Evaluation of Standards for Fracture Mechanics Testing - Tolerances for Test Equipment

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Abstract

Some frequently used standard methods for fracture mechanics testing are reviewed. Requirements as to load and displacement measurements are evaluated. The effects on inaccuracy in the testing fixture are calculated and compared with other error terms.

1. Introduction

Fracture mechanics testing is expensive owing to its complexity and the strict requirements regarding accurate testing equipment. Even with perfect equipment, a series of test exhibits a considerable scatter, inherent in the properties of the materials. This will not be considered here. However, even when strictly defined the material property fracture toughness is a random variable with a significant standard deviation. In other words a set of specimens, that are identical from a continuum mechanical point of view will yield a different set of results for the fracture toughness even when tested and evaluated in an identical fashion.

In practical work there are also errors in the specimen geometry, in testing fixtures and in measurements of force and displacement. Maximum values are specified in the standardised testing methods (1), (2) and (3).

If the sum of these experimental errors is small compared to the typical scatter in the fracture parameters of the material, one should consider the possibility of getting a better view of this scatter by easing the conditions imposed on the testing equipment and thus enabling more tests to be performed at the same expense.

It could also be valuable to investigate the relationship between various error terms as an independent problem. Imposing unnecessarily narrow limits may make some terms unreasonably expensive, whereas extending the limits may allow a large error to be decreased without cost.

A brief review of the requirements in the standard testing methods [1], [2] and [3] is made. A detailed analysis of the influence of errors in force and displacement measurements and the testing fixture is given in appendix 1-2. A change of the testing equipment is suggested to reduce the risk that the upper roller becomes oblique.

The three-point bend specimen is the only type of specimen that has been considered.

2. Load and displacements' measurements

With modern testing systems there is no difficulty in recording force within prescribed limits. Load cells are normally constructed to be insensitive to small side loads, and they are equipped with quick calibration checks. Annual calibration by a national calibration centre is recommended to certify the accuracy, reproducibility, linearity and hysteresis.

Errors in the load measurement affect the result of a fracture mechanics test proportionality, i.e. a certain percentage error gives the same percentage contributions to the error in *K*.

Errors from electronic devices are usually represented according to fig. 1. Here the two straight lines represent the envelope of the error conditions. The errors should be smaller than either 1% of the true measured value or 0.2% of the range. The loop *OA* shows a calibration loop, loading and unloading forces. From this loop relatively mean error, linearity and hysteresis can be evaluated. The only problem normally came across is when a small part of the loading range of the machine is to be used. When using small loads, it should also be noted that the load cell should be "put to zero" with respect to mass and inertia of the load fixtures.

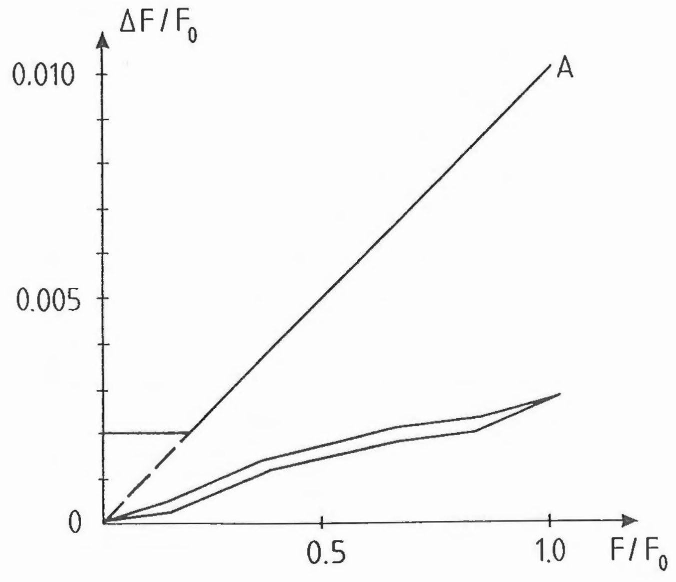

Fig. 1.Definitions of errors and error limits for a force transducer.

The limits prescribed by various standards (BS 5762 [1] and ASTM E-813 [2]) are shown in table 1.

Table 1. Error limits for displacement transducers and load cells in various standards.

