TIME-DEPENDENT BEHAVIOUR OF PRESTRESSED CONCRETE STRUCTURES

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Timo Turunen
Tampere University of Technology

SUMMARY

In this research an analysing model has been developed to describe the time-dependent behaviour of prestressed concrete structures. The deflection mode of prestressed structures is calculated, by taking into account the creep, shrinkage and strengthening of concrete and the relaxation of steel and the loading history of the structure. Long-term loading tests were carried out to test the working of this model.

1. INTRODUCTION

The long-term deformations of concrete, i.e. creep and shrinkage have been widely examined, but these results have been applied to real structures to restricted extent. For example only the midspan deflection of a prestressed beam has been calculated. In this research, the behaviour of the whole structure is examined.

The deflection mode of prestressed concrete beams differs a lot from that of reinforced concrete beams: camber (upward deflection) results from the prestress in the steel, since the entire length of the beam is subjected to negative bending due to prestress.

This analysis model for prestressed concrete structures is based on the iteration of the strains of the extreme fibers of a cross-section so, that the internal and external forces are equal. The internal forces are calculated using the real time stress-strain relationships. From the strains of the extreme fibers the curvature of the cross-section is calculated and the rotation and the deflection are integrated numerically using the trapezoidal formula. The model takes into account the creep, the shrinkage and the strengthening of concrete and the relaxation of steel and also the loading history of the structure.

2. STRESS-STRAIN ANALYSIS OF PRESTRESSED CONCRETE STRUCTURES

A computer program was prepared, because the calculation process requires a lot of iteration and the results of the previous stages are needed. Figure 1 shows in principle, how the deflections are calculated.

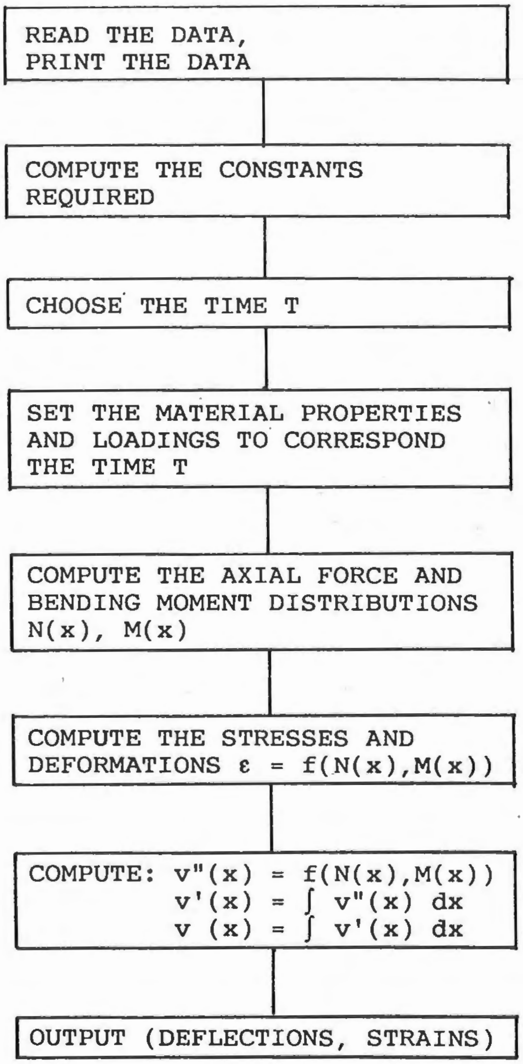

Fig. 1. Flow-chart of computer program.

This program is based on dividing the structure into a number of sections along its length. 10 to 30 sections have been used in the analysis. According to computing results, it is obvious, that in most cases 20 sections are enough to give sufficiently accurate results. The internal axial forces and bending moments acting in these sections can be calculated from the external loadings. Then it is possible to calculate the strains in the extreme fibers, by taking into account the stress-strain relationships of the concrete and reinforcement as well as the cracking, creep and shrinkage. These strains in the extreme fibers are linearly dependent on the second derivative of the deflection curve, if it is assumed that the deflections are so small that the effect of the first derivative on the curvature can be omitted. After this, it is possible to calculate the deflection values in these sections, by numerically integrating in two different stages.

2.1 Basic assumptions

When preparing the program the following basic assumptions have been taken into account.

