, and, therefore, it is a function of all the crack loading modes present, namely

$$G_c = G_c(G_I, G_{II}, G_{III}) \ . \tag{16}$$

By assuming some values for the ratios G_{II}/G_I and G_{III}/G_I, he used the model to predict the in situ strengths of the laminates, which he showed to be in good agreement with the experimental data.

There are numerous studies of the transverse cracking problem analysed by the shear lag approach. They all study the same differential equation (7), and the main difference of the studies is in the determination of the shear lag parameter ϕ. For example, Tan and Nuismer (1989) solved the matrix cracking problem using an approximate two-dimensional elasticity solution and fracture mechanics with the assumption $2a_0$=2d. However, they attained the same differential equation (eqn (7)) as was earlier obtained in the shear lag approaches. The treatment of the problem is quite similar to Flaggs (1985). The main difference between these two studies is that when Tan and Nuismer assumed a linear variation of the transverse shear stresses through the thickness of the laminate, Flaggs assumed these stresses to be constant over the ply-group and multiplied the stress field by a shear correction factor.

Strain energy release rate curve by FEM

A significant amount of work in the field of matrix-dominated cracks has been done by Wang and his co-workers (e.g., Wang and Crossman (1980), Crossman et al. (1980), Wang (1984), Wang (1986)). They have used classical fracture mechanics in conjunction with the finite element method in order to study matrix-dominated sublaminate cracks (matrix cracks, edge and interior delamination, longitudinal splitting). In this chapter their contribution to the transverse cracking is considered.

In all of the above mentioned methods authors have remained within the framework of ply-elasticity, where the properties of individual laminae are assumed to be homogeneous, and effective anisotropic material properties are used to describe the behaviour of a lamina. On this macromechanics level the true identities of the microdefects are lost. However, when we are using the classical fracture mechanics, some quantity for initial crack size must be determined. Wang (1984) used the 'effective flaw' concept where the contribution of the microstructural material flaws is retained by the quantity $2a_0$, which is an effective material property (fig. 5). The same concept is tacitly used in other studies involving the use of fracture mechanics theory.

In their articles Wang and Crossman (1980) and Crossman et al. (1980) used the 2-d generalized plane strain finite elements in order to study the propagation of a Griffith crack in the 90° -ply group of graphite/epoxy $[\pm 25/90_n]_s$-laminates. The energy balance criterion is

$$\frac{d}{da}(W-U)=\lim_{a\to 0}\frac{\Delta U}{\Delta a}=G_f(a) \quad , \tag{17}$$

where the strain energy release rate $G_f(a)$ associated with half crack length is determined by crack-closure procedure. The strain energy release rates due to mechanical and thermal loads can be expressed in the general form as

$$G_\varepsilon = C_\varepsilon (a/t) t (\varepsilon_0)^2 \tag{18}$$

and

$$G_T = C_T (a/t) t (\Delta T)^2 \quad , \tag{19}$$

where ϵ_0 is the far-field tensile strain, and t is a characteristic length. The shape functions C_ϵ and C_T are associated with mechanical and thermal loads, respectively, and they are independent of the applied loads and depend only on the lamination configuration and the crack size. The total strain energy release rate G_f is expressed as (Wang (1984))

$$G_f = [\sqrt{C_\varepsilon}\varepsilon_0 + \sqrt{C_T}\Delta T]^2 t \quad . \tag{20}$$

In the figure 7 there is shown a typical shape of functions C_ϵ and C_T. The functions can be calculated by the finite element method keeping the far-field tensile strain constant for different crack sizes. There are also exact solutions for G (Isida (1973)) and for K_I (Hilton and Sih (1971)) that give very similar results for the problem.

From the figure 7 we see that the value of C first increases as the crack grows, but when the crack tip is approaching the interface of the plies, it begins to decrease. This means that when the critical strain energy release rate is reached the crack will grow in unstable manner until the value of the arrest toughness G_a or the boundary of the constraining layer is reached.