Standard	Displacement limits	Load limits
BSI 5762	The linearity of the gauge shall be such that the deviation from true displacement is no more than 0.003 mm for displacements up to 0.3 mm and no more than 1% of the recorded value for larger displacements.	The same degree of linearity as for the displacement signal. The load shall be determinated with an accuracy of (+/-) 1%.
ASTM E813-87	Accuracy for gauges larger than those recommended in ASTM E-399 shall have an accuracy within (+/-) 1% of the full working range.	Accuracy of load measurements shall be within (+/-) 1% of full working range.

In Appendix 1 effects on *COD*-value and *J* of errors in force measurements are calculated. The effect on *COD* is

$$\Delta COD = \left| \frac{2PY^2(1-\upsilon^2)\Delta P}{B^2 W 2\sigma_y E} \right| \tag{1}$$

The effect on *J* is

$$\Delta J = \left| \left(\frac{2PS^2(1-\upsilon^2)f}{BB_N W^2 E} + \frac{2LV}{B_N b_0} \right) \Delta P \right| \tag{2}$$

If equation (1) and (2) are compared with equation (3) and (4) from [1], [2] and [3] one can see that the effect of the maximum error is greater when the load is small.

$$COD = \frac{P^2 Y^2 (1-\upsilon^2)}{B^2 W 2\sigma_Y E} + \frac{0.4(W-a)(V-V_e)}{0.4W + 0.6a + Z} \tag{3}$$

$$J = \frac{P^2 S^2 (1-\upsilon^2) f}{BB_N W^2 E} + \frac{2LPV}{B_N b_0} \tag{4}$$

Correct displacement measurements present a difficult problem. However, this has more to do with the method of measuring rather than the accuracy of the gauge.

The error limits of the displacement transducer are formulated in the same way as for the load cell. Actual values given in various standards [1] and [2] are shown in table 1.

In appendix 1 effects on the errors in displacement measurement on *COD* and *J* are calculated. The effect on *COD* is

$$\Delta COD = \left| \frac{0.4(W-a)\Delta V}{(0.4W+0.6a+Z)} \right| \tag{5}$$

The effect on *J* is

$$\Delta J = \left| \frac{2LP\Delta V}{B_N b_0} \right| \tag{6}$$

If equation (5) and (6) are compared with equation (3) and (4) one can see that the influence of a maximum error is becoming more and more important when the load is decreasing.

The displacement values obtained are, however, used in various ways. In linear fracture mechanics tests, a crack mouth gauge is used to control the linearity of the force vs. displacement curve. Here the relative error is insignificant, but the linearity of the transducer is of utmost importance. The relative P_Q - value is determined by the intersection with a 5% secant according to fig. 2. If the nonlinearity of the *P-COD*-curve is composed of a 5% non-linearity in the specimen and a 1% non-linearity in the transducer, the error in P_Q may easily be 5%. It should be noted that it is the linearity of the measuring range that is of interest.

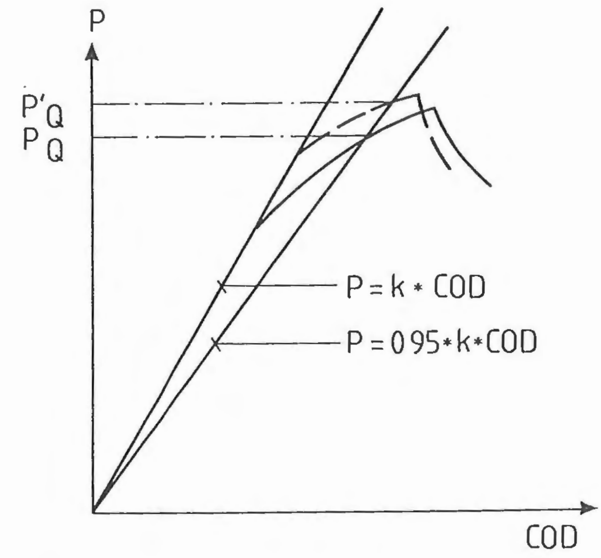

Fig. 2. Force vs. displacement for determination of the relative P_Q-value.

In the *COD*- and *J*-measurements, the displacement values affect the results linearly. Here it may be of some interest to consider the influence of the measured range. It is common to "clip" the transducer onto the test specimen in such a way that parts of the measuring range are used, see fig. 3.

