TRANSFER MATRIX VIBRATION ANALYSIS OF COMPOSITE BRIDGE DECK SLABS

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ABSTRACT

Present study details the transfer matrix vibration analysis of concrete deck slabs elastically supported by steel members of composite bridges. Transfer matrix method provides a concept for the approximate solution of a problem submitted and is actual for the solution of various types of composite bridges.

Generalized vibration analysis on the basis of the transfer matrix method is put forth. Transfer matrices dealing with dynamic analysis of the deck slabs of composite bridges are derived. Formulated are the requirements for development of the problem-oriented algorithms of the discrete streamline type, on the basis of transfer matrix method, adopting the vector processing facilities of modern computers. Direct time integration methods combined with the flow-chart of the transfer matrix method are applied. Numerical experiments dealing with the problems specified are presented and evaluated for the actual solutions of the natural mode vibration of the deck slabs of composite bridge structures. Conclusions for real applications are submitted and discussed.

INTRODUCTION

Most of the research work within the field of bridge engineering aims at the development of more economical and functional structures with an acceptable serviceability and safety. As a part of theoretical studies advanced computational models have been developed in order to obtain reliable informations on the behaviour of bridge structures in the limit state situations of exploitation. However, the solution of most large-scale linear and especially nonlinear transient structural problems is still not economically feasible when applying such models. This raised the question as to whether the use of vector processing computer systems might result in managing of advanced analyses of structural systems of composite bridge structures.

During the period of development of the finite element techni-

ques the transfer matrix method of discrete structural analysis
also advanced significantly.   In contrast to the finite element
method, the transfer matrix method is a solution procedure wor-
king with a discrete mesh simulated by movable finite elements.
The analysed discrete simulation is modelled during the motion
of elements over the structure.   The transfer matrix method has
the streamline character suitable for the vector processing
algorithms.   From such point of view the transfer matrix method
is a more efficient procedure in comparison with the finite e-
lement method.   There exists a significant difference in the
number of unknowns that must be determined in order to complete
the analysis.   Since these two numerical methods have different
strengths and deficiencies, the advantages may be gained by
combining both concepts for effective utilization of present
computer techniques.

This paper presents the derivation of transfer matrices suitab-
le for the solution of static and dynamic behaviour of the deck
slabs of composite bridge structures. Such plate members are e-
lastically supported by steel elements of composite bridges
whereas the elastic characteristics depend on the stiffness pa-
rameters of supports which are variable along the span of brid-
ge structure studied. Especially moving dynamic loads can ini-
tiate resonance vibrations of the deck slabs resulting in the
limit state behaviour of such members.   The discrete solution
approach of such problems is submitted.

THEORETICAL ANALYSIS

The determination of the transfer matrix for rectangular plate
member as shown in Fig. 1 will be performed first of all.   The
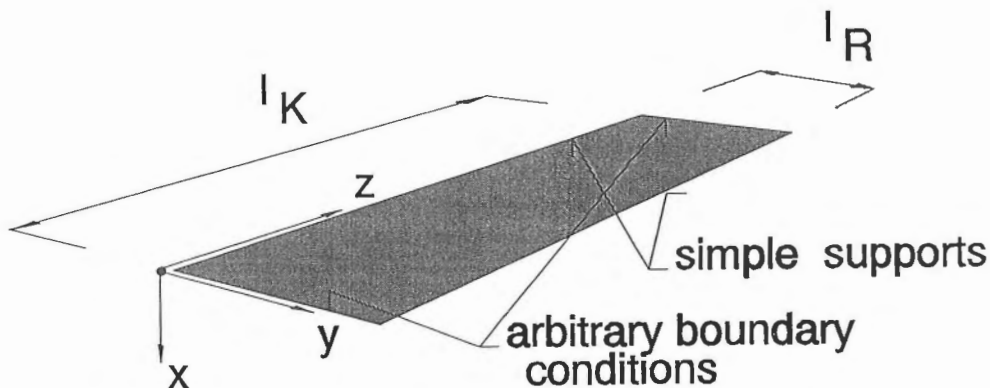differential equation of the general dynamic behaviour of a