1) After deformation due to the internal axial force and bending moment the cross-sectional planes still remain as planes (so called Bernoulli hypothesis).

2) The effect of the first derivative on the curvature of the deflection curve has been omitted.

3) Shearing deflections are small for ordinary proportions of prestressed structures and can be neglected.

4) The greater rigidity of the structure between cracks is not taken into account.

5) The stress-strain relationships for concrete and reinforcement are shown in figure 2.

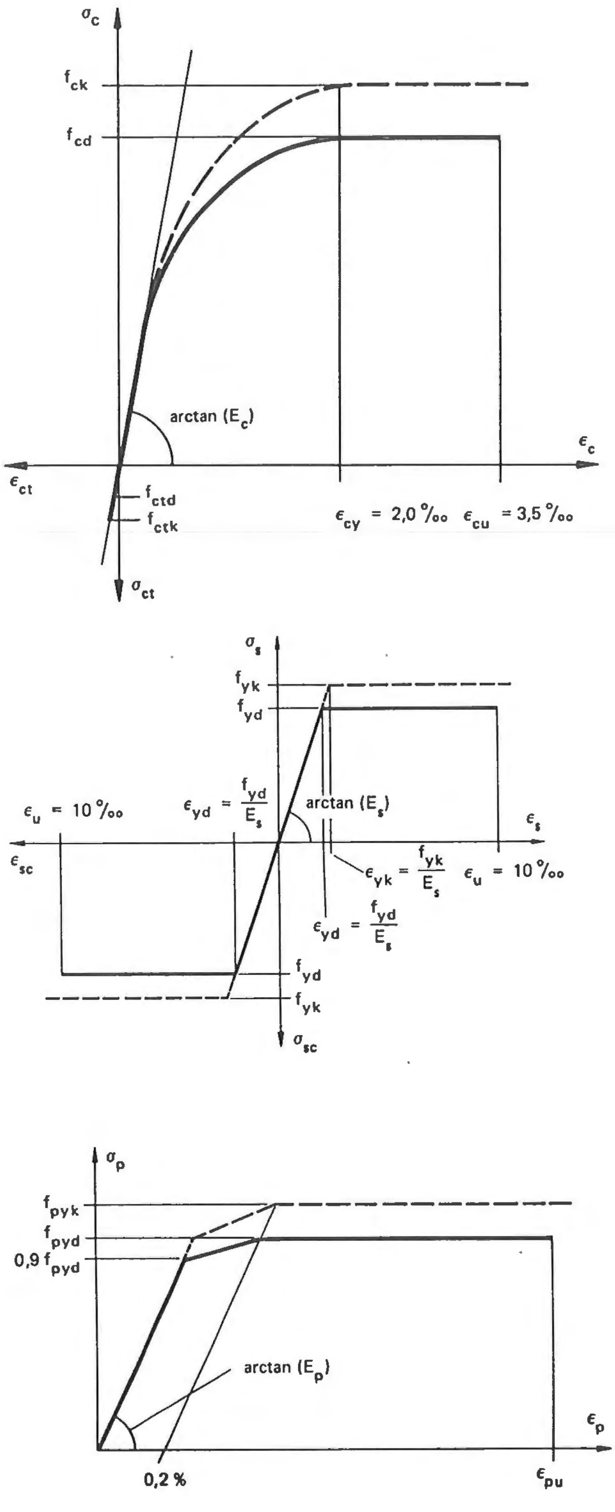

Fig. 2./1/ Stress-strain relationships for concrete and reinforcement.

The material properties of concrete have been taken accurately into account, by using the formula (1) up to the yielding point ε_{cy}.

$$\sigma(\varepsilon)=E_c\varepsilon\left[\left(\frac{\varepsilon}{\varepsilon_{cy}}\right)^2\left(1-\frac{2f_c}{E_c\varepsilon_{cy}}\right)+\left(\frac{\varepsilon}{\varepsilon_{cy}}\right)\left(\frac{3f_c}{E_c\varepsilon_{cy}}-2\right)+1\right] \tag{1}$$

where E_c = the modulus of elasticity for concrete
f_c = the compressive strength of concrete
ε_{cy} = the yielding compressive strain of concrete

This third degree polynomial shows more accurately the actual behaviour of the concrete using small values of strains. For service loads, the compressive strains of concrete, are always smaller than the yielding strains.

