ANALYSIS OF COMPOSITE FLEXURAL MEMBERS

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ABSTRACT

A brief review is made on presently existing methods for analysing the behaviour of composite flexural members, and on the difficulties in solving the resulting equations. Then, a set of formulas suitable for general applications is derived with the sectional equilibrium method. By applying physical principles of the Gauss-Seidel iteration method, the solving process for the simultaneous equations is converted into a series of iterations. In order to accelerate convergence, an algorithm is developed. This algorithm predicts the final slip pattern for the member by making use of previous iterations. Finally, results of example calculations are compared with tested or calculated results published elsewhere.

INTRODUCTION

Steel-concrete composite beams and one way slabs are composed of steel and concrete components interconnected by some means of interface connections. In general, properties of materials and load-slip relationship of connectors can be as complicated as shown in Fig. 1. The coexistance of the nonlinear behaviour of steel, concrete and connectors causes difficulties for the analysis of composite members. Experimental studies have been the most important and reliable means in research on the behaviour of composite members. In order to conduct more rigorous and systematic studies, there have been many methods proposed for setting up analytical models for composite flexural members.

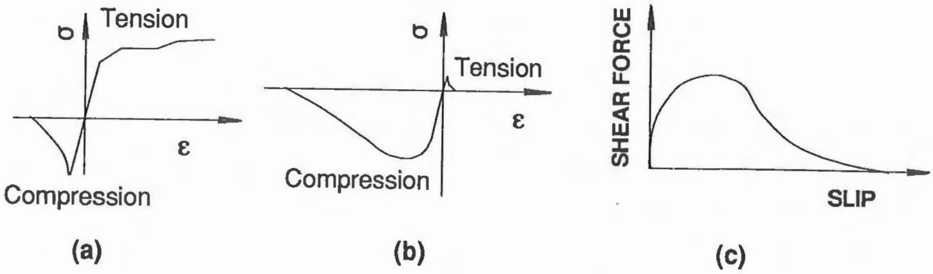

Fig. 1. General Material and Connector Behaviour: (a) Steel; (b) Concrete; and (c) Connector

Newmark et al (1951) derived a second order differential equation with respect to interface slips for composite beams. The equation was solved in closed form or solved numerically by the finite difference method. Formulations of that model were only capable of treating elastic material problems. Dabaon et al (1993) presented a similar second order differential equation capable of dealing with problems with nonlinear material and connector properties. Similar to the work of Yam and Chapman (1968), Zaremba (1988) derived a pair of coupled first order differential equations. The pair of coupled differential equations was solved by forward numerical integration method.

Ansourian and Roderick (1978) presented a numerical model for composite beams. This model used the conditions of sectional equilibrium. The relationships coupling discrete sections were set up by the specially defined slip strains. Thus a set of simultaneous equations could be derived. By solving these simultaneous equations, the problem was solved.

Daniels and Crisinel (1993), Daniels, et al (1990) used finite element method to set up a model for composite slabs. Modified beam elements were used to represent steel and concrete components. A step by step iteration process was used to solve the problem.

A calculation process using the sectional equilibrium conditions but avoiding the difficulties in solving the simultaneous equations has been developed. This is described here. The solution procedure has clear physical interpretations which provide the programming and debugging with ease and convenience.

Both beams and slabs are beam-like flexural members and there is no fundamental difference between the two members. Therefore, except that it is necessary to differentiate beams and slabs, flexural members will be referred as beams thereafter.

BASIC EQUATIONS

For the sake of generality, investigation is made for a composite beam of an arbitrary mono-symmetric cross-sectional shape as shown in Fig. 2. Under the action of external moment load *M(i)*, any cross section *i* must satisfy:

$$F_c(i) + F_s(i) = 0 \tag{1}$$

$$M(i) = M_c(i) + M_s(i) \tag{2}$$

$$F_c(i) = \int_{Ac} E_c \cdot \rho(i) \cdot [y - y_{co}(i)] \cdot dA \tag{3}$$

$$F_s(i) = \int_{As} E_s \cdot \rho(i) \cdot [y - y_{s0}(i)] \cdot dA \tag{4}$$

$$M_c(i) = \int_{Ac} y \cdot E_c \cdot \rho(i) \cdot [y - y_{c0}(i)] \cdot dA \tag{5}$$

$$M_s(i) = \int_{As} y \cdot E_s \cdot \rho(i) \cdot [y - y_{so}(i)] \cdot dA \qquad (6)$$