Fig. 1.   Analysed rectangular slab member

deck slab member located in the Cartesian coordinate system x-
-y-z is given by (see, for example, reference (1))

$$AB^4(v_x) + 2\frac{H}{x^2}F^2(v_x) + \frac{N}{x^4}D^4(v_x) + 2\frac{A}{x}B^3(v_x) -$$

$$- \frac{N}{x^2}B^2(v_x) + \frac{2}{x^4}(N+H)D^2(v_x) + \frac{N}{x^3}B(v_1) =$$

$$= Q - m\,C^2(v_x) - c\,C(v_x) \quad . \quad (1)$$

A, H and N are the plate stiffness parameters defined by e-
quations

$$A = EJ_x \quad , \quad N = EJ_y \quad , \quad H = GJ_T \quad , \quad (2)$$

with corresponding plate rigidities in flexural and torsional
modes of vibration of the plate member studied. Furthermore, D,
B,F and C are the additional differential operators expressed as

$$D = \partial/\partial y \ , \quad B = \partial/\partial x \ , \quad F = \partial^2/(\partial x\,\partial y) \ , \quad C = \partial/\partial t \ . \quad (3)$$

m is the element mass, v_x is vertical displacement, c is the pa-
rameter of internal damping and Q is the external dynamic load
assumed. Displacements and loads are the functions of time t. The
longitudinal and transverse dimensions of the plate studied are
l_x and l_R, respectively. Simple supports are assumed on the
longitudinal edges and arbitrary (elastic) boundary conditions
can be simulated on the shorter edges of the rectangular slab
member studied.

For determining of the transfer matrix of the problem, a series
expansion of the form

$$v_x = \sum_1^\infty R_n \sin(\lambda y) \quad , \quad (4)$$

is assumed for the vertical displacement v_x. Here, R_n is a func-
tion of coordinate z and the parameter λ is given by

$$\lambda = n\pi/l_R \quad , \quad (5)$$

Arbitrary transverse edge boundary conditions are specified by
the function R_n. By introducing a new variable $\mu = \ln z$, equation
(1) can be transformed into expression

$$AS^4(R_n) - 4AS^3(R_n) + (5A-2H\lambda^2-N)\,S^2(R_n) +$$

$$+(2A+4H\lambda^2+2N)\,S(R_n) - (2H\lambda^2+2N\lambda^2-N\lambda^4)\,(R_n) = 0 \ , \quad (6)$$

with operator $S = d/d\mu$ and by neglecting the right hand side of Eq.
(1).

When assuming a solution of the form $R_n = e^{s\mu}$, the four roots of
the characteristic equation of the problem studied are given by
expression

$$s_i = \pm \sqrt{0.5(\alpha+2\omega\lambda^2+1) \pm \overline{(0.25(1+\alpha+2\lambda\omega)^2)^2 -}}$$

$$\overline{- \sqrt{((\lambda^2-1)^2-\alpha)}} + 1 \ , \quad (7)$$

where

$$\alpha=N/A \ , \quad \omega=H/A \quad . \quad (8)$$

Recalling the variable transformation $\mu=\ln z$, the series solution for the displacements reduces to the expression

$$v_x = \sum_1^\infty (C_1 z^{s_1}+C_2 z^{s_2}+C_3 z^{s_3}+C_4 z^{s_4}) \ \sin(\lambda y) \ . \quad (9)$$

This type of solution, however, will yield terms of high magnitude even for moderate values of the roots s_1, s_2, s_3 and s_4. To produce a more accurate solution, a new term Z is introduced such that

$$Z = z/z_0 \ , \quad (10)$$

where z_0 is the distance to the outside edge of the plate studied in longitudinal direction l_x. Redefining the constants of equation (9)

$$A_1=C_1 z^{s_1}, \quad A_2=C_2 z^{s_2}, \quad A_3=C_3 z^{s_3}, \quad A_4=C_4 z^{s_4}, \quad (11)$$

the final form of the solution may be derived as follows

$$v_x = \sum_1^\infty (A_1 Z^{s_1} + A_2 Z^{s_2} + A_3 Z^{s_3} + A_4 Z^{s_4}) \ \sin(\lambda y) \ . \quad (12)$$

Taking account of substitutions

$$Y=\sin(\lambda y), \quad t_i=s_i-1, \quad W=AY/z^3, \quad (13)$$

then the individual state components for the determination of the corresponding transfer matrix between the edges of studied plate element can be written as

$$\sum_1^\infty (A_1+A_2+A_3+A_4) \ Y = v_k, \quad (14)$$

$$\sum_1^\infty (A_1 s_1+A_2 s_2+A_3 s_3+A_4 s_4) \ Y = \phi_k, \quad (15)$$

$$\sum_1^\infty AY(A_1 s_1 t_1+A_2 s_2 t_2+A_3 s_3 t_3+A_4 s_4 t_4)/z^2 = M_k, \quad (16)$$

$$\sum_{1}^{\infty} AY(A_1(s_1t_1-2\beta\lambda^2t_1-\alpha(s_1-\lambda^2)) +$$

$$+A_2(s_2t^2-2\beta\lambda^2t_2-(s_2-\lambda^2)) +$$

$$+A_3(s_3t_3-2\beta\lambda^2t_3-\alpha(s_3-\lambda^2)) +$$

$$+A_4(s_4t^2-2\beta\lambda^2t_4-\alpha(s_4-\lambda^2)))/z^3 = Q_k \ . \ (17)$$

where v_k, ϕ_k, M_k and Q_k are flexural displacements, corresponding slopes, flexural moments and lateral forces, respectively.