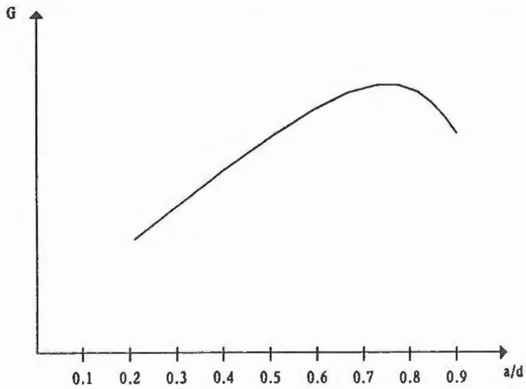

Figure 7. A typical shape of functions C_ϵ and C_T.

Now, the criteria for the onset of a transverse crack is

$$G_f(a_0) = G_c \quad if\ d > a_0$$
$$G_{f\max}(a) = G_c \quad if\ d \le a_0 \qquad , \qquad (21)$$

that is, if the thickness of the 90°-ply group is less than the effective crack size, the maximum value of the G curve is used (fig. 7). The condition (21) sets a maximum to the size of the effective flaw and, hence, a lower bound to the tensile strength of a laminate. This is not the case in the approaches that study the growth of a transverse crack spanning the entire thickness of the 90°-ply group (i.e., 2a=2d), which is the reason why, for example, the model created by Bailey et al. (1979) gives too low values for the onset of transverse cracking when d is large. Furthermore, when we are studying the growth of cracks spanning the entire thickness of the 90°-ply group by the Griffith criterion, the rate of strain energy release rate should be positive. However, the G-curve in figure 7 do not support this assumption. These considerations were given by Flaggs and Kural (1982). It should also be pointed out that the strain energy release rate curves are usually determined up to 0.9 for the ratio a/d. This is because of the difficulties occuring when two singularities, namely, the crack tip and the ply interface, are close to each other.

To obtain the solution for the problem, we must determine values for the critical strain energy release rate G_c and for the effective flaw size a_0. This is often the most difficult part of the analysis. Most of the studies of transverse ply failure are adressed to analyse the self-similar growth of a mode I crack. However, in composites the value of G_{Ic} for crack propagation in the direction of the fiber axis (L) is lower than in the direction perpendicular to the fiber axis (T). Wang (1984) referred to an experiment where the ratio G_{Ic}^L/G_{Ic}^T for a graphite/epoxy system detected by the splitting cantilever beam method was about 0.6-0.8. This demonstrates the directional dependence of G_c. In spite of this, only one value for G_c is used in the analyses.

As discussed before, in thin laminates there is no visible initial flaw size a_0 to be considered as a material parameter. This means that we must determine a_0 from the experiments in conjunction with proper analysis. One way to do this is to measure the tensile transverse strength σ_2^u of an unidirectional laminate and calculate the effective flaw size from the Griffith's formula

$$a_0 = \frac{G_c E_2}{\pi\left(\sigma_2^u\right)^2} \quad . \tag{22}$$

However, it must be remembered that failure in unidirectional laminates is likely to initiate from the surface and, for example, in the case of $[\pm 25/90_n]_s$-laminates we are studying an internal crack, i.e., a different problem. Bailey et al. (1979) found this to be an explanation why the strength of unidirectional laminates in tests was lower than that measured for thick cross-ply laminates.

The equation (22) gives a constant value for the initial crack size, which is also greater than the thickness of a single layer. That is why, in the [0/90/0]-laminate the initial flaw would be too large compared to the thickness of the 90°-ply.

Rothschilds et al. (1988) studied the effect of hygrothermal histories on the onset of matrix cracking. They compared the 2-d shear lag method by Flaggs (1985) and the method proposed by Wang (1984) for graphite/epoxy $[(\pm 45/90/0)_6/90_n]_s$-, $[\pm 45/90/0/90_n]_s$-, and $[\pm 0/90_n]_s$-laminates in dry conditions. Because of the inaccuracies of the results given by the 2-d shear lag model, they used the finite element method (Wang (1984)) to study the hygrothermal effects. Ilcewicz et al. (1991) also used the strain energy release method by FEM to analyse the matrix cracking in laminates with resin-rich interlaminar layers.

