NONLINEAR FEM-ANALYSIS AND DESIGN OF COMPOSITE BEAM-COLUMN

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ABSTRACT

This paper presents nonlinear FEM-analysis of concrete encased steel columns subjected to combined axial compression and bending. A materially and geometrically nonlinear model is used to analyse the ultimate strength of columns. The results of FEM-analysis are in a good agreement with test results.

Different combinations of axial and transverse loads are used in FEM-analysis in order to determine the M-N-interaction curve of the column. Comparison is made with the results of FEM-analysis and the design resistances based on the Finnish design code for composite structures. It was observed that the design method based on the code for composite structures underestimates the strength of columns due to overestimated eccentricities of the compressive axial force. A corrected design method is proposed. Results calculated according to this method are in a good agreement with the test results and the results of FEM-analysis.

INTRODUCTION

During the years 1990 and 1991 tests on the strength of composite columns were conducted at the University of Oulu. In the research report [8] test results are compared with the design strength based on the Finnish code for composite structures [15]. It was discovered that axial force eccentricities due to construction tolerances and geometric imperfections e_a and deflection of the column e_2 (second order effect) were too much emphasized. It is proposed in the report [8], that in the design method for concrete encased composite steel columns, eccentricities $(e_a + e_2)$ should be smaller.

Tests were carried out with five different combinations of axial and transverse loads in columns. Due to the practical limitations of test equipment, the bending moment

(M) caused by transverse loads was relatively high compared to the axial force (N), i.e. the eccentricity of the normal force ($e=M/N$) had a relatively high value.

In this study the columns tested are analysed by using a materially and geometrically nonlinear model. A nonlinear FEM-analysis using several eccentricities ($e=M/N$) of normal force is carried out. Different combinations of axial load (N) and bending moment (M) due to transverse loads are used in order to determine the M-N-interaction curve as a whole. In this way more information is gained on columns tested earlier especially under loading conditions when the normal force eccentricity is relatively small.

Analysis is carried out on the following basis:

A bending theory with the assumption of plane cross-sections remaining plane and normal during bending is used. The plasticity of reinforcement steel and the nonlinear stress-strain relation for concrete are taken into account. Tensile resistance of concrete has a significant meaning in estimating beam-column deflections and secondary bending moment due to the eccentric normal force. Tensile strain softening of concrete (i. e. the fact that after reaching the strength limit, tensile stress does not drop suddenly to zero but declines gradually with increasing strain) is taken into account. Tension stiffening of steel bars is neglected.

The above described model has been used recently in analysing deflections of non-prestressed and partially prestressed concrete beams [5], [6] (in the absence of a normal force the second order effect and geometric nonlinearity is neglected in these analyses). Results of these analyses are in a good agreement with test results. In this study the above described model is used for the reinforced concrete part of the composite column.

The geometric nonlinearity is taken into account. An ideally plastic material model is used for structural steel. Full composite action is assumed between structural steel and concrete.

Results of nonlinear FEM-analysis show good agreement with test results.

Comparison is made with the strengths obtained by the FEM-analysis and the design strengths based on the Finnish design code for composite structures. The observation of the earlier study [8] is confirmed. The design method based on the code for composite structures underestimates the strength of columns. In this study a corrected design method is proposed.

STRUCTURAL BEHAVIOUR OF A COMPOSITE BEAM-COLUMN

Consider the strength of a slender composite column under combined axial compression and bending in a plane which is parallel to the loads (Figure 1). In the ultimate limit state the strength of the column is equal either to the flexural buckling strength or the cross-sectional strength of the column depending on the slenderness and the loading condition of the column. If the eccentricity of the normal force ($e=M/N$) is small the flexural buckling is obvious to happen. When the eccentricity e is great the cross-sectional strength is dominant.

In Finland the strength of concrete columns is calculated on the basis of the cross-sectional strength of the column according to the code for concrete structures [10]. The stability of the column is taken into consideration in the calculation of the strength of slender composite columns and slender steel column in the code for composite structures [15] and in the code for steel structures [11].

An additional secondary bending moment due to the deflection of the column and an eccentric normal force (Figure 1) have to be taken into account in an exact analysis.

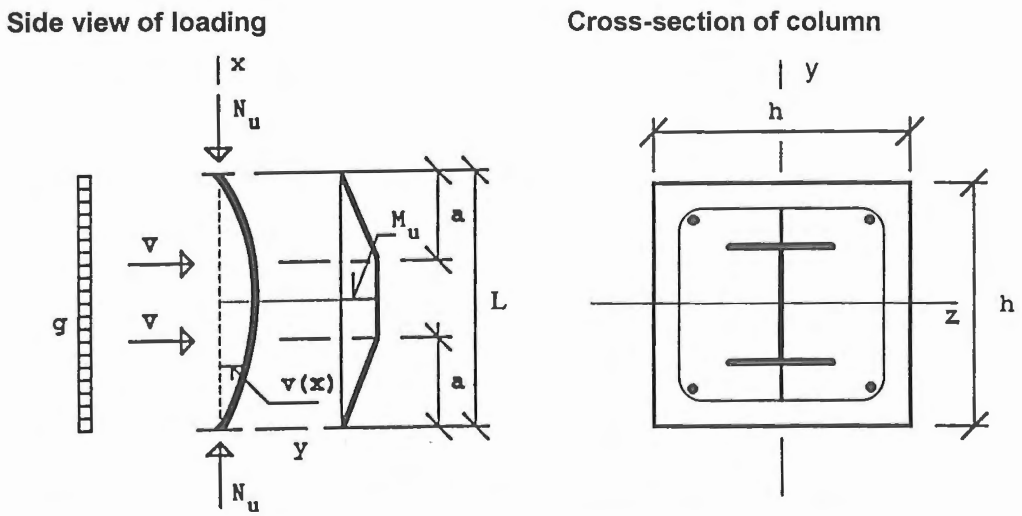

Figure 1. Composite column under combined compression and bending.

