LONG TERM DEFLECTION OF TIMBER BEAMS

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SUMMARY

The present paper summarises some of the main points of a recently published doctoral dissertation written by the author. A calculation method to predict the long term mechanical behaviour of timber beams in variable humidity and loading conditions was developed.

INTRODUCTION

The correct prediction of the deformations of timber structures is an important question in the design process. The mechanical performance of timber structures is usually regulated by the deflection of the member rather than by the actual strength. One reason being that the ratio of the strength of wood to its elastic modulus is very high as compared to other building materials. But perhaps a more important reason is that the increase of deformation with loading time is known to be very significant for wood.

From previous experience, the creep of wood in constant humidity conditions is of a moderate magnitude: for low stresses the elastic deformation can be multiplied roughly by a factor of 1.4 to account for a one year load duration. But if the humidity conditions are variable, the creep magnitude is known to be several times higher. This has been termed the mechano-sorptive effect. This effect has been known for a long time, but its quantitative prediction and impact on timber structures subjected to natural humidity and loading conditions is poorly understood.

Timber structures are subjected to a variable climate by nature. An outdoor sheltered environment could be considered as the most severe condition to which load carrying timber is used. An indoor climate is also variable, but usually with a lower cycle amplitude. To have a good estimate of the creep, it is thus important to understand the effect of variable moisture conditions on creep as well as to know the effect of the surrounding climate on the moisture content distribution history of the beam.

GENERAL DESCRIPTION OF THE MODEL

A model was derived in Toratti (1992) to predict the creep of wood in varying environment humidity when subjected to bending, tension or compression parallel to grain. This method is a combined transient moisture transfer analysis and a deformation analysis. The method is used to analyse test results in the study of constitutive equations as well as to model the creep of beams of different member cross sections and in different humidity environments.

The computation method is based on the finite difference method (explicit method) to compute the transient moisture content distribution and on the 'method of successive approximations' to compute the stress and strain distributions. These numerical methods have been introduced in early literature. Both of these numerical methods have been used before in a number of studies. In this study, these two numerical methods are combined. This is particularly important in the study of the long term behaviour of wood, since for wood the deformations under sustained loads have been found to be very sensitive to moisture content changes. The main principle in the model is that only a single cross section is analysed. The procedure is a time marching solution of an initial value problem. For every time step, first the two-dimensional moisture content field is determined. This is followed by the axial deformation analysis which uses the moisture content values, as governed by the constitutive relations, to calculate the stress and strain distributions parallel to grain in the cross section.

The input required to define the material behaviour in the analysis consists of material parameters involved in moisture transfer as well as parameters involved in the constitutive relations. The input needed for the moisture transfer analysis are the sorption isotherm equation, a moisture content dependent diffusion coefficient and a constant surface emissivity. The constitutive equations include a moisture content dependent elastic modulus, a normal creep model, a mechano-sorptive creep model and a shrinkage coefficient parallel to grain. The problem-dependent inputs are the cross section size, the loading and the relative humidity of the surrounding air as a function of time.

In the following a general description of the model is given. A more detailed explanation as well as the mathematical formulations and material parameters used, can be found in ref. Toratti (1992).

CONSTITUTIVE RELATIONS

The constitutive model and material parameters introduced in the following have been determined from creep tests of clear wood in bending. The constitutive equation of wood is assumed to be linear with respect to stress. This restriction leads to that the model, at present, can only be used at low stress levels. However, timber structures are designed such that the working stresses are at low levels where the material is known to behave linearly. Thus, this restriction imposes no disadvantages when the modelling of creep in real loading conditions is concerned.

The strain of wood when subjected to sustained loading and moisture content variation is assumed to consist of the following additive parts: elastic ε_E , normal creep ε_c , mechano-sorptive creep ε_{ms} and free shrinkage strains ε_u :

$$\varepsilon(t) = \varepsilon_E + \varepsilon_c + \varepsilon_{ms} + \varepsilon_u \quad (1)$$

Where: ε_E is the elastic strain
ε_c is the normal creep strain
ε_{ms} is the mechano-sorptive creep strain
ε_u is the shrinkage strain

The deformations are calculated only parallel to grain. A linear strain distribution, Bernoulli hypothesis, over the cross section is assumed. The strain increment at each time step is then calculated using the trapezoidal rule.