Variational analysis

Hashin (1985) analysed cracked cross-ply laminates by a variational approach. In this approach the region under consideration is a plane stress element of a laminate so that the x-direction of the RVE (representative volume element) is bound between two adjacent cracks spanning through the entire thickness of the 90°-plies and the thickness direction is bound between the top and bottom surface of the laminate (fig. 5). The stress tensor can be divided into two parts: the initial stress tensor of the undamaged state and the stress perturbation tensor caused by cracking. It is assumed that the stress perturbation in the x-direction is constant through the thickness of the plies, namely $\sigma_x^{(1)} = \sigma_x^{(1)}(x)$ and $\sigma_x^{(2)} = \sigma_x^{(2)}(x)$. This assumption was justified by the fact that the outer plies give a constraint to the crack opening displacement (COD) so that the crack tip stresses are finite.

The problem is formulated as a two dimensional boundary value problem, which is then solved using variational methods so that a statically admissible stress field is constructed and the constants involved are determined analytically by the principle of minimum complementary energy.

In his paper Hashin (1985) studied the stiffness reduction of a cracked laminate and the stress field between two adjacent cracks. Nairn (1989) extended his analysis to include thermal strains and gave a solution for the strain energy release rate. Varna (1991) extended Hashin's work by considering a nonuniform longitudinal stress in the 0° -layers.

The results obtained by 2-d variational analysis for the transverse cracking strain are in better agreement (Nairn (1989), Varna (1991)) with the experimental data than those predicted by the shear lag approach. The obvious reason for this is that the stress state in the cracked laminate determined by this 2-d approach is in better agreement with the reality than that obtained by shear lag analysis. However, the fracture mechanics criteria are evidently the same as in the classical work of Bailey et al. (1979), and this gives too low values for the onset of transverse cracking when the thickness of the 90°-ply group is high, as described previously. Varna and Berglund (1993) extended further their model to include nonuniform stress distribution in the thickness direction and the effective flaw size as a parameter in their model. This model contains several constants, which depend on each other, and which have to be found by numerical considerations.

Strain energy release rate curve by analytical methods

Dvorak and Laws (1987) derived the expressions for the onset of transverse cracking using the analytical strain energy release rate equation for a crack in an orthotropic medium. They considered the cases of thick and thin 90°-ply groups separately. Dvorak and Laws postulated that a microcrack will start to propagate as a Griffith crack when the microcrack becomes a crack nucleus of certain critical width 2a.

In the case of a thick ply they derived expressions for the onset of matrix cracking directly from the strain energy release rate expression for a crack in an orthotropic medium similar to that given by Griffith in the isotropic case. They arrived at the relationship $G^T/G^L = 2$ (the microcrack may grow in the longitudinal (L) or transverse (T) direction of the fiber axis) and concluded that the onset of failure of a thick 90°-ply group is determined by type T cracking because the experimental studies indicate that the ratio $G_c^L/G_c^T > 0.5$.

When the 90°-ply group is relatively thin, the presence of the adjacent layers is affecting the stress state at the tip of the crack nucleus. For this case, Dvorak and Laws introduced the shape functions ξ_i, which are functions of the ratio a/d for the different cracking modes. The idea is similar to that used by Wang, as discussed above. Now, the strain energy release rate equation for type L crack is

$$G^L = \frac{\pi a}{4}\left(\xi_I \Lambda_{11}^0 \sigma_1^2 + \xi_{II} \Lambda_{44}^0 \tau_{12}^2 + \xi_{III} \Lambda_{66}^0 \tau_{23}^2\right) , \tag{23}$$

where

$$\Lambda_{11}^0 = \Lambda_{44}^0 = 2(1/E_2 - \upsilon_{12}^2/E_1), \quad \Lambda_{66}^0 = 1/G_{12} \tag{24}$$

However, in the lack of the knowledge of ξ_i Dvorak and Laws considered the case of thin 90°-ply group and assumed that 2a=2d, $\xi_I = \xi_{II} = 1$, and that the contribution of mode III cracking is neglible, which means that the third term in the parenthesis of equation (23) vanishes. With these assumptions they arrived at a very simplified expression for the onset of transverse failure of thin 90°-ply group. The transition from thin to thick plies is then defined by the intersection of the curves determined by the expression (23) and the strain cut-off value for thick plies.

Dvorak and Laws also considered the transverse strength of a unidirectional laminate and postulated that the critical crack nucleus is most likely to be at the surface of the laminate. In this case the estimate for the ratio $(\sigma_2^{90})_\alpha/(\sigma_2^{u})_\alpha = (1.12\surd 2)$.