The structural behaviour of the concrete part of composite columns under consideration is in many ways equal to the behaviour of reinforced concrete beams. The tensile resistance of concrete has a significant role in the analysis of deflections of reinforced concrete beams (RC-beams).

Neglecting the tensile resistance of concrete leads to significant overestimation of the deflections of RC-beams. The actual behaviour of concrete is stiffer due to the capability of concrete to transmit stresses even after the beginning of cracking. To explain this stiffening effect, two different mechanisms have been proposed [6]:

1. Tensile strain softening of concrete:
   After reaching the strength limit, tensile stress does not drop suddenly to zero but declines gradually with increasing strain.

2. Tension stiffening of reinforcement steel bars due to the tensile resistance of the concrete layer surrounding the bar:
   The surrounding concrete is forced by bond stresses to extend simultaneously with the bar.

The tension stiffening implies the assumption that continuous tension-free cracks perpendicular to the steel bar form immediately at a certain spacing as soon as the concrete strength limit is reached, and that the concrete stress drops suddenly to zero.

After cracking a part of the force in the steel bar is assumed to be transmitted to the concrete between the cracks by means of bond stresses. The development of bond stresses requires a finite bond slip and the cross sections cannot be assumed to remain plane. The strain softening is neglected in the existing tension stiffening theory (i.e. in the theory a sudden drop of stress to zero is assumed).

It has become clear from recent fracture mechanics research [6] that in the RC-beams the concrete itself also exhibits tension resistance after cracking begins i.e. the strain softening. This phenomenon is explained by crack bridging at aggregate pieces and fragments that remain anchored at both surfaces of the crack. The cracks that start to form at the peak stress are discontinuous and do not become continuous until the strain increases and the stress is reduced to zero.

Therefore continuous tension-free cracks can be expected to form only after a large increase of strain in the steel bar occurs. This suggests that the strain softening of concrete should be the primary mechanism occurring first and tension stiffening of reinforcement bars should come into play only much later, after the concrete tensile stress is reduced to nearly zero.

In the analysis of deflections of reinforced concrete beams and partially prestressed RC-beams ([5] and [6]), only the strain softening of concrete is taken into account as the primary mechanism. The tension stiffening of the steel bar is neglected as a secondary mechanism which takes place after the strain softening of concrete.

In references [5] and [6] a good agreement is achieved between the theory and the test results. There is also an advantage of simplicity in restricting the attention to only the strain softening, since the cross sections may be assumed to remain plane, while for the latter mechanism (number 2 above) they cannot.

In this study of the structural behaviour of a composite beam-column the above described material model ([5] and [6]) is used for concrete part of the composite column. Nonlinear stress-strain relation for concrete in compression and plasticity of reinforcing bars are also taken into account.

In the concrete encased composite columns under consideration both the concrete part and the structural steel are located symmetrically about both axes of the cross section as indicated in Figure 1. Full composite action is assumed between structural steel and concrete. An ideally plastic material model is used for structural steel. Geometric nonlinearity and second order effects are taken into account.

FEM-ANALYSIS MODEL

Introduction

Calculations of the analysis were carried out by using a finite element program ABAQUS. The modified Riks-method was used in FEM-calculations to obtain the load-displacement relation of the columns. In this method the solution is sought for a proportional loading case including the possibility of an unstable behaviour. The basis of this method is to choose increments based on controlling the path length along the load-displacement curve, and thus to obtain the solutions regardless of whether the response is stable or unstable [1], [2]. The modified Riks-method is convienent to use in determining the load-displacement relation of a structure up to the ultimate limit state and beyond it.

A local unstability is not possible for the columns under consideration. When the eccentricity of compressive normal force $e=M/N$ is small a flexural buckling of the column is possible at the ultimate state. When the structural behaviour of columns is studied with the finite element method an initial effect must be added for the centrally loaded and ideally straight element model to take into account the possible loss of stability in the load-displacement path of the column. An initial imperfection corresponding to deformed shape of the column is used to model the geometrical imperfections and the small eccentricity of loads due to the construction tolerances and the residual stresses in the actual column. Presumably an imperfection in the form of the buckling mode would be the most critical. The initial deformed shape of a column may be obtained by buckling analysis or by applying a small transverse loading which will produce a deformed shape as that obtained from the buckling analysis.

In the tests [8] the columns were in a horizontal position (Figure 1). In this study the transverse self weight load of the structure will cause the needed perturbation in the perfect geometry of the model.

Element model

The columns were modelled with three node 3-dimensional beam elements. The I-section beam elements shown in Figure 2 were used to model the structural steel part (HE 100A structural steel member) of the columns. The concrete part was modelled with a rectangular cross section element. Steel ribs, welded in the structural steel, were used as additional shear connectors in the tests [8]. The ribs were modelled with eccentric rectangular elements. Part of the ribs have holes as shown in Figure 6. These ribs were modelled with narrower rectangular elements having the same cross-sectional area as the ribs. The elements for the concrete, the structural steel and the ribs have common nodal points. The integration points used in the cross sections of elements to take into account the material properties, strain and stresses, cracking of concrete and plastification of concrete and steel are shown in Figure 2.

