INVESTIGATION OF REVERSE WHIRL OF A FLEXIBLE ROTOR

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In this paper experimental investigations have been carried out to study the phenomenon of reverse whirl, occuring under certain conditions after a contact of a rotating shaft in a stator. After triggering reverse whirl shaft and stator vibrate as a joined system with frequencies higher than the rotating frequency of the shaft. By means of simple mechanical models the measured phenomena are explained theoretically and conditions of triggering reverse whirl and perimissible speed intervals of the shaft are discussed. The main result is, that it is impossible to pass any natural frequency of the contacting rotor-stator system, excited by the reverse whirl frequency.

1. INTRODUCTION

High-speed elastic rotors, running in a stator or a case with a small gap are endangered to come in contact to the stator. Usually the contact leads to synchronous rub of the rotor in the stator.

Under certain conditions, to be discussed in this paper too, reverse whirl occurs, i. e. a rolling of the shaft along the inner surface of the stator in the opposite direction of the shaft rotation. If the clearance between shaft and stator is smaller than the diameter of the shaft, the whirl frequency is higher than the rotating frequency of the shaft. After triggering reverse whirl at a certain minimum speed the whirl frequency is stable and increases with increasing shaft speed. Approaching natural frequencies of the joined rotor-stator system further increasing speed leads to a

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1) Lecture at TKK Helsinki, Sept. 1990

superposition of slip effects so that the whirl frequency stays constant in spite of increasing the shaft speed. This effect is connected with strongly increasing amplitudes of the vibrating system. The paper theoretically justifies the behaviour of the system and proves the correctness of the theoretical formulations by test runs with a special shaft configuration.

2. EXPERIMENTAL INVESTIGATIONS

2.1. THE TEST RIG

The testing equipment consisted of three major components:

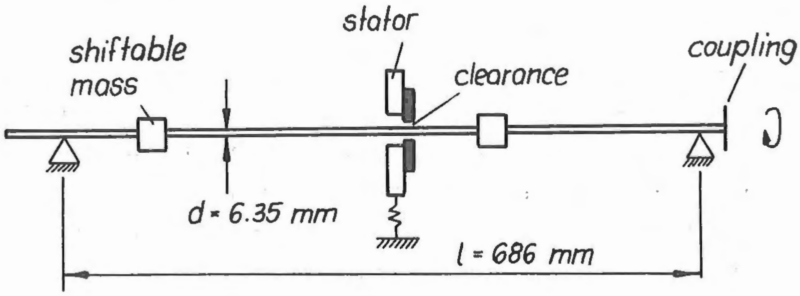


Fig. 1. Scheme of the test rig

1) flexible shaft, 6.35 mm in diameter and 890 mm in length with two shiftable masses (fig. 1). The shaft is driven by a speed-controlled electric motor with a maximum speed more than 2500 r.p.m., both connected by a flexible coupling.
2) An elastically supported brass stator near the middle of the shaft, carrying a plate with a hole, surrounding the shaft centrically with a small gap. The stiffness of the stator k_s could be adjusted by changing the length of 4 thin aluminium rods as shown in fig. 2. Additionally the foundation assembly could be mounted in different axial positions.
3) The instrumentation for measurement of the rotor and stator deflections, of the shaft speed and a frequency analyser.

This testing equipment practically allows the investigation of an arbitrary number of shaft-stator-combinations. The choice of combinations was limited by deliberations about the influence of the essential parameters:

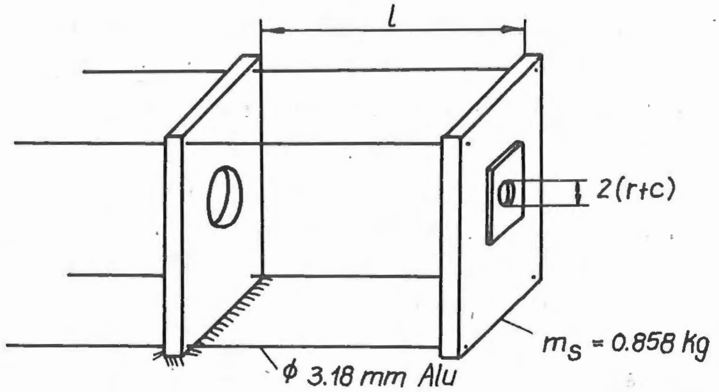


Fig. 2 Stator design with adjustable stiffness

The critial speed of the shaft, the stiffness of the stator and the clearance between shaft and stator.

In the main four test series were carried out:

1) Strongly asymmetric shaft (fixed position of the shiftable masses at one side of the stator), variable stator stiffness, radial clearance c = 0.87 mm.
2) Strongly asymmetric shaft as before, variable stator stiffness, c = 1.19 mm.
3) Variable shaft (position and number of masses) fixed stator stiffness k_s = 3.3 N/mm, c = 1.19 mm.
4) Symmetric shaft, masses in the middle close to the stator, k_s = 31.4 N/mm, c = 1.19 mm

In each case the stator position was in the middle of the shaft.

A comprehensive report about the first test series is given in /1/. The essential results of the test runs are presented and discussed in the following.