Analogous technique can be used and similar expressions may be derived for the analysis of the torsional behaviour of the plate member studied (see, for example, reference (1)).

Analogous analyses dealing with the determination of the nodal matrices of the slab members have been made in above publication (1) and are available there.

Both types of matrices are adopted in the algorithm of the transfer matrix method, taking into account the vector-oriented facilities of the approach submitted and are used for the numerical calculations presented below.

MIXED DISCRETIZATION IN TIME

Implementation of the combined Newmark-Wilson technique for direct time integration of corresponding equations of motion, into the algorithm of the transfer matrix method (see reference (1)), is given by the following set of operations:

A. Initial stage of solution

1. Calculation of displacements $r(t)$, velocities $\dot{r}(t)$ and accelerations $\ddot{r}(t)$ for the assumed loads and configurations of the plate member studied, using the transfer matrix approach.

2. The choice of the time step δt and of the parameters γ, β and $\delta\sigma=\Theta \ \delta t$.

3. Calculation of integrator operators

$$a_0=1/(\beta \ \delta\sigma^2), \quad a_1=\gamma/(\beta \ \delta\sigma), \quad a_2=1/(\beta \ \delta\sigma),$$

$$a_3=1/(2\beta), \quad a_4=\gamma/\beta, \quad a_5=(\gamma/(2\beta)-1)\delta\sigma \ . \quad (18)$$

4. Formulation of the effective stiffness as

$$\overline{k}(t) = k(t) + a_0 m + a_1 c(t) \qquad (19)$$

and its incorporation into corresponding transfer matrix of the problem studied.

Actual realization: The mass parameters in the nodal matrices are multiplied by the integration operator a_0. The parameters of damping $c(t)$, present in corresponding moduli of elasticity when using the hysteresis theory of internal and external damping in accordance with reference (1), are to be multiplied by the parameter a_1.

B. For each time step

5. Calculation of effective loads

$$\delta\overline{p}(t) = \delta p(t) + m[a_2\dot{r}(t) + a_3\ddot{r}(t)] + c(t)[a_4\dot{r}(t) + a_5\ddot{r}(t)] \quad . \quad (20)$$

6. With rigidities $\overline{k}(t)$ and loads $\delta\overline{p}(t)$, using the transfer matrix method, there are to be calculated the increments of displacements $\delta r(t)$.

7. Calculation of accelerations $\ddot{r}(t+\delta\sigma)$

$$\ddot{r}(t+\delta\sigma) = a_0\delta r(t) - a_2\dot{r}(t) - a_3\ddot{r}(t) + \ddot{r}(t) \quad . \quad (21)$$

8. Calculation of accelerations $\ddot{r}(t+\delta t)$

$$\ddot{r}(t+\delta t) = \ddot{r}(t) + [\ddot{r}(t+\delta\sigma) - \ddot{r}(t)]/\Theta \quad . \quad (22)$$

9. Calculation of velocities $\dot{r}(t+\delta t)$

$$\dot{r}(t+\delta t) = \dot{r}(t) + (1-\gamma)\ddot{r}(t)\delta t + \delta\ddot{r}(t+\delta t)\delta t \quad . \quad (23)$$

10. Calculation of dynamic deformations

$$r(t+\delta t) = r(t) + \dot{r}(t)\delta t + (0.5-\beta)\ddot{r}(t)\delta t^2 + \beta\ddot{r}(t+\delta t)\delta t^2 \quad . \quad (24)$$

C. Repeating of the calculations for the following time steps

Present approach allows the solution of the time response of the deck slabs subjected to various kinds of dynamic loads occurring. Analysed can be the influence of moving loads with impulsive, periodic or aperiodic forcing components and the corresponding time response of analysed deck slabs. The solutions of linear and non-linear problems of dynamic limit state response of plate and shell structures using present approaches are available, for example, in references (1),(2),(3) and (4).