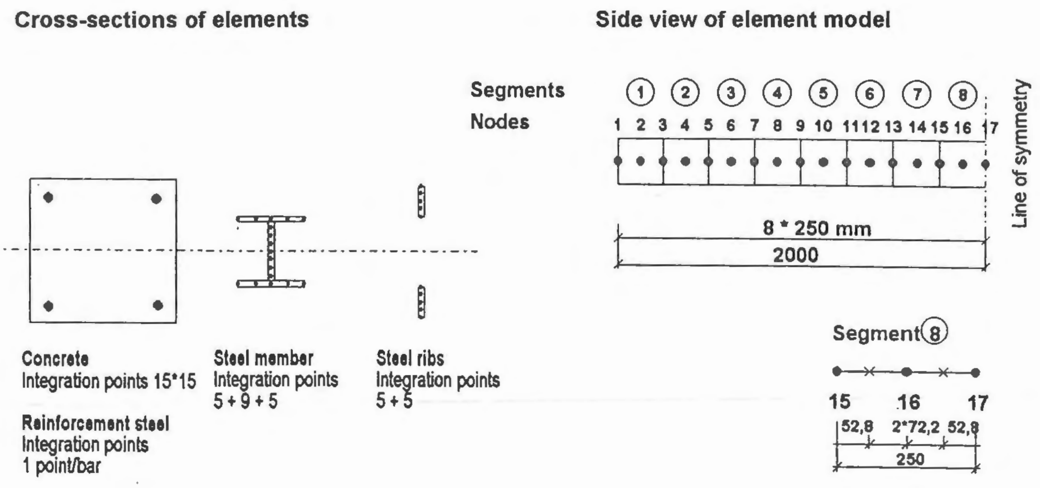

Figure 2. Element model of the composite column.

When analysing a reinforced concrete beam by the finite element method with beam elements the constitutive laws with strain softening are shown to lead to false sensitivity of results to the chosen finite element size [3], [4]. An element mesh which is too dense gives erroneous results, in which the curvature of the beam localizes into a segment of small length and failure occurs with little energy dissipation. Also it is known that curvature localization into segments whose lengths are shorter than the beam depth h cannot be correctly captured by the bending theory, since cross sections for such localizations do not remain plane, and a three-dimensional analysis is required. Applied forces or moments that are concentrated over a portion of the beam depth produce deformations that agree with the assumption of plane cross sections at a distance approximately h from the cross section of load application, but not any closer.

According to references [3] and [4] the finite element length, L_e, may not be smaller than approximately the beam depth h. At the same time, to include the effect of full curvature localization, the minimum element length must be used in the softening regions of beams.

In this study the strain softening constitutive laws are used for the concrete part of the columns. For this reason the element length 250 mm was chosen, the depth h of the columns being 240 mm. The element model is shown in Figure 2.

Material model

The stress-strain relation used in [5] is adopted for concrete in tension:

For $\varepsilon \le \varepsilon_{tp}$: $\sigma = E_c \varepsilon$ (1)

For $\varepsilon_{tp} < \varepsilon < \varepsilon_{tf}$: $\sigma = f_t' - (\varepsilon - \varepsilon_{tp})(-E_t)$ (2)

For $\varepsilon \ge \varepsilon_{tf}$: $\sigma = 0$ (3)

where σ , ε = uniaxial stress and strain

f'_t = direct tensile strength

E_c = modulus of elasticity of concrete

E_t = tangent strain softening modulus of concrete

ε_{tp} = strain at peak tensile stress

ε_{tf} = strain at zero stress after strain softening.

The tangent strain softening modulus is obtained from the formula

$$E_t = \frac{-0,48 E_c}{0,39 + f_t'} \tag{4}$$

in which E_C, f'_t, and E_t are in MPa.

According to reference [10] $E_c = 5000\sqrt{K} = 29155$ MPa, where K is the cubic strength of concrete. For the columns under consideration the cube strength of concrete is 34 MPa. The stress-strain relation in figure 3 for concrete in tension is obtained from equations (1) - (4).

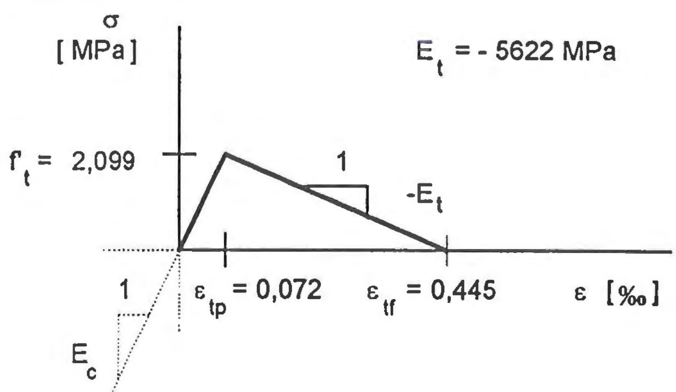

Fig 3. Stress-strain relation for concrete in tension.

For concrete in uniaxial compression, the well known expression is used [12]:

$$\sigma = \frac{E_c \varepsilon}{1 + \left(\frac{E_c \varepsilon_{cp}}{\sigma_{cp}} - 2 \right) \left(\frac{\varepsilon}{\varepsilon_{cp}} \right) + \left(\frac{\varepsilon}{\varepsilon_{cp}} \right)^2} \tag{5}$$

in which σ_{cp} = peak stress (compression strength f_{ck})

ε_{cp} = strain at peak stress

For the FEM-analysis the stress-strain relation of equation (5) is modified to a curve that is composed of several linear parts (Figure 4). In the curve the compression strength [10] σ_{cp} = 23,8 MPa, for the concrete cube strength 34 MPa, is reached at the strain value ε_{cp} = -2.0 ‰. As the strain exceeds ε_{cp}, concrete will exhibit compressive strain softening.

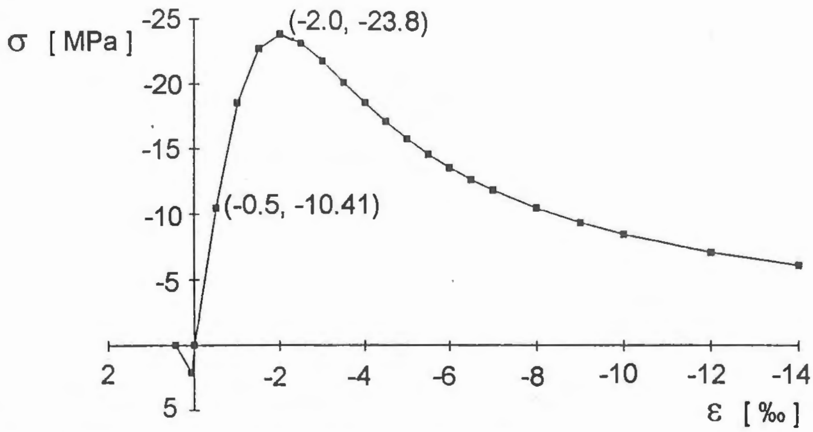

Figure 4. Stress-strain relation for concrete in compression.