2.2. TEST RESULTS (1)

Fig. 3 shows the shaft configuration and the frequencies measured at the shaft near the stator position versus the rotating frequency f_r of the shaft. The rotor at first shows the expected behaviour. The measured vibrations are unbalance excited. In the interval around the critical speed the

rotor contacts the stator in a synchronous rub (fig. 4a). At higher speeds the shaft is running smoothly again without any contact to the stator. Such kind of "normal" behaviour corresponds to the straight line $f = f_r$ in fig. 3.
Beginning with a certain frequency between 6 and 8 Hz however it is possible by hitting the shaft with a wooden stick

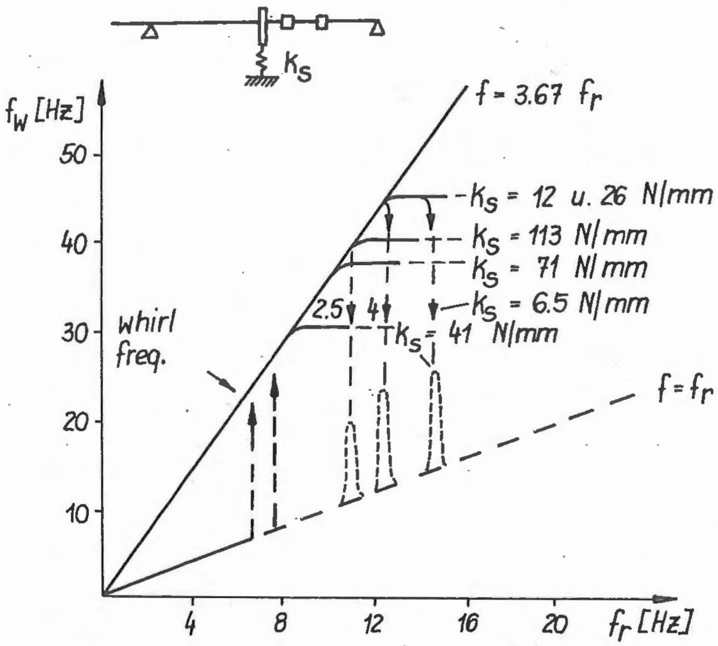


Fig. 3 Test results (1). Frequencies of reverse whirl versus rotational frequency of the shaft, r/c = 3.67

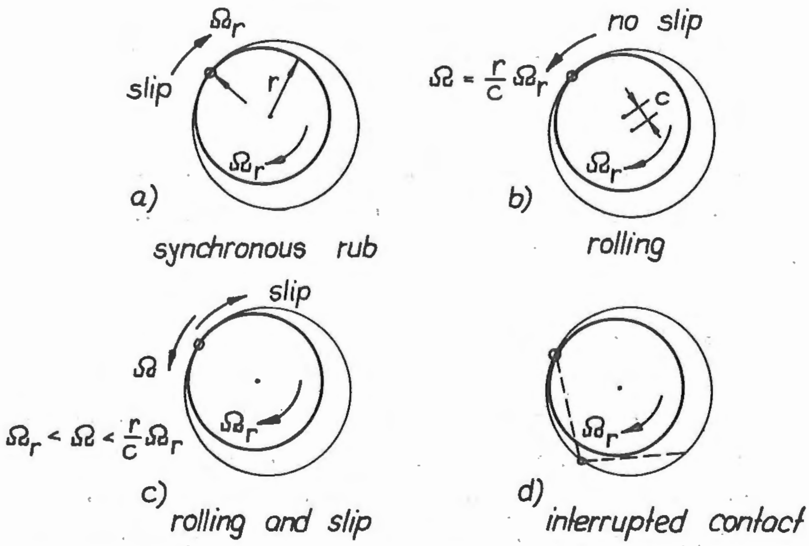


Fig. 4 Possible behaviour of the shaft after contacting the stator

to trigger a higher frequency, the shaft and the stator are vibrating with. This whirl frequency is exactly $f_w = f_r r/c$ with r-the radius of the shaft. It increases resp. decreases with the rotor speed along the straight line $f = 3.67\ f_r$. The shaft is now rolling along the inner surface of the stator in the opposite direction of the shaft rotation. With a further increasing of the rotor speed the whirl frequency doesn't increase proportionally and approaches a constant value (different branches in fig. 3).

The limiting frequency depends on the stator stiffness k_s, but a rule could not be clearly seen. Approaching the limiting frequency the deflections of shaft and stator increase resonantly. It could be supposed that near the limiting frequency the reverse whirl (fig. 4b) is superposed by slip (fig. 4c).

2.3. DELIBERATIONS OF MODELING

To clarify the measured phenomena simple mechanical models of the test rig were developped and the parameters of the models estimated. Rotor and stator were idealised as an undamped single-degree-of-freedom system each (fig. 5a).

A third model describing the behaviour with reverse whirl joins the submodels to one SDOF-System (fig. 5b). The natural frequencies of the three models were determined by a free decay test. In order to determine 4 parameters k_r k_s, m_r, m_s, one parameter of each subsystem in neccessary to know. Therefore the brass stator was weighed, $m_s = 0.858$ kg. The rotor stiffness could be calculated with the shaft diameter and length:

$$k_r = 48\ EI/l^3 = 249\ \text{N/mm}.$$

With $k_r = m_r \omega^2_o$ and the measured natural frequency $f_o = 16.4$ Hz follows a reduced rotor mass $m_r = 0.235$ kg. Table 1 contains the stator stiffness $k_s = m_s\ (2\pi f_s)^2$ (3rd column) corresponding to eight different lengths of the stator rods.