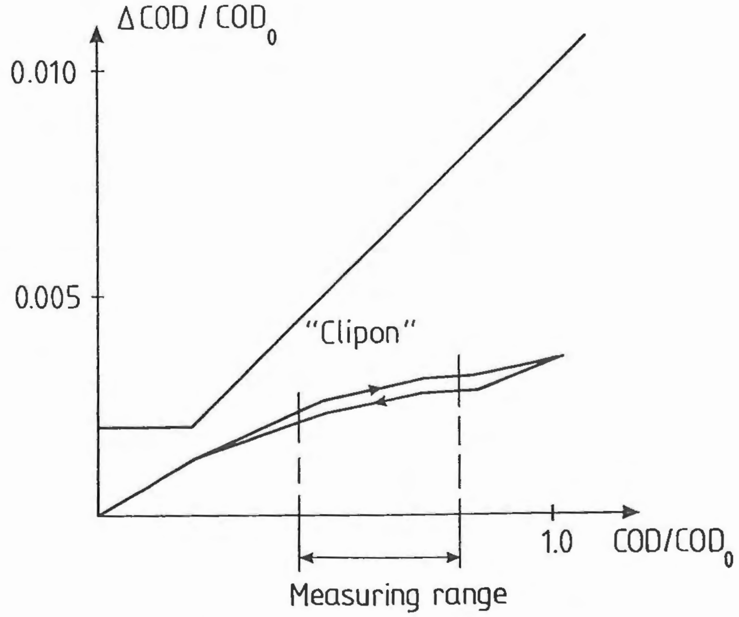

Fig. 3. Definitions of errors and error limits for a displacement transducer.

The measuring range may be of interest with regard to the segments of the absolute lengths that are involved. A normal measuring range is 0.5-1.0 mm of a transducer with a total range of 2.0-2.5 mm. A 1% error over this range means that errors should be in the range of 0.001 mm.

Recommended practice is to calibrate the displacement measuring system, regarding stability, both before and after a fracture toughness test over the measuring range with a micrometer. The knife edge and the shape of the gauge are crucial

When the crack length measurements are made by a Partial Unloading method, special precautions have to be taken.

2.1 Conclusions about the load and displacement measurements

To conclude, the requirements as to load measurement's devices are realistic and can be fulfilled with standard equipment. Displacement measurements are more crucial and the fulfilment of requirements, particularly regarding linearity, has to be carefully verified for the equipment used.

1. The requirements on load and displacement measurements are necessary.
2. The displacement measurement equipment should be calibrated regarding linearity both before and after a fracture mechanics test.
3. An appropriate range should be used during the test.

3. Loading device

In the standard fracture mechanics testing methods [1], [2] and [3] the demands on the test fixture vary. In BS 5762:1979 [1] and ASTM E813-87 [2] very little is mentioned about, for example the demands for the rollers to be parallel with each other. On the other hand, in EGF recommendations for determining the fracture resistance of ductile materials [3], the dimensions of the fixture are specified. Since it is possible for the testing laboratory to choose the test fixture and the demands on it, it seems reasonable to make an evaluation of errors, demands and testing expenses.

In this section four types of errors in the test fixtures are discussed. These errors (see fig. A1-A7) are:

1) The upper roller is oblique to the three-point bend specimen.

2) The rollers are not parallel to each other.

3) The load is not applied in the right place.

4) The rollers are pressed into the specimen.

3.1 Oblique upper roller

If the upper roller is oblique to the test specimen, the load will be applied to one of the edges of the specimen instead of a line perpendicular to the specimen. This will cause a twist of the specimen. The deformation causes higher stresses in part of the crack tip. This situation is calculated in appendix 2. This sort of error will affect the result of a fracture mechanics test considerably (5%).

3.2 Non-parallel rollers

In the EGF recommendations it states that "the axes of the rollers must be parallel to within 1 degree of each other". Appendix 2 treats a situation where this recommendation is not fulfilled. The result of the calculation is shown in equation (7).

$$\Delta d = \frac{D_2^2 D_3^2}{4D_1} \tag{7}$$

Δd in equation (7) is the quotient between the displacement in a perfect test fixture and in one with non-parallel rollers. From equation (7) it is obvious that if the rollers are not parallel do not affect the results of fracture mechanics test much.

