UNDAMPED CANTILEVER PIPES ASPIRATING FLUID MAY BE STABLE

By Antti Pramila
University of Oulu, Oulu, Finland

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SUMMARY

The perplexing result that undamped pipes aspirating fluid are inherently unstable, i.e. they lose their stability at infinitely small flow velocity is reconsidered. It is shown that slight changes in the assumptions on the flow direction just before the inlet - the free end of the pipe - cause essential differences to the stability behaviour. For example, if the flow is assumed to be along the undeformed axis of the pipe just outside the free end, pipes aspirating and discharging fluid have the same critical velocity which is nonzero even when there is no damping.

INTRODUCTION

The literature on vibration and stability of pipes conveying fluid is quite extensive, *vide*, the references of Paidoussis (1987). The problem has received its share of attention mainly because of its special significance in the context of stability theory of nonconservative systems. Namely, the flow velocity necessary to cause flutter in a cantilevered pipe is so high that it is unlikely to be encountered in usual engineering practice. The phenomenon itself belongs, however, to everyday life. The large lateral force that must be exerted by one holding a fire-hose at high discharge rate, is well known as is the snaking motion of a garden hose if held at some distance from the free end.

In almost all studies the flow has been from the fixed end towards the free one. The first paper dealing with the dynamic stability of a pipe aspirating fluid was the one of Paidoussis&Luu (1985). They obtained the rather unexpected result that the system is inherently unstable, i.e. it loses its stability at infinitely small velocity if there is no damping present. Similar results were later obtained by Pramila&al. (1988) using the FEM and employing the same basic assumptions concerning the fluid flow.

Dupuis&Rousselet (1991) commented recently on the same result obtained by Sällström (1990) by using the so called exact finite elements. In their authors' reply Sällström&Åkesson (1991) mention that they are unaware of physical experiments verifying the abovementioned phenomenon. This and the fact that pipes aspirating fluid have been successfully used in the practice since the time immemorial, has led to the following considerations.

We will show that the stability behaviour of the system depends very much on the assumptions made concerning the fluid flow at the free end. All the derivations are first made for the classical problem, where the pipe is discharging fluid, in order to keep the text as assimilable as possible.

ENERGY FLOW FROM FLUID TO PIPE

Consider a straight cantilever pipe of length L, mass per unit length m_p, bending stiffness EI, discharging fluid, mass per unit length m_f, tangentially from its free end with axial velocity U. The case in which the pipe is aspirating fluid is obtained from this case simply by letting U be negative.

It has been shown by Paidoussis (1970) that the net energy received by the pipe from the fluid during one vibration cycle is

$$W=-\int_0^T m_f U(\dot{w}^2+Uw'\dot{w})\Big|_{x=L}\,dt. \qquad (1)$$

where w denotes the transverse deflection and a prime indicates differentiation with respect to the spatial co-ordinate x and a dot indicates differentiation with respect to time. This expression is valid provided that the shear force Q at the free end equals to zero, i.e. the fluid leaves the pipe tangentially. Hence, the boundary conditions at the free end are $w''=0$ and $w'''=0$.

If the velocity is small the first term in the integrand dominates and W is negative. Thus the motion of the pipe will be damped. However, when the velocity becomes greater and if the product $w'\dot{w}\big|_{x=L}$ is negative over most part of the vibration cycle, W may become positive, i.e. pipe is gaining energy from the fluid flow. Hence the pipe may become dynamically unstable and lose its stability by flutter.

When U is negative, W is positive even with infinitesimally small velocities and thus flutter instability occurs.

The flow field can be entirely different at the free end depending on if the pipe is discharging fluid or aspirating fluid, see e.g. Batchelor (1967). The fluid which is forced out of the pipe usually emerges as a concentrated tangential jet. The flow due to suction at the open end of the pipe is approximately the same as the flow due to a point sink. The streamlines can in this case be like in Figure 1a or in Figure 1b depending on the location of the sucking end, close to the free surface or deeply in the fluid. An everyday experience confirms this qualitative difference: a match can be extinguished by blowing, but not by sucking.