The reinforcement steel and the structural steel are assumed as elastic - plastic, characterized by modulus of elasticity E_s and uniaxial yield stress f_y (Figure 5).

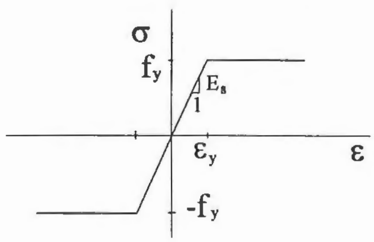

Figure 5. Stress-strain relation for structural steel and reinforcement steel.

Verification of analysis model

The model used in FEM-analysis is verified by comparing the test results in reference [8] and the results of FEM-calculations.

During the years 1990 and 1991 tests on the strength of 16 composite columns were conducted at the University of Oulu. The tests were carried out with different combinations of the axial and transverse loads in columns. Also the effect of additional shear connectors, ribs welded in the steel member, was studied in the tests. The loading arrangement and the dimensions of the columns for both test series are shown in Figure 6.

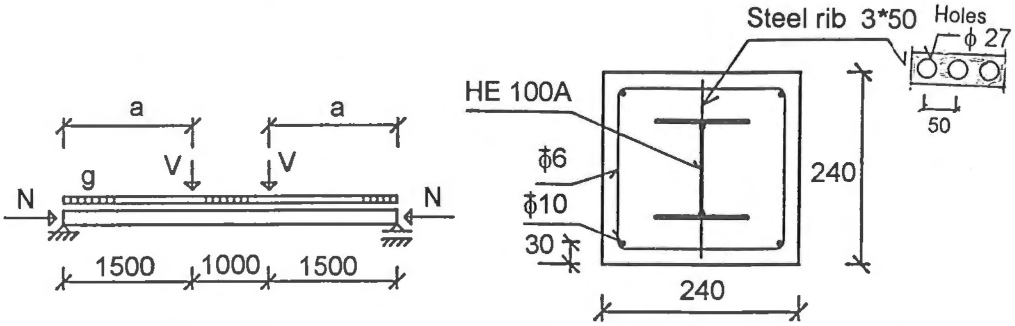

Figure 6. Loading arrangement and dimensions of the columns in the tests [8].

The strength of materials of 8 columns in the test series 1 were on average:

Cube strength of concrete	$K = 34$ N/mm^2
Structural steel, I-beam HE 100A	$f_y = 293$ N/mm^2
Structural steel, ribs 50*3 mm	$f_y = 293$ N/mm^2
Reinforcement steel bars B400H	$f_y = 565$ N/mm^2

and the strength of materials of 8 columns in the test series 2 were on average:

Cube strength of concrete	$K = 33$ N/mm^2
Structural steel, I-beam HE 100A	$f_y = 311{,}2$ N/mm^2
Structural steel, ribs 50*3 mm	$f_y = 293$ N/mm^2
Reinforcement steel bars B400H	$f_y = 634$ N/mm^2

When loaded to ultimate load the concrete crushed in the compression zone in all columns in the tests, usually in the cross section near the mid-length of the columns. Before that the plastification of the steel ribs, and the reinforcement and the structural steel were observed. The test results showed that the holes in the ribs or the absence of the bond between the ribs and the concrete has no significant effect on the strength of columns.

The results of the tests [8] and the results of the FEM-calculations are compared in Table 1. The results are also plotted in *M*-*N*-coordinations in Figure 7.

		Ultimate limit state											
		Test results				Results of FEM-analysis				FEM/Test			
Test n:o	e_0 [m]	N [kN]	V [kN]	M(L/2) [kNm]	v(L/2) [mm]	N [kN]	V [kN]	M(L/2) [kNm]	v(L/2) [mm]	$\frac{N_{FEM}}{N_{test}}$	$\frac{V_{FEM}}{V_{test}}$	$\frac{M_{FEM}}{M_{test}}$	$\frac{v_{FEM}}{v_{test}}$
1/4	0,6	107,5	43	73,47	53,7	110,9	44,36	74,91	46,74	1,032	1,032	1,020	0,870
1/3	0,6	95	38	65,56	56,4	99,33	39,74	67,52	47,57	1,046	1,046	1,030	0,843
2/2	0,588	100	39,2	68,72	67,20	106,1	41,63	70,69	47,36	1,061	1,061	1,029	0,705
1/6	0,2	352,5	47	93,76	56,9	332,3	44,31	82,76	40,03	0,943	0,943	0,883	0,704
2/1	0,19	307,5	38,9	79,78	59,3	322,2	40,79	78,48	43,79	1,048	1,048	0,984	0,738
2/3	0,195	296	38,5	79,92	64,1	314,0	40,85	78,21	43,79	1,061	1,061	0,979	0,683
1/2	0,1	540	36	82,31	46,5	599,1	39,94	86,18	38,78	1,109	1,109	1,047	0,834
1/1	0,1	570	38	78,16	31,5	548,0	36,54	81,25	42,48	0,961	0,961	1,040	1,349
2/4	0,093	540	33,3	83,01	55,3	583,4	35,98	83,20	44,66	1,080	1,080	1,002	0,808
2/6	0,053	775	27,6	75,99	40,5	915,4	32,6	78,84	29,25	1,181	1,181	1,038	0,722
1/7	0,06	550	22	63,21	49,1	769,2	30,77	75,06	33,45	1,399	1,399	1,187	0,681
2/5	0,055	720	26,3	69,94	37,90	816,9	29,84	75,31	33,52	1,135	1,135	1,077	0,884
2/8	0,046	925	28,1	82,17	39,8	1010	30,68	74,79	25,35	1,092	1,092	0,910	0,637
1/8	0,036	950	23	64,11	27,8	1054	25,29	64,28	22,20	1,109	1,109	1,003	0,799
2/7	0,038	927	23,69	72,01	35,9	1015	25,94	67,54	25,08	1,095	1,095	0,938	0,699
1/5	0,35	154,3	36	65,96	56,8	170,2	39,71	69,94	42,28	1,103	1,103	1,060	0,744
FEM/Test			average value (excluding test 1/7)							1,070	1,070	1,003	0,801
$\left\lvert\frac{\text{FEM} - \text{test}}{\text{test}}\right\rvert$ *100			average value (excluding test 1/7)							8,32 [%]	8,32 [%]	4,65 [%]	24,3 [%]

