EXPERIMENTAL INVESTIGATION OF THE PLUNGER TYPE WAVE MAKER

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SUMMARY: This article describes the preliminary model test results of a plunger type wave maker conducted at the Ship Laboratory of the Technical Research Centre of Finland. The main goal of the research project was two-fold: First to study the wave making characteristics in a 130 m long towing tank with an aid of model techniques. Second to design a new wave-making device and effective absorbing dampening construction for a 40x40 m^2 wide testing basin.

INTRODUCTION

This paper presents the preliminary model test results of a plunger-type wave maker. The main aim of the study was to find out the main characteristics of an oscillating body having a triangle shaped cross-section. The tests included the measurements of the wave amplitudes for various wave maker strokes. In order to study the hydrodynamic forces acting on the body and the hydraulic actuator dimensions needed, the total force required to move the body was measured, too.

The wave maker study described in this paper was one part of the project to design a new wave maker for a 40 x 40 m^2 wide ice tank (former manoeuvring basin), thus enabling open water testing with or without drifting ice in waves. In the first phase, however, the main characteristics of 'laboratory waves' were studied by modelling the existing towing tank wave maker in the scale of 1 to 5. An other purpose was also to study the possibility to renew later the towing tank wave maker to achieve higher wave amplitudes.

Three different body cross-sections were tested using the wave angular frequency range of 1..8 rad/s in full scale ie. in the towing tank conditions. To avoid any reflections during the testing phase the duration of single measurement was arranged so that any secondary distortions could not disturb the measurement. This was achieved by having short measuring period in respect to the travelling time of the reflected wave from the end of the flow flume. All tests were carried out by using regular waves.

This work has been carried out in a co-operation with the Hydraulics Laboratory of the Helsinki University of Technology, which has offered the flow flume for the measurements. The developed model wave maker with the PC-microcomputer based control unit enables now, after this towing tank study, basic 2- dimensional hydrodynamic testing in a flow flume. Thus various coastal engineering problems like the stability of breakwaters etc. can be studied using suitable scaling dimensions.

THEORY OF A PLUNGER TYPE WAVEMAKER.

In laboratory test tanks, water surface waves are created by causing a forced oscillation of the water particles at one end of the tank. In most cases, this is done mechanically with different kinds of waveboards and plungers. One major principle in wavemaker design is to try to get the forced oscillation to match the natural water particle oscillation in a wave as well as possible. Usually wavemakers work satisfactorily at a limited, specific frequency range. At frequencies outside this range, horizontal accelerations at the wavemaker surface at various depths are inadequate, causing distortion of the wave shape.

The majority of wavemaker theories concern piston- or flap-type wavemakers /4/. Especially studies on vertically oscillating plungers seem rare. Besides there may be lesser need for theories of this kind one reason has been the difficulty to present a theory that would in some manner cover the variety of plunger shapes in use. Actually it has proved difficult even to find an analytical solution of the generated wave for a body of any shape. Ursell /9/, however, has analyzed waves generated by an oscillating circular cylinder and found that the generated wave height only depends on a dimensionless parameter kr, k being the wave number and r the radius. Wang /10/ used a similar reasoning when extending the theory to plungers of more or less triangular shape, using Lewi's conformal transformation. In Wang's study a two parameter transformation was used that does not yield exactly the triangular shapes intended.

Wang, too, found that the wave height only depended on the dimensionless parameter kd, where d is the breadth of the plunger at the water surface. The plunger geometry, Wang described by two parameters; the sectional area coefficient and the breadth at the water surface.

For prismatic plunger Galvin /3/ found an approximative solution for the wave generators in shallow water. He stated that the height of waves generated by displacement type wave generators equals approximately to $2\pi S/L$ times an appropriate dimension of the wave generator. The terms S and L are the stroke of the wave generator and the wave length, respectively. His assumptions, however, are valid for the ratio of water depth and wave length beeing less than 0.05.

Wu /11/, unlike Wang, took into account the effect of water depth. He used the boundary collocation method to solve the Laplace equation together with the combined free surface boundary condition, the kinematic boundary condition at the wavemaker and the bottom boundary condition. In this method the test channel is divided horizontally into a number of segments each containing one node point at each end. The nodes on the plunger side of the channel are required to satisfy an equation formed of the Laplace equation and the boundary conditions. The unknowns in this equation are found by simultaneously solving the linear combination of functions for the segments in the sense of the least square.