The elastic strain depends on the elastic compliance, which is a function of the current moisture content.

The normal creep, which is due to the deformation induced by the duration of load, is given in a Kelvin series form which consists of six Kelvin elements having different retardation times. The parameters of the viscoelastic creep function were derived from experiments performed in constant relative humidity. The normal creep model used here is independent of moisture content and inversely proportional to the elastic modulus. Based on the experimental evidence available to the author, the creep of wood at different constant moisture contents seem to be of a similar magnitude and no moisture content dependency could be incorporated in the model.

The mechano-sorptive creep strain, which is due to the additional creep deformation caused by the coupled effect of variable moisture and load duration, is computed using the creep limit formulation. This strain component, being less known than the other strain components and being of high importance for wood in variable humidity conditions, is explained in the following with more detail. In a constant stress case this takes the form:

$$\varepsilon_{ms}(t) = J^{\infty}\sigma\left[1 - \exp(-c\int_{0}^{t}|du(t')|\right] \quad (2)$$

Where: J^{∞} is the limit mechano-sorptive compliance
u is the moisture content
c is a material parameter

The limit compliance, which is an important parameter when estimating creep in variable humidity conditions, was found to be best given as proportional to the elastic compliance. In a variable stress, applying the Boltzman superposition principle, it takes the form:

$$\varepsilon_{ms}(t) = J^{\infty}\int_{0}^{t}\left[1 - \exp(-c\int_{t'}^{t}|du(t'')|)\right]d\sigma(t') \quad (3)$$

The expression above is analogous to the Kelvin element model used to describe viscoelasticity, when the moisture accumulation integral is replaced by the time variable. In the analysis, discrete moisture content increments at defined time steps are used and the model can then be formulated as follows:

$$\varepsilon_{ms}(t) = J^{\infty}\int_{0}^{t}\left[1 - \exp(-c\left\{\sum_{0}^{t}|\Delta u| - \sum_{0}^{t'}|\Delta u|\right\})\right]d\sigma(t') \quad (4)$$

The mechano-sorptive strain increment is then calculated by the trapezoidal rule from:

$$\Delta\varepsilon_{ms} = J^{\infty}(1 - \exp(-c|\Delta u|))(\frac{\Delta\sigma}{2} + \sigma_n^{hist}), \quad (5)$$

where the term σ^{hist}, is updated after each time step.

$$\sigma^{hist,update} = (\sigma^{hist} + \frac{\Delta\sigma}{2})\exp(-c|\Delta u|) + \frac{\Delta\sigma}{2} \qquad (6)$$

The shrinkage strain, the reversible moisture induced strain, is assumed to be effected by the total strain. Wood then shrinks and swells more in a state of compression strain and less in a state of tension strain than when not loaded. This behaviour can be explained by a change of the structure geometry when wood is deformed under load: A compression strain increases the crookedness of the cell walls and on the other hand a tension strain increases the alignment of cell walls to a more parallel and straight formation. Since the shrinkage is of much higher magnitude perpendicular to the cell wall, this would result in a higher shrinkage value when wood is compressed and less when tensioned as compared to an unstressed state. This strain component does not actually cause creep, but in cyclic humidity conditions it accounts for (and is the only accepted explanation of) the oscillation of the creep curve of a bended member.

$$\varepsilon_u(t) = \{\alpha - b\varepsilon(t)\}[u(t) - u(0)] \qquad (7)$$

Where: α is the shrinkage coefficient
b is a material parameter
$\varepsilon(t)$ is the current total strain, excluding the shrinkage strain itself

ANALYSIS ALGORITHM

The cross section of the modelled member is spatially discretized into a number of nodes in the height and width directions. The computations carried out for each node of the cross section follows the algorithm below:

1. Compute the moisture content distribution over the cross section. Transient moisture transfer analysis by the finite difference method using an explicit scheme.
2. Update material properties according to the moisture content distribution.
3. Compute stress distribution.
   - Initially (t=0) according to elastic theory.
   - First iteration values from previous time step.

4. Compute strain increment at the nodes during the elapsed time step using current stress distribution, current moisture content and moisture content increment during the time step.

5. Calculate stress increment at each node during the elapsed time step. Return to 3 and re-estimate the strain increments, until the change of the stress increment per iteration is less than 0.1 %.

