ANALYSIS OF THE FIRE RESISTANCE OF STEEL AND COMPOSITE COLUMNS

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SUMMARY

A simple computer program for the structural analysis of eccentrically loaded columns in fire conditions is presented. In the program a pin-ended column is studied. The spatial discretization in the axial direction is carried out by uniform difference mesh. One dimensional material models developed for Eurocodes and in Technical University of Brunswick are used. Forces and moments in the cross-section are integrated using finite element discretization and Gaussian quadrature formula. The validity of the approach is established by comparison with available test results and analytically derived data of proven accuracy.

INTRODUCTION

Calculation of the fire resistance of column is carried out in various steps. It involves calculation of the fire temperatures to which the column is exposed, the temperatures in the column cross-sections and deformations and strength of the column during the fire exposure.

In the thermal analysis temperature distribution in the cross-section is determined. At free surfaces of the column heat is transferred by convection and radiation. These phenomena are complex and difficult to model, but approximate formula can be used. Convective heat transfer is calculated by using Newton's law and radiation heat flux from surface is calculated using by Stefan-Boltzmann equation. Heat conduction is described by the heat balance equilibrium equation and with the well-known Fourier-law. Applied to composite structures some simplifications /1/ are necessary;

- water vaporizes once it has reached its boiling point,
- consumption of energy for vaporizing the water and other similar peculiarities are taken into account in a simplified way by suitable design values for the specific heat-capacity of concrete with up to 200 °C.

Program used in the present paper for temperature calculations is LIPA-program developed by SUBNIC OY /2/. In LIPA-program finite element method in connection with conditionally stable time integration scheme is used to calculate the temperature distribution in the cross-section.

STRUCTURAL ANALYSIS

In VTT Fire Technology Laboratory a simple computer program (about 1000 FORTRAN 77 lines) called SWEAT is developed. This program predicts the fire resistance of eccentrically loaded rectangular reinforced concrete infilled steel hollow sections. The program can be used also for reinforced concrete or bare steel columns.

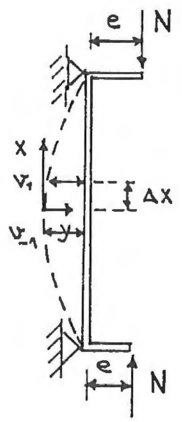

Fig. 1. Finite difference mesh used in load-deflection analysis in SWEAT-program.

For the structural analysis of columns finite difference method is used /3/ in the program. The program is used for simple case of pin-ended column. The spatial discretization in the axial direction is carried out by uniform difference mesh (fig. 1), with mesh size Δx . In an arbitrary finite difference node the deflection is v_i .

Equilibrium equations

Equilibrium equation for pin-ended column is as follows

$$N_i^j = N_o$$
$$M_i^j = N_o(e + v_n^j - v_i^j) \tag{1}$$

where N_o is external normal force,
v_i^j deflection at node i at approximation iteration step j
v_n^j deflection at the end of the column at approximation iteration step j
e eccentricity of the external load,
N_i^j, M_i^j moment and normal force at approximation iteration step j

Here origin of the co-ordinate system is assumed to follow the deformed column at midspan. Because the problem is geometrically nonlinear, normal force is increased incrementally, and to find out if an equilibrium state is possible for an axial load and a given eccentricity an iteration process of successive approximations is used inside the force increment step. Reasonable starting values for deflections are the deflections of the previous equilibrium state.

The problem is also materially nonlinear and therefore one has to calculate iteratively the magnitude of curvature κ_i^j and strain e_i^j at central axis of the column cross-section. This is done by using modified Newton-Raphson method in a following manner:

$$\begin{pmatrix} N_i^j - N_i^{j,k} \\ M_i^j - M_i^{j,k} \end{pmatrix} = \begin{bmatrix} \frac{\partial N_i^{j,k}}{\partial \varepsilon} & \frac{\partial N_i^{j,k}}{\partial \kappa} \\ \frac{\partial M_i^{j,k}}{\partial \varepsilon} & \frac{\partial M_i^{j,k}}{\partial \kappa} \end{bmatrix} \begin{pmatrix} \delta \varepsilon_i^{j,k+1} \\ \delta \kappa_i^{j,k+1} \end{pmatrix} \tag{2}$$

where $N_i^{j,k}, M_i^{j,k}$ are normal force and moment in finite difference node i at k:th Newton-Raphson iteration step during j: th approximation iteration step.