3.3 Incorrectly applied load

This sort of errors can be divided into two separate cases, the first being when the load is not applied in the middle of the specimen's span. The other case is when the span is not exactly $4W$. Both these cases are calculated in appendix 2. Equations (8) and (9) show that their influence on the fracture toughness is small.

$$\Delta d = \frac{64W^3 + 48\Delta W \cdot W^2 + 12\Delta W^2 \cdot W + \Delta W^3}{64W^3} \tag{8}$$

$$\Delta d = 16\left(\left(0.5 + \frac{\xi}{4W}\right)^2 \cdot \left(0.5 - \frac{\xi}{4W}\right)^2\right) \tag{9}$$

Δd is quotient between the displacement for a perfect fixture and one with an error.

3.4 Rollers pressed into the test specimen

When the load used in a fracture mechanics test is large the rollers could be pressed into the specimen. This means that the measured displacement contains an error due to the plastic deformation of the test specimen. In this calculation, made in appendix 2, the rollers are assumed to be very stiff and hard. The result of this calculation is given by equation (10).

$$r - f = \left(r - (4P \cdot r / \pi B) \cdot \left(\left(1-\upsilon_1^2\right)/ E_1\right) + \left(\left(1-\upsilon_2^2\right)/ E_2\right)\right)^{1/2} \qquad (10)$$

3.5 Conclusions about the test fixture

To conclude, the requirements regarding the testing fixture are realistic and can be fulfilled with standard equipment. The most critical part of the testing fixture is the upper roller. If this roller for any reason is oblique to the test specimen, the fracture toughness measured would be greatly affected. This problem could easily be solved

1. To prevent the upper roller from becoming oblique to the test specimen the roller should be guided.

4. Acknowledgement

The authors would like to thank Prof. Hans Andersson, Swedish National Testing and Research Institute, for fruitful discussions. The project was financially supported by Nordtest and Swedish National Testing and Research Institute.

5. References

[1] Methods for crack opening displacement (*COD*) testing, BS (British Standard) 5762:1979.

[2] Standard test method for *J* - a measure of fracture toughness, ASTM (American Society for Testing and Materials) E813-87.

[3] EGF (European Group of Fracture), Recommendations for determining the fracture resistance of ductile materials. 1st draft, June 1987.

[4] Formelsamling i hållfasthetslära, Ed. Hedner G., Publikation nr. 104, Institutionen för hållfasthetslära KTH Stockholm 1986. (In Swedish)

[5] Handbok i maskinelement, Ed Folkesson A., Institutionen för maskinelement KTH Stockholm 1980. (In Swedish)

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Appendix 1

Calculation of errors (ΔJ) in J caused by tolerances in load (ΔP) and displacement (ΔV) measurements. For nomenclature see appendix 3.

From [2]

$$J = \frac{K^2(1-\upsilon^2)}{E} + J_{pl} \tag{1}$$

$$K = PS / \left(\left((BB_N)^{1/2} W^{3/2}\right) f(a/W)\right) \tag{2}$$

$$J_{pl} = \frac{2A_{pl}}{B_N b_0} \tag{3}$$

$$A_{pl} = LPV \qquad L = \text{constant} \tag{4}$$

(1), (2), (3) and (4) $\Rightarrow$

$$J = \frac{P^2(1-\upsilon^2)f}{BB_N W^2 E} + \frac{2LPV}{B_N b_0} \tag{5}$$

$$\Delta J = \left|(\delta J / \delta P)\Delta P + (\delta J / \delta V)\Delta V\right| \tag{6}$$

$$\delta J / \delta P = 2PS^2(1-\upsilon^2)f / BB_N W^2 E + 2LV / B_N b_0 \tag{7}$$

$$\delta J / \delta V = 2LP / B_N b_0 \tag{8}$$

(7) and (8) in (6) $\Rightarrow$

$$\Delta J = \left|\left(2PS^2(1-\upsilon^2)f / BB_N W^2 E + 2LV / B_N b_0\right)\Delta P + \left(2LP / B_N b_0\right)\Delta V\right| \tag{9}$$

Calculations of errors (ΔCOD) in COD due to the tolerances in load (ΔP) and displacement (ΔV) measurements. For nomenclature see appendix 3.