NUMERICAL EXPERIMENTS WITH DISCUSSION

Studied is a rectangular reinforced concrete slab with dimensions 2980 x 590 x 10 mm, as shown in Fig. 2. In longitudinal direction, in distance 15 mm from the lower edge of the slab are positioned longitudinal reinforcing steel rods having diameter 8 mm and located 120 mm apart in the transverse direction. Young's moduli of elasticity were measured and given by the values 26.5 and 40.0 GPa for concrete and 210 GPa for steel reinforcing rods used.

The panel was assumed to be elastically supported on the shorter edges by line rubber supports in transverse direction. The elastic characteristics of the rubber supports used were found to have the constant values 4000 N/m.
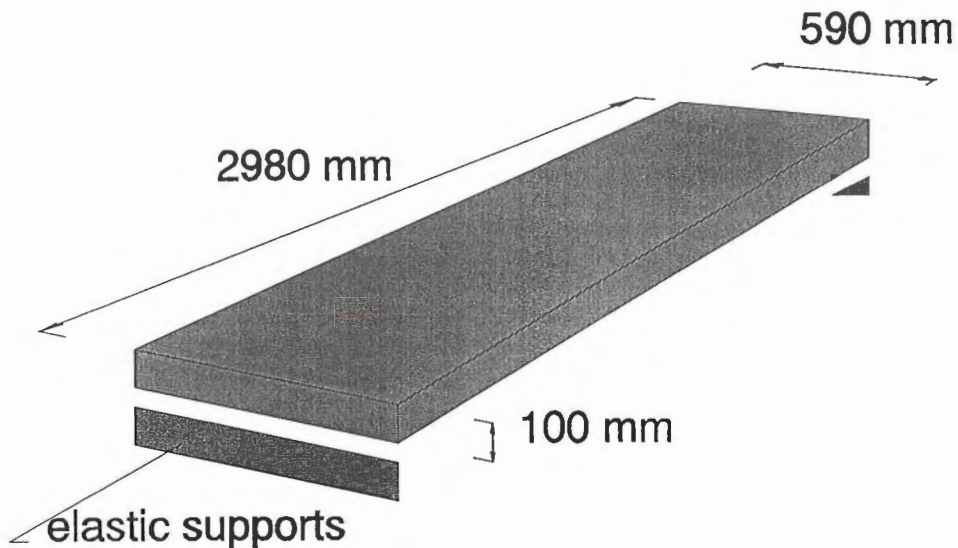
Fig. 2. Geometry of the studied slab member

Present problem was investigated numerically and experimentally in accordance with the development and erections of several new types of cable-stayed bridges on the basis of composite structures in Czechoslovakia.

Due to fabric imperfections the distinct variability of the moduli of elasticity of concrete has been found in the interval 26.5 until 40.0 GPa. Therefore, two above mentioned values have been used for the numerical analyses performed.

Assumed elastic supports simulate the boundary conditions of the slab when located on the load-bearing steel elements used in composite bridge structures.
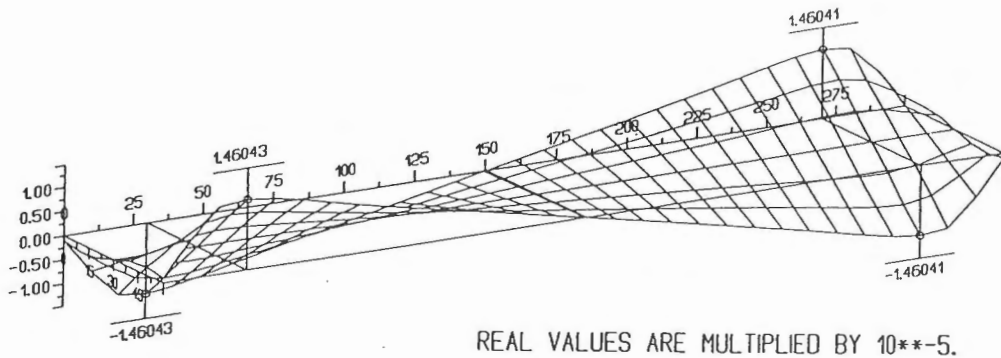
1.-TH MODE OF VIBRATION FOR FR: 4.296 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-5.