Table 1. Comparison of the test results and the results of FEM-calculations.

At the ultimate state the axial load *N* according to the FEM-analysis is on average 7.0 % higher than the axial load in the tests, while the bending moment *M* is on average 0.3 % higher than the bending moment in the tests. The deflection of columns obtained by FEM-analysis is on average 80.1 % of the deflections in the tests. The test results of the column 1/7 are excluded in the comparison, while the difference is very large between the test results of column 1/7 and the test results of the columns 2/5 and 2/6. Those columns were all tested with the same eccentricity of the normal force e_0 = 0.06. The difference indicates that the test of column 1/7 was unsuccesful.

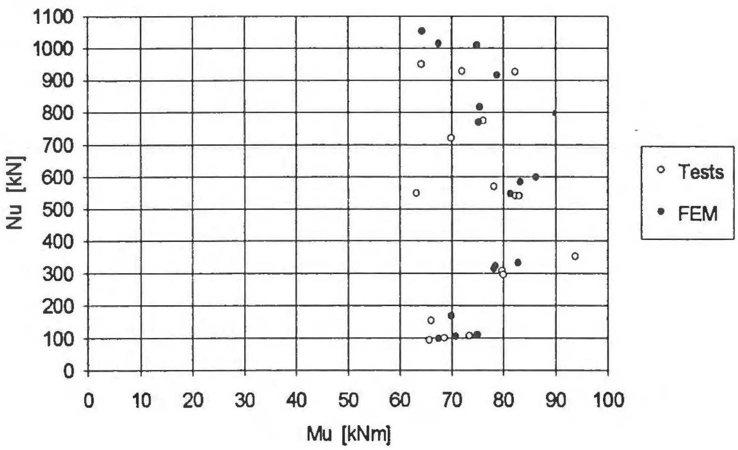

Figure 7. Strengths observed in tests [8] and strengths calculated by FEM.

The load-deflection relation of columns 1/4, 1/6, observed in the tests and obtained by the FEM-analysis, are plotted in Figure 8. The model of the columns used in the FEM-analysis is slightly stiffer than the actual columns.

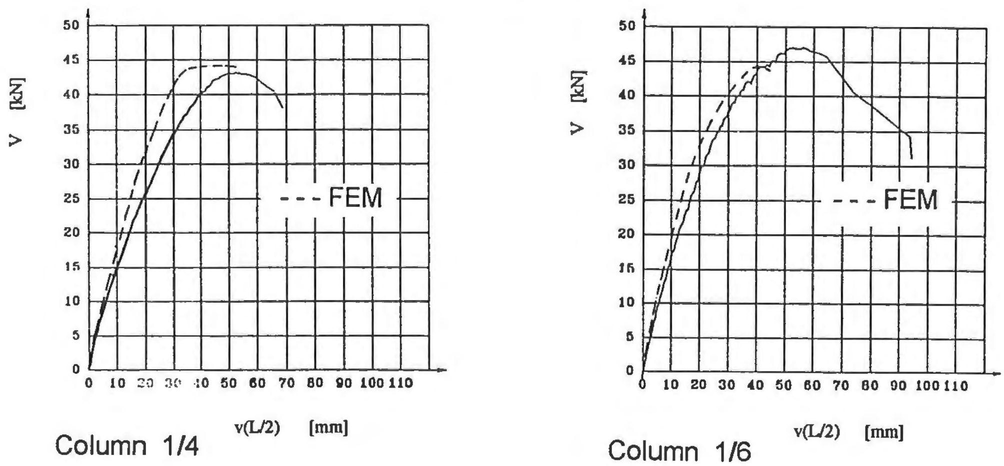

Figure 8. Load-deflection relation of the columns.

The comparison above shows that the strength of columns obtained by the nonlinear FEM-analysis using the model described in this paper are in a good agreement with the strengths observed in tests. The model is also relatively good in predicting the deflections of the columns.

EVALUATION OF COMPOSITE COLUMN DESIGN

Introduction

A nonlinear FEM-analysis of the concrete encased composite column under combined compression and bending was carried out using the model described and verified earlier in this paper. The *M-N* interaction curve of columns was determined in the analysis by using twelve loading cases with different combinations of axial and transverse loads.

In loading case 11 (Table 2) only self-weight is acting on the column causing an initial deformation which is assumed to simulate the initial imperfections in the actual column. In this case the strength of the column is reached at the ultimate state by the loss of stability in a flexural buckling of the column. In loading case 12 the geometry of the column is ideally straight, and the strength of the column is equal to the compressive crushing strength of the column.

In the analysis were used the dimensions and materials of the columns in test series 1 in reference [8]. The tests [8] showed that the steel ribs with holes, used as additional shear connectors in the columns, have no significant effect on the strength of the columns. In this study the ribs were not used.