Inserting equations of stress resultants and strain

$$\begin{aligned} N_i &= \int_{A_i} \sigma dA \\ M_i &= \int_{A_i} \sigma y dA \\ \varepsilon_{tot} &= \varepsilon + \varepsilon_{th} + \kappa y \end{aligned} \tag{3}$$

where ε_{tot} is total strain

ε force induced strain at central axis of the cross-section

ε_{th} is thermal strain

κ force induced curvature strain

to the matrix equation (2) one gets a matrix equations (4, 5) from which strain and curvature corresponding certain moment and normal force N_i^j, M_i^j can be solved iteratively.

$$\begin{pmatrix} \delta \varepsilon_i^{j,k+1} \\ \delta \kappa_i^{j,k+1} \end{pmatrix} = \begin{bmatrix} \int_A \frac{\partial \sigma}{\partial \varepsilon} dA_i & \int_A y \frac{\partial \sigma}{\partial \varepsilon} dA_i \\ \int_A y \frac{\partial \sigma}{\partial \varepsilon} dA_i & \int_A y^2 \frac{\partial \sigma}{\partial \varepsilon} dA_i \end{bmatrix}^{-1} \begin{pmatrix} N_i^j - N_i^{j,k} \\ M_i^j - M_i^{j,k} \end{pmatrix} \tag{4}$$

$$
\begin{aligned}
\varepsilon_i^{j,k+1} &= \varepsilon_i^{j,k} + \delta\varepsilon_i^{j,k+1} \\
\kappa_i^{j,k+1} &= \kappa_i^{j,k} + \delta\kappa_i^{j,k+1}
\end{aligned} \tag{5}
$$

Discretization of the column

The curvature κ at certain point may be approximately calculated as follows:

$$\kappa = -v_i'' = -\frac{v_{i+1} - 2v_i + v_{i-1}}{(\Delta x)^2} \tag{6}$$

As in the middle of a pin-ended column $v_1 = v_{-1}$ and $v_0 = 0$ are valid, it follows that

$$v_1 = -\frac{(\Delta x)^2 \kappa_o}{2} \tag{7}$$

so it is possible to calculate the deflection v_1 in the next node if the curvature is calculated using equations (4) and (5). In general it follows from equation (6) that

$$v_{i+1} = 2v_i - v_{i-1} - (\Delta x)^2 \kappa_i \tag{8}$$

Using this equation it is possible to calculate deflection at each point from the results of earlier calculated deflections. Thus deflection at the ends of the column is determined. This deflection is the midspan deflection if the origin of coordinates is changed.

Axial displacement is calculated using the well known von Karman formula for strain:

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 \tag{9}$$

From this equation axial displacement of each point can be solved with a forward difference scheme using deflections calculated from (7) and (8):

$$u_{i+1}=u_i+\Delta x\ \varepsilon_i-\frac{1}{2}\frac{(v_{i+1}-v_i)^2}{(\Delta x)^2} \tag{10}$$

Integration of normal force and moment

Temperature field of the column cross-section calculated at different time steps by using LIPA-program is read to the SWEAT-program as input data. One quarter or one half of the column cross-section is divided to elements with the same element mesh as in the thermal analysis. Moment and normal force are calculated at each element k by using Gaussian quadrature formula /4/ as follows:

$$\begin{aligned} N &= \sum_k \frac{A_k}{4}\sum_i \sum_j H_i H_j \sigma(\Theta_{ij},\zeta_j,\eta_i) \\ M &= \sum_k \frac{A_k}{4}\sum_i \sum_j H_i H_j y(\eta_i)\,\sigma(\Theta_{ij},\zeta_j,\eta_i) \end{aligned} \tag{11}$$

where $H_i,\ H_j$ are weight coefficients of quadrature formula,

Θ_{ij} is temperature in Gaussian integration point i,j

and $\zeta_j=\frac{z_j-z_c}{a}$, $\eta_i=\frac{y_i-y_c}{b}$ are natural local co-ordinates (see fig.3) of the Gaussian integration points.