From [1]

$$\Delta COD = \frac{K^2(1-\upsilon^2)}{2\sigma_Y E} + \frac{0.4(W-a)V_p}{0.4W+0.6a+Z} \tag{10}$$

$$K = \frac{YP}{BW^{1/2}} \qquad Y = F(a/W) \tag{11}$$

$$V_p = V - V_e \tag{12}$$

(10), (11) and (12) $\Rightarrow$

$$COD = \frac{Y^2P^2(1-\upsilon^2)}{B^2W2\sigma_Y E} + \frac{0.4V(W-a)}{0.4W+0.6a+Z} + \frac{0.4V_e(W-a)}{0.4W+0.6a+Z} \tag{13}$$

$$\Delta COD = \left| \frac{\delta COD}{\delta P}\Delta P + \frac{\delta COD}{\delta V}\Delta V \right| \tag{14}$$

$$\frac{\delta COD}{\delta P} = \frac{2PY^2(1-\upsilon^2)}{B^2W2\sigma_Y E} \tag{15}$$

$$\frac{\delta COD}{\delta V} = \frac{0.4(W-a)}{0.4W+0.6a+Z} \tag{16}$$

$$\Delta COD = \left| \frac{2PY^2(1-\upsilon^2)}{B^2W2\sigma_Y E}\Delta P + \frac{0.4(W-a)}{0.4W+0.6a+Z}\Delta V \right| \tag{17}$$

Equations (1) - (17) are also valid for the errors in [1]

Appendix 2

About errors in fracture toughness caused by an oblique upper roller, see figure A1 and appendix 3 for nomenclature.

If the load is applied on one of the specimens edges instead of on a line perpendicular to the specimen the test is not made in a proper way. This case could be divided into two separate cases, one symmetrical case (A) and a case where a torque is acting on both specimen halves. In this calculation the crack is not considered to affect the twisting.

The cross-section of a specimen is deformed according to figure A2. The value of β is calculated from equation (1).

$$\beta = \frac{2M_v W}{GK} = \frac{24P(1+\upsilon)}{0.69EW^2} \tag{1}$$

If the hole cross section of the specimen is exposed to plastic loading the load can be written as

$$P = \frac{\sigma_{Y} W^2}{32} \tag{2}$$

The load P can be normalised with a load parameter $\alpha\,(0 < \alpha < 1)$ as

$$P = \frac{\alpha\sigma_{Y} W^2}{32} \tag{3}$$

(3) in (1) leads to

$$\beta = \frac{3\alpha\sigma_{Y}(1+\upsilon)}{4\cdot 0.69} \tag{4}$$

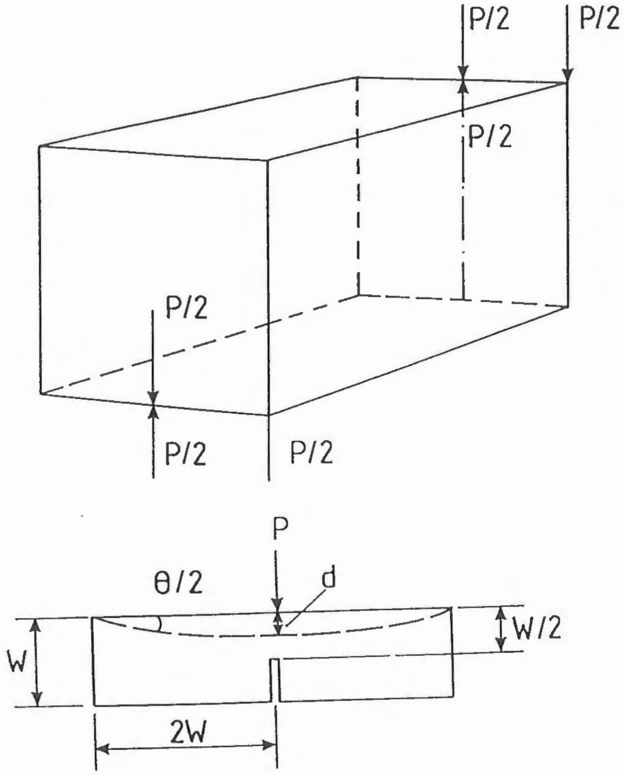

Fig. A1. The test specimen when the upper roller is oblique.