In the case depicted in Figure 1a we can assume that the fluid just entering the pipe has a momentum vector which is vertical. The transverse component is equal to zero, because the flow field is axisymmetric. In the case of Figure 1b the flow field has spherical symmetry and therefore the momentum of the fluid just entering the pipe can be assumed equal to zero. Here the inertia effect of surrounding fluid is taken into account by substituting m_p+m_a for m_p, m_a being the added mass per unit length. Of course, the situations described above are approximate. In reality, the motion of the pipe disturbs the flow field and makes it very complicated.

According to the principle of impulse and momentum the flow exerts a transverse force

$$F_z=m_f U(\dot{w}+Uw') \tag{2}$$

to the end of the pipe, if the velocity of the fluid changes from being tangential in the pipe to being in x-direction just outside the free end of the pipe, see Figure 2. This could require controllable vanes in a case of a pipe discharging fluid, but corresponds quite well the situation of Figure 1a in a case of a pipe aspirating fluid.

If the fluid velocity in x-direction is also equal to zero at the free end (corresponding to Figure 1b in suction case) we obtain in addition to the transverse force also a longitudinal force

$$F_x=m_f U^2 \tag{3}$$

causing tension to the system consisting of the pipe and the fluid. In pipes discharging fluid this kind of situation arises, e.g. if the free end is closed and fluid is forced out through holes drilled circumferentially around the perimeter of the pipe.

In the case of Figure 2 the work done by the force F_z to the pipe during one cycle of vibration is

$$W_F = \int_0^T m_f U(\dot{w}+Uw')\dot{w}\Big|_{x=L} \, dt. \qquad (4)$$

Thus, the total energy gained by the pipe during one period of oscillation, $W+W_F$, according to equations (1) and (4), equals zero!

This means that the problem is a conservative one and the system can lose its stability only by divergence. If the effect of gravity and damping are neglected for simplicity, the traditional equation of motion

$$EIw'''' + m_f U^2 w'' + 2m_f U\dot{w}' + (m_f+m_p)\ddot{w} = 0 \qquad (5)$$

is obtained. However, the boundary conditions at the free end are now $w''=0$ and $EIw'''=m_f U(\dot{w}+Uw')$. Because divergence is a static instability mechanism equation (5) can now be simplified for stability studies by dropping the terms containing time derivatives, i.e. we obtain

$$EIw'''' + m_f U^2 w'' = 0 \qquad (6)$$

from which we can immediately deduce (compare (6) with the buckling equation) the critical velocity to be

$$U_{cr}=\frac{\pi}{2L}\sqrt{\frac{EI}{m_f}} \qquad (7)$$

which is independent from mass ratio and sign of U, i.e. the critical velocity for a pipe discharging fluid and aspirating fluid are equal. It is also considerably smaller than the critical velocity obtained using the traditional flow model assuming tangential discharge. This is natural, because the transverse force tries to increase the deflection, vide Figure 2.

If the system is assumed to be of the type depicted in Figure 1b, the system cannot loose stability with any velocity, because velocity disappears from the terms not containing time derivatives of w in the equation of motion, because axial tension given in Equation (3) adds a term $-m_f U^2 w''$ to the equation of motion (5).

EXTENDED HAMILTON'S PRINCIPLE AND FLOW ASSUMPTIONS

The extended Hamilton's principle, see McIver (1973), states that the motion of the system is such that

$$\delta\int_{t_1}^{t_2}(T-V)\,dt+\int_{t_1}^{t_2}\int_{S_o}\rho\mathbf{U}\cdot\delta\mathbf{r}(\mathbf{u}-\mathbf{U})\cdot\mathbf{n}\,dS\,dt=0 \qquad (8)$$

where T and V are the kinetic and potential energies of the pipe plus fluid therein, ρ is the density of the medium flowing throughout the open boundary S_o, **U** is the velocity vector of the fluid, $\delta\mathbf{r}$ is the virtual displacement, **u** is the velocity vector of the boundary and **n** is the unit outward normal vector of the boundary.