In the formulas above it is assumed that shape functions of the temperature field are linear Serendip-type:

$$\Theta_{ij}=\sum N_k\Theta_k \tag{12}$$

where Q_k are temperatures in the element nodes.

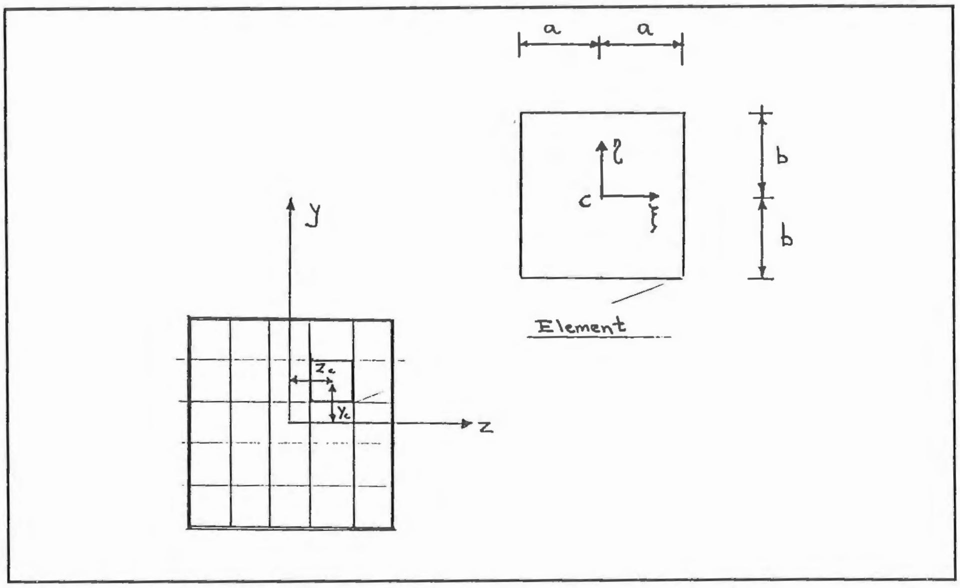

Fig. 2. Element mesh of the cross-section and natural local co-ordinates of the elements.

$$
\begin{aligned}
N_1 &= \frac{1}{4}(1-\zeta_j)(1-\eta_i)\\
N_2 &= \frac{1}{4}(1+\zeta_j)(1-\eta_i)\\
N_3 &= \frac{1}{4}(1+\zeta_j)(1+\eta_i)\\
N_4 &= \frac{1}{4}(1-\zeta_j)(1+\eta_i)
\end{aligned}
\qquad (13)
$$

MODELS FOR MECHANICAL PROPERTIES

Eurocode models

In Eurocode No. 2, Design of Concrete Structures /5/ and in Eurocode No.4, Design of Composite Structures /6/ a simple one dimensional nonlinear elastic model for strength and deformation properties of uniaxially loaded concrete in compression at elevated temperatures is presented (Fig.3). Values for model parameters, relation between compressive strengths and strain, are given in Eurocode No. 2 as a function of the concrete temperature for both siliceous and calcareous concretes.

Strength and deformation properties of steel at elevated temperatures according to the Eurocodes are characterized by a set of stress-strain relations as shown in the Fig. 4. In the Eurocode model yield plateau at different temperatures is reached at 2 % strain.

Brunswick model

A nonlinear elastic one dimensional material model developed at Technical University of Brunswick (TUBS) /7, 8/ may be used for stress-strain calculations of both siliceous concrete and different steel qualities. Stress-strain curves for structural steel and steel siliceous concrete are shown in Figures 5 and 6. Coefficients of the model for siliceous concrete and several structural and reinforcing steel qualities are given in /8/.