The results of FEM-analysis are used to evaluate the validity of the design method in the Finnish code for composite structures by comparing the strengths of columns calculated according to the code with the strengths obtained by the FEM-analysis.

Design method of code for composite structures

According to the code for composite structures [15] ultimate compressive (N_u) and bending (M_u) capacity of a composite column must fullfill following conditions:

$$N_d \leq N_u = kN_p \tag{6}$$

$$M_d \leq M_u \tag{7}$$

where N_u is the ultimate compressive capacity of the column
M_u is the ultimate (plastic) bending capacity, when N= 0
N_p is the ultimate plastic compressive capacity
N_d *and* M_d are the design axial force and the design bending moment

The factor k is given by the formula

$$k = k_1 - (k_1 - k_2 - 4k_3)\frac{M_d}{M_u} - 4k_3\left(\frac{M_d}{M_u}\right)^2 \tag{8}$$

While acting combined with the axial load, the bending moment M_d reduces the compressive strength of the column according to the formulas (6) and (8).

In the case of the columns under consideration factors $k_2 = 0$ and $k_3 = 0$ and the formula (8) reduces to the form

$$k = k_1\left(1 - \frac{M_d}{M_u}\right) \tag{9}$$

The factor k_1 is also used in the code for steel structures [11] in calculating the compressive strength of an axially loaded steel column.

$$N_{Rc} = f_{cd}A = k_1 f_y A \tag{10}$$

where A is the cross-sectional area of the column
f_y is the yield strength of the steel

With factor k_1 the possible instability, buckling of the column, is taken into account. Also, in factor k_1 are included possible inaccuracies in the geometry of the column and initial stresses for example due to welding. This method of calculation is based on large test programme made by the organisation of the European Convention for Constructional Steelwork (ECCS).

According to the code for composite structures [15], [16] in the calculation of a concrete encased composite column, the eccentricities of the normal force are taken into account as for the concrete column in the code for concrete structures [10]. Then the design bending moment due to the loads is

$$M_d = M_0 + \Delta M_d \tag{11}$$

where $M_0 = N*e_0$ is the primary bending moment due to the loads by first order theory

N is the normal force due to the loads

e_0 is the primary eccentricity

$\Delta M = N*(e_a + e_2)$ is the secondary bending moment due to the eccentricity $(e_a + e_2)$ of the normal force

$e_a = \frac{L}{500} + \frac{h}{20}$ is the eccentricity, which takes into account the initial geometrical imperfection of the column $\left(\frac{L}{500}\right)$ and the construction tolerances $\left(\frac{h}{20}\right)$

$e_2 = \left(\frac{\lambda}{145}\right)^2 * h$ is the eccentricity which simulates the deflection of the column at the ultimate state.

In the design method based on the code for composite structures [15] the design bending moment (M_d) is composed of the primary bending moment (M_0) due to the loads and of the secondary bending moment (ΔM_d) due to the eccentric normal force.

In this method the eccentricity of the normal force is taken into account twice:

- The formula (8), used in the calculation of the strength of the column under combined compression and bending, is based on the theoretical analysis and tests of the columns [7], [17]. The equation takes into account the effect of the initial geometric imperfections and the construction tolerances and the deflection of the column and also the initial stresses.

- On the other hand, the eccentricities $(e_a + e_2)$, which are placed on the normal force due to the loads, take into account the construction tolerances and the geometric imperfection and the deflection of the column. The secondary bending moment (ΔM_d) due to the eccentric normal force is used in the formulas (11) and (8) in the calculation of the strength of column. This leads to an underestimation of the strength of the concrete encased composite columns.

The comparison of the column strengths obtained by FEM-analysis with the strengths calculated according to the Finnish code for composite structures [15] is presented in the Table 2.

Load case	e_0	e_1	FEM				Code [15]				$\frac{\text{Code}}{\text{FEM}}$			
			N [kN]	V [kN]	M_0 [kNm]	M [kNm]	N [kN]	V [kN]	M_0 [kNm]	M [kNm]	$\frac{N_{[15]}}{N_{FEM}}$	$\frac{V_{[15]}}{V_{FEM}}$	$\frac{M_{0\,[15]}}{M_{0\,FEM}}$	$\frac{M_{[15]}}{M_{FEM}}$
1	∞	∞	0	36,27	57,59	57,59	0	37,79	59,89	59,89		1,042	1,040	1,040
2	0,6	0,636	89,42	35,79	56,89	61,50								
	0,435	0,459					130,4	37,79	59,89	68,37				
3	0,2	0,212	267,8	35,70	56,74	67,42	225,2	29,70	47,75	62,42	0,841	0,832	0,842	0,926
4	0,1	0,106	503,1	33,54	53,51	73,48	327,3	21,00	34,70	56,01	0,651	0,626	0,648	0,762
5	0,06	0,065	704,6	28,19	45,49	71,27	396,8	15,06	25,79	51,64	0,563	0,534	0,567	0,725
6	0,04	0,044	899,1	23,98	39,17	65,38	445,3	10,93	19,60	48,59	0,495	0,456	0,500	0,743
7	0,03	0,033	1059	21,18	34,97	58,37	475,8	8,333	15,70	46,68	0,449	0,393	0,449	0,800
8	0,02	0,023	1260	16,81	28,42	51,78	507,3	5,645	11,67	44,70	0,403	0,336	0,411	0,863
9	0,01	0,012	1480	9,870	18,01	41,48	547,2	2,244	6,566	42,19	0,370	0,227	0,365	1,017
10	0,005	0,007	1608	5,363	11,24	32,81	567,4	0,515	3,973	40,92	0,353	0,096	0,353	1,247
11	0	0,002	1792	0	3,200	18,40	590,2	-1,43	1,055	39,49	0,329		0,329	2,146
12	0	0	1973	0	0	0	599,1	0	0	39,01	0,283			∞

Table 2. Column strengths calculated by FEM compared with the strengths according to the code for composite structures [15].