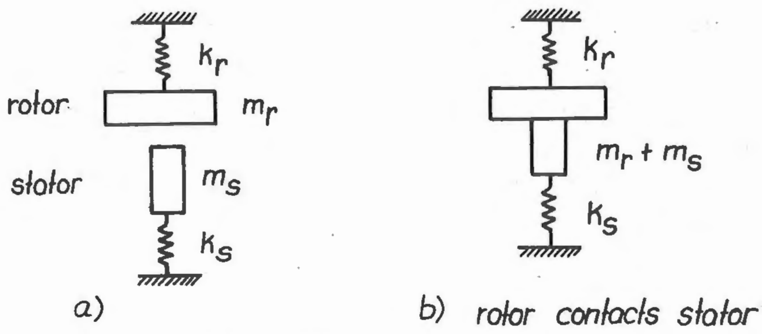


Fig. 5 Simple models of the test rig

L(mm)	f_s(Hz) measured	k_s(N/mm)	$f_{1\Sigma}$(Hz) measured	$f_{1\Sigma}$(Hz) calculat.
177.8	8.65	2.5	10.9	10.9
152.4	10.95	4.1	12.3	12.3
127.0	14.00	6.7	14.3	14.6
101.6	18.85	12.1	18.2	18.4
76.2	27.4	25.5	25.2	25.5
63.5	34.6	40.6	31.6	31.7
50.8	45.8	71.2	41.3	41.3
38.1	57.7	113.0	48.0	51.8

Table 1 Parameters of the models with different stator stiffness

These models could be verified by measuring the natural frequencies of the system (rotor and stator in contact) and comparing them with theoretical values according to

$$f_{1\Sigma} = \sqrt{\frac{k_r + k_s}{m_r + m_s}} \Big/ 2\pi$$

The columns 4 and 5 in table 1 confirm a very good coincidence. This simple models already allow to systematize the measured results of the test runs.

1. The lowest whirl frequency is always higher than the lowest natural frequency f_o of the rotor.

2. The upper limiting frequency of reverse whirl is always lower than the lowest natural frequency of the joined system (rotor + stator), if this is higher than f_o (for k_s = 41, 71 and 113 N/mm, fig. 6).
3. In the case of very soft stator supports, between 2.5 and 26 N/mm, it is not possible to trigger reverse whirl underneath the lowest natural frequency of the joined system. Here either the dominating frequency of the stator (and rotor) switches over to the rotating frequency of the rotor (k_s = 2.5 ... 6.5 N/mm) or the reverse whirl frequency is limited by the second natural frequency of the joined system, which was measured near $f_{2\Sigma}$ = 48 Hz at the non-rotating shaft. This natural frequency was nearly independent of the stator stiffness k_s.

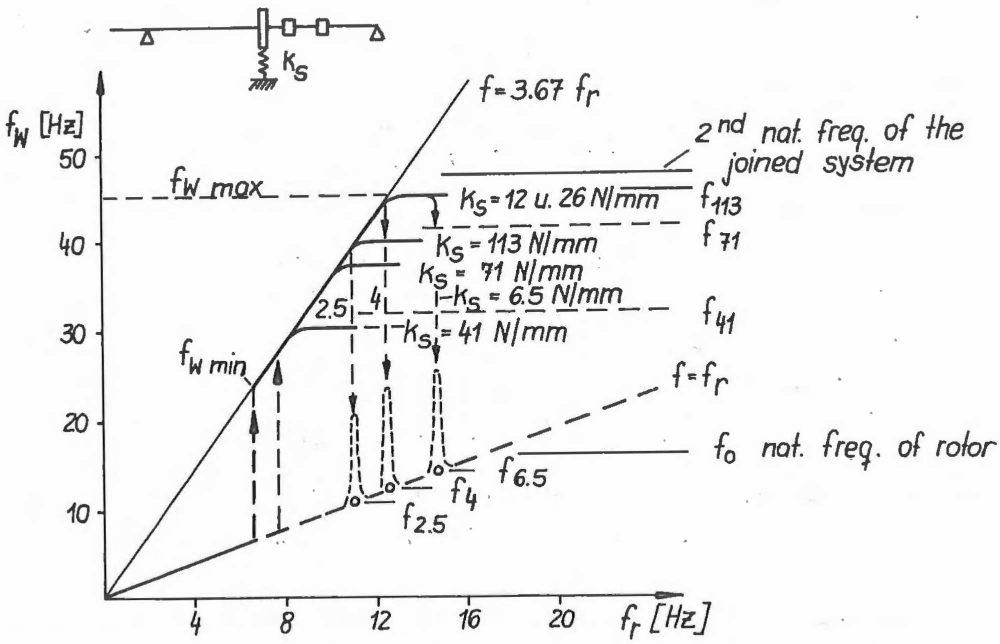


Fig. 6 Test results of fig. 3 with natural frequencies of the joined system

2.4. TEST RESULTS (2)

Based on the test results (1) and their explanation further test runs were planned and carried out. The diameter of the stator drill hole was enlarged to c = 1.19 mm. The other parameters were not changed. The results in principle showed the same behaviour (fig. 7). In consequence of a smaller r/c now higher rotor speeds at the same whirl frequency were possible. A new result was that the reverse whirl stays stable during running through the unbalance excited natural frequency of the joined system. The reverse whirl in this case is superposed by the resonance vibrations with the rotating frequency. This could be seen clearly from the amplitude spectrum of the stator or rotor vibrations. Above the resonances the reverse whirl dominated again. The upper limiting frequency was about 45 Hz. Approaching this frequency a deflection of the shaft according to te second eigenmode could be clearly seen.

