OPTIMIZATION OF THE FORCE RESPONSES OF CONTROLLED SUSPENSION FOR GYROSCOPIC ROTOR

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SUMMARY

The problem of determining the law of controlling the non-contact suspension for a gyroscopic rotor ensuring minimization of the integral quadratic deviation in transient processes with the least expenditure of force resources is solved.

Controlled non-contact rotor suspensions provided by electromagnetic forces are used in various branches of modern engineering on a wider scale [1, 2]. The suspension control should be provided so that the required quality of transient processes be ensured with the least expenditure of the force resources. The problems of optimal one-dimensional suspension control were considered, for example, in [3,4]. In the given paper the problem of optimal multi-dimensional suspension control of a gyroscopic rotor is solved.

A balanced rigid rotor having mass M, equatorial J_1 and axial J_3 moments of inertia and rotating in suspension at the constant frequency ω is considered. We shall determine the rotor position in a fixed system of axes 0xyz by the coordinates x_c, y_c, z_c of the mass centre C and by the angles of rotation φ_x, φ_y about the axes x, y. The angle $\varphi_z = \omega t$. Let us denote the vector of the generalized coordinates by $\boldsymbol{q} = \mathrm{col}(x_c, y_c, z_c, \varphi_x, \varphi_y)$. Let the suspension state be assigned by the vector of the finite number of currents or charges π ,whereas the elements of the vector of the generalized forces $\boldsymbol{F} = \mathrm{col}(F_1, \ldots, F_5)$ are considered as familiar functions of the variable q , q , π

$$\boldsymbol{F} = F(q \ , q \ , p) \tag{1}$$

The equations of the rotor movements in the suspension under the assumption of smallness of displacements q can be represented in the form

$$Mx_c = F_1, \qquad My_c = F_2, \qquad Mz_c = F_3; \tag{2}$$

$$J_1\varphi_x + \omega J_3\varphi_y = F_4, \qquad J_1\varphi_y - \omega J_3\varphi_x = F_5 \tag{3}$$

$$\mathrm{D}(q, \pi) = \boldsymbol{u}, \tag{4}$$

where $\boldsymbol{u}$ is the vector of the control voltages on the windings of the electromagnets or on the electrodes; D is in the general case the non-linear differential matrix operator [1,2].

Using decomposition of the complete system (2)-(4) into mechanical (2),(3) and electromagnetic (4) subsystems, we shall solve the problem of control in two stages : at first we shall find the forces $\boldsymbol{F} = \boldsymbol{F}^\circ$, ensuring the required rotor movement, and then we shall determine the vectors $\pi = \pi^\circ$ and $\boldsymbol{u} = \boldsymbol{u}^\circ$, realizing the forces $\boldsymbol{F}^\circ$, [2]. We shall find the solution under the assumption that the vector of the system state can be accurately measured and that the noise of measurements and external disturbances are not available (the influence of these factors is discussed at the end of the article).

At the first stage the controlled objects are three systems (2) with one degree of freedom and scalar output variables $y = q_j$, $j = 1,2,3$, and the system (3) with two degrees of freedom and the two-dimensional output variable $\boldsymbol{y} = \mathrm{col}\,(\varphi_x, \varphi_y)$.The forces $\boldsymbol{F}$ control the objects. Let us reduce each of the systems to the normal form and add the equation of the output variable

$$\boldsymbol{x} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{f}, \qquad \boldsymbol{y} = \mathbf{C}\boldsymbol{x} \tag{5}$$

where

$\boldsymbol{x} = \mathrm{col}\,(q_j, q_j)$, $\boldsymbol{f} = F_j / M$ in case of the objects (2), and
$\boldsymbol{x} = \mathrm{col}\,(\varphi_x, \varphi_x, \varphi_y, \varphi_y)$, $\boldsymbol{f} = \mathrm{col}\,(F_4 / J_1, F_5 / J_1)$ in case of the object (3);
A, B, C, are the constant matrices.

It is required to find the forces $\boldsymbol{f} = \boldsymbol{f}^\circ$ (or $\boldsymbol{F} = \boldsymbol{F}^\circ$), which during change-over of the system (5) from the arbitrary initial state $\boldsymbol{x}(0)$ into a zero one minimize the functional

$$\int_0^{\infty}[\, \mathbf{y}^T(t)\,\mathbf{y}(t) + \rho \mathbf{f}^T(t)\,\mathbf{f}(t)\,]\,dt$$

where ρ is the positive weighting scalar.