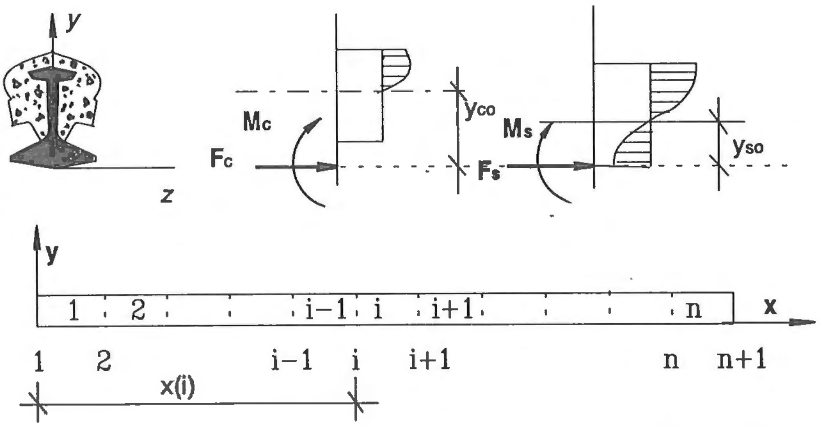

Fig. 2. A beam divided into n elements and the stress distribution in a section

where $\rho(i)$ is the curvature of the beam at section i; E_c, E_s are tangent moduli of concrete and steel, respectively; Ac, As are the section area of concrete and steel components, respectively.

Equations (1) to (6) are in a general form. While using numerical integration, materials with arbitrary constitutive properties and sectional shapes can be processed. Some factors like residual stresses and prestressings can be incorporated by revising the materials' constitutive laws.

In cases of complete interaction, equation $y_{co}(i)=y_{so}(i)$ holds. So there are 6 unknowns $F_c(i)$, $M_c(i)$, $F_s(i)$, $M_s(i)$, $\rho(i)$ and $y_{so}(i)$ *or* $y_{co}(i)$ with 6 equations. The problem can be solved for any individual section. However, in the cases of incomplete interaction, inequality $y_{co} \neq y_{so}$ holds. In such cases it is not possible to solve the equations merely at individual sections.

Suppose that the beam is divided into n segments of beam elements by $n+1$ sections as shown in Fig. 2. Denote slips of section i and section $i+1$ as $s(i)$ and $s(i+1)$ and define the slip strain ε_{sp} as:

$$\varepsilon_{sp}(i) = \frac{s(i) - s(i+1)}{x_{i+1} - x_i} \qquad (7)$$

Assume that the concrete part and the steel part are only connected by connectors and the connectors are concentrated at element ends. Denote the connection force in connector i as $Fconn(i)$. The following relationship must exist:

$$Fc(i) = \sum_{j=1}^{i} Fconn(j) \qquad (8)$$

or $$Fconn(i) = Fc(i) - \sum_{j=1}^{i-1} Fconn(j) \qquad (8')$$

On the other hand, the connector force $Fconn(i)$ is a known function $\varphi[s(i)]$ which can be obtained from connector tests or is defined by design specifications.

$$Fconn(i) = \varphi[s(i)] \qquad (9)$$

If the commonly used Kirchhoff or Bernoulli-Navier assumption (cross-sections remain plane in bending) is adopted (Fig. 3.), the following eq. (10) can be derived:

$$\Delta\varepsilon(i) = \frac{s(i) - s(i+1)}{x_{i+1} - x_i} = \varepsilon_{sp(i)}$$

$$\Delta\varepsilon(i) = \rho(i) \cdot \Delta y = \rho(i) \cdot (y_{c0}(i) - y_{s0}(i)$$

$$\therefore \qquad \frac{s(i) - s(i+1)}{x_{i+1} - x_i} = \rho(i) \cdot [y_{c0}(i) - y_{s0}(i)] \qquad (10)$$

