ANALYSIS ON THE STRUCTURAL BEHAVIOUR OF COMPOSITE SADDLE-SHAPE ROOF SHELLS

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INTRODUCTION

This paper discusses the structural behaviour of cable network and contour beam of suspension roof with roof cladding structure. The action of the cladding of roof is commonly neglected when calculating inner forces and displacements for the bearing part of a suspension roof, i.e. cable network and contour beam. Joint work of the network, contour beam and supports with a cladding yields substantial changes in the stress-strain state of whole structure. Our interests in Tallinn Technical University lie in the work of composite suspension roofs, both diamond- shaped (Fig 1) and oval (Fig 2), including those without tie-rods and counterforts (inside horizontal equilibrium), where plywood or steel plates (Fig 1b, 1c) or a layered timber shell (Fig 2e) are used as a cladding. Major attention was paid on the statical aspects, but at the same time dynamic characteristics, like natural frequences and damping factors. The value and distribution of wind load were also determined for a structure presented on Fig 2. Some results of experimental investigations and numerical analysis are described below.

MODEL TESTS

To study the actual work of described systems, different models were built and tested. Model parameters of the first group systems (Fig 1, type 1-without cladding, type-2 with cladding from plywood plates and type 3 with steel plates) were as follows. Dimensions in plan 2a = 2b = 2.5 m; edge element consisted of steel tube of 33.5 mm diameter; t = 3.2 mm and four corners were supported in the verical direction, steel tie-rod diameter was 12 mm, f = 0.49 m; 8 high strength wire cables of 1.6 mm diameter were used in each direction; cable pitch was about 278 mm $\beta = f_v/f_x = 210/280 =$ 0.75. In the initial state cable network surface was obtaind by cable prestressing with mutual sliding, followed by fixing network knots and mounting panels. Roof plates were constructed of three layer plywood t = 4mm or steel plates t = 0.35 mm, with the edges strengthened by 8 mm ribs. Plates were attached to network knots and contour beam through central steel knot elements by bolts (Fig 1c).

Models were loaded both by evenly all over the surface (live load 0.25-2 kPa,dead weight 0.25-0.5 kPa) and by unevenly distributed or local loads (0.08-0.16 kN) and dead weight of the contour (0.41 kN/m). The load was suspended to the network knots and the weight

of the contour beam along the edge element. Both, cladding and horizontal tie-rod structures and those without cladding and tie-rods were examined.



Fig 1 Structural schemes. T-tie-rod; C-cables; W,S-plywood or steel plates; CB-contour beam; PEC-pin-ended column; FBfixed bearing; M-freely movable bearing.

Model of the **second group** structures (Fig 2, type 4 without shell, type 5 with shell), in every case without tie-rod, is a hyparshaped suspension roof inside an incleaned elliptical contour, where central points of cross-sections of the contour joints were located on a spatial curve, formed by crossing two surfaces: hypar

$$z = f_{y} \cdot x^{2}/a^{2} - f_{y} \cdot y^{2}/b^{2}$$

and an elliptical cylinder

$$x^2/a^2 + y^2/b^2 = 1$$
,

where a = 2,13 m and b = 2,663 m. The surface part of the roof model consists of a 3-layered timber shell made from boards, that rests on prestressed cable network and is connected to the contour and cable network. High strength wire cables of 3.0 mm were used in each direction. The prestressing forces were $T_x^0 = 0.86$ kN/m and $T_y^0 = 2.04$ kN/m.The contour consisted of straight sections of steel tubes and rested on three plain supports. Hinge joints were used to connect the columns of the supports and contour tube and foundation. This supporting enables free displacements of the supported parts in the directions, perpendicular to the planes of the supports. Longitudinal ribs with the cross-section of 1.6×20 mm for fastening cables and the timber shell were welded to the inner side of the contour tube. Other parameters are shown on Fig 2.



Fig 2 Structural scheme.