Thermal expansion

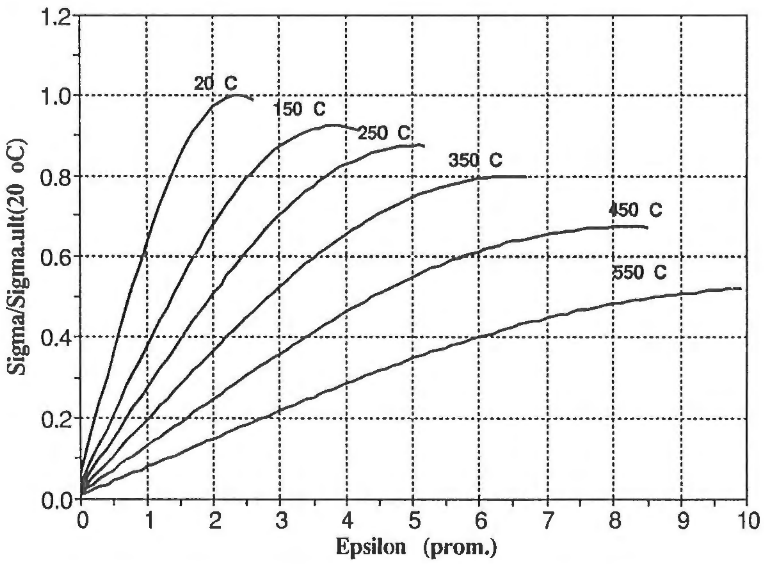

Fig. 3. The stress-strain relations of siliceous concrete according to Eurocode model /5/.

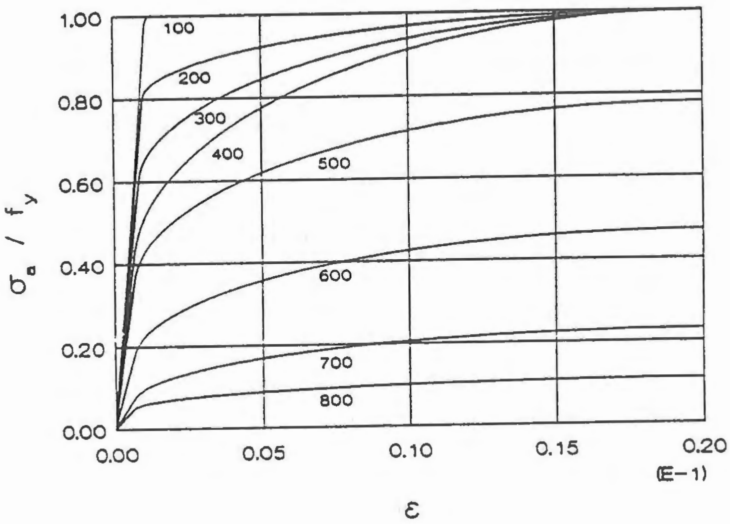

Fig. 4. The stress-strain relations of structural steel according to Eurocode model /6/.

Thermal expansion of both concrete and steel is calculated by using equation (14):

$$\epsilon_{th} = \sum_{k=0}^{3} e_k \Theta^k \tag{14}$$

Coefficients e_k given either in Eurocode or in Brunswick model are used.

EXAMPLES

Euler buckling at ambient temperature

In the SWEAT-program load is increased stepwise. Calculated axial force-displacement is dependent on the load increment which can be seen in Fig. 7, where midspan deflection of an elastic column at ambient temperature has been calculated by using two different load increments.