Fig. 3. Mode No. 1 for elasticity modulus 26.5 Gpa

1.-TH MODE OF VIBRATION FOR FR: 5.267 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-5.
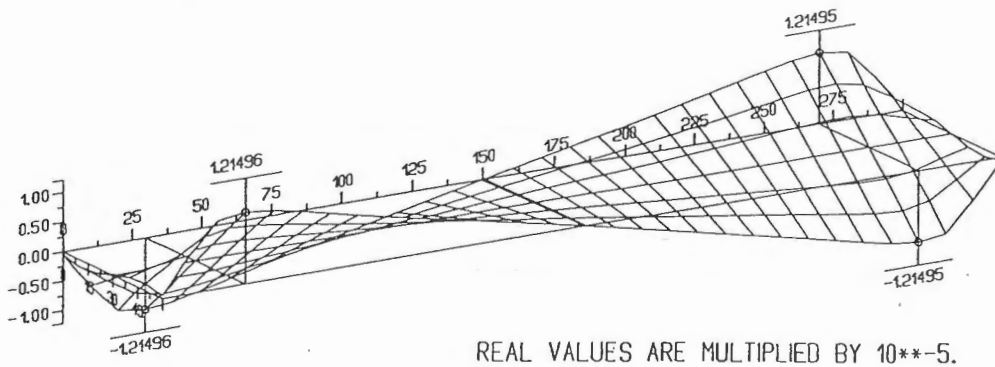
Fig. 4. Mode No. 1 for elasticity modulus 40.0 GPa
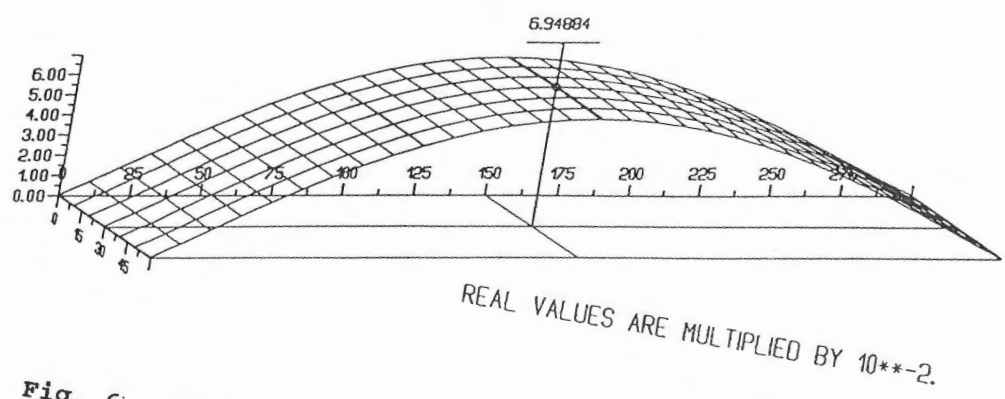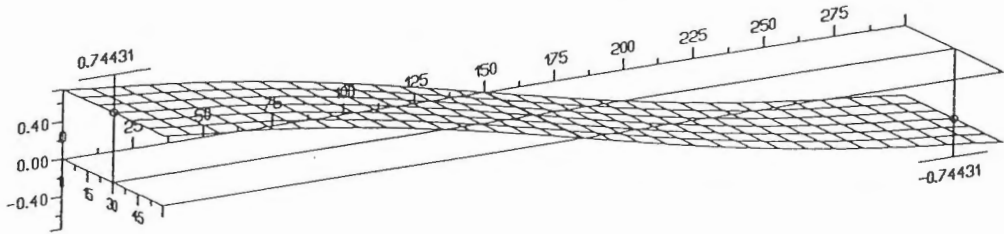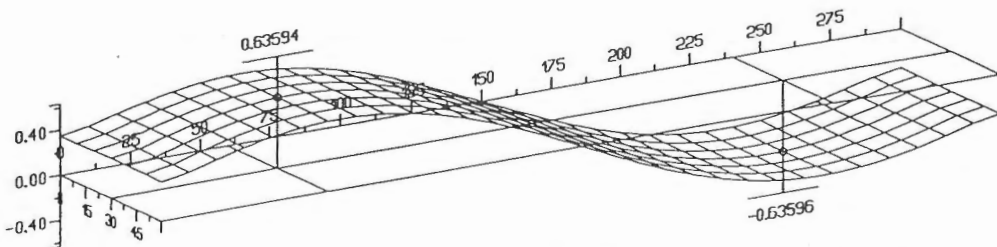
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2.-ND MODE OF VIBRATION FOR FR: 10.05 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-2.
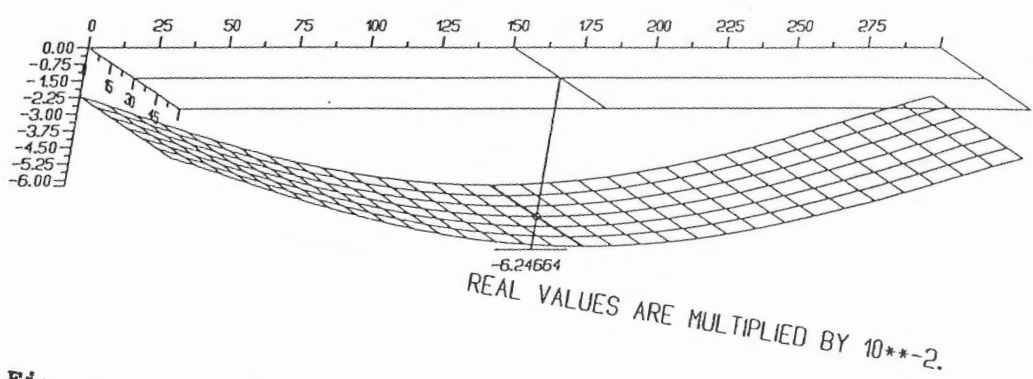
Fig. 5.   Mode No. 2   for elasticity modulus 26.5 GPa