Different gravitational loads were used for loading of models (see Fig 2 and 3), whereas they were uniformly distributed all over the roof surface and distributed on half or on quarter the surface (0.9 - 1.62 kPa). Additional loads were used to simulate the weight of the contour 0.45 kN/m for an empty tube and 2.89 kN/m for tube filled with concrete. The concrete filling was used to shorten the bending moment in the contour and to decrease vertical displacement of the front freely suspended part of it, however, it is sufficient to use a filling only in some parts of the contour tube. These parts will resist to forces as composite steel-concrete members. The statical work of the roof was examined and dynamic parameters were determined, also a 1:100 scale roof model was tested in a wind tunnel to determine the coefficients of wind pressure (Fig 4).

This particular structure was designed for the acoustic screen of the song festival tribune in Tartu, Estonia, by design organisation "Eesti Projekt". Professor V.Kulbach and author of this paper are designers of that roof structure. The scale of model is 1:10.



Fig 3 Views of models. a-without cladding; b-with a cladding from timber shell.



Fig 4 Roof model in wind tunnel.

Structural parameters of all studied types of roof models were as follows:

Table 1

Roo: typ	E e	t,	tcladding	Σt _r	I., m ⁵	$I_e \cdot t_s$, m^5	
1	1/1.	73·10 ⁵ a	-	1/30458	a 2.5.10 ⁻⁸	·2a 1.45 · 10 ⁻¹³	a ²
2	=	$t_{s,1}$	$t_{pp} = (1/630) 2a$	1/16004a	=J _{c,1}		
3	=	$t_{s,1}$	t _{sp} =(1/7150)2a	1/6863a	=J _{e,1}		
4	=	$t_{s,1}$	-		45.7.10	⁸ d _m	
5	=	t.,1	$t_{ts}=1/530d_m$	$1/19615d_{m}$	45.7·10 ⁻⁸ d	$m89.5\cdot10^{-13}d_{m}^{2}$	
t_s $t_{pp} = 1_c$ $t_{sp} = 2a$ d_m $t_u = 2t_r$	= red = thi = ine = thi = di = ar el = thi = sun cla	duced t ckness ertia m ckness agonal ithmet liptic ckness mmary m dding,	chickness of s of plywood p moment of cont of steel play lenght of sq ic mean of tw al roof Fig 2 of timber sho reduced thickr reduced to s	teel membric lates, cour tubes tes, uare roof o main dia), ell, ness of st teel.	rane, , (Fig 1), ngonal leng eel networ	gths of an k and timber	

As we can see, all types of studied systems had relatively thin roof structure and contour with high slenderness.

Apart from the above factors influencing statical and dynamical behaviour of the roof, we have to consider the following: the value of prestressing forces in cables, shear stiffness and pliability of connections of a cladding, roof geometry, type of contour supports.

TEST RESULTS

Some data obtained when testing models of roof structure are given below.

1. First group of models

Figures 5 and 6a show the diagrams of deflections along main diagonals, Fig 6b increase in central sections of bearing and damping cables. All structures without tie-rod were exposed to a twice redused load. On graphs indexes have following meaning: T - tie-rod; W - plywood plates; S - steel plates; gc - dead weight of the contour; F2 - concentrated load 0.16kN; p1, p2, p3, p4, p5, p6 - live load, correspondingly 0.25, 0.5, 0.75, 1, 1.5 and 2 kN/m².