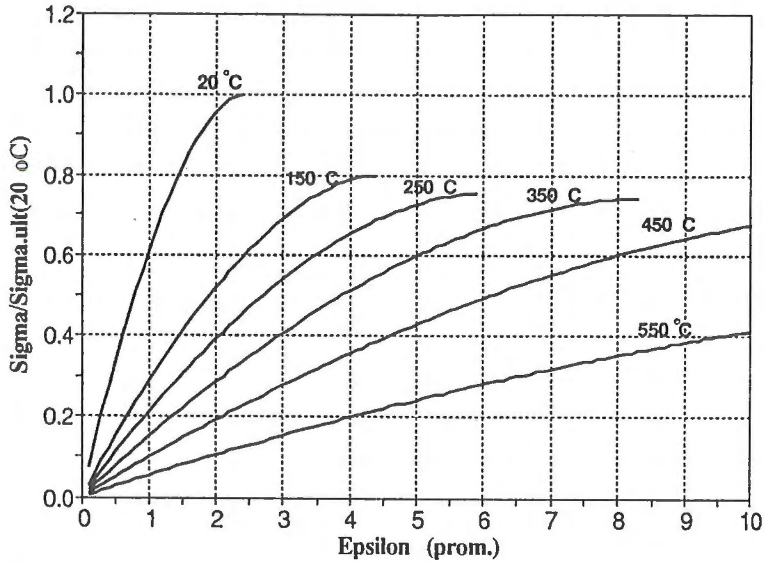

Fig. 5. The stress-strain relations of siliceous concrete according to Brunswick model /7, 8/.

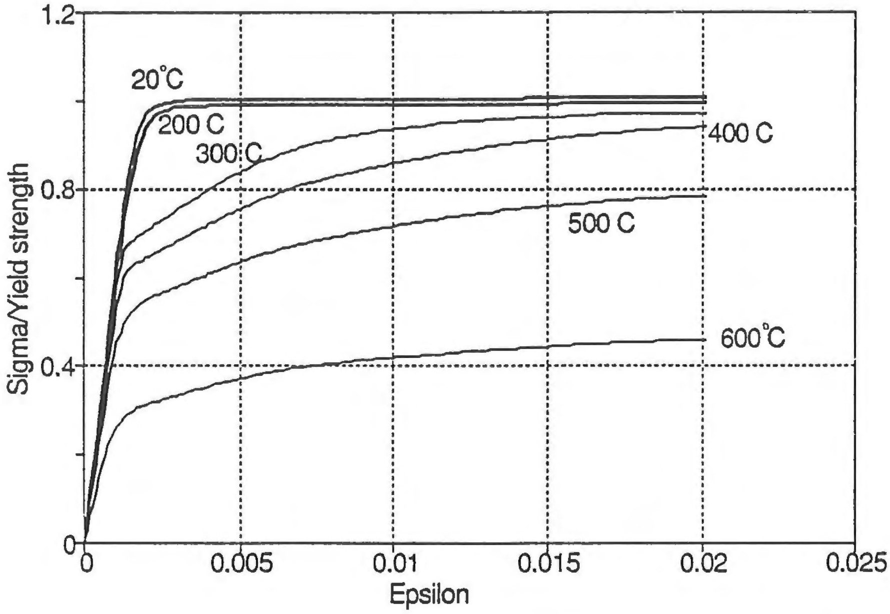

Fig. 6. The stress-strain relations of structural steel according to Brunswick model /7, 8/.

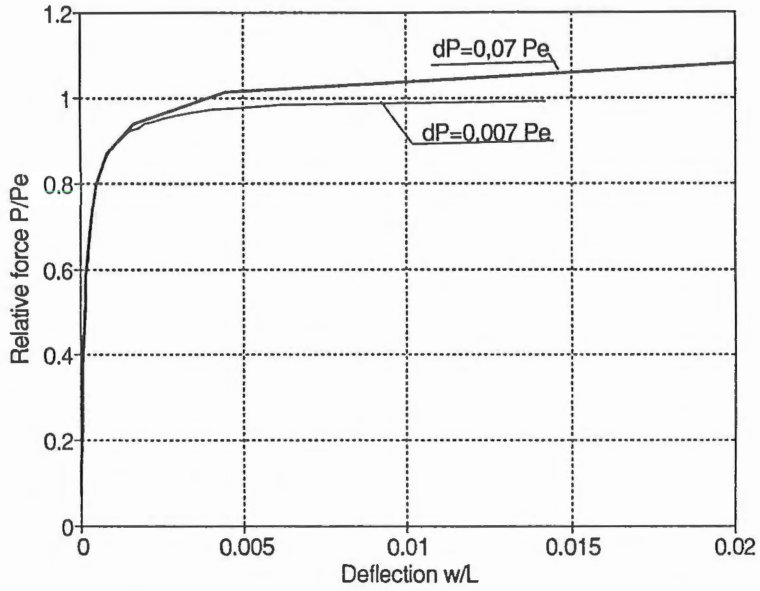

Fig. 7. Force-displacement curve of a pin-ended column, P_e = Euler buckling load.