When the traditional assumption of tangential flow at the free end is made, the last integral simplifies to

$$-\int_{t_1}^{t_2} m_f U(\dot{w}+Uw')\delta w\Big|_{x=L}\, dt \qquad (9)$$

leading to natural boundary conditions $w''=w'''=0$ at the free end.

When small deflections are assumed $\delta\mathbf{r} = \delta w\mathbf{k}$ and velocity vector **U** at the outlet is now assumed to be along the x-axis, i.e. $\mathbf{U} = U\mathbf{i}$, $\delta\mathbf{r}$ and **U** are perpendicular to each other and thus the last term in equation (8) disappears. Thus, the stability behaviour obtained using the extended Hamilton's principle as a starting point leads to the same conclusion obtained using energy considerations, because it yields the same equation of motion (5) and same boundary conditions $w''=0$ and $EIw''' = m_f U(\dot{w}+Uw')$.

When the extended version (8) of Hamilton's principle is employed as the basis for the FEM discretisation, the last integral causes an additional term to the gyroscopic matrix **G** and to the stiffness matrix **K** in the system of equations of motion

$$\mathbf{M}\ddot{\mathbf{a}} + \mathbf{G}\dot{\mathbf{a}} + \mathbf{K}\mathbf{a} = 0 \qquad (10)$$

making both of them unsymmetric. However, when the assumption is made that the flow just outside the end of the pipe is along the undeformed axis, **G** remains skew-symmetric and **K** remains symmetric. When these changes were made to the computer program system described by Kangaspuoskari&al. (1991) the numerical results agreed well with the analytical one given in equation (7).

CONCLUSIONS

The perplexing result that undamped cantilevered pipes aspirating fluid are inherently unstable, i.e. they lose their stability at infinitely small flow velocity has been reconsidered. It has been shown that slight differences in the simplifying assumptions on the flow direction at the free end lead to significantly different stability behaviour. If it is assumed that the flow entering (leaving) the pipe is along the undeformed axis of the pipe just prior to the inlet(just after the outlet), the critical velocity is nonzero and finite. With this assumption the critical velocity of pipe aspirating fluid and discharging fluid become the same. If the resultant momentum of the fluid is zero at the free end (flow field is spherically symmetric for a pipe aspirating fluid or fluid is discharged radially from a pipe having a closed end), the critical velocity is infinitely large. However, the actual flow field around the inlet of fluid aspirating pipe is very complex. Therefore, further studies are needed to clarify the detailed stability behaviour of these kind of systems which represent idealizations of e.g. marine risers.

REFERENCES

BATCHELOR, G.K. 1967 An Introduction to Fluid Dynamics. Cambridge, England:Cambridge University Press.

DUPUIS,C.&ROUSSELET, J. 1991 Discussion on "Fluid conveying damped Rayleigh-Timoshenko beams in transverse vibration analyzed by use of an exact finite element;Part I: Theory; Part II: Applications". *Journal of Fluids and Structures* 5, 597-598.

KANGASPUOSKARI, M., LAUKKANEN, J.&PRAMILA, A. 1991 The effect of feedback control to critical velocity of cantilever pipes aspirating fluid. *Journal of Fluids and Structures* (submitted).

MCIVER D.B. 1973 Hamilton's principle for systems of changing mass. *Journal of Engineering Mathematics* 7,249-261.

PAIDOUSSIS, M.P. 1970 Dynamics of tubular cantilevers conveying fluid. *Journal of Mechanical Engineering Science* **12**, 85-103.

PAIDOUSSIS, M.P.& LUU, T.P. 1985 Dynamics of a pipe aspirating fluid such as might be used in ocean mining. *Journal of Energy Resources Technology* **107**, 250-255.

PAIDOUSSIS, M.P. 1987 Flow induced instabilities of cylindrical structures. *Applied Mechanics Review* **40**, 163-175.