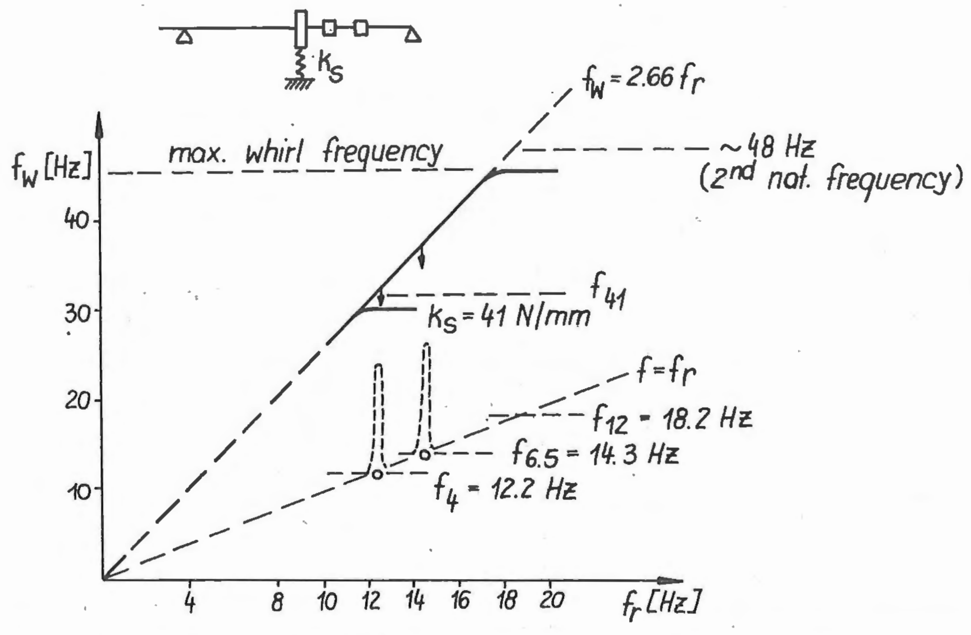


Fig. 7 Test results (2). Reverse whirl frequencies versus rotational frequency of the shaft, r/c = 2.66

3. THEORETICAL INVESTIGATIONS

The test results are to be explained theoretically as follows. Firstly: Upper and lower limiting frequency of reverse whirl. Secondly: conditions of triggering reverse whirl.

3.1. LIMITING FREQUENCIES OF REVERSE WHIRL

By way of determination of the limiting frequencies the kind of excitation of the joined system was studied (fig. 8).
Fig 8a) shows the forces acting at rotor and stator, fig 8b) the forces at the rotor.

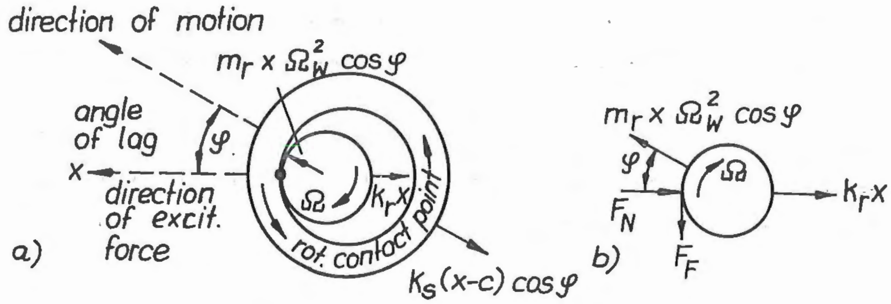


Fig. 8 Forces at the joined system (a) and at the shaft (b)

The exciting force is transmitted from the rotor to the stator. The rolling shaft in the stator demands a positive normal force F_N (fig. 8b). The approach of the whirl frequency to a resonance of the joined system causes a phase shift between exciting force and deflection. From fig 8a) follows for the normal force

$$F_N = (m_r \Omega_W^2 \cos^2\varphi - k_r)x \qquad (1)$$

That means, the normal force can be zero at phase angles smaller than $\pi/2$. The energy of motion of the stator. coming from the driving moment of the motor, is to be transmitted by the friction force F_F between shaft and stator. Another condition is

$$\mu F_N \geq F_F > 0$$

The consequence is, that the rolling of the shaft already for small phase angles can be superposed by a slip, $\mu F_N = F_F$. From this reason slip already occurs in a certain distance to the critical whirl frequency. In the limiting case the whirl frequency reaches the resonance frequency of the joined system. In consequence of the very small damping this case could not be run.

The conclusion is, that resonances of the joined system, excited by reverse whirl cannot be passed.

If the lowest natural frequency is smaller than the lower limiting frequency of reverse whirl (fig. 6, k = 2.5 to 26 N/mm) then the frequency of reverse whirl is limited by the second natural frequency of the joined system.

The lowest whirl frequency can be estimated as well by equation (1). At frequencies far below the resonance yields $\varphi \to 0$. In this case follows

$$\Omega_{min} > \sqrt{k_r/m_r} = \omega_o$$

with the natural angular frequency ω_o of the rotor.

The frequency of reverse whirl cannot be lower than the natural frequency of the rotor. With the two different values of r/c follow the minimum rotor speeds at which a triggering of reverse whirl is possible. Referring to a natural frequency f_o = 16 Hz yields

$$f_{rmin} > 4.4 \text{ Hz} \quad \text{at } r/c = 3.67$$
$$f_{rmin} > 6 \text{ Hz} \quad \text{at } r/c = 2.66$$

3.2. TRIGGERING OF REVERSE WHIRL

ZHANG /2/ developed a trigger condition of reverse whirl from theoretical deliberations (fig. 9). In the moment before the contact a point on the shaft surface has a peripheral speed v = (r+c). The condition of ZHANG demands a velocity v* of the expected contact point in the opposite direction of v (fig. 9b)

$$v^* = -\omega^*(r+c)$$

with

$$\omega^* > \omega_o \left[(\vartheta/\mu) + \sqrt{(\vartheta/\mu)^2+1}\right] > \omega_o$$

$\vartheta = b/2\sqrt{m_r k_r}$ is the damping ratio and $\omega_o = \sqrt{k_r/m_r}$ the natural frequency of the rotor. As ϑ is very small, yields $\omega^* \approx \omega_o$.
With f_o = 16 Hz follows