2.-ND MODE OF VIBRATION FOR FR: 17.28 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-2.

Fig. 6.   Mode No. 2   for elasticity mopdulus 40.0 GPa

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3.-RD MODE OF VIBRATION FOR FR: 20.57 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-1.

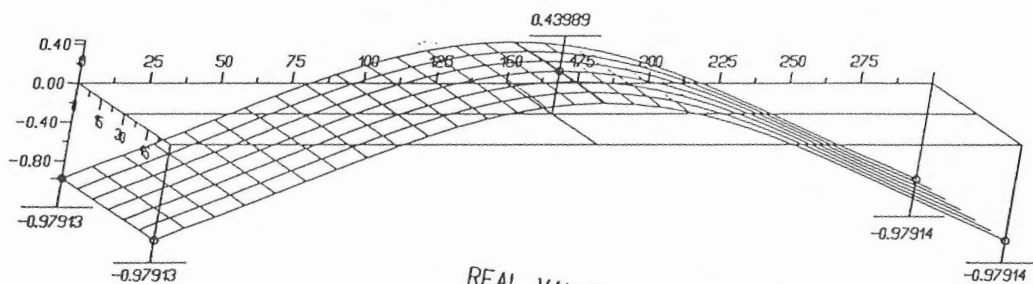**Fig. 7. Mode No. 3 for elasticity modulus 26.5 GPa**

3.-RD MODE OF VIBRATION FOR FR: 51.23 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-1.

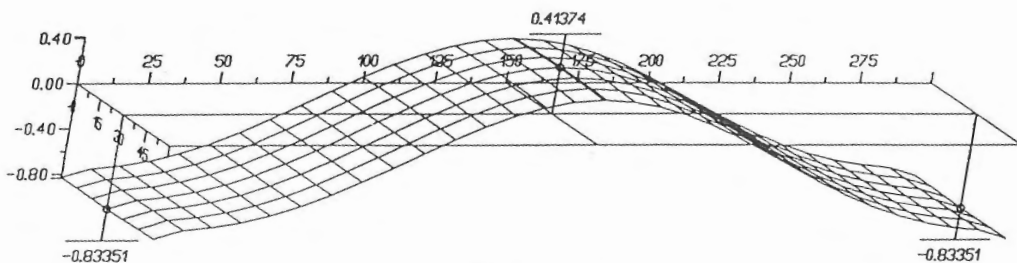**Fig. 8. Mode No. 3 for elasticity modulus 40.0 GPa**

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4.-TH MODE OF VIBRATION FOR FR: 34.48 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-1.

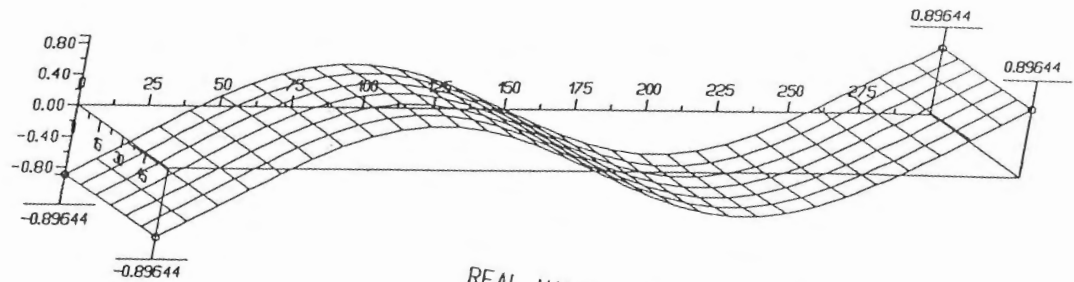Fig. 9.   Mode No. 4   for elasticity modulus 26.5 GPa