The tensile stresses of concrete are taken into account only if the tensile side is not cracking. After the cracking has occurred, the crack is assumed to reach neutral axis immediately.

6) The creep of concrete has been taken into account, by using the value E_{cc} for the modulus of elasticity in the elastic range.

$$E_{cc}=\frac{E_c}{1+\phi} \tag{2}$$

where E_c = the modulus of elasticity for concrete in the short-term loading
ϕ = the creep factor

At the same time the compressive strain has been multiplied by a factor $1 + \phi$, thus the stress-strain relationship for the concrete is shown in figure 3.

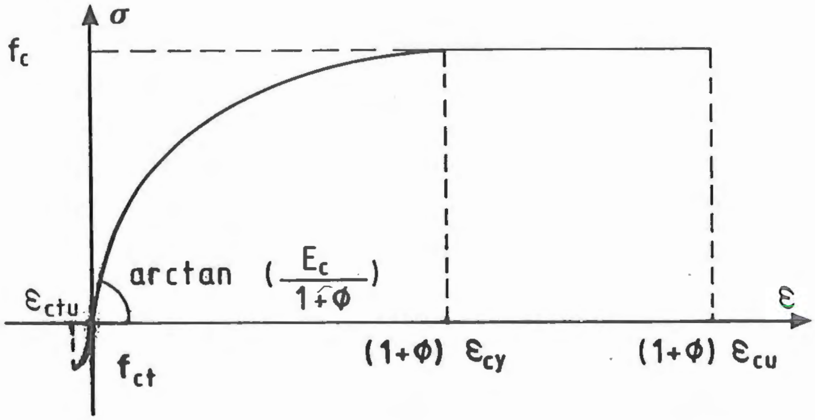

Fig. 3. The stress-strain relationship for the concrete after the creep has taken place.

7) The effect of the shrinkage of concrete has been taken into account, first by assuming the strains of the extreme fibers to be equal to the "free" shrinkage of concrete. "Free" shrinkage is the expected average shrinkage strain if the member were not externally or internally restrained.

The actual strains of the extreme fibers are iterated (by taking the reinforcement into account) so, that the sum of the internal forces is equal to zero.

8) The strength of concrete is assumed to increase with age according to table 1.

Table 1./2/ Factors for increase in compressive strength of concrete with age.

concrete age (days)	3	7	28	90	360
normal PC	0.4	0.65	1	1.20	1.35
rapid PC	0.55	0.75	1	1.15	1.20

9) The relaxation of steel has been taken into account by multiplying the modulus of elasticity for steel by a factor (figure 4)

$$k = 1 - \frac{a}{100} \tag{3}$$

where a = the magnitude of relaxation [%]

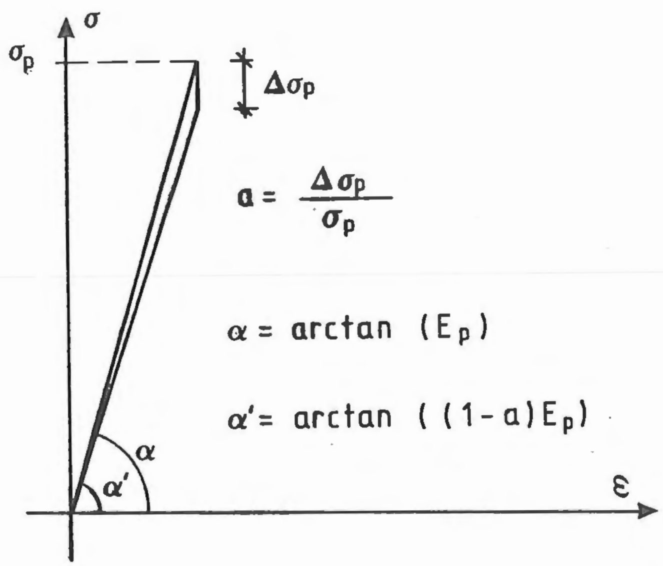

Fig. 4. The stress-strain relationship for steel after the relaxation has taken place.

2.2 Change in loading conditions

The loading history of a prestressed precast structure can be divided into three stages:

First stage begins from the transfer of prestress force and ends at the erection of the structure.

Second stage lasts from the superimposing of dead load to the final stage.

In the final stage are also the live loads acting.