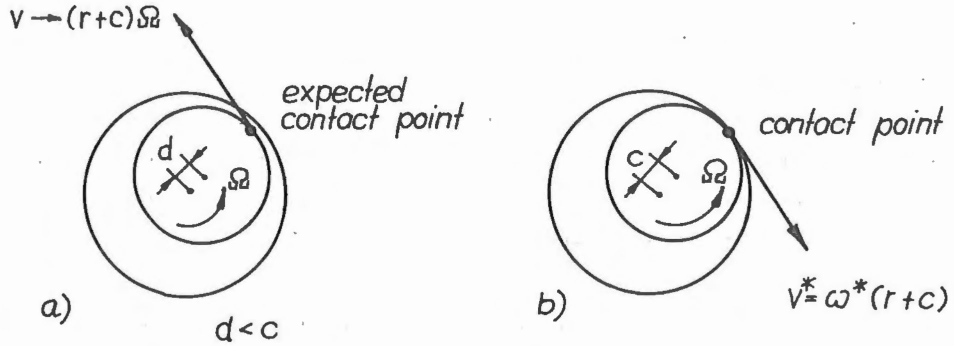


Fig. 9 Reverse whirl trigger condition.
a) before contact b) in the moment of contact

$$v^* \approx \omega^*(r+c) = 16 \cdot 2\pi \cdot 3.97 \text{ mm/s} \approx 400 \text{ mm/s}$$

The reverse whirl could be triggered in each case at frequencies smaller 10 Hz, i. e. the maximum peripheral speed before the contact is

$$v < 10 \cdot 2\pi \cdot 3.97 = 250 \text{ mm/s}$$

The shaft consequently must be accelerated by the triggering hit up to a velocity of v + v* = 650 mm/s. By way of estimation of the neccessary force a constant (mean) value of the force during the blow is supposed. In this case the radial velocity of the center of the shaft increases linearly with a mean value of 325 mm/s. If the clearance is c = 0.8 mm the acceleration takes a time of t = 0.8/325 = 2.46 ms. The acceleration itself is

$$a = 2c/t^2 = 1.6 \text{ mm}/2.46\ 10^{-6} \text{ s} = 264 \text{ mm/s}$$

The mean value of the force on the reduced mass m_r of the shaft during the time t finally is

$$\bar{F} = m_r\ a = 0.235 \text{ kg } 264 \text{ mm/s} = 62 \text{ N}$$

A hit to the shaft with a wooden stick lightly produces this relativ small force. A conclusion is, that narrow clearances need greater forces to trigger reverse whirl.

4. EXPERIMENTAL CHECKING OF THE THEORETICAL RESULTS BY MEANS OF DIFFERENT SHAFT CONFIGURATIONS. COMPREHENSIVE SOLUTION OF THE PROBLEM

Further tests should clarify whether it is but possible to pass a resonance of the joined system or to find out a sub-interval between the first and second natural frequency which allows a stable (safe) operation with reverse whirl.

Therefore the shaft configuration had to be chosen so, that the lowest natural frequency $f_{1\Sigma}$ of the joined system is as low as possible and the second $f_{2\Sigma}$ as high as possible. As long as the stator is situated in the middle of the rotor, this condition is fulfilled by a rotor with the shiftable masses very near to the middle of the shaft. Table 2 gives a survey about the natural frequencies of different configurations with a soft stator support. As the second natural frequency was not higher than 59 Hz, the overhanging part of the shaft was cut off (three rows at the bottom of table 2).

rotor configuration	f_0	$f_{1\Sigma}$	$f_{2\Sigma}$
	12,2	10.5	57
	15.6	12.1	57
	24.8	12.3	59
	25.0	13.3	59
	17.2	12.7	54
	12.2	10.5	89
	14.9	11.3	49
	13.4	10.9	56

Table 2 Natural frequencies of different rotor configurations, k_s = 3.3 N/mm

4.1. TEST RESULTS (3) AND (4)

Test runs with the three rotor configurations mentioned above didn't give any new results (fig. 10). In each case

$f_{1\Sigma}$ was the upper limiting frequency of reverse whirl. For this reason finally for the configuration with $f_{2\Sigma}$ = 89 Hz a stator stiffness was calculated which allowed to trigger reverse whirl below the first natural frequency $f_{1\Sigma}$ of the joined system. Additionally should be valid

$$f_{1\Sigma} < f_{2\Sigma} \cdot r/c$$

The corresponding stiffnes was 31.4 N/mm, the natural frequency $f_{1\Sigma}$ = 27 Hz respectively. The results of the test runs (3) are presented in fig. 11. At first the rotor was run up to the minimum speed which allowed triggering reverse whirl (about 7 Hz). At about 10 Hz rotational frequency slip occured and the vibrating frequency of the joined system approached 27 Hz, connected with very large deflections.