6. Compute section forces using the stress distribution and compare if they match with the external loads N, M. If precision or convergence requirements are not satisfied return to 3. and adjust the strain distribution according to the difference of the internal and external loads considering the current stiffness of the cross section.

7. Program output for the current time:
   - Strain distribution $\varepsilon(x,y)$
   - Deflection from the curvature of a beam
   - Stress distribution from the given sections $\sigma(x,y)$.

8. Proceed to next time step by adding a time increment and return to 1.

The output from the analysis consists of the following distributions over the cross section at every time step:

moisture content, $u(x,y)$,
strain, $\varepsilon(x,y)$,
stress, $\sigma(x,y)$.

The strain and stress distributions are directed parallel to grain, normal to the cross section plane. The curvature values derived from the strain distributions are used to compute the creep of the member. Because the analysis is carried out only to a single cross section of the member, it can only scope with the load acting on this cross section. Thus, the deflections determined are of beams subjected to a constant moment over the span where the curvature is constant.

The value of the time step is chosen in a way that the stability of the computations is ensured. The explicit method of solution is convenient computationally, but suffers from the above requirements for stability. More accurate results are obtained when using a large number of nodes to model the cross section, but the stability time step is relative to the second power of the node spacing and this leads to smaller time steps. Better methods of solution, such as the implicit method, which is unconditionally stable could have been used. However, the creep analysis gives better accuracy of the integration's the smaller the time steps, and it was found that the time steps dictated by the explicit method of moisture transfer performed well also in the creep analysis. For this reason the explicit method of solution was chosen for the transient analysis of moisture transfer.

COMPARISONS OF THE MODEL AND TEST RESULTS

In general, the modelling method derived performed well in comparison to the test results. It was able to scope with the different specimen sizes and different relative humidity environments. The test results showed usually a relatively high scatter in creep between matched specimens. However, since the modelled creep values were in the mid-range within the test result scatter, it was not necessary nor appropriate to change the material parameters of the models, when making comparisons to different experiments with different specimens sizes or different humidity environments. Fig. 1 shows an example where the model has been compared to a test sample of five 45 x 45 mm^2 sized specimens.

In addition to the experimental work carried out by the author, the model has also been compared to experiments carried out in other foreign research institutions in joint publications as well as to those test results available from earlier literature where the relative humidity conditions are cycled. The manner of proceeding was to determine the material parameters for the constitutive equations from the first set of experiments carried out by the author and then keep them fixed in the comparison against other experimental results, where the cross section of the specimens or the relative humidity cycles are different. The good agreement between computed results and test results in almost all cases indicate that the calculation method is correct and that the material parameter values used are in the correct range.

Fig. 2 shows a comparison of the modelled creep to test results where four pine glulam beams of size 95 x 176 mm^2 have been loaded to a bending stress of 8.2 MPa and subjected to an outdoor humidity environment but protected from direct rain. The modelled results were computed using the monthly mean