In the following the wave maker dynamics is described with an aid of the similarity of forced and dampened spring-mass-system (Fig. 1). However, only the basic equation of motion and some essential parameters are shown. The more detailed discussion of the body motion dynamics is presented in refs./2/ and /7/.

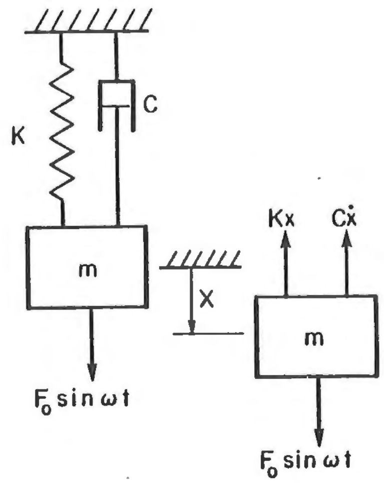


Fig. 1. The spring mass system.

The dynamics of a plunging type wave maker can be described by a dampened spring-mass system with a single degree of freedom in the direction of the imposed harmonic external force. According to Fig.1 we can consider a system with a mass, m and a spring constant, K. The mass is moved by an external force of amplitude F_O, and it is linearly damped with a damping coefficient, C. Assuming further the angular frequency of linear vertical movement beeing, ω, the equation of motion thus consists of an inertia force, a damping force and a restoring force term all resisting the external force. Thus the equation of motion can be described as

$$m\ddot{x} + C\dot{x} + Kx = Fo\sin\omega t \qquad (1)$$

In above equation the mass term includes both the mass of the body and the added mass. The added mass is the mass of water which when accelerated would produce an inertia force equal to the vertical resultant of all fluid pressures caused by the actual acceleration of water particles relative to the body.

The steady-state oscillation is generally represented by the following equation:

$$x = X\sin(\omega t - \alpha) \qquad (2)$$

where X is the amplitude of oscillation and the angle $(\omega t - \alpha)$ is the phase angle. Term α is the lagging phase angle between the motion and the external force.

Differentiating x in above equation with respect to time and substituting in eq. 1. we get:

$$-m\omega^2 X(\omega t - \alpha) + C\omega X\cos(\omega t - \alpha) + KX\sin(\omega t - \alpha) = F_o\sin\omega t \qquad (3)$$

The coefficients of $\sin\omega t$ and $\cos\omega t$ can be assumed to be zero (orthogonal to each other). Thus we get from eq. 3:

$$-m\omega^2 X\cos\alpha + C\omega X\sin\alpha + KX\cos\alpha = F_o \qquad (4)$$

$$m\omega^2 X \sin\alpha + C\omega X \cos\alpha - KX \sin\alpha = 0 \qquad (5)$$

Solving X and α the following formula presents the amplitude of oscillation and the phase angle of motion /2/:

$$X = \frac{F_o}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}} \qquad (6)$$

$$\tan\alpha = C\frac{\omega}{K - m\omega^2} \qquad (7)$$

From the general force-deflection equation of a spring-mass system,

$$F = KX_s \qquad (8)$$

we get for the deflection, $X_s = F_o/X$. So the magnification factor, X/X_S can be written as:

$$\frac{X}{X_s} = \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{2\zeta\frac{\omega}{\omega_n}\right\}^2}} \qquad (9)$$

For the phase angle we get:

$$\tan\alpha = \frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \qquad (10)$$

where

$$\omega_n = \sqrt{K/m} \qquad (11)$$

is the natural frequency of the system and

$$\zeta = C/Cc \qquad (12)$$

is the damping factor which states the ratio of the dampening to its critical damping value, C_C ($=2\sqrt{Km}$).

The solutions of equations above are shown graphically in Fig.2 for $0 \leq \zeta \leq 1.0$, where all curves approach the static deflection at very low frequencies. At higher frequencies the dampening effect is negligible but near the natural period, ie. near the resonance point the damping has a profound effect on the deflection.