Note that with no index there is no load, i.e.no plates or no tierod. 1) with evenly all over the shell surface distributed loads:

- Plywood plates used for under-roof structure caused a considerable redistribution of inner forces, as a result, increases of inner forces in all bearing cables were almost equalized. Increments of inner forces in central cables decreased by 70% with the application of tie-rod, but without tie-rod by 50% as compared to the model without cladding, see Figure 5 and 6.
- Steel plates had a still greater equalizing effect on inner forces between bearing cabls. It is interesting to note that increase of inner forces in central bearing cables was even smaller than in edge cables.
- When no horizontal tie-rod was applied stiffening cables could to some extent replace it and then inner forces showed an increase for all types of claddings. Approximately 15% decrease of maximum increments of inner force was observed for all types of boardings.
- When tie-rods were applied, inner forces of tightening cables, i.e. prestressing forces were reduced nearly to zero. Plate cladding improved the condition: when plywood plates were applied - 50% and with steel plates even 80% of prestressing forces remained (Figure 6c, d). All types of cladding constructions reduced vertical displacements of central point of cable network as compared to those with no cladding with the application of tie-rod and steel plates - by 55%, without tierod and steel plates - by 35%.
- Plywood plate cladding developed an increase of vertical deflections of edge elements.
- Without tie-rod a decrease of about 40% of contour crawling was observed in case of batten boarding, but with other types it was slightly smaller.
- By step-by-step increase of load nonlinearity of network and contour deformations is observed by the absence of horisontal tie-rod.
- Bending moments caused by live and dead load on the vertical contour plane exceed about five times those observed on the horizontal plane, but contour parameters are still determined through the load caused by cable prestressing and considerable bending moments were observed mainly on the horizontal plane.
- 2) With the application of nonsymmetrical loads:
- both plate types lead to a substantial redistribution of inner forces in cables, resulting in more even distribution of inner forces. At the same time a 50-60% reduction of absolute increase in inner forces within cables is observed as compared to cable network with no plates. It should be noticed that when loading the model covered with steel plates, inner forces show their greatest absolute value in edge cables not in central ones as



Fig 5 a - diagrams of vertical displacements of central points of bearing cables by evenly distributed load; b- vertical displacements caused by nonsymmetrical loads.

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Fig 6 a - vertical displacements caused by concentrated loads; b - diagrams of increase in inner forces of central bearing cable sections when load is evenly distributed all over the span.

was the case with panels and plywood panel cover. This is observed both with symmetrical and nonsymmetrical loading. By the absence of horizontal tie-rod increments of inner forces in the stiffening cables are also reduced by the panels.

a)

- Plates reduce knot displacements in the vertical direction. No substantial differences are distinguished here between timber and steel panel coverings; displacements are reduced by 20-40%.
- 3) Under the action of concentrated loads:
- Under the action of local loads a particularly notable effect of plates (Fig 6a) on the reduction of local deflections of roofing is observed, which is also significant to avoid major deformation and to maintain an integral boarding.
- 4) Maximum stresses developed in plywood panels when tie-rods were applied and the load p4 was 1.84 MPa, with no tie-rod and p2 -3.53 MPa, but in steel plates considerable stresses - tension up to 250 MPa and compression up to 128 MPa were observed at the corner and in the area close to the contour.
- 5) When tie-rod is applied, a good deal of prestressing force is preserved due to panels, which is mainly observed with steel plates. This leads also to an increase of dynamic stability. Steel plates reduce the amplitude of shock-load vibrations up to 30-70%, increase frequency by 30-50% and accelerate vibration dampening. Inner forces of contour without tie-rod exeeded about 5-10 times and roofing displacements several times those present with tie-rod. But the structure still has working capacity and is characterized by interesting features.

It should be noted that in case of thrustless contour aerodynamic sensitivity is considerably reduced, since in the loading process rigidity of the structure increases, while by the application of tie-rod it decreases.

2. Second group of models

Fig 7 and 8 show results of statical tests, when loading A was uniformly distributed all over the surface, B - dead load + uniformly distributed live load on the half of the roof surface. As a rule, number 1 means cable network (without shell), number 2 - network + timber shell connected to the network and the contour. The diagrams in Fig 7 demonstrate increments of the inner forces in cables, both in carrying (section 1-1, Fig 2) and damping (section 2-2) cables, that occur by roof loading.





Fig 8 Roof deflections in section 2-2 (Fig 2).