The solution of the problem has the form [5]

$$\mathbf{f}^{\circ} = -\,\rho^{-1}\,\mathbf{B}^T\mathbf{P}\,\boldsymbol{x} \qquad (6)$$

Here $\mathbf{P}$ is the positively defined symmetric n x n matrix, where n is the dimensionality of the state vector $\boldsymbol{x}$, being a unique solution of the non-linear matrix Rikkati equation

$$\mathbf{C}^T\,\mathbf{C} - \rho^{-1}\mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P} + \mathbf{A}^T\,\mathbf{P} + \mathbf{P}\mathbf{A} = 0. \qquad (7)$$

The main difficulty in solving similar problems is to find the solution of (7) which is equivalent to the system n(n+l)/2 of non-linear algebraic equations with respect to the elements of the matrix $\mathbf{P}$. For n > 2,as a rule, numerical methods [5] are used. Here we have the case when the problem has an analytical solution.

With reference to the objects (2) three elements $P_{11} = z_1$, $P_{12} = z_2$, $P_{22} = z_3$ of the matrix $\boldsymbol{P}$ satisfy the system of equations

$$z_2^2 = \rho, \qquad z_3^2 = 2\,\rho\, z_2, \qquad z_2 z_3 = \rho\, z_1,$$

having obvious solution

$$z_1 = \sqrt{2}\ \rho^{1/4}, \qquad z_2 = \rho^{1/2}, \qquad z_3 = \sqrt{2}\,\rho^{3/4},$$

Taking into account of (6), we obtain that the optimal force response of the objects (2) has the form

$$F_j^{\circ} = -\,M\,(\,\omega_{\circ}^2\, q_j + 2\xi\omega_0\, q_j\,), \qquad j = 1,2,3, \qquad (8)$$

where $\omega_0 = \rho^{-1/4}$ is the natural frequency of the translational rotor movements; $\xi = \sqrt{2}\,/\,2$ is the damping parameter. From the relation ρ =

ω_o^{-4} the physical sense of the weighting parameter ρ is clear.

In case of the object (3) the problem is to determine ten elements $P_{11} = z_1$, $P_{12} = z_2, \dots$, $P_{22} = z_5, \dots$, $P_{44} = z_{10}$ of the matrix **P**, satisfying the system of equations

$$z_2^2 + z_4^2 - \rho = 0,$$

$$z_2 z_5 + z_4 z_7 - \rho z_1 - h \rho z_4 = 0,$$
$$z_2 z_6 + z_4 z_9 = 0,$$
$$z_2 z_7 + z_4 z_{10} - \rho z_3 + h \rho z_2 = 0,$$
$$z_5^2 + z_7^2 - 2 \rho z_2 - 2 h \rho z_7 = 0,$$

$$z_5 z_6 + z_7 z_9 - \rho z_3 - h \rho z_9 = 0,$$
$$z_5 z_7 + z_7 z_{10} - \rho (z_4 + z_6) + h\rho (z_5 - z_{10}) = 0,$$
$$z_6^2 + z_9^2 - \rho = 0,$$

$$z_6 z_7 + z_9 z_{10} - \rho z_8 + h\rho z_6 = 0,$$
$$z_7^2 + z_{10}^2 - 2\rho z_9 + 2h\rho z_7 = 0,$$

where $h = \omega J_3/J_1$ is the gyroscopic parameter. Having expanded the functions $z_s(h)$, $s = 1,\dots, 10$, into a power series, one can make sure that the following equalities are correct

$$z_1 = z_8, \quad z_2 = z_9, \quad z_3 = z_7 = 0, \quad z_4 = -z_6, \quad z_5 = z_{10}. \tag{10}$$

Substituting (10) into (9) results in the system of four equations which has an analytical solution

$$z_1 = 4z_6^3 / \rho^2 h^3 + h z_6, \qquad z_2 = 2z_6^2 / \rho h^2, \qquad z_5 = 2z_6 / h,$$

$$z_6 = [(\rho^4 h^8 / 64 + \rho^3 h^4 / 4)^{1/2} - \rho^2 h^4 / 8]^{1/2}. \tag{11}$$

Taking into account the fact that in accordance with (6)

$$f_1^{\rho} = -\rho^{-1} (z_2 \varphi_x + z_5 \varphi_x + z_6 \varphi_y + z_7 \varphi_y).$$

$$f_2^\rho = -\rho^{-1} (z_4 \varphi_x + z_7 \varphi_x + z_9 \varphi_y + z_{10} \varphi_y).$$

and with regard to (10) and (11), we obtain the following result. The optimal force responses of the object (3) have the form

$$F_4^o = -J_1 (k_1 \varphi_x + k_2 \varphi_x - k_3 \varphi_y),$$
$$F_5^o = -J_1 (k_1 \varphi_y + k_2 \varphi_y - k_3 \varphi_x), \qquad (12)$$

where

$$k_1 = \sqrt{h^4/16 + 1/\rho} - h^2/4, \quad k_2 = \sqrt{2k_1}, \quad k_3 = h\sqrt{k_1/2}. \qquad (13)$$

The first terms in (12) are the moments of the elastic forces, the second ones are those of the dissipative forces, the third terms are the moments of the radial correction forces.