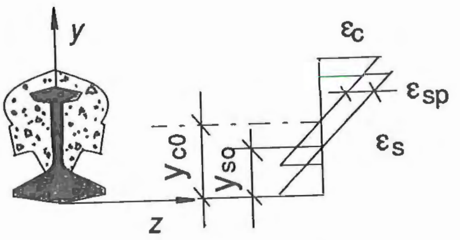

Fig. 3. Strain distribution over the cross-section

Now for each section there are 2 unknowns *S(i), Fconn(i)* and 3 equations (8), (9) and (10) have been added. So the problem can be solved but the simultaneous equations coupling all the $n+1$ sections are involved. Note that when steel and concrete adopt general nonlinear properties, the integrations in eq.(1) to eq.(6) can not have closed forms. They can be numerically integrated only after the locations of neutral axis are known. If the load-deformation function φ of the connector is also nonlinear and if there are more complications like changes in sectional dimensions along the axis, etc., to solve this set of simultaneous numerical equations can be extremely difficult. It is experienced that the difficulty of solving the problem is mainly because all sections are coupled. If a way can be found to decouple all the sections then the solution will be easier.

SOLVING THE EQUATIONS

Gauss-Seidel iteration (moment distribution) method is normally used in the analysis of statically indeterminate frame structures (West 1980). This method has the advantage of avoiding solving simultaneous equations. In order to decouple the present nonlinear simultaneous equations, the physical principle of the Gauss-Seidel iteration method can be applied here.

a. Assume complete interaction for the whole beam: concrete and steel have the same neutral axis. From equation (1) to (6) *Fc(i)*, $\rho(i)$ and the location of neutral axis are obtained for every section. This corresponds to the 'clamping' stage in Gauss-Seidel iterations.

b. For each section there is an unbalanced force *Fc(i-1)-Fc(i)*. This unbalanced force tends to cause section *i* to slip. If the 'clamp' is released for this section, section *i* will slip. When section *i* has a slip increment $\Delta s(i)$, neutral axis of section *i* and section *i-1* are changed according to eq.(10). From eq. (1) to (6) the new *Fc(i)* and $\rho(i)$ are obtained. Eq. (11) specifies the condition for a stable state of section *i*.

$$Fc(i-1) - Fc(i) - Fconn(i) = 0 \tag{11}$$

This is the 'releasing' stage for section *i*.

c. Apply step b. for every section consecutively and repeat along the beam untill balance is reached for all sections. The balance criterion of a section is as follows:

$$Fc(i) - \sum_{j=1}^{i-1} Fconn(j) \leq (\text{specified accuracy}) \cdot Fc(i) \tag{12}$$

When eq. (12) is satisfied for all sections, the process is converged. $\rho(i)$, $y_{c0}(i)$ *and* $y_{s0}(i)$ are obtained for all values of *i*.

After the convergence, deflection *y(x)* of the beam can be integrated as:

$$y(x) = \int_0^x \left[\int_0^\eta \rho \cdot d\xi \right] \cdot d\eta + \int_0^x C1 \cdot dx + C2 \tag{13}$$

where C1 and C2 are constants determined by boundary conditions of the beam.

This is a routine and simple process. Each step has a clear physical meaning and the satisfactions of the equations are easy to be checked. So when anything goes wrong in the calculation, it is very easy to find out the reason. This is very useful for debugging in programming.

ACCELERATING THE CONVERGENCE

The process converges fast in cases when the stiffness of connectors is close to the axial stiffness of concrete component. Whenever the connector stiffness is very low or the connection level is low, the process converges very slowly. The reason for this is as follows: When a section is released, all the rest sections are still fixed. Axial stiffness of the concrete part prevents the section from moving to its 'expected' place. So the slip increment in one round of iteration is very limited. In the case of low connection level, slips can only propagate from the ends to the central part of the beam. In order to make this method practically valuable, an algorithm accelerating the convergence of the process has to be found. In view of this the authors have developed an algorithm to accelerate the convergence. Various applications show that it is efficient and generally applicable.