According to Neville /3/ and Scordelis /4/ the creep functions corresponding to different loading times should be taken into account by the superposition principle. This means that the deformations due to a change in loading conditions at time t' < t are at a given time t independent from both already existing and possible further deformations.

At a given time t_o the loading causes deformation $\varepsilon_{ce}(t_o)$ in concrete, which is calculated by using value $E_c(t_o)$ for the modulus of elasticity for concrete. At time t the deformation is $\varepsilon_c > \varepsilon_{ce}(t_o)$ assuming no change in loading conditions. During this interval the stresses in concrete decrease.

It is assumed that the long-term loading increases at time t_1. The effect of the increase of loading on deformations is taken into account by using so called "permanent" deformations. First one must calculate the deformation $\varepsilon_c(t)$ caused by the loadings superimposed at time t_o, using formula (4) for the modulus of elasticity for concrete.

$$E_c = \frac{E_c(t)}{1+\phi(t,t_o)} \tag{4}$$

where $\phi(t,t_o)$ denotes the creep factor of loading time t_o at time t

Values for the creep factor $\phi(t,t_o)$ are given, for example, in reference /5/.

From that deformation is "returned" to the ε-axis by using formula (5) for the modulus of elasticity for concrete.

$$E_c = \frac{E_c(t)}{1+\phi(t,t_1)} \tag{5}$$

Because of the similarity of stress-strain curves the following formula is valid (figure 5):

$$\frac{E_c(t)}{1+\phi(t,t_o)}\varepsilon_c(t) = \frac{E_c(t)}{1+\phi(t,t_1)}\varepsilon_{ce}(t) \tag{6}$$

By solving $\varepsilon_{ce}(t)$ from equation (6) and taking into account the following relationship

$$\varepsilon_c(t) = \varepsilon_{ce}(t) + \varepsilon_{cc}(t) \tag{7}$$

we get the following formula for the "permanent" deformation:

$$\varepsilon_{cc}(t) = \frac{\phi(t,t_o) - \phi(t,t_1)}{1+\phi(t,t_o)}\varepsilon_c(t) \qquad (8)$$

Using value $\varepsilon_{cc}(t)$ as basis, the deformation due to all long-term loadings is calculated using value (5) for the modulus of elasticity for concrete.

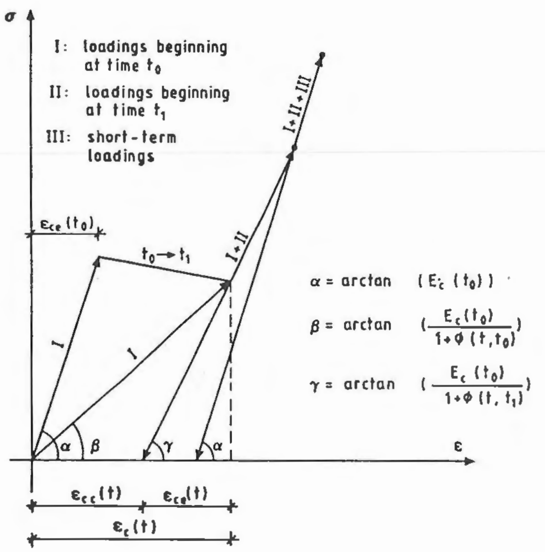

Fig. 5. The calculation model for the creep deformations. (In this picture the smaller values than ε_{cy} in the compressive side of the concrete are replaced by a straight line.)

The effect of short-term loadings on deformations is taken into account analogically.

3. TESTING THE THEORETICAL MODEL AND COMPUTER PROGRAMS

Long-term loading tests were carried out to test the working of this model. The aim of these tests was to study the development of the deflection mode, along the time, of a prestressed structure. Three Variax-5 hollow slabs, with 8 tendons, length 12 metres, were used as test samples.

The loads shown in figure 7, were added to the weight of the slabs. This additional loading was added about four weeks from the transfer of prestressing, and remained constant permanently.