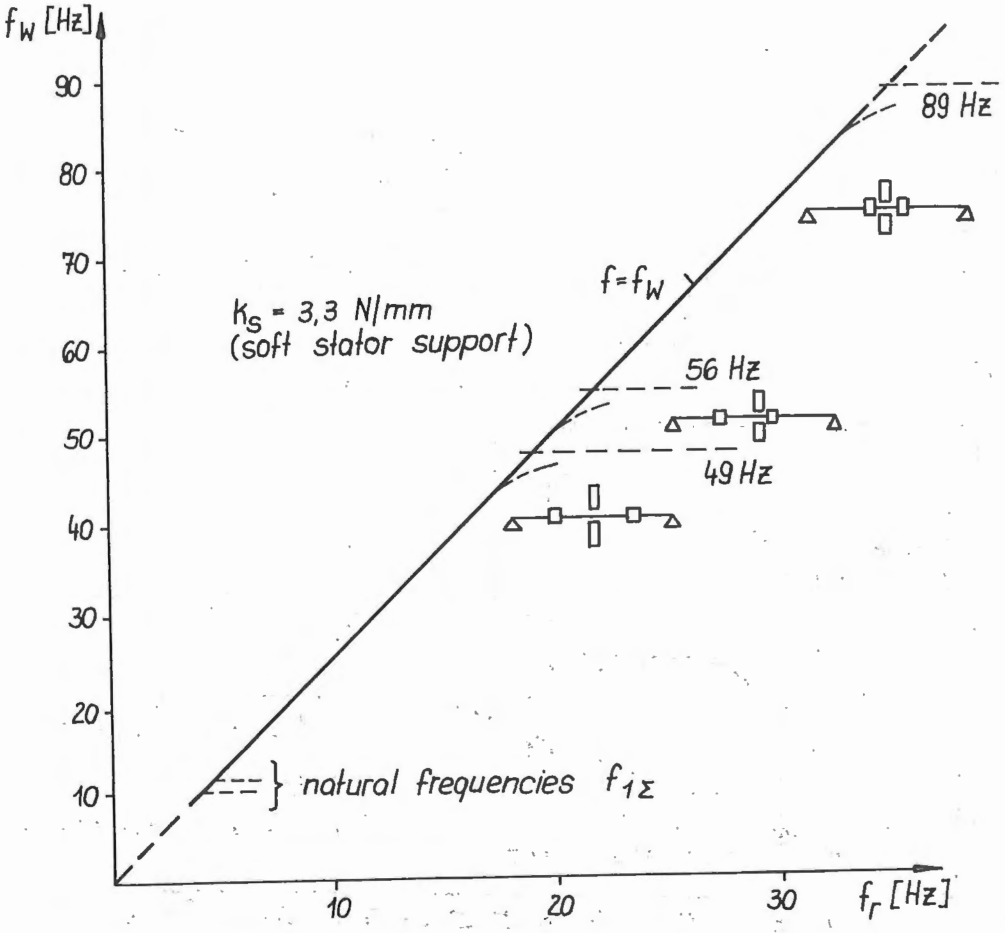


Fig. 10 Test results 3). Reverse whirl frequencies of different rotor configurations, k_s = 3.3 N/mm

If reverse whirl is not triggered, then the rotor passes the critical frequency 12.2 Hz in the usual way. After synchronous rub (fig. 4a) selfcentering of the shaft leads to a smooth operation. If the rotor is hit in the interval between 11 and 18 Hz then the joined system immediately vibrates with 27 Hz and cannot leave this frequency except by decreasing the rotational speed.

Rotational frequencies f_r higher than 19 Hz again allowed to trigger reverse whirl frequencies $f_w = f_r r/c > 47$ Hz.

It is supposed, that at this speeds already the second mode of the system predominates. Further increasing of the rotational frequency is connected with the effects measured before: At 27 Hz the natural frequency $f_{1\Sigma}$ of the joined

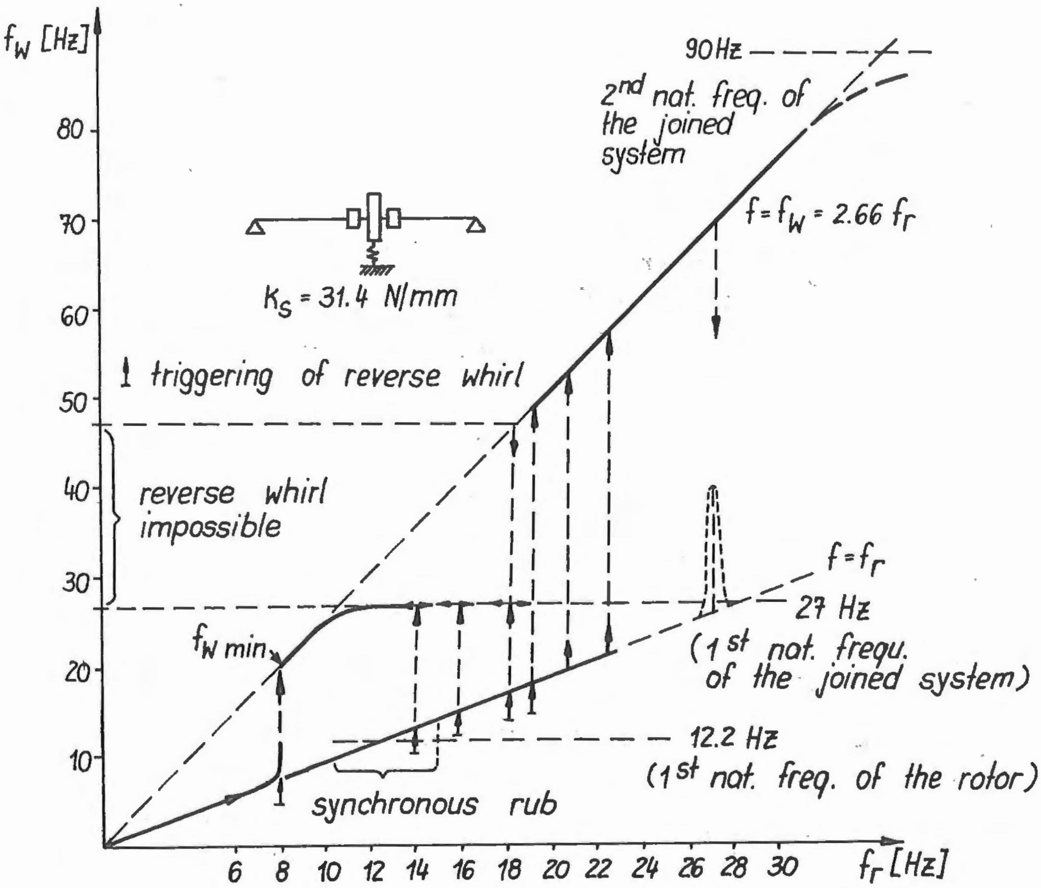
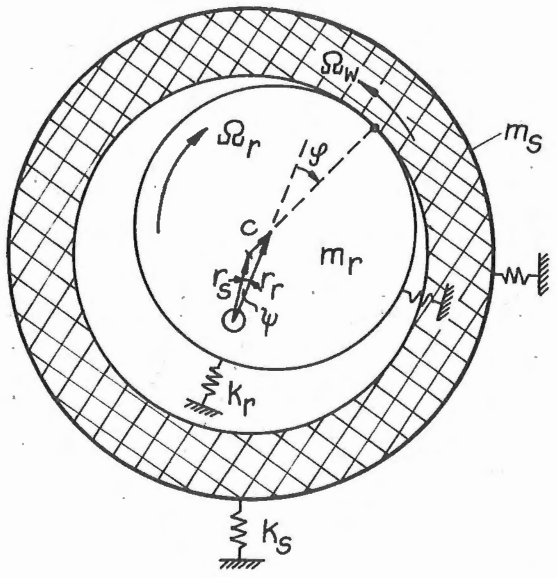