In the table $e_0 = \frac{a * V}{N}$ is the eccentricity of the normal force due the transverse loads V

$e_1 = \dfrac{(a*V)+M_g}{N}$ is the eccentricity of the normal force due the transverse loads V and selfweight g

N is the compressive axial load at the ultimate limit state i.e. the compressive strength

V is the transverse load at the ultimate limit state

$M_0 = M_V + M_g$ is the primary bending moment due the transverse loads V and selfweight g

$M = M_V + M_g + [v(L/2)*N]$ is the maximum bending moment due to the loads of the column at the ultimate state i.e. the bending strength of column.

According to the code for composite structures [15]:

$$M = M_V + M_g + (e_a + e_2)\cdot N$$

The strengths are plotted in the M_0-N-coordinations in the Figure 9.

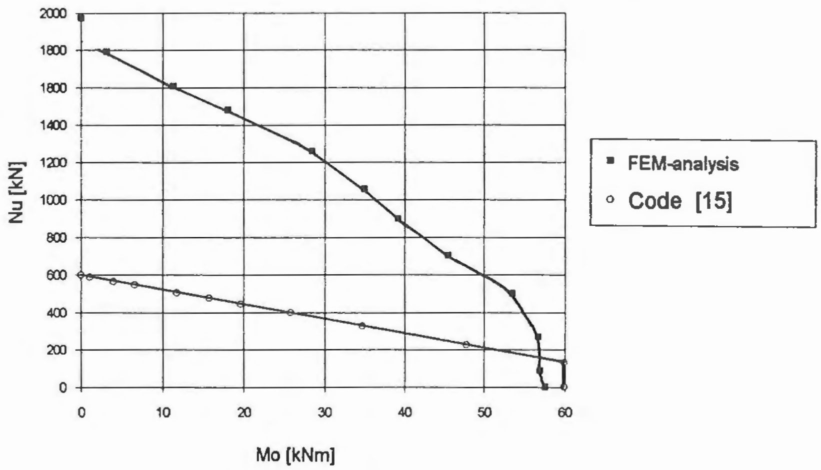

Figure 9. The M_0-N-interaction curves determined by the FEM-analysis and the code for composite structures [15].

One can see from Table 2 and Figure 9, that for the small eccentricity e_0, the strengths calculated on the basis of the code for composite structures are much lower than the strengths obtained by FEM-analysis. These results confirm the observation of the earlier study [8]. The design method based on the code for composite structures underestimates the strength of columns due to overemphasizing the normal force eccentricities $(e_a + e_2)$.

Modifications to the design method

If the eccentricities $(e_a + e_2)$ are neglected in the design method based on the code for composite structures, the calculated strengths of the columns agree closely with the test results [8] and the strengths obtained by FEM -analysis, as shown in Table 3 and Figure 10.

The neglecting of the eccentricities e_a and e_2 is supported by the facts, that already into factor k_1 in formula (8) the effect of construction tolerances and geometric imperfections and initial stresses are included, as when calculating the strength of the steel columns. Also the effect of the eccentricity due to the deflection of the column is included in formula (8), as the formula is adjusted to the test results and theoretical analysis, which takes into account second order effects.

When the eccentricities are neglected, the design method would also have the same basis as the method in the ECCS model code [7] (Commentary 16.4.2 and 16.5.3 Method A).

Load case	e_0	e_1	FEM			Code [15]			$\frac{\text{Code}}{\text{FEM}}$		
			N [kN]	V [kN]	M [kNm]	N [kN]	V [kN]	M [kNm]	$\frac{N_{[15]}}{N_{FEM}}$	$\frac{V_{[15]}}{V_{FEM}}$	$\frac{M_{[15]}}{M_{FEM}}$
1	∞	∞	0	36,27	57,59	0	37,79	59,89		1,042	1,040
2	0,6	0,636	89,42	35,79	56,89						
3	0,2	0,212	267,8	35,70	56,74	298,3	40,03	63,27	1,114	1,12	1,115
4	0,1	0,106	503,1	33,54	53,51	508,6	33,81	53,88	1,011	1,008	1,007
5	0,06	0,065	704,6	28,19	45,49	698,3	28,13	45,40	0,991	0,998	0,998
6	0,04	0,044	899,1	23,98	39,17	864,0	23,21	38,03	0,961	0,968	0,971
7	0,03	0,033	1059	21,18	34,97	987,0	18,81	32,56	0,932	0,888	0,931
8	0,02	0,023	1260	16,81	28,42	1133	15,24	26,06	0,899	0,907	0,917
9	0,01	0,012	1480	9,870	18,01	1353	8,840	16,23	0,914	0,896	0,901
10	0,005	0,007	1608	5,363	11,24	1484	4,785	10,39	0,923	0,892	0,924
11	0	0,002	1792	0	3,200	1650	-0,17	2,947	0,921		0,921

Table 3. Strengths calculated by FEM and strengths calculated on the basis of the code for composite structures [15], case $M_d = M_V + M_g$.

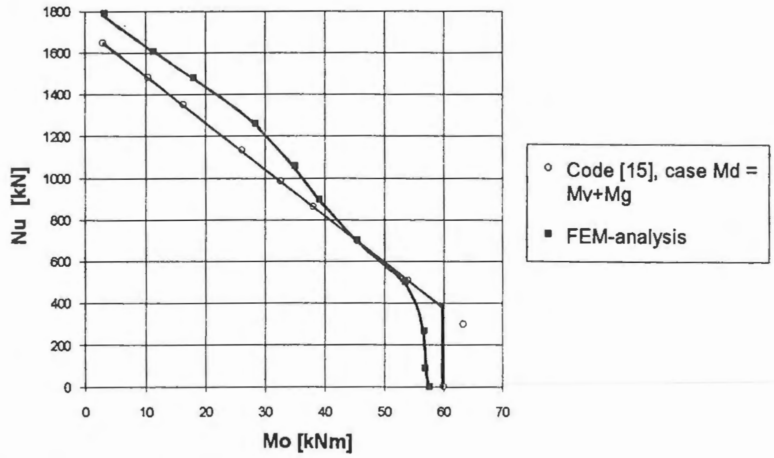

Fig. 10. The M_0-N-interaction curves determined by the FEM-analysis and the codes for composite structures [15], when in formula (8) the bending moment $M_d = M_V + M_g$.