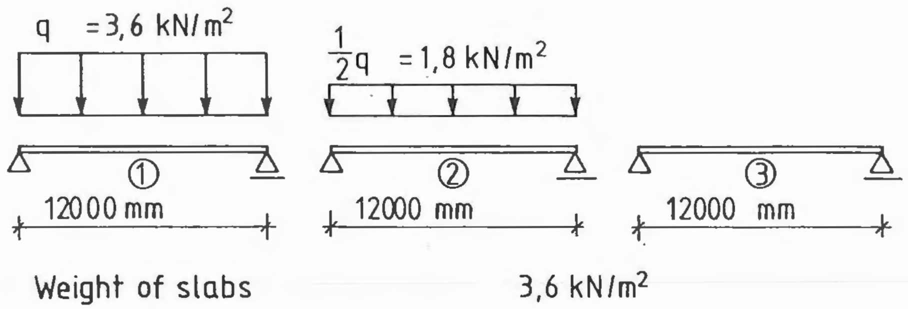

Fig. 7. Loading of the slabs.

The additional load q, for the slab 1, is the maximum live load to this type of slab, according to design recommendations for the Variax-slabs. The additional load 0.5 q, for the slab 2, is half for that of slab 1. Slab 3 is shown loaded purely by its own weight. The test was continued for one year.

Within each slab there were 8 tendons of steel wire, diameter 12.7 mm, with an average value for prestressing of 1043 MPa. It was intended to achieve a strength-class of 50 Mpa, and within the transfer of prestressing it was intended to achieve a cubic strength of 35 MPa. After 28 days the cubic strength was seen to be 69 MPa and at the end of half a year it was seen to be 77 MPa.

Figures 8, 9, 10 illustrate deflection modes, of each slab in different moments. The comparisons between the theoretical and measured deflection modes, before and after the effect of the additional loads were made on the 28th day. Further comparisons were made after 50 days and finally after 177 days.

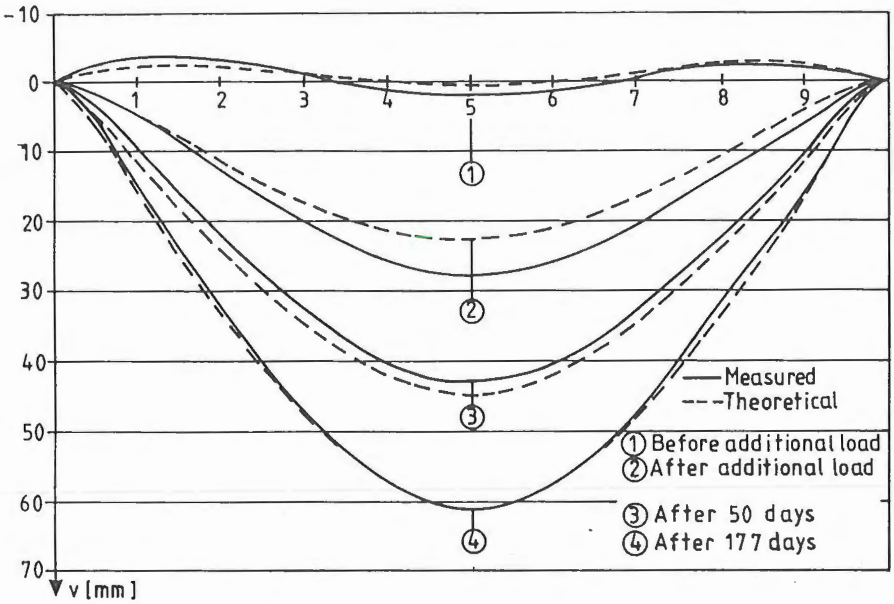

Fig. 8. Deflection modes for slab 1. The horizontal axis denotes the 10 sections measured.