Failure criteria based on strength of materials

In the strength based approaches the constraint effect is explained by an assumption that the failure probability is related to the volume of the stressed material.

Shear lag models

Fukunaga et al. (1984) studied the failure of graphite/epoxy cross-ply laminates by the means of the statistical strength analysis with shear lag model assuming the existence of the interlaminar shear layer and taking the thermal residual stresses and Poisson effect into consideration. They used the Weibull strength theory

$$\frac{\sigma_2^1}{\sigma_2^0} = \left(\frac{V_0}{V_1 \delta_1} \right)^{1/m} , \qquad (25)$$

where σ_2^0 is the 90°-ply strength predicted for material volume V_0, σ_2^1 is the 90°-ply strength for material volume V_1, m is a material constant, and δ reflects the effect of stress nonuniformity on the 90°-ply strength. When the onset of transverse cracking is considered, $\delta_1 = 1$, which means that the shear lag approach is not used to determine the first cracking event of the 90°-ply.

Taking into account residual thermal stresses and Poisson effect, the equation for the transverse cracking strain is

$$\bar{\varepsilon}_x = \frac{1}{Q_{22}\left(1 - \dfrac{Q_{12}A_{12}}{Q_{22}A_{22}}\right)} \left[\sigma_2^0 \left(\frac{V_0}{V_1} \right)^{1/m} - \sigma_x^{(2)T} \right] , \qquad (26)$$

where Q_{ij} and A_{ij} are the reduced and in-plane stiffnesses of the laminate, respectively, and $\sigma_x^{(2)T}$ is the thermal residual stress in the 90°-ply. In the determination of the ultimate failure of the laminate they showed that by choosing the ratio G^i/d_0 (2d is the thickness of the 90°-ply) and assuming proper values for σ_2^0 and m, a curve that fits the experiments can be determined.

DISCUSSION

The criteria for the onset of the matrix cracking have been introduced so far only for very simple laminates and loading conditions. The theories are usually formulated to predict the transverse strength of a 90°-ply group of a thin cross-ply or $[\pm\theta/90_n]_s$-laminates. This is in sharp contrast to reality where substantially large composite parts with complicated lay-up patterns are used as shell structures to utilize the most of the features of the composites. Further, it should be also noted that the laminates with thick 90°-ply groups are rare in applications, and in the case of cross-ply laminates the stacking sequences like [0/90/0/90/0/90...] are common practice.

We feel that the concepts of the linear fracture mechanics have been applied somewhat inadequately to the theories presented here. Most of the strength predictions use the energy balance criterion (eq. 3) as presented by Aveston and Kelly. Further, by setting the size of the initial flaw to be the same as the thickness times the width of the transverse ply, the criterion for the onset of the transverse crack propagation is determined by the growth of a full transverse crack that already exist. To overcome this dilemma some authors (Dvorak & Laws, Lim & Hong, Varna & Berglund) have postulated that in thin 90° -ply the crack grows in the width direction, i.e. the size of the initial flaw is d times some length in the width direction. The idea is that the size of the initial flaw in the width direction does not have any effect on the stress intensity of the crack tip and consequently to the onset of crack propagation. This seems physically questionable for small crack lengths (in transverse direction) although the cracks tend to grow along the fiber direction (Sierakowski and Chaturvedi 1986). Further, we feel that the concept of macroscopic initial flaw is somewhat questionable, because they are not detected in actual laminates and the use of this parameter only shows the lack of rigorous micromechanical models. In addition, many researchers have used the Weibull theory to explain the variation of the strength as a function of volume. However, both the Weibull theory and the initial flaw size should be verified with unidirectional laminates, where the volume effect exists, but where the constraint effect is absent.