The principle of the algorithm is to apply the extrapolation technique to the iteration process. The algorithm predicts the final slip pattern by making use of the previous iteration results. It goes in the following way: Beginning from iteration k let the program iterate further m round. Record the values of unbalanced forces $F_{unbalanced}(i) = Fc(i) - \sum_{j=1}^{i-1} Fconn(j)$ and slips $s(i)$ for every section at iteration k and iteration $k+m$. The changes in $s(i)$ during this m iterations are:

$$\Delta s(i) = [s(i)]_{k+m} - [s(i)]_k \quad i=1,2, \dots n+1, \tag{14}$$

The changes of the unbalanced force, $\Delta F_{unbalanced}(i)$ in this m iterations are:

$$\Delta F_{unbalanced}(i) = [F_{unbalanced}(i)]_{k+m} - [F_{unbalanced}(i)]_k, \qquad i=1,2, \ldots n+1, \qquad (15)$$

The present unbalanced forces are:

$$F_{unbalanced}(i) = [F_{unbalanced}(i)]_{k+m}, \qquad i=1,2, \ldots n+1$$

Then, the expected final slips $s(i)$ are:

$$s(i) = [s(i)]_{k+m} + \left| \frac{\Delta s(i)}{\Delta F_{unbalanced}(i)} \right| \cdot F_{unbalanced}(i), \; i=1,2, \ldots n+1, \qquad (16)$$

In this way, a pattern of slips along the beam is predicted. The process can go on with iteration from this predicted pattern of slips. This process can be repeatedly used untill the convergence of the iteration. Sometimes in the calculation process $\Delta F_{unbalanced}(i)$ may approach zero. In such cases just omit the second term on the right side of eq. (16).

Eq. (16) works like a penalty function. It 'pulls' the slips to their equilibrium state. The value inside the absolute sign determines how much the 'fine rate' will be. Because it makes use of previous iteration results the algorithm is robust and efficient.

Now the problem is to choose a suitable iteration interval m. If m is too small the process may jump back and forth (finally it will converge, though) because iteration process can not give a good prediction for convergence. If m is too large the process converges slower because less accelerating algorithms have been used. Experience of the authors shows that a value of m=5 is good for many cases except for cases of unevenly distributed connections. A value of m=10 is satisfactory for most cases.

Experience shows that the developed algorithm is very efficient. The authors have experienced a case of nonconvergence after 10000 iterations without applying the accelerating process. With the help of this algorithm, convergence can be reached within

100 iterations for one load step for most cases. One other advantage of this algorithm is that it only slightly adds extra work for the computer.

For the sake of an easy illustration, slip was chosen as the predicted variable in the above description. In fact, in view of programming, slip is not a good choice. Because if slip is the predicted variable, in the application process coupling of neighboring sections must be involved. A good choice of the predicted variable should satisfy two conditions: 1. It should not cause the failure of the process no matter how big a prediction will be; 2. It should not couple other sections. $\Delta y(i) = y_{co}(i) - y_{so}(i)$ suits for condition 2 well but does not satisfy condition 1. The best choice may be the slip strain $\varepsilon_{sp}(i) = \rho(i) \cdot \Delta y(i)$. This variable does not involve neighboring sections and, no matter how big the predicted ε_{sp}(i) is the program can reach it by adjusting $\rho(i)$ and $\Delta y(i)$ (at least theoretically so).

COMPARISONS BETWEEN TEST AND CALCULATED RESULTS

In order to check the validity of this method, comparisons are made between the analysed results and some test results published elsewhere. The sketches of the related member sections are shown in figures 4, 5 and 6.

Fig. 4 shows the tested and calculated load-deflection curves for ZHL-1. ZHL-1 is a 4.9m span composite beam, consisting of a profiled I-section and a concrete slab which was tested by Lu and Zang (1989).

Daniels and Crisinel (1993) developed a finite element program to analyse composite slabs. In Fig. 10 of their paper, calculated M-δ curves are shown for a composite slab with different spans. In order to compare the method developed by them and the one described here, calculation is made for the same slab using the same data they have used.

Fig. 5 shows the calculated results. By comparing, it is seen that curves in Fig. 5 are almost identical with those in Fig. 10 of their paper. This attests the validity of these two different analysis procedures.