CONCLUSIONS

For the analysis of a double symmetric concrete encased composite column under combined compression and bending a model can be used, in which:

- for the concrete part of the column constitutive laws [5], [6] are used that take into account the nonlinear stress-strain relation of concrete in compression and the tensile strain softening of concrete in tension.
- an elastic - plastic material model is used for the reinforcement steel and for the structural steel.
- the bending theory with the assumption of plane cross sections remaining plane and normal during bending is used
- full composite action is supposed between the structural steel and the concrete
- the geometric nonlinearity is taken into account.

The strengths of columns obtained by a nonlinear FEM-analysis using the above described model are in a good agreement with the test results. The model is relatively good in predicting the deflections of a column under the loads. However, the stiffness of the element model of columns is slightly greater than the actual stiffness of the columns.

The design method based on the code for composite structures [15], [16] for concrete encased composite columns under combined compression and bending underestimates the strength of the columns compared to the test results [8] and the results of FEM-analysis presented in this paper.

The reason for this is that on the normal force due to the loads are placed eccentricities (e_a+e_2), which take into account construction tolerances and the geometric imperfection and deflection of the column. The design bending moment used in the calculation of the strength includes the secondary bending moment $N^*(e_a+e_2)$ due to the eccentric normal force in addition to the primary bending moment caused by the loads.

That leads to a situation in which these eccentricities (e_a+e_2) are taken into account twice, because the effect of them is also included in the formula used in the calculations of the strength of the columns.

If the eccentricities (e_a+e_2) placed on the normal force due loads are neglected in the design method, the calculated strengths are in a good agreement with the test results [8] and the results of the FEM-analysis presented in this paper. In this case the design method based on the code for composite structures would also have the same basis as the method in the ECCS model code [7] (commentary 16.4.2 and 16.5.3 Method A).

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REFERENCES

/1/ ABAQUS Version 4.8, Users' manual, Example Problems manual, Hibbitt, Karlsson & Sorensen Inc.

/2/ ABAQUS Version 4.8, Theory manual, Hibbitt, Karlsson & Sorensen Inc.

/3/ Bazant Zdenek P., Pan Jiaying and Pijaudier-Cabot Gilles, Softening in Reinforced Concrete Beams and Frames, Journal of Structural Engineering Vol 113 Number 12 December 1987

/4/ Bazant Zdenek P., Pijaudier-Cabot Gilles, and Pan Jiaying, Ductility, Snapback, Size Effect, and Redistribution in Softening Beams or Frames, Journal of Structural Engineering Vol 113 Number 12 December 1987

/5/ Bazant Zdenek P. and Oh Byung H., Deformation of Progressively Cracking Reinforced Concrete Beams, ACI Journal May-June 1984.

/6/ Chern Jenn-Chuan, You Chii-Ming, Bazant Zdenek P., Deformation of Progressively Cracking Partially Prestressed Concrete Beams, PCI Journal January-February 1992.

/7/ European Convention for Constructional Steelwork (ECCS), Composite Structures, The Construction Press 1981.

/8/ Hyttinen Esko, Hyttinen Ville, (Observations of the Strength of Composite Columns, University of Oulu, Laboratory of Structural Engineering) Havaintoja puristetun ja taivutetun liittopilarin toiminnasta, Oulun yliopisto, Rakennetekniikan laboratorio, Julkaisu 33, Oulu 1993.

/9/ Outinen H., Pramila A., (The use of Finite Element Method, Tampere University of Technology) Lujuusopin elementtimenetelmän käyttö, Tampereen teknillinen korkeakoulu Opintomoniste 110A, Tampere 1988.

/10/ RIL 131-1990, (The Finnish Code for Concrete Structures) Betoninormit, Helsinki 1990.

/11/ RIL 173-1988, (The Finnish Code for Steel Structures) Teräsrakenteet, Rajatilamitoitusohjeet 1988, Helsinki 1988.

/12/ Saenz, Luis P., Discussion of "Equation for the Stress-Strain Curve of Concrete" by Prakash Desayi and S. Krishnan, ACI JOURNAL, Proceedings V. 61, No. 9 Sept. 1964, pp. 1229-1235.

/13/ Scordelis Alex C., Computer Models for Nonlinear Analysis of Reinforced and Prestressed Concrete Structures, PCI Journal November-December 1984.

/14/ Timoshenko S. P., Gere J. M., Theory of Elastic Stability, McGraw-Hill Book Company, New York 1961.

/15/ TRY/by 26, (The Finnish Code for Composite Structures) Liittorakenteet, Suunnitteluohjeet 1988, Helsinki 1988.

/16/ TRY/by 36, (Commentary on the Finnish Code for Composite Structures) Liittorakenteiden sovellusohjeet, Helsinki 1991.

/17/ Yam Lloyd C. P., Design of Composite Steel-Concrete Structures, 1981.

/18/ Zienkiewicz O. C., The Finite Element Method, Third edition, McGRAW-HILL Book Company (UK) Limited, London 1977.

/19/ Zienkiewicz O. C. and Taylor R. L., The Finite Element Method, Fourth edition, Volume2 Solid and Fluid mechanics, Dynamics and Non-linearity, McGRAW-HILL Book Company (UK) Limited, London 1991.

/20/ Johnson R. P. and Buckby R. J., Composite Structures of Steel and Concrete, Volume 2: Bridges, London 1986.

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