Some authors claim that the application of LEFM to the matrix cracking problem eliminates the use of adjustable parameters like those used in the strength based theories. Generally, this is not true. The large discrepancy in the measured values for G_c, the concept of initial macroscopic flaw size a_0, value of residual stresses (indicated by the value of stress free temperature ΔT), and the introduction of non-measurable material constants leaves, in fact, many parameters left to be speculated. There seems to be a tendency to fit the obtained criteria by choosing some of these parameters so that the theory under consideration gives best correlation to the measured data. Therefore, we have compared some of the methods presented here for the cross-ply laminates. The criteria are divided in two groups. In the first group there are the criteria, where the size of the initial flaw is the same as the thickness of the transverse ply, and in the second group there are the criteria, where a_0 is a parameter. These groups are presented in table 1, and the material parameters are shown in table 2. The results for the transverse cracking strain are presented in figures 8-9, where also the measured values are plotted. The results for the 2-d shear-lag and 2-d elasticity are taken from the references. All the other results are calculated by the authors. The finite element results are obtained by the ABAQUS-program using parabolic generalized plain strain elements with crack tip singularity.

Table 1. The criterion under consideration.

Group 1.			Group 2.		
Method	Legend	Ref.	Method	Legend	Ref.
shear-lag	+	Bailey et al. 1979	2-d shear-lag	+	Flaggs 1985
shear-lag	__	Lim & Hong 1989	FEM	*	Wang 1984
2-d elasticity	x	Tan & Nuismer 1989	experimental data	--	Flaggs & Kural 1982
analytical energy release rate	-.-	Dvorak & Laws 1987 [1]			
variational	o	Nairn 1989			
variational	--	Varna 1991			
experimental data	*	Flaggs & Kural 1982			

1) the results are obtained from expression of thin plies without strain cut-off

The general form of the energy release rate equation in group 1 is

$$G_f = \left(\varepsilon_2^{cr}\right)^2 E_1 d \sqrt{f(C_{ij}, d_0, b, d)} \quad , \tag{27}$$

where C_{ij} and d_0 are material parameters. The difference between the methods under consideration is in the form of the function f. Because the shape of G is determined by the strain, the value of $\sqrt{f}$ and the choice of G_c shifts the position of the criteria relative to the experimental data. We like to emphasize that the comparison is not mentioned to be quantitative. By choosing the values of G_c and a_0 otherwise, any method can be shifted closer to the experimental data.

Table 2. Material properties for T300/934.

Parameter	Value	Parameter	Value
E_1	138 GPa	α_1	0.09 μ/°C
E_2	11.7 GPa	α_2	28.8 μ/°C
G_{12}	4.56 GPa	ΔT	-147 °C
G_{23}	4.18 GPa	G_c	228 J/m^2
υ_{12}	0.29	b	0.264 mm
υ_{23}	0.40	d	n * 0.132 mm (n=1,2,4,8)
a_0	d (n=1,2), 0.33 mm (n=4,8)	G/d_0	9.7E+13 Pa/m

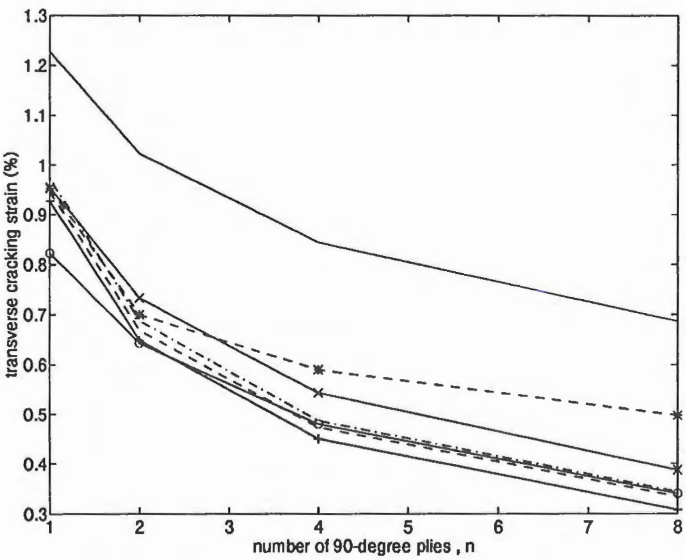

Figure 8. Transverse cracking strain of a $[0_2/90_n]_S$ T300/934 graphite/epoxy laminate family by several criteria as $a_0 = d$.