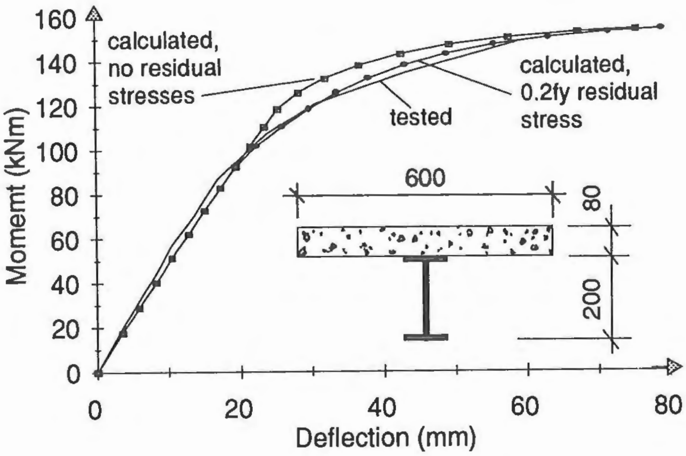

Fig. 4. Comparison between calculated and tested results of ZHL-1

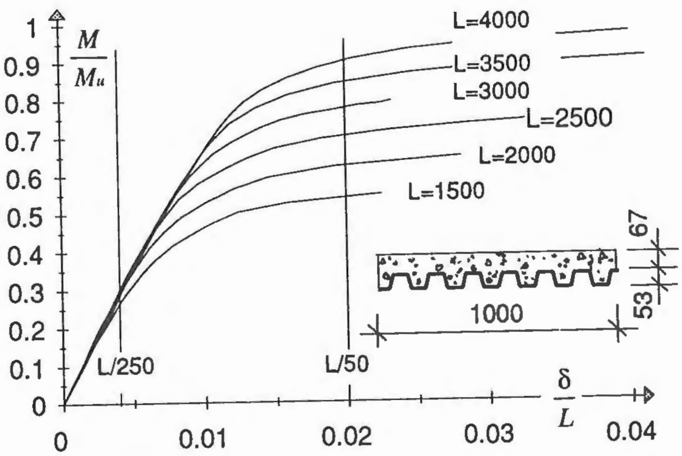

Fig. 5. Calculated moment-deflection curves for the slabs analysed by Daniels and Crisinel (1993)

Fig. 6. shows the moment-deflection curve of a composite slim floor beam tested at the Helsinki University of Technology by Lu and Mäkeläinen (1994). The beam has a span of 5.8m with 148mm diameter holes in the webs. The edges of the holes are bent inside to act as shear connectors. The result highlights the effects of residual stresses.

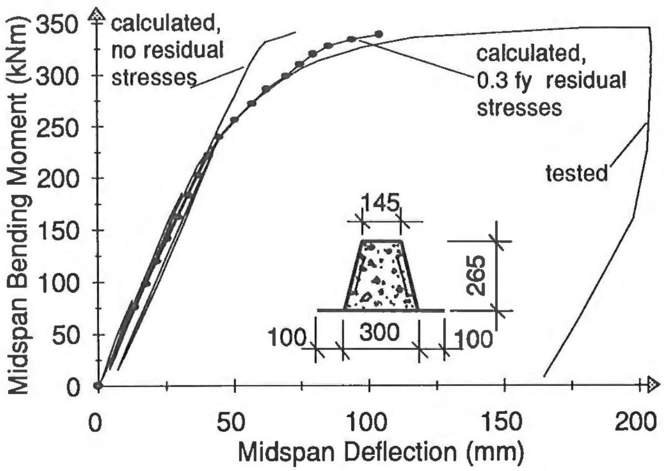

Fig. 6. Comparison between calculated and tested results of CS-1

CONCLUSIONS

Computational model is an important tool for researchers investigating composite structures. It is common that researchers develop their own models. Over decades some methods have been used and many successful models have been presented. For the method using sectional equilibrium conditions, there will be a set of simultaneous equations to be solved. Because of the complex behaviour of composite member, the process of solving the equations can be very difficult. Programming of this process will also be troublesome. In some cases, numerical difficulties and convergence can be serious problems. This paper makes an effort to set up a unified formulation and solving process for general applications. It is concluded that:

1. Formulas in this process are in a general form. They are applicable to any continuous material and connection constitutive laws. Parameters like residual stresses, prestressing, end anchorages, etc., can be routinely incorporated in the process.
2. By introducing the principle of Gauss-Seidel iteration method, the equation solving process is simplified. The simplification makes the equation solving a unified routine process. It can easily avoid numerical difficulties. The physical interpretation guarantees its convergence.
3. The algorithm applied to accelerate the convergence is robust and efficient. Because of this algorithm, the idea of applying the Gauss-Seidel iteration becomes practically feasible.
4. The simplicity and uniformity of the process and its physical meaning make compiling and debugging of the program easy. This can reduce much work of the developers.
5. Sample runs of the process verify the applicability and generality of the process.

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