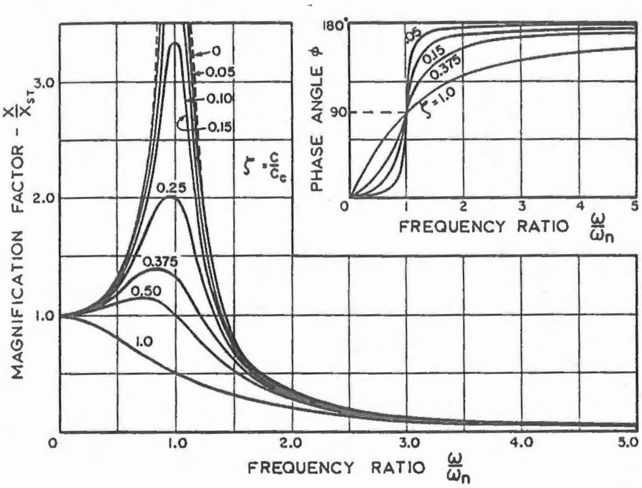


Fig. 2. Solutions for a forced linear spring-mass system /2/.

TEST PROCEDURE

The towing tank wave maker

The towing tank wave maker consists a wedge-shaped plunger which oscillates through a lever arm as shown in Fig. 3.

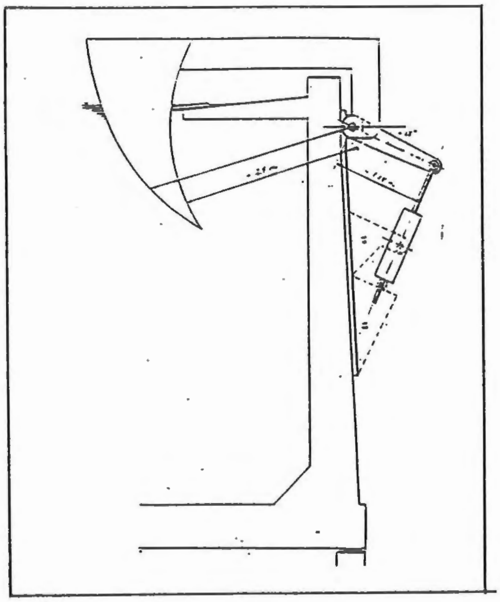


Fig. 3. A schematic representation of a towing tank wave maker.

The wave maker is powered by an electro-hydraulic servo actuator system which consists of a hydraulic cylinder for motion, dampening cylinders and pressure accumulators. The length of the lever arm is 2.5 m thus the radius of the oscillation line is 2.5 m round the bearing point.

The cross-section of an oscillating wedge is theoretically a triangle when the plunger is assumed to oscillate only vertically without any rotation. The existing body form has been derived from a triangle by obtaining the angular movement round the bearing point. So the basic wave maker theories developed by Biesel, Havelock and Ursell /1/, /9/ have been used when estimating the ratio of wave amplitude achieved and the stroke of the body.

The angular frequency range of the existing wave maker is abt. 1..8 rad/s. The maximum wave height observed is abt. 0.3 m which could be seen from Appendix 1. The data of Appendix 1 include also the model test results. The original full scale data are on the right hand side column of the appendix 1. These full scale data are based on the calibration measurements carried out after the installation of the existing towing tank wave maker.

Selected test criteria

Two basic goals were set for the tests: First to try to achieve a maximum full scale wave height of 0.5 m. Secondly to increase the wave heights especially in the case of long waves having angular frequency of 1..2 rad/s. The linearity of waves and the wave asymmetries were also studied. Thus the demand of 5 % for second order harmonic sine wave amplitude ratio was set.

In order to verify the performance of the model, the test was carried out as a scale model of a previously run full scale calibration test of the actual wave maker. Thus the distances from the wave maker to the three wave gauges corresponded to the distances in the calibration test. Also the same stroke amplitudes and angular frequencies were used, and one of the plunging bodies was made as a scale model of the original plunger.

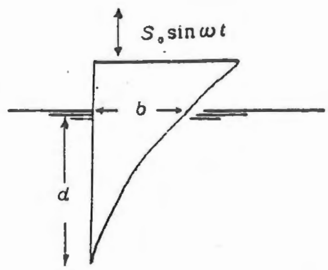
As the width of the channel was not scaled, it was found most accurate to make the comparisons using the wave gauge closest to the wavemaker. The main reason for this was disturbances caused by friction of the channel walls. The friction caused an oblique cross-waves to develop especially with short waves, which in the case of very steep waves resulted a wave breaking in the node points of progressing wave crests and oblique disturbances.