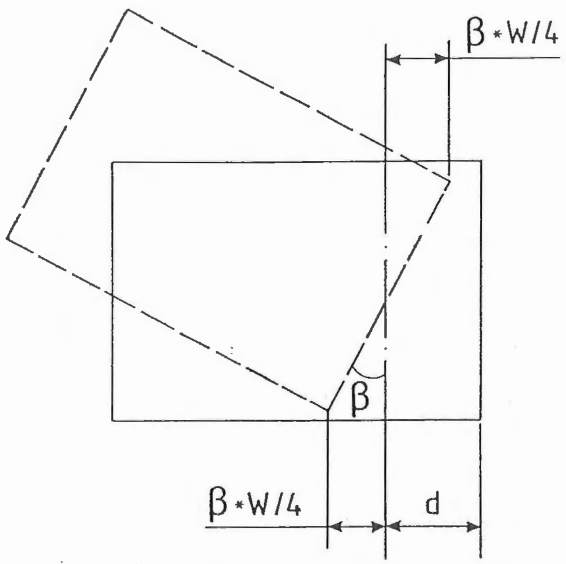

Fig. A2. A cross section of the test specimen when the roller is oblique.

$\upsilon = 0.3$ and $\sigma_Y / E = 0.002 \Rightarrow$

$$\beta = 2.8 \cdot 10^{-3}\alpha \tag{5}$$

The displacement d is given by equation (6)

$$\frac{d}{2W} = \frac{\Theta}{2} = \frac{64P}{EW^2} \tag{6}$$

If P is normalised the same way as above.

$$COD = \frac{128\sigma_Y W^2}{32EW} = \frac{4\alpha\sigma_Y W}{E} \tag{7}$$

Equation (4) and (7) are compared with each other, which gives

$$\frac{\beta W}{4d} = \frac{1.4\alpha\sigma_Y WE}{16\alpha\sigma_Y WE} = 0.088 \tag{8}$$

This leads approximately to an 8% overestimation of the *COD*-value at one edge. An 8% overestimated *COD*-value leads to an 4% overestimated *K*-value.

Calculation of errors in fracture toughness value if the rollers are non parallel

In this calculation the distances between the rollers are assumed to be correct along one edge of the test specimen. To simplify the calculations the twisting of the test specimen is neglected. For nomenclature see figure A3 and appendix 3.

The distances between the rollers are $2W$ along one edge of the test specimen. Along the other edge the distance are X_1 and X_2. X_1 and X_2 are calculated according to

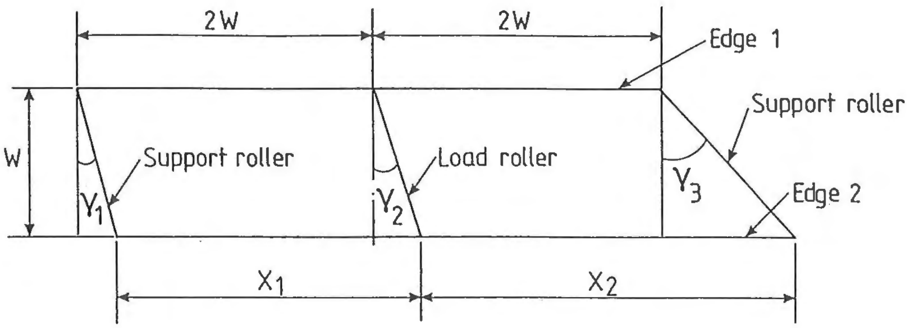

Fig. A3. Non-parallel rollers.

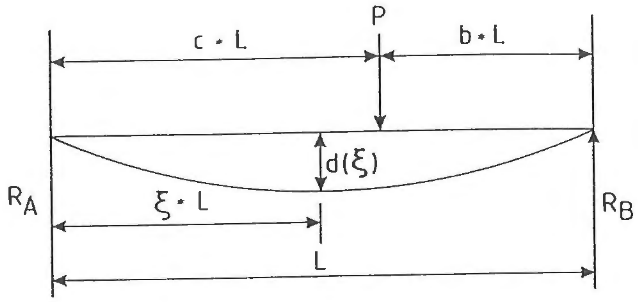

Fig. A4. Displacement of the test specimen when the rollers are non-parallel.