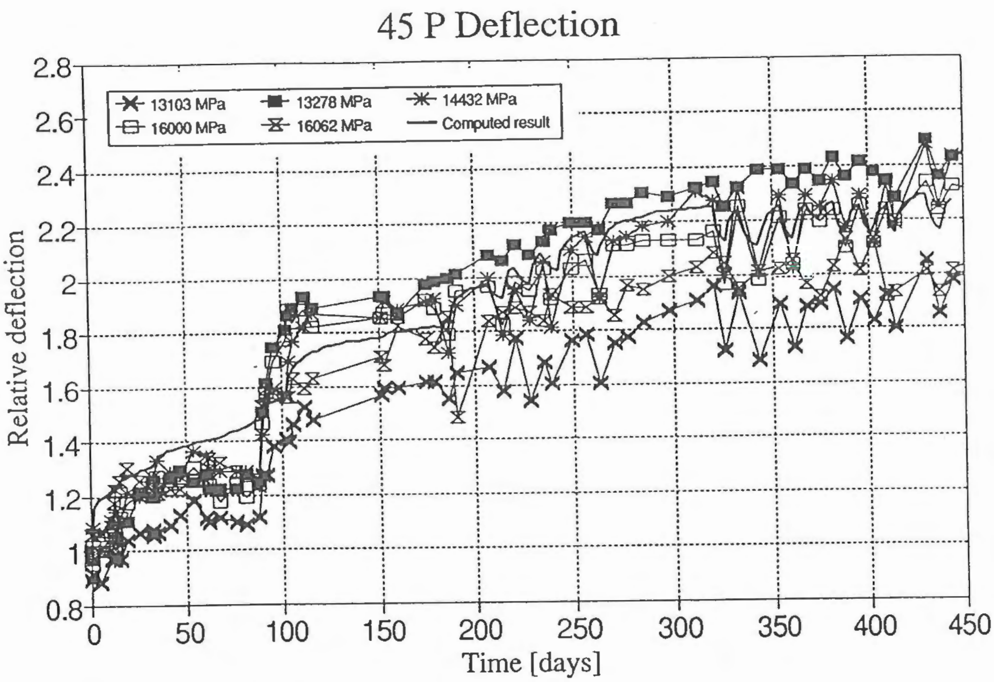

Fig.1. Creep test results of five 45 x 45 mm^2, span 1200 mm, beam specimens subjected to cyclic humidity conditions as compared to the model.

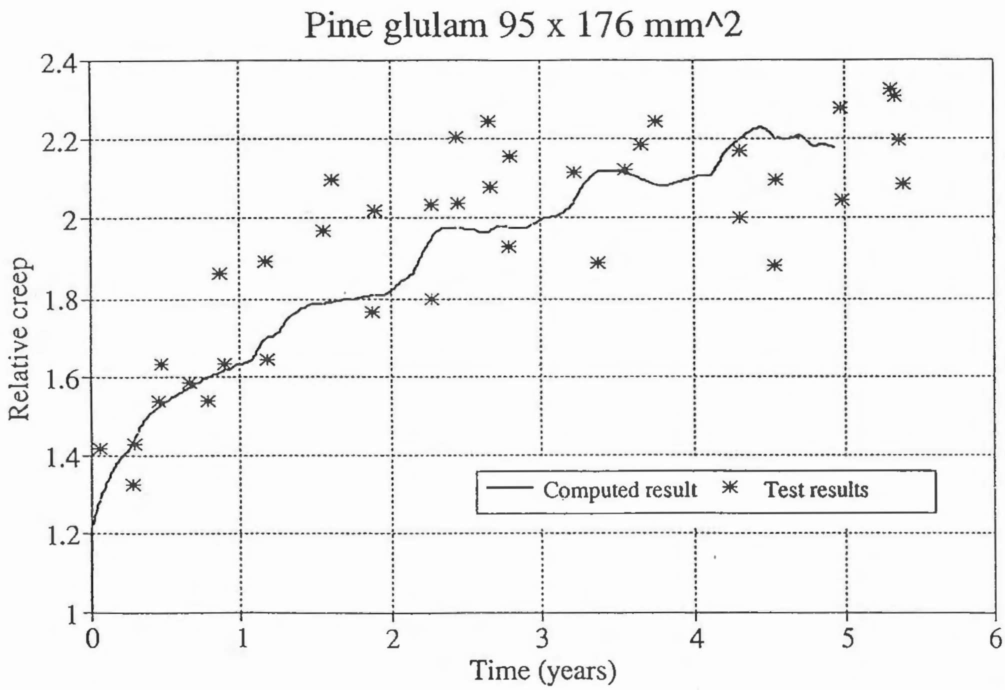

Fig. 2. Comparison of computed results to test results of glulam beams subjected to an outdoor and sheltered humidity environment.

relative humidity values as the environment input data. Here the model agrees very well with the test results. However, for the cases where the beams were not protected from direct rain the model did not agree or it could not be used since the moisture distribution history cannot be calculated with reasonable accuracy. These beams obtained high magnitudes of creep as compared to those of fig.2.

CONCLUSIONS

The mechano-sorptive as well as the normal creep compliance's were given as values relative to the elastic compliance at a reference moisture content. These relations seem appropriate since the creep appears in most cases to be relative to the elastic deformation value at low stress levels. The creep limit type mechano-sorptive formulation was found good in estimating the strain which is caused by a variable moisture content and the Boltzman superposition principle appeared to be valid for normal as well as for mechano-sorptive creep at the tested low load levels. The stresses used in the experiments were 10 MPa and below. For clear wood this is a stress level of about 15 %. This level corresponds to about the maximum working stress value on timber structures.