Also the wave height records of the full scale test appeared more reasonable at the first wave gauge, because some of the measurements further away may have been affected by reflections.

The model tests were conducted in the scale 1 to 5 in the glass walled flow flume of the Hydraulics Laboratory of Helsinki University of Technology. The cross-section of the flume is 1.1 x 1.3 m^2 and its effective length is 55 m. Figure 4 shows the measuring principle of the test facilities. The dimensions of plunger shapes tested are shown in table 1.

Table 1. The full scale dimensions of plunger-cross-sections tested.

Wedge	d [m]	b [m]
A (original)	1.63	0.975
B	1.70	1.10
C	1.60	1.10



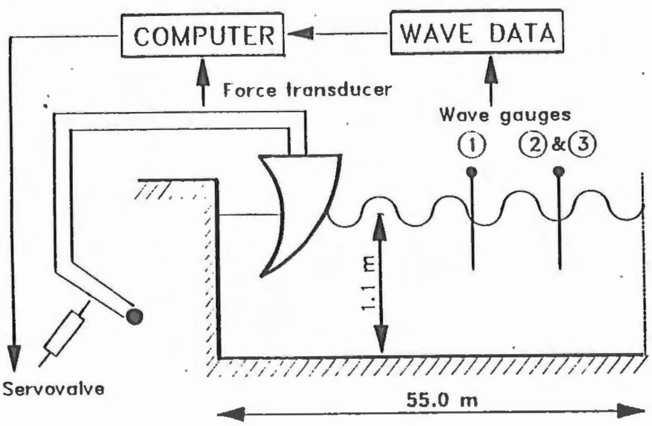


Fig. 4. The measuring principle.

The wave maker was controlled by PC/AT-microcomputer, which was equipped with AD/DA-converter thus enabling both the controlling and measuring phases. The waves were recorded in sequenses of ten or fifteen seconds, each record consisting of 1024 measuring points. Depending on frequency, the waves were thus described by 35..191 points each.

The time history of the plunger motion was recorded using the same sampling frequency. In these tests the stroke amplitude is defined as a half the length of the arc described by a point on the back side of the plunger during one complete cycle of oscillation.

The force required to move the wedge up and down was measured by a force transducer. The transducer was connected between the wedge and the hydraulic actuator, thus the measured forces represent the values of total forces. The measurements of forces will not be discussed in this paper, but published later in the VTT- Research Notes.

RESULTS

Wave heights.

From each record, the gained wave height was calculated as the arithmetic mean of a number of successive zero downcrossing waves, defined in accordance with the IAHR-recommendation /5/.

The recorded wave heights, transformed to full scale, are presented in Appendix 1, together with the original full scale wave recordings. It appears, that the wave heights in the model tests with the original plunger shape match the full scale measurements reasonably well especially on frequencies below 6 rad/s. It is therefore believed that also other plunger geometries perform in accordance with Froude's model law, and that additional scale effects are negligible.

The modified plungers, B and C, seem to produce considerably greater wave heights. Fig. 5 shows amplitude/stroke ratios plotted against frequency, which is made dimensionless as kb. Amplitudes are taken to be half the wave height. The plotted co-ordinates are connected with analytically determined best fit lines of a power function, in order to visualize differences. Thus, it is not claimed that these curves would represent an approximation of the amplitude ratios at different frequencies. This is also due to the fact that the shape of any best fit curve depends on the number of measurements and the choice of stroke amplitudes at each frequency. However, a clear tendency can be seen. Plungers B and C give a wave height about 40 percent greater than the original plunger shape A.