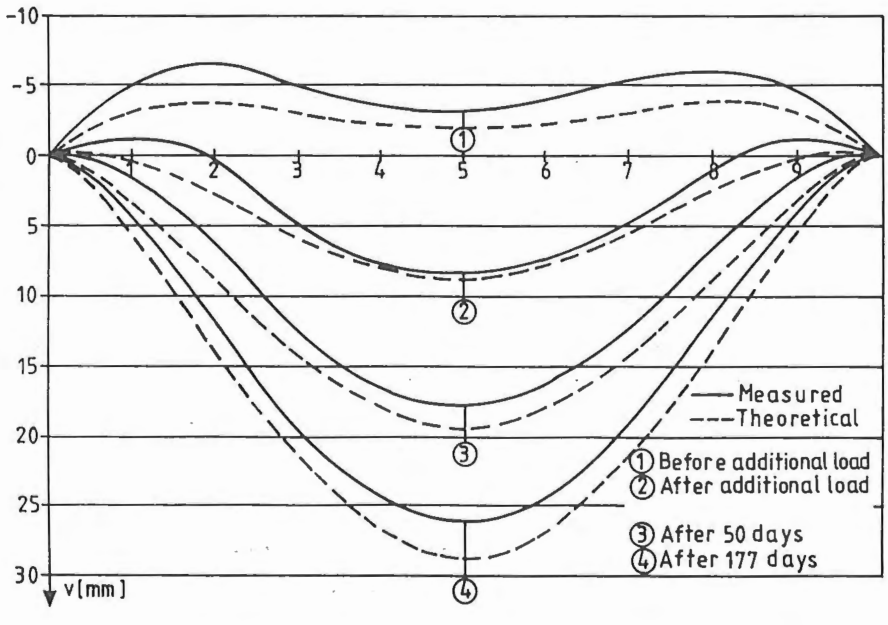

Fig. 9. Deflection modes for slab 2.

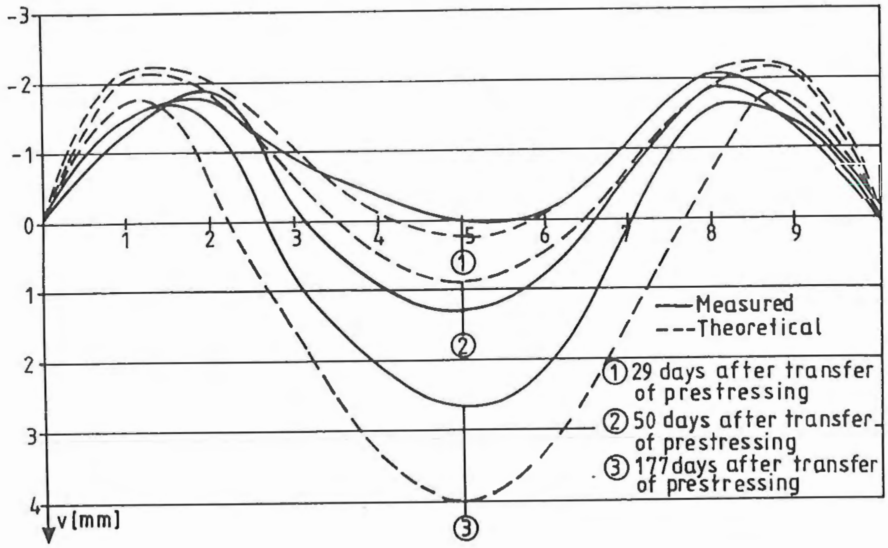

Fig. 10. Deflection modes for slab 3.

The non-defined data for the material and environmental properties have been taken so, that the measured deflection in the midspan of slab 1 corresponds to those calculated after 177 days. All other calculations have been carried out using the same parameters.

If one compares the differences between the calculated and measured deflections, figures 8, 9 and 10, in the test, it can be seen that this difference in all cases, is less than 10 % of the maximum allowed deflection (48 mm) for a slab.

Figure 11 shows the theoretical and measured deflection modes 177 days after the transfer of prestressing. It is clear, that with the help of this theoretical model, it is possible to describe the time-dependent behaviour of a prestressed structure, quite accurately.