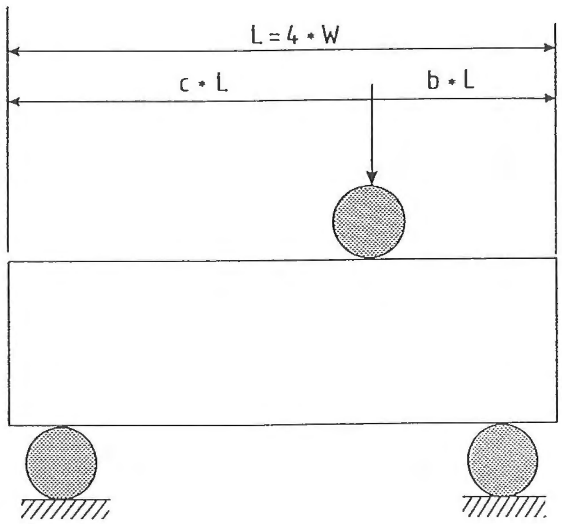

Fig. A5. The test specimen when the load not is correctly applied.

$$X_1 = 2W + W\tan\gamma_2 - W\tan\gamma_1 = W(2 + \tan\gamma_2 - \tan\gamma_1) \tag{9}$$

$$X_2 = 2W + W\tan\gamma_3 - W\tan\gamma_2 = W(2 + \tan\gamma_3 - \tan\gamma_2) \tag{10}$$

In figure A4 the nomenclature used to calculate the displacement of a bent beam is shown. The figure is taken from [4] and so are the equations used.

$$d(\xi) = \frac{PL^3 b(((1-b^2)\xi) - \xi^3)}{6EI} \tag{11}$$

$$d(c) = \frac{PL^3 c^2 b^2}{3EI} \tag{12}$$

along edge 1

$$c = b = 0.5 \tag{13}$$

$$L = 4W \tag{14}$$

$$d(2W) = \frac{64 \cdot 0.0625 PW^3}{3EI} \tag{15}$$

along edge 2

$$L = X_1 + X_2 = W(4 + \tan\gamma_3 - \tan\gamma_1) \tag{16}$$

$$c = \frac{X_1}{L} = \frac{2 + \tan\gamma_2 - \tan\gamma_1}{4 + \tan\gamma_3 - \tan\gamma_1} \tag{17}$$

$$b = \frac{X_2}{L} = \frac{2 + \tan\gamma_3 - \tan\gamma_2}{4 + \tan\gamma_3 - \tan\gamma_1} \tag{18}$$

$D_1 = 4 + \tan\gamma_3 - \tan\gamma_1$

$D_2 = 2 + \tan\gamma_2 - \tan\gamma_1$

$D_3 = 2 + \tan\gamma_3 - \tan\gamma_2$

$$dW(2 + \tan\gamma_2 - \tan\gamma_1) = \frac{P(WD_1)^3\left(\frac{D_2}{D_1}\right)^2\left(\frac{D_3}{D_1}\right)^2}{3EI} \tag{19}$$

Equation (19) divided with equation (15) gives

$$\Delta d = \frac{D_2^2 D_3^2}{4D_1} \tag{20}$$

Calculation of errors in the fracture toughness values due to an incorrectly applied load.

For nomenclature see figure A5, appendix 3 and [4].

Case 1

The load is not applied in the middle of the test specimen

$$L = 4W \tag{21}$$

$$c = \frac{2W + \xi}{4W} \tag{22}$$

$$b = \frac{2W - \xi}{4W} \tag{23}$$

$$d(c) = P(4W)^3\left(0.5 + \frac{\xi}{4W}\right)^2\left(0.5 - \frac{\xi}{4W}\right)^2 \tag{24}$$

which should be compared with the perfect case

$$d(0.5) = \frac{p(4W)^3}{48} \tag{25}$$

(24) divided by (25) gives

$$\Delta = 16\left(0.5 + \frac{\xi}{4W}\right)^2\left(0.5 - \frac{\xi}{4W}\right)^2 \tag{26}$$

Case 2, see figure A6. The test specimen span is not $4W$.

$$L = 4W \pm \Delta W \tag{27}$$

$$d(c) = \frac{P(4W \pm \Delta W)^3}{48EI} \tag{28}$$

which should be compared with the perfect case

$$d(0.5) = \frac{P(4W)^3}{48EI} \tag{29}$$

(28) divided by (29) gives

$$\Delta = \frac{(4W \pm \Delta W)^3}{(4W)^3} \tag{30}$$

Errors in the fracture toughness values due to the fact that the rollers are pressed into the test specimen.

