THE SHEAR FAILURE CONDITION IN HOLLOW-CORE SLAB UNITS LOADED BY VERTICAL AND TRANSVERSE SHEAR FORCE COMPONENTS

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Rakenteiden Mekaniikka, Vol. 24
No 2 1991, ss. 22 - 29

ABSTRACT: Hollow-core slab units are widely used as decking systems in Finland. They were originally designed to be supported on walls, which provide a non-flexible support. In such cases the web of the slab is subject to a vertical shear stress and a longitudinal normal stress, caused by the shear force and the prestressing force of the slab, respectively. These slabs have increasingly been used lately supported by beams and girders, which causes the shear failure condition to be met on much lower load levels than is the case when they are assembled on non-flexible supports.

This paper deals with the failure condition of concrete when two shear stresses and one normal stress are applied to the web of the slab. The failure condition of N.S. Ottosen is used, because it is well applicable to different stress states. The results are expressed as scaled with respect to the tensile strength of concrete.

INTRODUCTION

Prestressed hollow-core slabs are a common part of decking systems, and are nowadays being used increasingly in composite floors. When assembled on beams, a portion of the slab becomes an effective part of the beam cross-section and the slab becomes inevitably affected by the composite interaction between the beam body and the slab units after the groutings and other concreting has been finished and the concrete in the joints has gained strength.

For all loads acting after the interaction ability, the web of the slab is affected by an additional transverse shear force component, due to the longitudinal shear flow of the composite beam, which balances the changes in the normal stress resultants in the direction of the beam axis. The shear flow of the composite action must be transferred through the web sections of the hollow-core units (Fig. 1) and thus the stress state of the webs of the slab is governed by two shear stresses, one due to the shear force of the slab and the other due to the shear flow of the beam. As both of these tend to increase the tensile stresses in the web, the failure condition is met at an earlier stage of loading than would be the case if the slabs were supported on non-flexible surfaces.

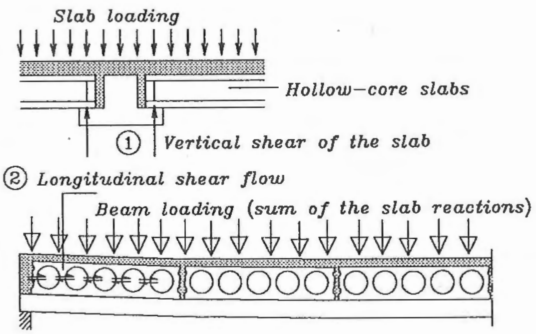

Fig. 1 Shear forces acting on hollow-core sections: 1) vertical shear due to slab loading, 2) longitudinal shear flow due to composite beam behaviour

It is not the purpose of this paper to consider the theory of composite beams, but it can be stated that composite behaviour always ensues when at least one interface can be found in the cross-section of the beam where the strains of the interacting parts are compatible (i.e. they are equal or the difference is known and is smaller than that caused by free slip).

FAILURE CONDITION OF CONCRETE UNDER MULTIAXIAL STRESS STATE

The failure of concrete is governed by the state of the stresses in it, one of the most versatile failure functions being that developed by N.S. Ottosen /1/. This is expressed in terms of invariants of the stress state, and thus the invariants must be determined from the principal stresses or directly by means of stress tensors. The normal stress tensor yields the invariants $I_{1\sigma}$, $I_{2\sigma}$, $I_{3\sigma}$ and the deviatoric tensor yields the invariants $J_{2\sigma}$, $J_{3\sigma}$ /2/.

The failure function /2/ is normally written as

$$F(\sigma_{ij}) = AJ_{2\sigma}/f_c^2 + \lambda\sqrt{J_{2\sigma}}/\left|f_c\right| + BI_{1\sigma}/\left|f_c\right| - 1 \qquad (1)$$

where the function λ depends on the deviatoric angle θ (for the essentials of octahedral stress theory, see /2/).

$$\lambda = K_1 \cos\{\frac{1}{3} \text{arc} \cos(K_2 \cos 3\theta)\} \text{ with } \cos 3\theta \geq 0 \tag{2a}$$

$$\lambda = K_1 \cos\{\frac{\pi}{3} - \frac{1}{3} \text{arc} \cos(-K_2 \cos 3\theta)\} \text{ with } \cos 3\theta \leq 0, \tag{2b}$$

where $\cos 3\theta = (3\sqrt{3}/2)(J_{3\sigma}/\sqrt{J_{2\sigma}^3})$. (2c)

A, B, K_1, K_2 are parameters that are calibrated so that the following four failure states satisfy the criterion:

- uniaxial compressive strength $|f_c|$,
- uniaxial tensile strength given by the ratio $r_f = f_{ct}/|f_c|$,
- biaxial compressive strength $|f_{2c}|$, when two equal components σ_2, σ_3 act on a plane stress state,
- a failure state at the compressive meridian $\sigma_{oct}/|f_c|$, $\tau_{oct}/|f_c|$.