4.-TH MODE OF VIBRATION FOR FR: 79.51 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-1.
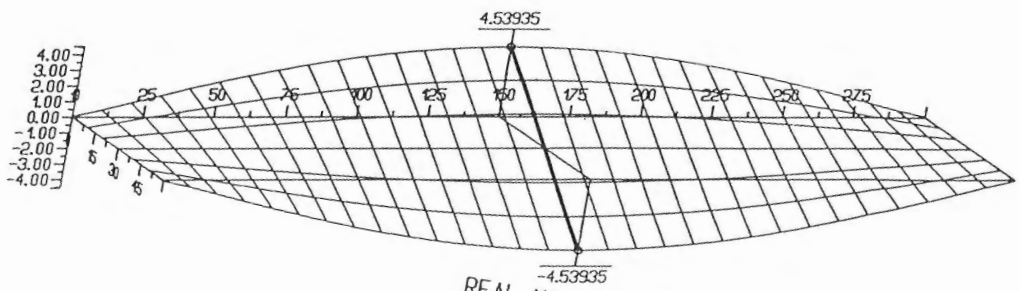
Fig. 10.   Mode No. 4   for elasticity modulus 40.0 GPa

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5.-TH MODE OF VIBRATION FOR FR: 63.54 Hz.

Fig. 11. Mode No. 5 for elasticity modulus 26.5 GPa

5.-TH MODE OF VIBRATION FOR FR: 93.98 Hz.

Fig. 12. Mode No. 5 for elasticity modulus 40.0 GPa

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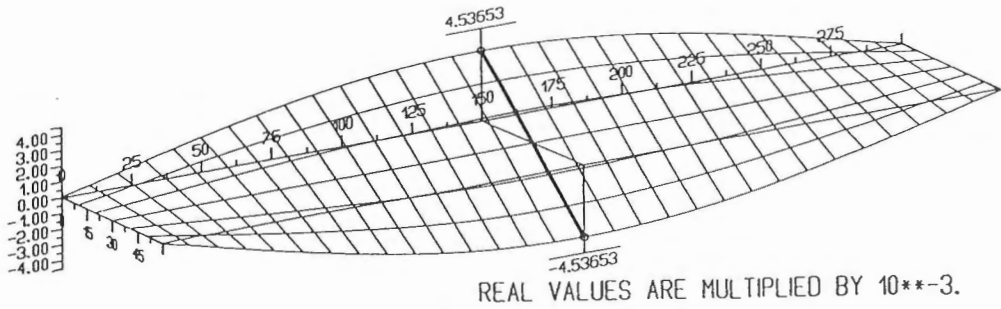6.-TH MODE OF VIBRATION FOR FR: 70.02 Hz.

Fig. 13.   Mode No. 6   for elasticity modulus 26.5 GPa
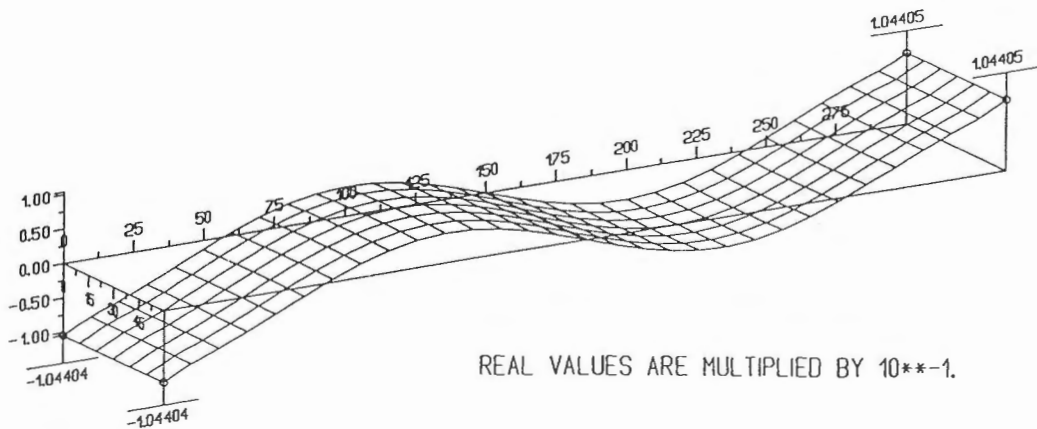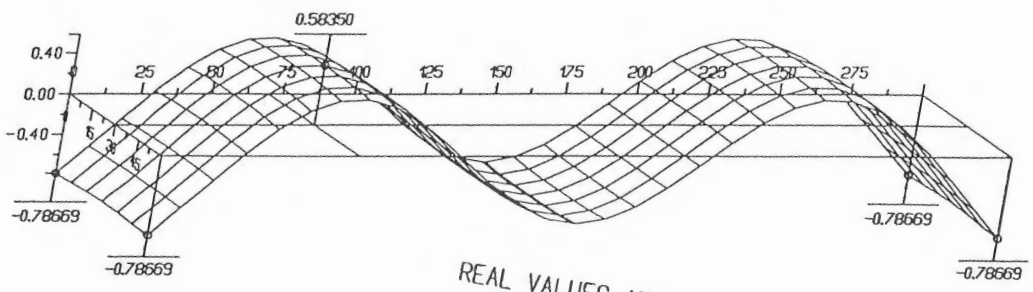
6.-TH MODE OF VIBRATION FOR FR: 105.4 Hz.