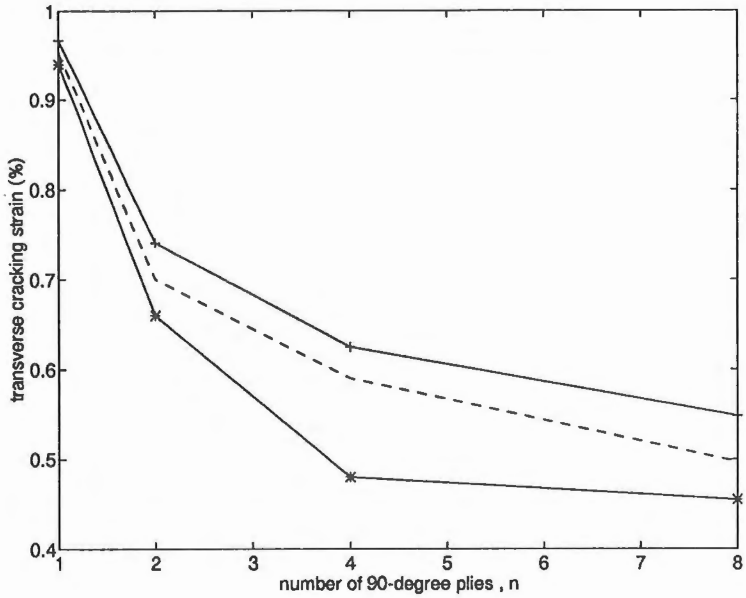

Figure 9. Transverse cracking strain of a $[0_2/90_n]_S$ T300/934 graphite/epoxy laminate family by several criteria as a_0 (n=1,2) = d, a_0 (n=4) = 0.625 d and a_0 (n=8) = 0.3125 d.

CONCLUSIONS

The problem of matrix failure concerning matrix cracking and delamination has been examined. A critical survey shows that the criteria reviewed are applicable only to very limited types of specimen geometry, material configurations, environmental conditions, and loading types. Also, the so-called in situ strength problem is still lacking physical explanation. The authors feel that the thorough understanding of the curing process and the micromechanical behaviour of the composites would be valuable in the analysis of matrix failures. Also, the theory of the fracture mechanic may need modifications when applied to the composites.

REFERENCES

AVESTON J., COOPER G. A., KELLY A., Single and Multiple Fracture. Conference Proceedings, National Physical Laboratory: The Properties of Fibre Composites, I.P.C. Science and Tecnology Press, 1971, pp. 15-26.

BAILEY J. E., CURTIS P. T., PARVIZI A., On the Transverse Cracking and Longitudinal Splitting Behaviour of Glass and Carbon Fibre Reinforced Epoxy Cross Ply Laminates and the Effect of Poisson and Thermally Generated Strain. Proc. R. Soc. Lond. A., 366 (1979), pp. 599-623.

CHAMIS C. C., Micromechanics Strength Theories. In Broutman L. J., Knock R. H. (eds): Composite Materials, 5. Academic Press, New York 1974.

CROSSMAN F. W., WARREN W., WANG A. S. D., LAW G. E., Initation and Growth of Transverse Cracks and Edge Delamination in Composite Laminates Part 2. Experimental Correlation. J. Composite Mater., supplementary volume 14 (1980), pp. 88-108.

DVORAK G. J., LAWS N., Analysis of Progressive Matrix Cracking in Composite Laminates II. First Ply Failure. J. Composite Mater., 21 (1987), pp. 309-329.

FLAGGS D. L., KURAL M. H., Experimental Determination of the In Situ Transverse Laminate Strength in Graphite/Epoxy Laminates. J. Composite Mater., 16 (1982), pp.103-115.

FLAGGS D. L., Prediction of Tensile Matrix Failure in Composite Laminates. J. Composite Mater., 19 (1985), pp. 29-50.

FUKUNAGA H., CHOU T-W, PETERS P. W. M., SCHULTE K., Probabilistic Failure Strength Analyses of Graphite/Epoxy Cross-Ply Laminates. J. Composite Mater., 18 (1984), pp. 339-356.

GARRETT K. W., BAILEY J. E., Multiple Transverse Fracture in 90° Cross-Ply Laminates of a Glass Fibre-Reinforced Polyester. J. Mater. Sci, 12 (1977), pp. 157-168.

HART-SMITH L. J., A Scientific Approach to Composite Laminate Strength Prediction. Composite Materials: Testing and Design (tenth volume), ASTM STP 1120, G. C. Grimes ed., American Society for Testing and Materials, Philadelphia, 1992, pp. 142-169.