The last condition involves the invariants σ_{oct} and τ_{oct}

$$\sigma_{oct} = I_{1\sigma}/3, \quad \tau_{oct} = \sqrt{2J_{2\sigma}/3}. \tag{4}$$

Tensile stresses are considered positive and if the principal stresses are applied, they are always arranged so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$ holds good arithmetically.

Five sets of parameters can be readily applied to the failure function. These are based upon the experimental data found in references /1/, /3/, /4/, /5/ and /6/ (the parameters based on /5/ and /6/ are also summarised in reference /2/).

Table 1. Parameters of the failure function of Ottosen

f_{ct}/f_c	A	B	K_1	K_2	
0,087	1,2259	3,3699	11,9298	0,9914	/1,3,4/
0,121	0,8653	2,6322	9,6653	0,9801	/1,3,4/
0,145	0,6252	2,1386	8,1620	0,9647	/1,3,4/
0,121	2,1868	2,8460	9,1854	0,9962	/5/
0,121	1,4426	2,7148	9,4710	0,9900	/6/

DETERMINATION OF THE INVARIANTS

We will now discuss the values of the invariants of the stress state in the case when two shear stresses σ_{13}, σ_{23} and one normal stress σ_{11} are applied to the concrete (fig. 2). Following the eigenvalue procedure for the dermination of the principal stresses σ_1, σ_2, σ_3, $I_{1\sigma}$ is found to be equal to σ_{11}. Now, as $\sigma_{oct} = \sigma_{11}/3$, the deviatoric stress state, in which the diagonal components s_{ii} are

$$
\begin{aligned}
s_{11} &= \sigma_{11} - \sigma_{oct} = 2\sigma_{11}/3, \\
s_{22} &= - \sigma_{oct}, \\
s_{33} &= - \sigma_{oct}
\end{aligned}
\tag{5}
$$

yields the invariants $J_{2\sigma}$, $J_{3\sigma}$.

$$J_{2\sigma} = \sigma_{11}^2/3 + \sigma_{13}^2 + \sigma_{32}^2, \tag{6a}$$

$$J_{3\sigma} = 2\sigma_{11}^3/27 - 2\sigma_{11}\sigma_{32}^2/3 + \sigma_{11}\sigma_{13}^2/3. \tag{6b}$$

When the failure condition is applied, it is relevant to use relative values for the stress components, proportioned to the tensile strength of concrete, f_{ct}

$$
\begin{aligned}
\eta_P &= \sigma_{11}/f_{ct}, \\
\eta_{hc} &= \sigma_{13}/f_{ct}, \\
\eta_{vl} &= \sigma_{32}/f_{ct}.
\end{aligned}
\tag{7}
$$

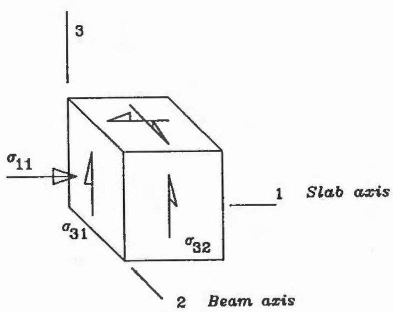

Fig. 2 Stress components for failure consideration

It must be noted that the normal stress σ_{11} is taken to be negative when compression is considered. For convenience, the following abbreviations are used when applying the failure function

$$\begin{aligned}
F_1 &= \sqrt{J_{2\sigma}}/f_c = r_f\sqrt{\eta_{hc}^2 + \eta_{vl}^2 + \eta_P^2/3}, \\
F_2 &= J_{2\sigma}^{3/2}/f_{ct}^3 = (\eta_{hc}^2 + \eta_{vl}^2 + \eta_P^2/3)^{3/2}, \qquad (8) \\
F_3 &= J_{3\sigma}/f_{ct}^3 = 2\eta_P^3/27 - 2\eta_P\eta_{vl}^2 + \eta_P\eta_{hc}^2.
\end{aligned}$$