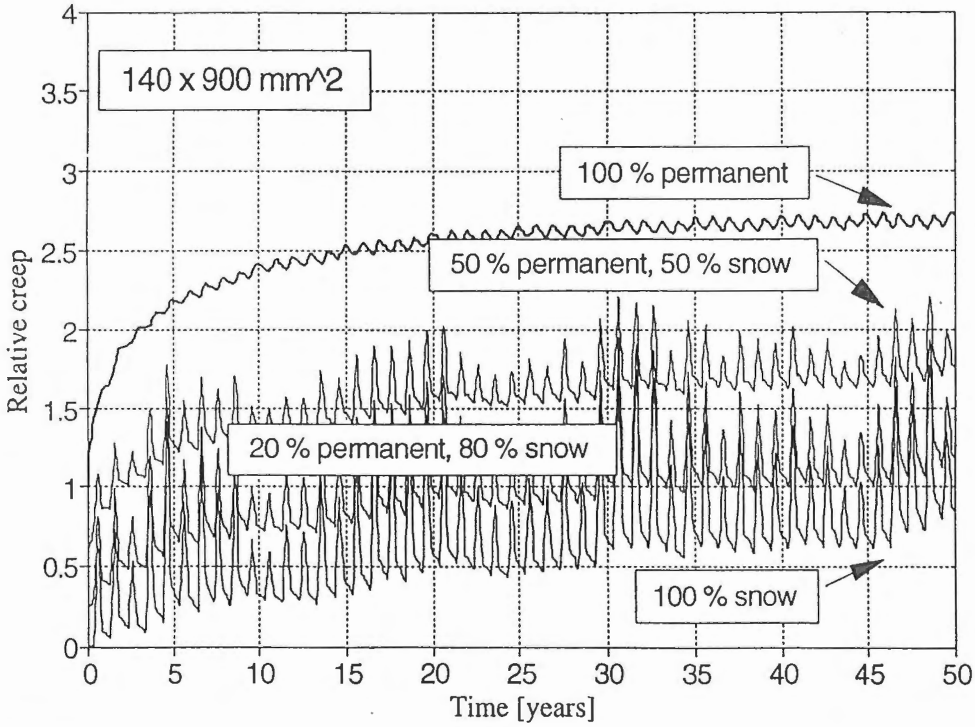

Fig.3 Estimated creep for a 50 year period in outdoor conditions of 140 x 900 mm^2 size glulam beams.

The creep of timber structures in bending is not proportional to the increment of the average moisture content when different cross section sizes are compared. This is due to the differences in the fluctuation of the moisture content distributions in different size cross sections. To study the size effect a computational analysis is required, as for instance the one described here, to evaluate the effect of the environment on the mechanical behaviour.

Fig. 3 shows the estimated creep behaviour of 140 x 900 mm^2 sized glulam beams when loaded with different snow load - permanent load ratios and subjected to an outdoor relative humidity environment. The snow loads are assumed to be triangular in the time scale and the maximum snow load value has been simulated simulated for each year based on a 85 year database of registered snow depth values in Umeå Sweden. The simulation was necessary since the ultimate snow load is random by nature and should be treated by a probabilistic basis.

DISCUSSION

The main assumptions of the model can be briefly summarised as follows:

- The deflection of the member is assumed to be only due to bending. Only a single cross section of the beam is analysed and the Bernoulli hypothesis of cross sections remaining in plane is assumed valid.
- Here no account has been taken on the variable temperature environment. It has been experimentally shown that the temperature which is within the natural range does not influence the creep behaviour significantly. The temperature does though have some impact on the moisture transfer properties of wood.
- The constitutive equations are taken to be linear with respect to stress. At working stress levels, this has been experimentally found to be so.

At present, the model is being compared with experimental results carried out in France and Germany, where full size timber and glulam beams have been loaded for several years in an outdoor environment. The results from these comparisons will be published shortly. Further theoretical work is also underway to refine the model inorder to predict the long term strength of timber as well in variable humidity conditions. This is part of an initiating co-operative research project on the size effect of the long term strength of wood to be carried out by Finland together with four other European countries.