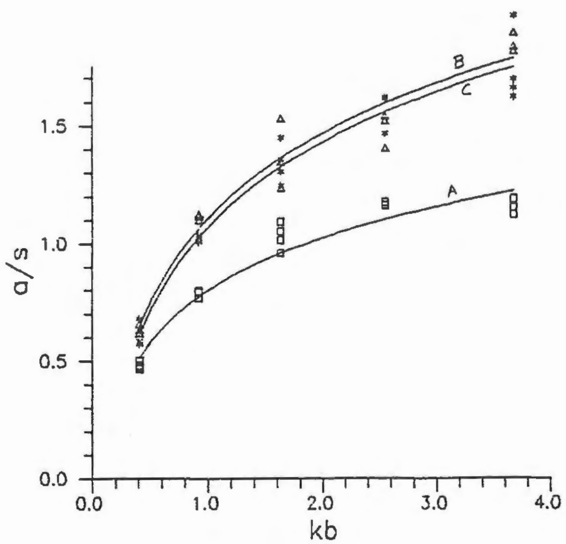


Fig. 5. Wave amplitude to stroke ratios with three different plunger shapes.

Regularity of a wavetrain.

As a measure of the grade of regularity of the wavetrain, the standard deviation of the determined wave heights was calculated from each wave record. The percentage of deviation from mean wave height at different frequencies is shown in Fig. 6. In all cases, a relatively great deviation can be noted at small wave heights. This is caused by the adhesion effect between the water and the glass side walls, that creates small diagonal waves interfering with the actual generated waves. Because it cannot be concluded that these disturbances exist in the actual wave tank, less attention should be paid to these values.

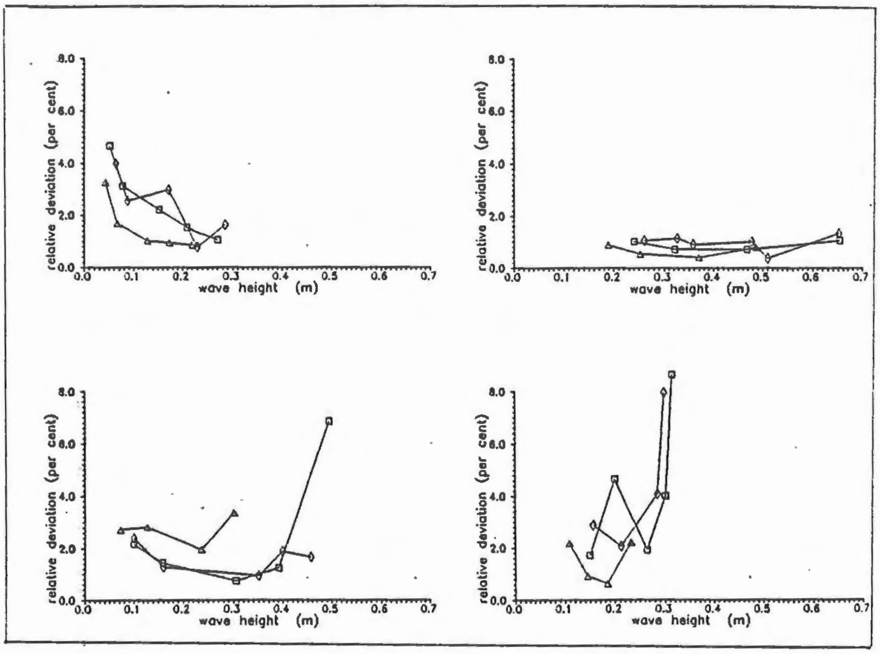


Fig. 6. Percentage of standard deviation of recorded wave height at different frequencies. a) 2 rad/s, b) 3 rad/s, c) 4 rad/s and d) 5 rad/s.

At greater wave heights, a sharp bend upwards occurs, which can be interpreted as the breaking point. At angular frequencies of 3 rad/s and less, breaking waves did not occur.

For all of the plungers 3 rad/s seemed to be the optimal working frequency in the sense of wave regularity. This was also the frequency at which the greatest waves were recorded.

Wave shape

Although the oscillation of the plunger can be considered strictly sinusoidal, the formed wave differs in shape from a pure sine wave. As most wave theories still assume sinusoidal waves or an irregular seastate composed of sine formed waves, attempts are made to create such waves or at least to know how much the created waves differ from sine waves. Figure 7 shows a recorded wave compared with a sine of the same wave length and the same height.