Axially loaded steel columns at elevated temperatures

In a paper by Vandamme and Janns /9/ a series of full scale tests made at the University of Ghent as well as tests made by Olesen at Aalborg University on centrally loaded steel columns at elevated temperatures are described.

Figure 8 shows the stress results of 29 tests against the theoretical results obtained by SWEAT-program with using lengthwise 22 finite difference nodes and the Eurocode steel model. In figure 9 cumulative distribution of the relations of test result and calculated result is shown.

Also all the 60 test results (Vandamme & Janss /9/, Olesen /9/ and Knublauch et al. /10/) have been calculated by using an elementary limit state method of Eurocode N:o 3, Design of Steel Structures /11/, with factor 1,2 and also by using the method presented in Finnish design code B7 /13/ (nearly the same as ECCS-method). Both methods adopt the European buckling curve c. Cumulative distribution of the relation of test results and calculated results (with material safety factor 1) are also shown in Fig. 9. As it can be seen a simple method of Eurocode N:o 3 gives almost the same cumulative distribution (25-30 % unsafe) as a more general finite difference method SWEAT with Eurocode steel model. When material safety factor is 0,9, as Eurocode N:o 2 assumes, 46 % of the test results are on the unsafe side when the simple method of Eurocode No.2 is used. When calculating with standard B7 (ECCS) method, 97 % of the results are on safe side.

Composite columns

In the following, results of the SWEAT-program have been compared to the results calculated with LIPA-program, which adopts the European buckling curve c in the analysis of composite columns. In the analysis LIPA needs functions for compression strength of concrete and yield strength of both structural steel and reinforcement.

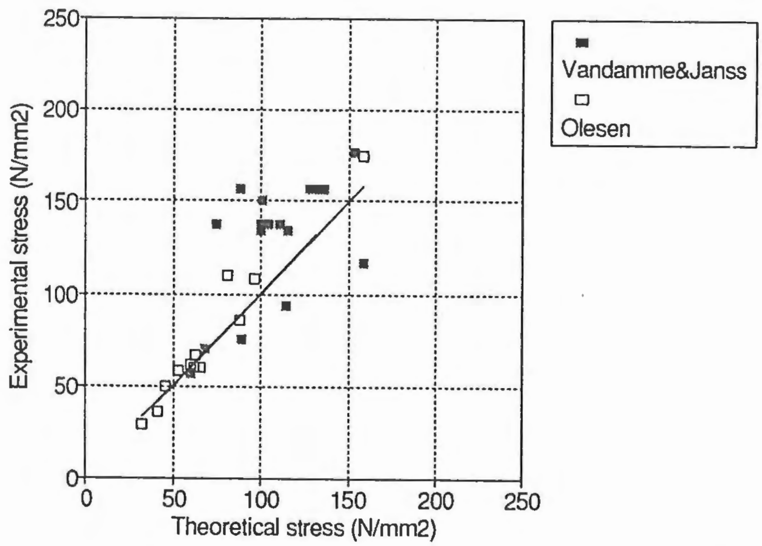

Fig. 8. Stress results of tests against theoretical results calculated with finite difference method.