PRAMILA, A. et al. 1988 Vibration and stability of axially moving material. In *Proceedings of the 3rd Finnish Mechanics Days* (ed. M.A. Ranta), pp.339-348, Otaniemi: Helsinki University of Technology.

SÄLLSTRÖM, J.H. 1990 Fluid conveying Rayleigh-Timoshenko beams in transverse vibration analyzed by use of an exact finite element; Part II: Applications. *Journal of Fluids and Structures* **4**, 573-582.

SÄLLSTRÖM, J.H.&ÅKESSON, B.A 1991 Authors' reply. *Journal of Fluids and Structures* **5**, 598-600.

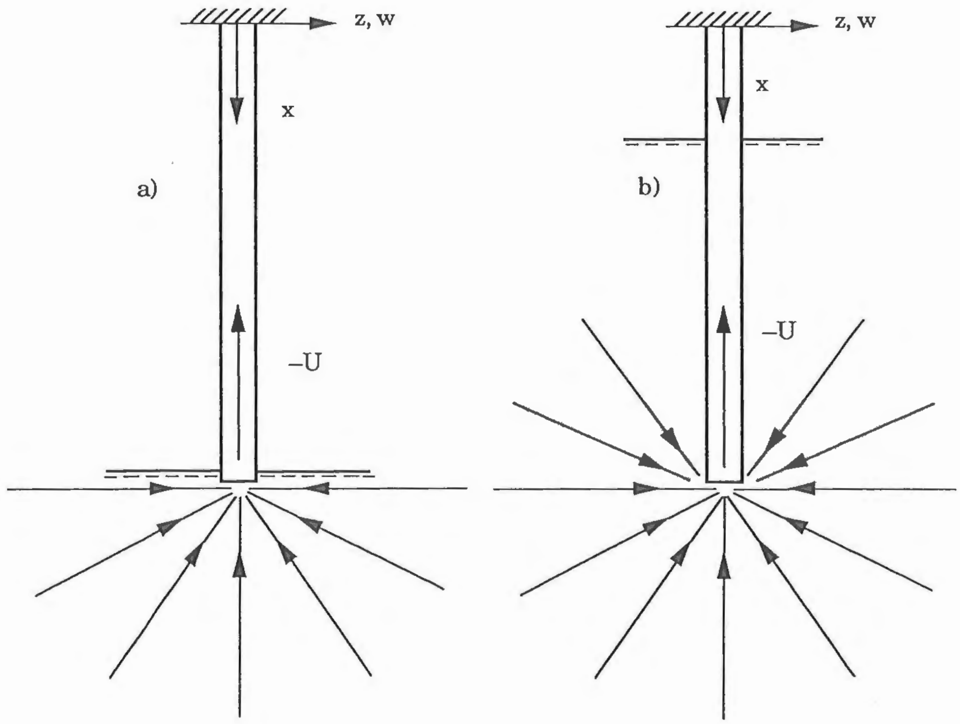

Figure 1. Streamlines of the fluid sucked by the pipe.

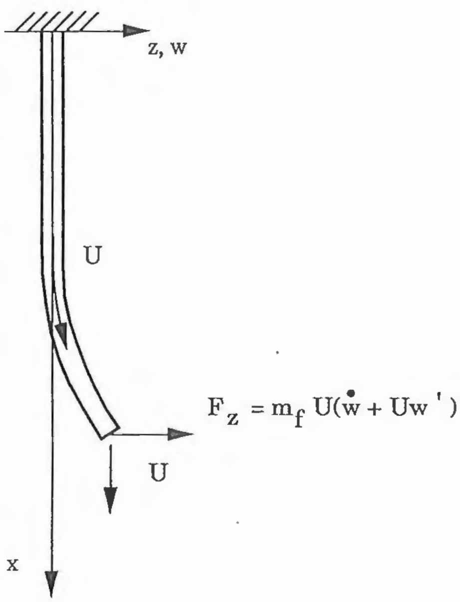

Figure 2. Transverse force caused by the change of flow direction.