HASHIN Z., Failure Criteria for Unidirectional Fiber Composites. J. Applied Mech., 47 (1980), pp. 329-334.

HASHIN Z., Analysis of Cracked Laminates: A Variational Approach. Mech. Mater., 9 (1985), pp. 121-136.

HILTON P. D., SIH G. C., A Laminate Composite with a Crack Normal to the Interface. Int. J. Solids Struct., 7 (1971), pp. 913-930.

ILCEWICZ, L. B., DOST, E. F., McCOOL, J. W., GRANDE, D. H., Matrix Cracking in Composite Laminates with Resin-Rich Interlaminar Layers. Composite Materials: Fatigue and Fracture (Third Volume), ASTM STP 1110, T. K. O'Brien, ed., American Society for Testing and Materials, Philadelphia, 1991, pp. 30-55.

ISIDA M., Method of Laurent Series Expansion for Internal Crack Problems. Methods of Analysis and Solutions of Crack Problems, G. C. Sih ed., Noordhoff, Holland, 1973, p. 56.

LIM J. W., HONG C. S., Prediction of Transverse Cracking and Stiffness Reduction in Cross-Ply laminated Composites. J. Composite Mater., 23 (1989), pp. 695-713.

NAIRN J. A., The Strain Energy Release Rate of Composite Microcracking: A Variational Approach. J. Composite Mater., 23 (1989), pp. 1106-1129. (Errata 24 (1990) p.233)

PARVIZI A., GARRETT K. W., BAILEY J. E., Constrained Cracking in Glass Fibre Reinforced Epoxy Cross-Ply Laminates. J. Mater. Sci, 13 (1978), pp.195-201.

REIFSNIDER K. L., HENNEKE E. G., STINCHCOMB W. W., DUKE J. C., Damage Mechanics and NDE of Composite Laminates. Mechanics of Composite Materials. Recent Advances. Pergamon, 1983. pp. 399-420.

ROTHSCHILDS R. J., ILCEWICZ L. B., NORDIN P., APPELGATE S. H., The Effect of Hygrothermal Histories on Matrix Cracking in Fiber Reinforced Laminates. J. Engng Mater. Tech., 110 (1988), pp. 158-168.

SIERAKOWSKI, R. L., CHATURVEDI S. K., Crack-Growth Beahiviour of Polymer-Matrix Composites. Composites '86: Recent Advance in Japan and United States, K. Kawata ed. Proc. Japan-U.S. CCM-III, Tokyo, 1986. pp. 257-265.

TALREJA R., Damage and Failure of Composites. Lectures at the Luleå University of Technology, 1993.

TALREJA, R., Modeling of Damage Development in Composites Using Internal Variables Concepts. Wang A. S. D., Haritos G. K. (eds): Damage Mechanics in Composites. ASME 1987.

TAN S. C., NUISMER R. J., A Theory for Progressive Matrix Cracking in Composite Laminates. J. Composite Mater., 23 (1989), pp. 1029-1047.

VARNA J., Transverse Cracking in Thin Cross-Ply Laminates. Institute of Technology, Dept of Mech Eng, Linköping, Sweden, 19 (1991), p. 66.

VARNA J., BERGLUND L., Two-dimensional Transverse Cracking in $[0_m/90_n]_s$ Cross-ply Laminates. Eur. J. Mech., A/Solids, 12 (1993), pp. 699-723.

WANG A. S. D., Fracture Mechanics of Sublaminate Cracks in Composite Materials. Composite Tech. Review, 6 (1984), pp. 45-62.

WANG A. S. D., On Fracture Mechanics of Matrix Cracking in Composite Laminates. Proceedings of International Symposium On Composite Materials and Structures. Technomic Publishing Co, 1986, pp. 576-584.

WANG A. S. D., CROSSMAN F. W., Initation and Growth of Transverse Cracks and Edge Delamination in Composite Laminates Part 1. An Energy Method. J. Composite Mater., supplemenary volume 14 (1980), pp. 71-87.

Juha Isometsä, M.Sc. (Eng.) University of Oulu,
Hannu Lahtinen, Lic.Tech. Engineering Mechanics Laboratory