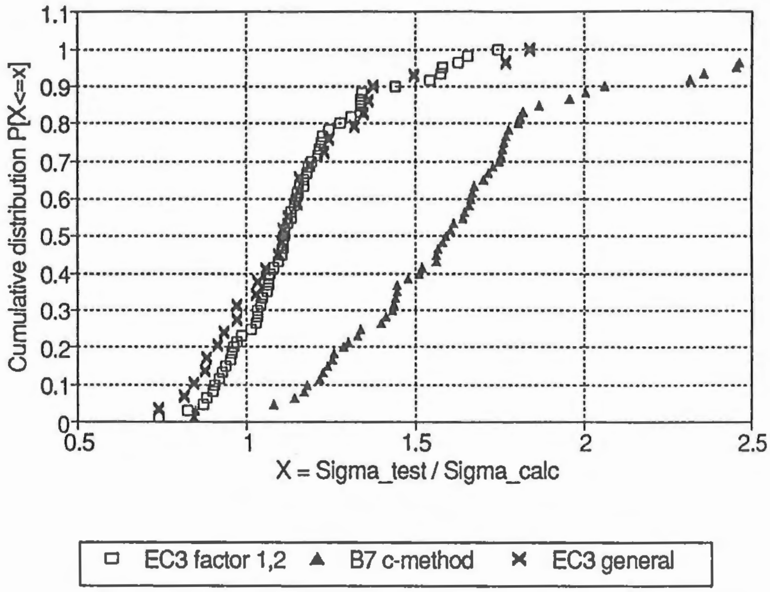

Fig. 9. Cumulative distribution of relations between test stress and calculated stress.

Temperature dependence of tangent modulus (for zero stress) derived from the Brunswick model has been used for both concrete and steel. Thermal properties have been those used in the Finnish Fire Technical Design Manual for Composite Columns /12/.

Calculated results have also been compared to the results of STABA-program /1/ developed in Technical University of Brunswick. STABA-program is based on finite element method. In STABA-program thermomechanical properties of steel and concrete are based on the Brunswick model.

Following composite column has been studied,

-	square hollow section	300 x 300 x 12,5 mm,
-	steel grade	Fe 52
-	standard fire resistance:	A30
-	reinforcement:	8 ø 20 mm grade A500H

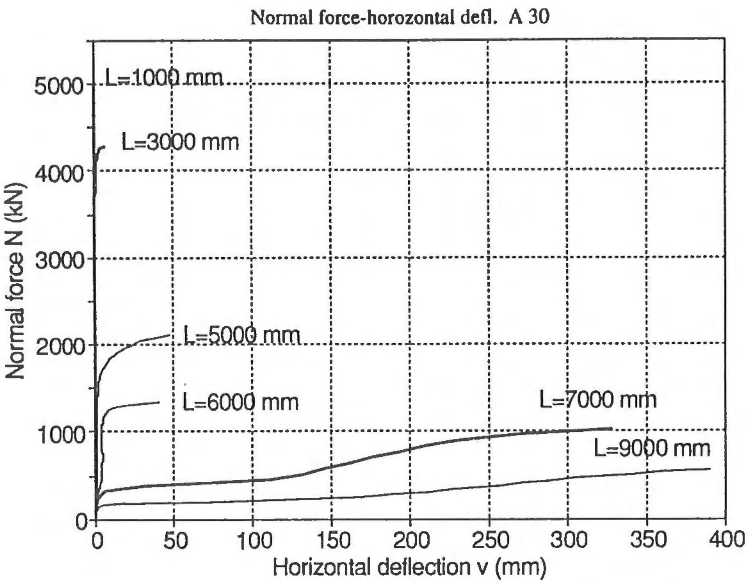

Fig. 10. Normal force-deflection curves of a square hollow section 300x300x12,5 calculated with SWEAT.

Following values of material parameters have been used:

- compression cylinder strength of concrete: 34 N/mm^2
- yield strength of structural steel: 355 N/mm^2
- yield strength of reinforcement: 500 N/mm^2

In Fig. 10 normal force N-deflection v curves calculated with SWEAT-program for columns of different length are drawn. Temperature field calculated by LIPA-program corresponding time t=30 min has been used as input data for SWEAT-program. In fig. 11 axial load capacities of the column for a fire grading A 30 derived from fig. 10 are shown as a function of the buckling length. Capacities calculated by LIPA and STABA programs using Brunswick model are also shown in Fig. 11.