These allow the failure function to be written as

$$F(\sigma_{ij}) = AF_1^2 + \lambda F_1 + B\, r_f\, \eta_P - 1, \qquad (9a)$$

where λ is the same as in equation (1) and the angle θ is calculated briefly from

$$\cos 3\theta = (3\sqrt{3}/2)F_3/F_2. \qquad (9b)$$

A short code for a computer program was written for use with equations (7) ... (9). While σ_{11} (or η_P) was kept constant, different sets of η_{hc}, η_{vl} that satisfy the failure condition $F(\sigma_{ij}) = 0$ were studied. The results are shown in the diagrams of Fig 3, for which the following data were used:

$r_f = 0{,}074$ (= 1/13,5)
parameters A, B, K_1, K_2 according to Table 1, row 5.

DISCUSSION

As can be easily seen from Table 1, the parameters applied in Fig. 3 are not 'exactly valid', because the ratio of the uniaxial strengths r_f does not belong to any of the rows of Table 1. It is not possible to obtain the parameters connected with $r_f = 0{,}074$ easily, because the exact points required in the failure condition (i.e. the biaxial strengths) are not obtainable using simple test equipment.

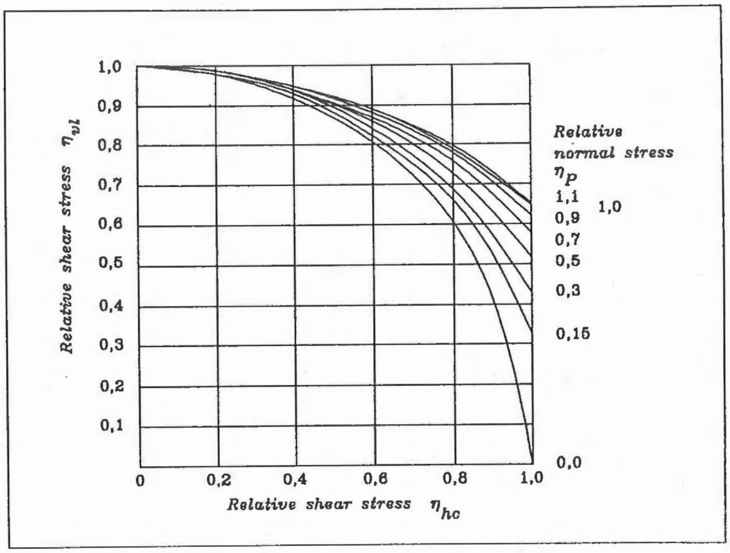

Fig. 3 Shear failure curves for different values of the normal stress σ_{11}

At boundaries, where $\eta_{vl} = 0$ or $\eta_{hc} = 0$ the calculation repeatedly yields values $\eta_{hc} = 1{,}32$, when $\eta_{vl} = 0$, or vice versa. Consequently the curves in Fig. 3 were scaled with respect to the maximum stresses of the boundaries. All the time the compressive normal stress η_P is non-zero, it must be noted that it is correct to obtain values of $\eta_{vl} > 1$ or $\eta_{hc} > 1$ if only one shear stress is applied. In any case, in order to reach a more universal and conservative solution, it is better to scale all the values to give strengths $\eta \leq 1$.

The innermost curve in Fig. 3 also deserves special consideration. For this diagram, the normal stress $\eta_P = 0$, and thus the curve must be symmetric with respect to a line drawn from origin with an inclination of 45 degrees. It is seen that this condition is satisfactorily met. The outermost curve represents a failure state when there is considerable compression applied together with the shears. Because the curves for $\eta_P = 1$ and $\eta_P = 1{,}1$ are quite close to each other, it can be assumed that the outermost curve approximately represents the failure envelope for all stress cases $\eta_P > 1{,}1$.

The failure curves are straightforward to use. Failure in pretensioned hollow-core slabs takes place in a region where there is always some prestress present. For a good approximation, it is assumed that an average of η_P = 0,25 .. 0,3 gives a sound, conservative basis for any calculation, although it has not been possible to check these considerations against representative test data, and only a few basic checks have been made. Further verification and any calibration required will follow as soon as a satisfactory amount of test data can be collected.

ACKNOWLEDGEMENTS

Special thanks are expressed to DELTATEK OY who kindly forwarded their test data concerning the phenomenon discussed and made it possible to make some preliminary checks concerning the theory.

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