The dynamic parameters were determined in the conditions of damped free vibrations of the structure. Test results are presented in Fig 9. Distribution of wind pressure coefficients is presented in Fig 10, that were determined in a wind tunnel by different orientations of the model.



Fig 9 Load support reaction relationship. a - scheme; P - summary load; C4, C2, C3 - reaction forces of the columns 4, 2 and 3 for loading A and B.

Conclusions

The examined type 4 and 5 of the structure is freely deformable in direction X and Y (Fig 2). The reason for its behaviour lies here. Usually counterforts or tie-rods in direction Y are used and that is the reason why by loading prestressing forces in damping cables decrease. In this case, because of free deformation of the contour in direction Y damping cables act as tie-rods preventing large displacements and consequently their tension grows.



Fig 10 Mass frequency relationship. p - uniformly distributed load on the roof surface.



Fig 11 Distribution of wind pressure.

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Relatively small forces are sufficient to prestress (to create) the initial network surface. By loading in cables tension forces in both directions increase except for some outer cables. At the same time increments of tension in damping cables exceed those in carrying cables approximately by 40%.

The bearing capacity (strength, stiffness) of the roof will be considerably increased when the network is covered with timber shell, formed 2 - 3 layers of nailed boards and connected with the nodes of the network and with the contour (see Fig 7 - 10). The influence of cladding on statical and dynamical properties of the roof depends on the stiffness that a cladding adds to the structure.

For example, by using steel plates as cladding (type 3) tension stiffness increased 24 times as compared to uncovered (pure) cable network. It caused the levelling of tension increments of the carrying cables (usually central cables are loaded more than outer ones) and the maximal value decreased about 6.5 times. The same levelling took place by damping cables where prestressing forces decreased only 20%, while usually, by thrust contour, tension in damping cables is redused nearly to zero. But it is the tension forces that remain under loading in damping cables that quarantee roof stifness and prevent large kinematic displacements.Deflection of the central point of the roof decreased about 2.4 times. By thrustless (without tie-rod) contour decrease the tension of the maximum stressed carrying cable was 3.6 times and deflection -1.26 times. Thus, a cladding has a smaller effect on the system by a thrustless contour.

By type 5 (thrustless contour) tension stiffness of roof surface went up only 1.6 - 2 times with the shell. Tension increments of the carrying cables were reduced twice, deflection of the roof surface - 1.7 times and increases of tension of the damping cables 1.8 times (Fig 2). At the same time relative stiffness of the contour in this case was substantially bigger and geometry more favourable than by type 3.

Because of the suffness of timber shell horizontal displacements of the contour in direction X decrease about 25% and in Y - about 40%. But at the same time upword displacements of the front part of the contour increase. The latter undesired factor can be compensated with an additional mass of front part the contour. Moderate filling of concrete into the middle of the front part of the tubular contour brings about a decrease of the bending moments and displacements, but it is accompanied by an increase of cable forces.

Loading on the lower (back side) half of the roof surface proves most unfavourable.

We could observe, that the cladding, connected to the contour and network brings about a decrease of the amplitude and a rise in frequences of vibrations. For example, frequency of the first mode by type 3 rises 1.3 times and maximum amplitude reduces 3.6 times by a thrust contour. By a thrustless contour these changes are correspondingly 1.5 and 2 times (type 3). By type 5 frequency increases by 10 - 15% and amplitude decreases 1.5 - 2 times. The lowest frequency of natural vibrations of the type 5 structure is to a great extent determined by the mass of the freely suspending front part of the contour. Thus, to design a real structure it is necessary to consider the fact that the increase of contour stiffness with the simultaneous increase of its own weight does not necessarily lead to an increase of dynamic stiffness of a roof. Restrained changes in the network weight do not substantially influence the frequencies of the first two modes. No significant influence of the presence of cladding or the changes in loads on the damping characteristics was observed. The value of logarithmic decrement varies between 0.13 and 0.25. Natural frequency was for roof without shell 2.45 Hz and with shell 3.0 Hz. It is for real structure 0.78 and 0.95 Hz.