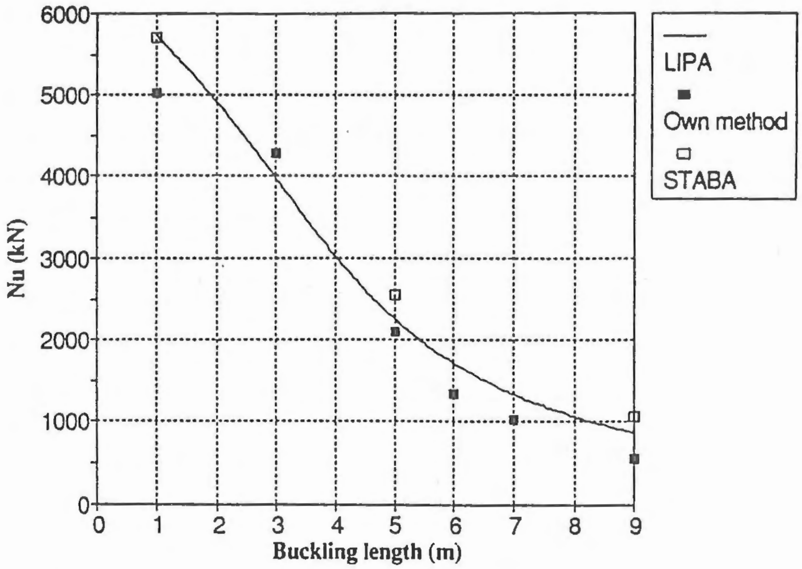

Fig. 11. Buckling capacities of concrete filled square steel hollow section 300x300x12,5 calculated with LIPA, STABA and finite difference program SWEAT.

REFERENCES

1. Hass R., Assessment of the Fire Design of Concrete Filled Hollow Section Columns. Expert's report No. G 91 401. Braunschweig 1991. 35 p.

2. Fire Technical Program for Composite Structures LIPA. Subnic Oy. 6 pp.

3. CEB/FIP Manual of Buckling and Instability. London 1979, Comité Euro-International du Béton (CEB), Fédération Internationale de la Précontrainte (FIP). 87 pp.

4. Zienkiewicz, O.C. The Finite Element Method in Engineering Science. London 1970, McGraw-Hill. 521 pp.

5. Eurocode No. 2, Design of Concrete Structures, Part 10: Structural Fire Design, Draft August 1992. Commision of the European Communities. 54 p.

6. Eurocode No. 4, Design of Composite Structures, Part 10: Structural Fire Design, Draft April 1990. Commision of the European Communities. 82 p.

7. Hass R., Meyer-Ottens C. & Quast U. Verbundbau Brandschutz Handbuch. Berlin 1989, Ernst-&Sohn. 287 p.

8. Richter E., Spannungs/Dehnungs-Linien zur Berechnung des Trag- und Verformungsverhaltens von Konstruktionsbauteilen unter Feuerangriff.Sonderforschungsbereich 148, Arbeitsbericht 1984-1986, Teil I Band A, Braunschweig 1987, TU-Braunschweig.

9. Vandamme, M. & Janns, J. Buckling of Axially Loaded Steel Columns in Fire Conditions. IABSE Periodica 3/1981, August 1981.

10. Knublauch, E., Rudolphi, R. & Stanke, J. Theoretische Ermittlung der Feuerwiderstandsdauer von Stahlstützen und Vergleich mit Versuchen, 2. Teil: Bestimmung der kritischen Stahltemperatur. Der Stahlbau 8/1974, pp. 249-254.

11. Eurocode No. 3, Design of Steel Structures, Part 1.2: Fire Resistance, Draft 1992-11-04. Document CEN/TC N 232 E. British Standards Institution. 61 p.

12. Fire Technical Design Manual for Composite Columns with Concrete Filled Hollow Steel Sections 1992. Helsinki 1992, The Finnish Constructional Steelwork Association. 43 pp. (in Finnish)

13. Suomen rakentamismääräyskokoelma, Teräsrakenteet B7, Ohjeet 1988. Helsinki 1988, Ympäristöministeriö. 40 p.

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