Fig. 11 Test results (4). Reverse whirl in two frequency intervals

system is excited by the unbalance of the rotor and this frequency superposes reverse whirl. If the whirl frequency passes over 80 Hz, additional slip occurs, which is to be expected approaching the second natural frequency $f_{2\Sigma}$ at 89 Hz. Further facts of interest are, that at rotational frequencies over 27 Hz the reverse whirl occured without triggering by a hit and that with decreasing rotor speed of the whirling system towards the interval 11 - 18 Hz the whirl frequency suddenly changed from 47 to 27 Hz connected with dangerous amplitudes.

4.2. INFLUENCE OF FRICTION COEFFICIENT

At least the friction coefficient μ was changed by dribbling oil on the whirling shaft. This measure didn't change the vibrations. Triggering reverse whirl now seemed to be a little more complicated. But this might have been a subjective impression. A detectable influence of the friction coefficient on the described behaviour could not be proved. This behaviour can be derived from further theoretical deliberations.



$$r_r = r_S \exp(j\psi) + c \exp(-j\varphi)$$

Fig. 12 Model of the joined system

4.3. THEORETICAL TREATMENT OF THE PROBLEM

A theoretical explanation of the steady state behaviour of the system in contact is given in [5]. The underlying one-degree-of-freedom model takes into account a small damping. Starting from the model in fig. 12 the acting forces at rotor and stator are drawn in fig. 13. ϑ_r and ϑ_s are the damping rates of rotor and stator and $\omega_s = \sqrt{k_s/m_s}$ is the natural frequency of the stator.

The equilibrium of forces yields in complex notation

$$m_r r_r (\Omega_W{}^2 + j\vartheta_r \omega_o \Omega_W - \omega_o{}^2 = (F_N + jF_r)\exp(-j\varphi)$$

$$m_s r_s (\Omega_W{}^2 + j\vartheta_s \omega_s \Omega_W - \quad {}_s{}^2) = -(F_N + jF_r)\exp(-j\varphi - j\psi)$$

With the geometric relation

$$r_r = r_s \exp(j\psi) + c \exp(-j\varphi)$$

we have three vector equations for six unknowns r_r, r_s, φ, ψ, F_T and F_N, which can be solved analytically [5], [6].

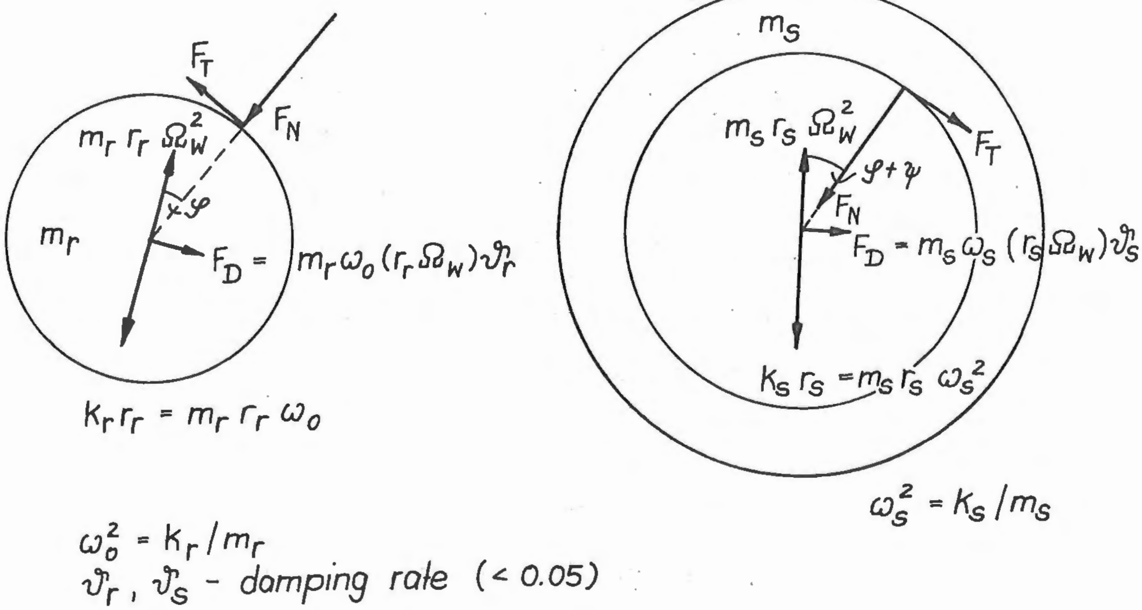


Fig. 13 Forces, acting on rotor and stator

Fig. 14, 15, 16 grafically show the results of a numerical example with m_r/m_s = 0.487, ω_o/ω_s = 0.366, ϑ_r = 0.02, ϑ_s = 0.01

These figures show exactly the measured behaviour: The frequency interval between $_r$ and is the interval of possible whirl, as the difference ψ - φ is very small or zero: The exciting force can be transmitted to the stator without slip.
The "impossible interval" in fig. 15 is characterised by theoretically negative normal forces, that means reverse whirl is impossible.