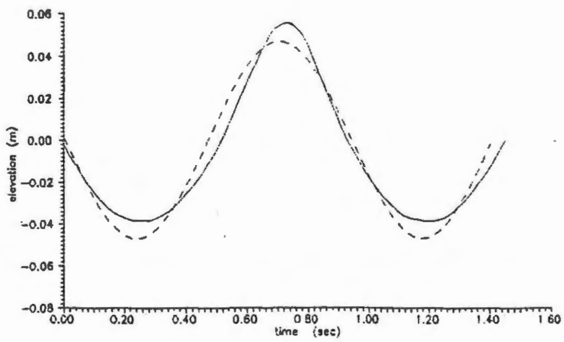


Fig. 7. Recorded wave compared to pure sinusoidal wave. Plunger C. $\omega = 3 rad/s$, second harmonic ratio is 17.3 %.

For a more analytical approach the wave records were Fourier-analysed and dissolved into a series of harmonic sines. The ratio of the amplitudes of the main wave and its second harmonic was taken as a measure. Figure 8 show this ratio plotted against wave height for the different plungers at different frequencies. In the case of plunger A the ratio is clearly more dependent on the frequency than in the case of plungers B and C. The results of B and C are considerably similar to each other, although model C behaves slightly better at high frequencies.

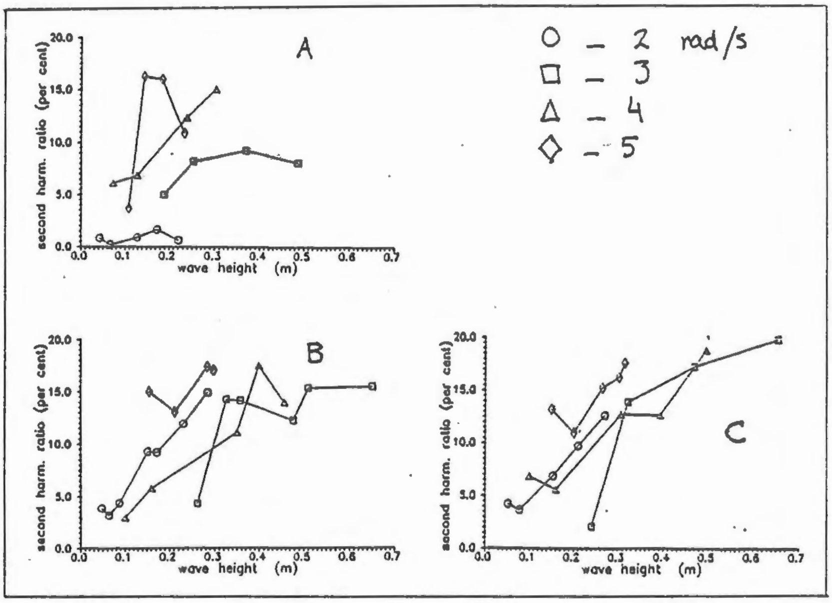


Fig. 8. Ratio of second to first harmonic sine wave amplitudes (per cent).

The wave shapes were further examined by determining the horizontal and vertical asymmetries as defined in Figure 9.

A horizontal asymmetry greater than 0.5 indicates a sharp crested wave with a flat trough. In the sense of horizontal asymmetry plungers B and C again seem to perform equally well, while waves produced with plunger A are somewhat less asymmetric. The plots resemble very much those of the second harmonic wave amplitude ratios.

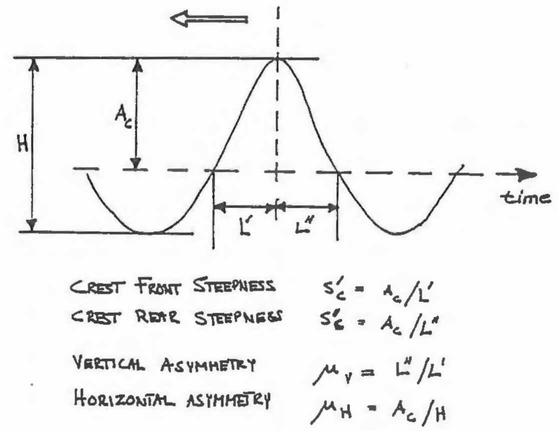


Fig. 9. Wave asymmetry parameters.