We have to consider that the effect of the cladding reduces when considering that part of the loading (the weight of the cladding) will be applied to roof before connecting it to the contour. It is not reasonable to support the shell under construction. Secondly, particularly by timber shell an aftereffect occures during maintenance and that also reduces the above mentioned effect. A study of the latter case is under way.

NUMERICAL ANALYSIS

We developed a special method for analysis of suspension saddleshaped roofs, based on nonlinear theory (large displacements, also yielding of connections is possible), enabling to consider tension-compression and shear stiffness of the cladding. Equations and results of analysis are given in (Ref 1).

In Fig 12 some calculation and test results of a roof model are demonstrated. As we can see, calculated results adequately coincide with those obtained from tests.

Some results using a universal programm system TPS-10 for analysis of type 4 and 5 will also discussed here. The analysis was carried out in co-operation with the Finnish firm PI Consulting Ltd, computer implementation by eng O-P. Joronen. TPS-10 is based on the FEM and its version which we could use is based on geometrically linear theory. We had to neglect the naturally nonlinear behaviour of the structure. It can be done for determining the natural frequencies, if we assume the amplitude of vibrations to be relatively small. At the same time linear theory is easier to handle for such complex structures and expensive computer time can be significally economized.

Straight beam elements were used for modelling the contour, cable network and side supports. The back supports and the timber shell were modelled through triangular plate elements. All joints between elements, exept for pin-jointed support-columns were made rigid. In the calculation scheme the structure with natural dimensions was described. The real network with a pitch of 1.5 x 1.5 m was replaced with a network of 4.5 x 4.5 m. To preserve the actual stiffness of the network, the diameter of cables was increased accordingly. Two structural alternatives were examined:



- Fig 12 a diagrams of vertical displacements of convex diagonal; b - diagrams of increase in inner forces of bearing cables sections (convex diagonal); dotted line calculation results.
- cable network without a timber shell, and contour without concrete filling;
- cable network with timber shell and contour filled with concrete.

The version of TPS-10 used was not determined for analysing geometrtrically nonlinear problems. And as could expect the differences between the calculated and experimental values, by network, without the shell, were significant. For example, calculated vertical displacements of the network were 2.5 times greater, increments of carrying cables 50% and increments of damping cables 30% larger than by experiment. But by the timber shell roof and cable network where stiffness of roof surface was 1.6 - 2 times greater, the differences amounted to 1.7 times, 30% and 20%. So this program allowed us determine with the sufficient approximation the stress-straine state of the composite roof and its contour.

We analysed the work of the roof by 3 various types of back supports (Fig 13)., It is interesting to note that by the first and second type (Fig 13c 2 and 3) large horizontal forces (abot 1800 kN), caused by deformation of the flat arc 1 between them,

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arose between these supports. So we had to use type 3 and 4 of the back supports.



Fig 13 a - variants of back supports; computer diagrams, b normal forces; c, d - bending moments; e - torsion moments of the contour when loading B.



Fig 14 Computer diagrams. a - horizontal deformation of the contour; b - horisontal deformation of the cable newtwork

Assuming linear behaviour of the system then TPS-10 enables to determine natural frequencies of the complex structure (consisting of contour, cable net and timber shell) with acceptable accuracy.

CONCLUSION

The suspension roof cladding - timber shell, steel panels, etc., connected to the network and the contour increases rigidity and bearing capacity of the roof and contributes to a more even distribution of the inner forces between cables and additional utilization of cladding material are of major importance. The cladding raises the dynamic stability, i.e. reduces the amplitude of vibrations and increases natural frequency.

APPLICATION

At present a suspension roof with the described composite shell (type 5), where 2a = 42 m and 2b = 54 m is under construction in Tartu, Estonia (Fig 14).



Fig 15 Acoustic screen of Tartu song festival tribune under construction.

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