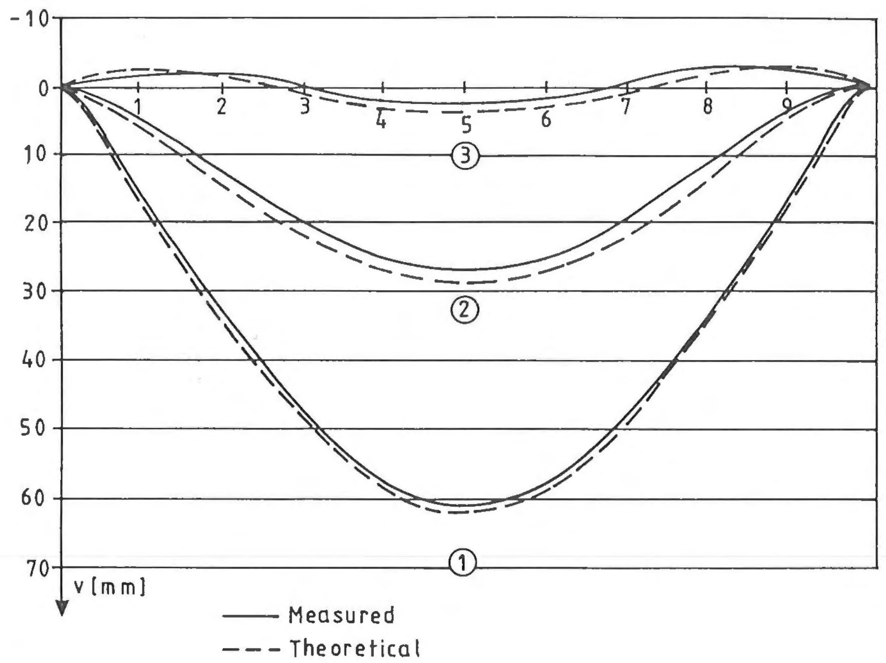

Fig. 11. Deflection modes 177 days after the transfer of prestressing.

4. APPLICATION EXAMPLE

As an example, the deflection mode of an I beam, is researched using the theoretical model in different loading stages. In figure 12 is shown the midspan cross-section and the loadings of the beam.

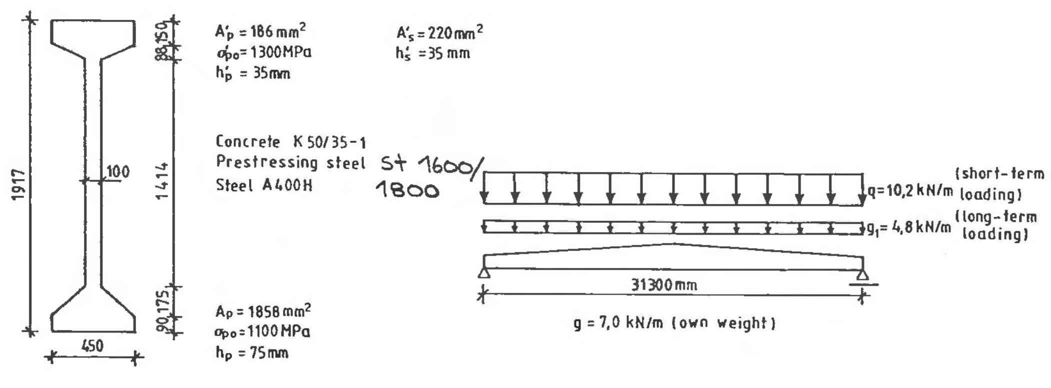

Fig. 12. Cross-section and loadings of the I beam.

In figure 13 is shown the deflection mode of the I beam in different loading stages. The deflection due to long-term loadings is always upward; only the adding of the short-term loading will cause downward deflection.

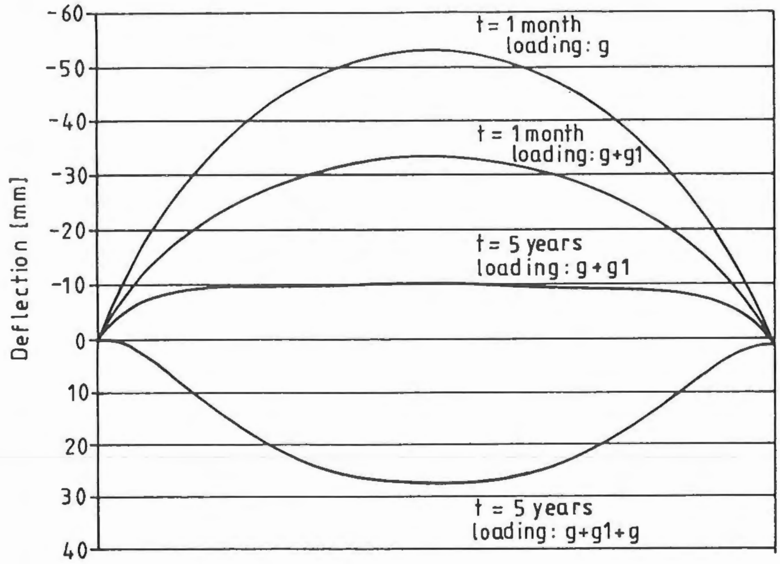

Fig. 13. The deflection mode of the I beam in different loading stages. Dry environmental conditions.

5. REFERENCES

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