Vertical asymmetry is a measure to compare front and rear crest steepnesses. A value of more than 1.0 means that the front of the wave is steeper than the rear. Surprisingly, plunger B seems to create very asymmetric waves at frequencies 2 and 3 rad/s. At 3 rad/s the asymmetry occurs when the wave height exceeds 0.5 metres, but at 2 rad/s also the smallest waves have an asymmetry of about 1.2 and it increases fast with increased wave height. This is explained by the sharp angle which the plunger's front side forms with the water surface. The forced acceleration of water particles is greater than the natural acceleration on the surface of these long waves. The characteristics of A and C are much more stable, keeping for mostly within the limits 0.8..1.2. The average vertical asymmetry of waves made with plunger B is, however, clearly greater than 1.0, while in the case of A it is somewhat less than 1.0.

<u>Comparison of measured and theoretical wave heights</u>

In this study, the measured amplitude/stroke ratios were compared with calculated results according to Wang's theory

/10/. The sectional area coefficient, σ is described as the ratio between the cross section area and the product of the breadth and height of the cross section:

$$A = \sigma \cdot b \cdot d \qquad (13)$$

Because the plungers here were hinged at a pivot point and oscillated not vertically as in Wang's theory, the sectional area coefficient was determined considering the distribution of horizontal displacements, rather than the shape of the plunger itself. Using this assumption, plunger A has a sectional area coefficient very close to 0.5. For plungers B and C the coefficients are more difficult to determine, as they are not constant with depth, which is assumed in theory.

The wave amplitude/stroke ratios of plunger A is plotted in Fig. 10 together with Wang's theoretical values correspon-ding to triangular plungers of d/b ratios of 1.5 and 2.0 and a sectional area coefficient of 0.5. The measured values fit between these two curves at all frequencies. From the geometry, the corresponding d/b-value of plunger A can be estimated to be about 1.7.

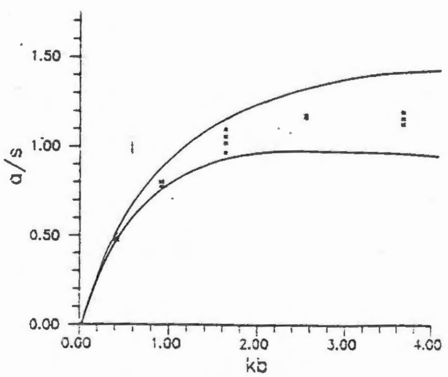


Fig. 10. A comparison between measured and theoretical a/s-ratios, Plunger A (Wang, $\sigma = 0.5$).

The a/s ratios of plungers B and C, were compared with the plungers having sectional area coefficients 0.4 (Fig. 11). Both B and C produce proportionally greater waves than the theoretical curves indicate. From the geometry it is clear that these plungers cannot be compared with the theoretical, horizontally moving plungers with constant sectional area coefficients.

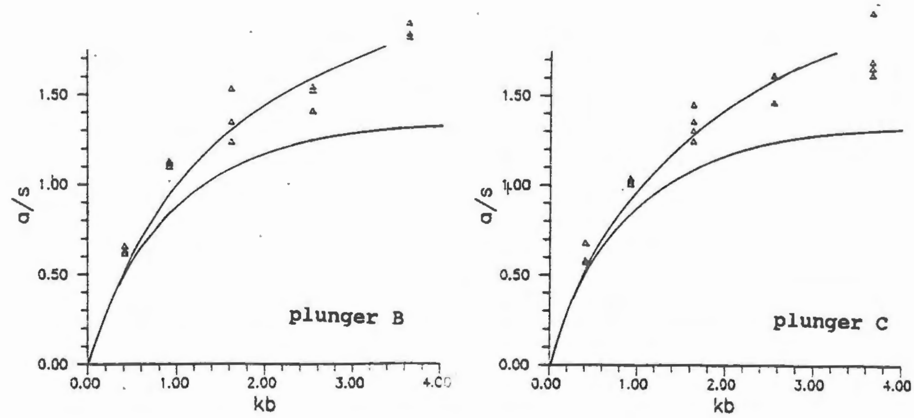


Fig. 11. A comparison between measured and theoretical a/s-ratios (Wang, $\sigma = 0.4$).