For nomenclature see figure A7 and [5].

$$n = \left(\frac{4P \cdot r \left(\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \right)}{\pi B} \right)^{1/2} \tag{31}$$

$$r - f = r - \left(r^2 - \frac{4P \cdot r \left(\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \right)}{\pi B} \right)^{1/2} \tag{32}$$

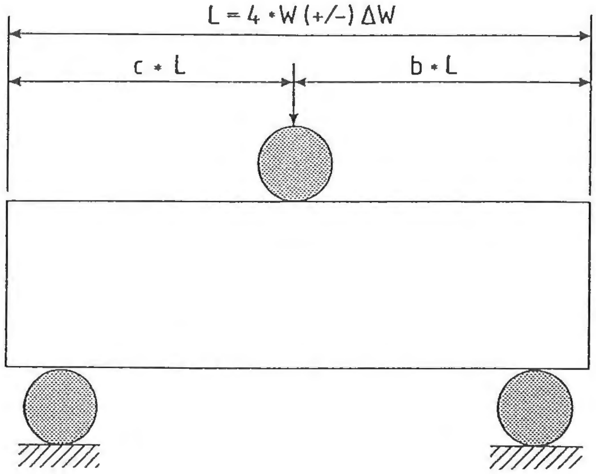

Fig. A6. The test specimen when the length of it is not 4*W.

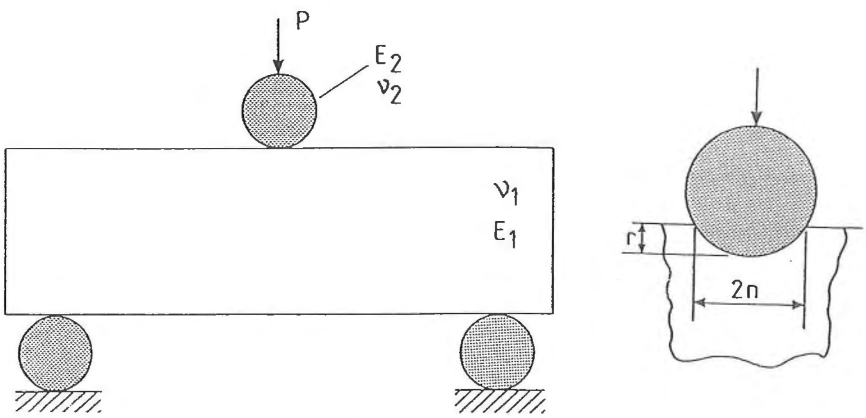

Fig. A7. The test specimen when the rollers are pressed into the test specimen .

Appendix 3

Nomenclature

J = a mathematical expression used as materials parameter

J_{pl} = the plastic part of J

S = specimen span

B_N = net thickness

a_0 = original crack size

b_0 = original uncracked ligament

A_{pl} = area under the load-displacement curve

d = displacement

α = load parameter

β = angel due to oblique upper roller

c = distance from the left roller to the upper roller

b = distance from the right roller to the upper roller

r = distance the rollers are pressed into the test specimen

Δ = indicates an error

COD = crack opening displacement

K = stress intensity factor

ν = Poisson's ratio

E = Young's modulus

σ_s = 0.2 % proof stress

W = with of the test specimen

V = total clip gauge displacement

V_e = elastic component of the total clip gauge displacement

V_p = plastic component of the total clip gauge displacement

Z = distance between location of the clip gauge and test specimen surface

Y = stress intensity coefficient

B = test specimen thickness

P = applied load

$f = D\text{-}r$

D = roller diameter

EI = bending stiffness

γ = angle between rollers