Fig. 14.   Mode No. 6   for elasticity modulus 40.0 GPa

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7.-TH MODE OF VIBRATION FOR FR: 103.5 Hz.

Fig. 15. Mode No. 7 for elasticity modulus 26.5 GPa

7.-TH MODE OF VIBRATION FOR FR: 146.1 Hz.

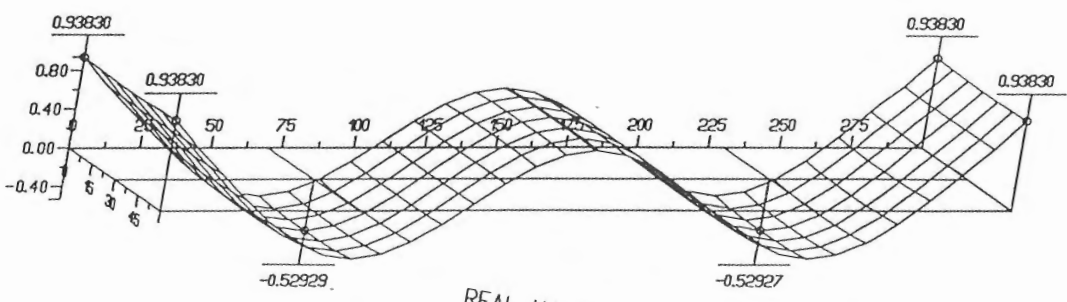Fig. 16. Mode No. 7 for elasticity modulus 40.0 GPa

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8.-TH MODE OF VIBRATION FOR FR: 139.5 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-2.

Fig. 17. Mode No. 8 for elasticity modulus 26.5 GPa

8.-TH MODE OF VIBRATION FOR FR: 187.3 Hz.

REAL VALUES ARE MULTIPLIED BY 10**-2.

Fig. 18. Mode No. 8 for elasticity modulus 40.0 GPa

Theoretical concepts mentioned above have been used for numerical calculations of natural vibrations of studied panel. Calculated was the spectrum of natural frequencies with corresponding modes of vibration for both values of the moduli of elasticity assumed above. Present analysis submit the imagination about dangerous resonance regions of vibration of the panel in flexural and torsional modes of oscillation as well as about the influence of the variability of the material parameters on the resulting behaviour in limit state situations when the panel is submitted to dynamic loads.

The results obtained are illustrated in the set of figures 3 until 18. For the values of the elasticity moduli assumed, the spectrum of natural frequencies and modes of vibration in flexural and torsional oscillations is presented. For both studied values of the moduli of elasticity the first three found modes and corresponding frequencies are valid for the rigid body motions of the panel studied. There follows then the flexural modes of vibration (No. 4 and 5) with additional torsional mode (No. 6). Seventh mode corresponds to the flexural behaviour of higher order similarly as the eighth mode corresponding to higher torsional mode of the simulation studied.

The differences between the values of natural frequencies and shapes of individual modes allow to study the influence of the variability of material parameters on the dynamic response and behavioour of the slab panels located on elastic supports occurring in bridge engineering.

CONCLUSIONS

If any predictions about possible future trends in the discrete analysis of bridge structures are to be made, they must be based on the assumption that research and development in the branch will be responsive to the needs of engineering community.

Because the efficiency of the available solution procedures is highly case-dependent, an optimally efficient computer program must contain a number of options and consequently requires an increasing degree of user sophistication. The quality of structural analysis can be greatly improved if more efficient solution procedures are derived and introduced into a computer code, together with a way to select automatically from the available options the procedures that are suited for each particular case.

Present paper, hopefully, can contribute to these strivings and introduce some new approaches for the solution of engineering problem studied.

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