A non-rotating rotor (h=0) has natural frequency of the angular vibrations $\Omega_0 = k_1^{1/2} = \rho^{-1/4}$. As before, the physical sense of the weighting parameter ρ is explained by the relation $\rho = \Omega_0^{-4}$. For a rotating rotor as the gyroscopic parameter h increases from zero to infinity the elastic k_1 and dissipative k_2 coefficients in accordance with (13) must asymptotically decrease to zero, whereas the coefficient of radial correction k_3 must asymptotically increase from zero to the value of k_1 for $h = 0$. Consequently, the optimal controller is characterized by a non-permanent adjustment depending on the rotational frequency ω.

Characteristic equation

$$\lambda^4 + 2 k_2 \lambda^3 + (k_2^2 + 2k_1 + h_2)\lambda^2 + 2 (k_1 k_2 + hk_3)\lambda + k_1^2 + k_3^2 = 0$$

corresponds to the closed system (3), (12).

Its roots in the limiting cases $h = 0$ and $h \to \infty$ take the following values:

$$\lambda_{1,2} = \lambda_{3,4} = (-1 \pm j)\rho^{-1/4}/\sqrt{2} \qquad \text{for } h = 0,$$

and

$$\lambda_{1,2} \approx - jh, \qquad \lambda_{3,4} \approx \rho^{-1/2}/ h \qquad \text{for } h \to \infty,$$

where j is the imaginary unit.

For unchangeable h the quality of the transient processes in the optimal suspension is completely determined by the weighting parameter ρ. As ρ decreases the speed of response of the suspension increases but the values of the forces $\boldsymbol{F}^o$ also increase. For $\rho = 0$ the values of $\boldsymbol{F}^o$ will be infinitely great. In this connection the problem is solved by the trial method: the value of ρ is chosen so that the speed of response of the suspension should be similar to that required and the forces $\boldsymbol{F}^o$ have acceptable values.

At the second stage of solving the problem of the rotor suspension control for a familiar force response of the suspension $\boldsymbol{F} = \boldsymbol{F}^o (q, q)$ by means of (1) and (4) one can find the variables of the suspension state $\pi = \pi^o$ and the control voltages $\boldsymbol{u} = \boldsymbol{u}^o$, realized the forces $\boldsymbol{F}^o$, [2].

As to the external disturbances and noise of measurements which were not taken into account in the problem of synthesis let us note the following. If the disturbances are controlled, they will enter into (1), π^o , $\boldsymbol{u}^o$ and, consequently, will automatically be counteracted by the suspension. The required quality of counteracting uncontrolled disturbances is achieved by the corresponding choice of the weighting parameter ρ. The noise of measurements restricts the bandwidth of the differential links used for calculating the speeds $\boldsymbol{q}$. Therefore, in case of non-optimal but simpler controller with permanent adjustment in which $k_3 = 0$, k_1, k_2 are the constants, there always exists a threshold rotational frequency at which the suspension loses its stability. In physical sense this means that at the threshold rotational frequency because of the errors in calculating the speeds high-frequency nutational rotor vibrations at the frequency h are not damped but on the contrary are replenished from the direction of the suspension. The suspension with the optimal controller has no disadvantage of that kind because damping nutational vibrations is carried out not by dissipative forces but by forces of radial correction.

REFERENCES

1 Martynenko Yu.G. Rigid body motion in electric and magnetic fields.- M. : Nauka, 1988.- 308 p.

2 Zhuravlyov Yu.N. Dynamics of mechanical systems with active magnetic bearings. Mashinovedeniye, 1988 No.5.- P.70-76.

3 Voronkov V.S.,Pozdeev O.D. Optimization of the stabilization system of magnetic suspension. Izv.Vuzov.Priborostroeniye, 1979, No.9.- P.53-57.

4 Zhuravlyov Yu.N., Khmylko N.V. Dynamic optimization of linear control system by active magnetic bearings. lzv.Vuzov, Elektromekhanica, 1987, No.12. - P.74-82.

5 Kvakernaak H.,Sivan R. Linear optimal control systems.- M.: Mir, 1977. - 650 p.

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