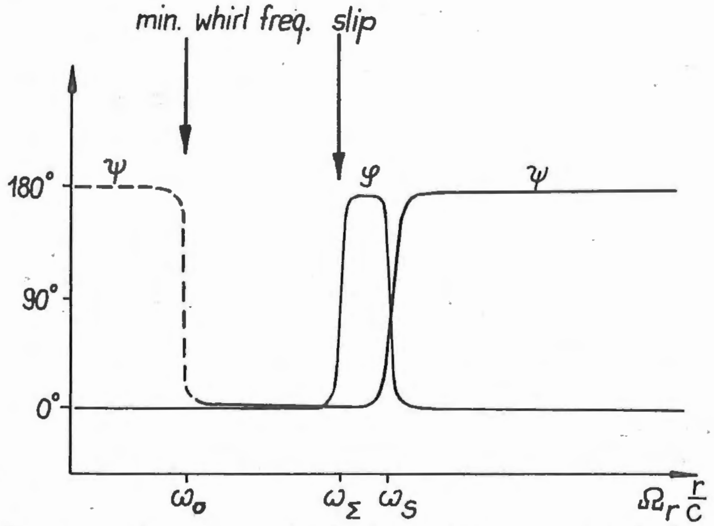


Fig. 14 Phase angles φ and ψ depending on whirl frequency

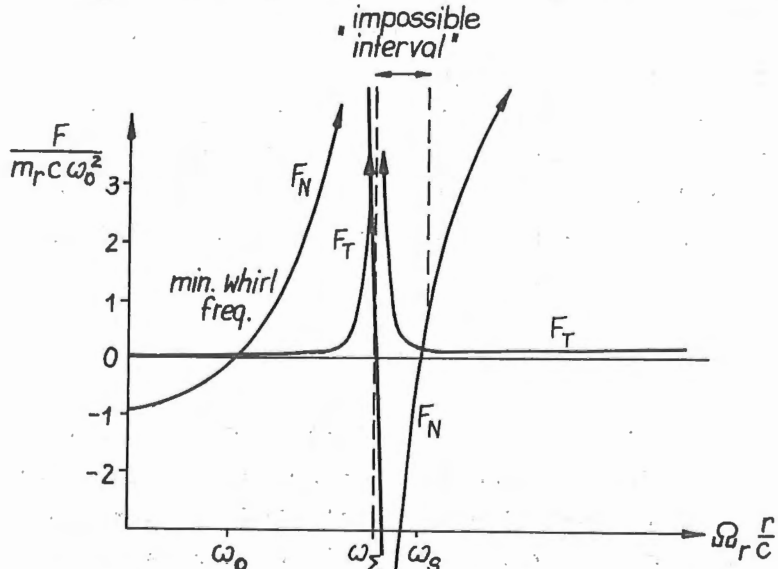


Fig.15 Normal and tangential forces versus whirl frequency

Fig. 16 explains, why the influence of friction coefficient is neglectible. The neccessary coefficient in the "whirl interval" between ω_0 and ω_Σ is so small, that a lubricated contact doesn't influence the general behaviour.

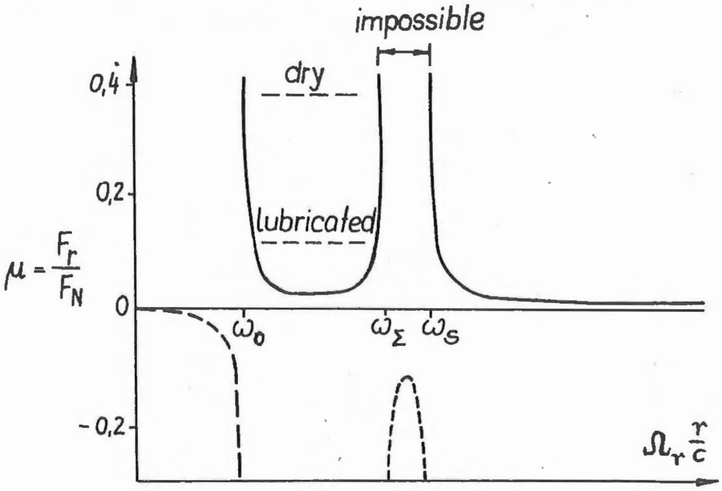


Fig. 16 Friction coefficient $\mu = F_T/F_N$ required to maintain steady reverse whirl

5. CONCLUSIONS

1. The effects in connection with reverse whirl of a rotor in a stator can be described by a simple theory.
2. It is impossible, to run through any resonance of the joined system, excited by reverse whirl, neither with increasing nor with decreasing speeds. In a broad frequency interval the system is permanently in resonance.
3. For real shafts with the possibility of reverse whirl inadmissible speed intervals must be definitely excluded.
4. Further investigations are neccessary to define the limiting values of r/c.

6. ACKNOWLEDGMENT

The author would like to thank Prof. S.H. Crandall, Ford Professor of Engineering at Massachusetts Institute of Technology, Cambridge USA who initiated to carry out the experimental investigations in his laboratories.

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