Force requirements

In order to evaluate the force needed to drive the plungers, an attempt was made to determine their added masses and damping coefficients. The damping was assumed linear. The test was done as a zero-force test by disconnecting the power unit from the wavemaker and letting the plungers swing freely on the water surface. The dampened oscillation of the plungers was recorded. The mass of the oscillating part of the wavemaker was measured in each case, and its mass centre was determined. For simplicity, also the spring coefficient, k, was taken to be constant, although it is evident from the geometry that especially for plungers B and C this is not true. The spring constant was thus described as

$$k = A_w \rho g \qquad (14)$$

where A_w is the water plane area of the plunger at mean water level. With these assumptions made, the following values of coefficients a, c and k were obtained:

Table 2. Added mass, a; damping coefficient, c; and spring constant, k for the different plunger shapes.

plunger type		A	B	C
a	[kg/m]	497.7	562.5	398.1
	[kg]	5474.7	6187.5	4379.6
c	[kg/s/m]	1421.3	1825.1	2459.7
	[kg/s]	15634.3	20075.0	27056.4
k	[kg/s*s/m]	9406.0	10890.0	11138.4
	[kg/s*s]	103466.0	119789.0	122522.7

CONCLUSIONS

The plunger type wave maker characteristics were studied experimentally. The effect of a plunger cross-sectional shape on generated waves was studied using three model wedge cross-sections in the scale of 1 to 5.

The modified plungers, B and C, seemed to produce considerably higher waves compared to the original body form A. The wave heights greated were approximately 40 % higher with the plungers B and C. The wave horizontal asymmetries, however, were greater for the plungers B and C.

The analyzed values of vertical asymmetries showed plungers A and C beeing quite stable in a sense of the linear wave shape and the regularity of the wave train. Plunger C created very asymmetric waves at the frequency range 2 to 3 rad/s. For all the plungers tested the frequency of 3 rad/s was the optimal working frequency when the highest waves were observed. Thus the waves generated by plunger B were out of the design limits in the sense of the linear and 'pure' wave formation.

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APPENDIX 1. Average zero downcrossing wave heights

freq. rad/s	stroke ampl.		Plunger shape A	B	C	original
1	0.147				0.059	0.049
	0.148			0.038 ?		
	0.201					0.067
	0.202				0.070	
	0.205			0.085 *		
	0.331				0.138	
	0.410			0.155 *		
2	0.042			0.047		
	0.044				0.051	
	0.045		0.042			0.042
	0.048			0.063 *		
	0.064			0.078		
	0.065				0.077	
	0.066		0.066			0.066
	0.070			0.087 *		
	0.129			0.152		
	0.132					0.121
	0.133		0.128			
	0.134				0.1531	
	0.138			0.172 *		
	0.179					0.273
	0.182		0.173		0.209	
	0.186			0.230 *		
	0.238		0.220		0.269	
	0.240			0.285 *		
	0.058					0.100
3	0.115			0.240		
	0.117					0.202
	0.118		0.188		0.240	
	0.121			0.261 *		
	0.155			0.326		
	0.158					0.262
	0.159		0.252			
	0.160				0.321	
	0.161			0.358 *		
	0.228			0.477		
	0.233		0.369			
	0.234			0.510 *		
	0.235				0.468	
	0.241	-0.2865	0.372			
	0.315			0.651 *		
	0.316		0.486		0.653	

4				0.100 *		
	0.0338				0.0993	
	0.0345					0.074
	0.0346		0.072	0.105		
	0.0573			0.160 *		
	0.058		0.128	0.158	0.159	0.120
	0.117					0.226
	0.118		0.237	0.289	0.305	
	0.133			0.350 *		
	0.153			0.399 *		
	0.157				0.391	
	0.158		0.301	0.457 *		0.283
	0.159			0.376		
	0.232			0.466 *	0.494	
	0.045			0.156 *		
5	0.046		0.108	0.140	0.149	0.118
	0.060			0.212 *		
	0.061					0.150
	0.062		0.144	0.191	0.200	
	0.088			0.285 *		
	0.090				0.264	0.223
	0.091			0.253		
	0.092		0.184			
	0.105				0.301	
	0.117				0.314	0.276
	0.118		0.232	0.298		
6	0.041			0.155 *		
	0.042		0.0966	0.159	0.165	0.113
	0.053		0.121	0.196 *		
	0.054			0.198	0.175	
	0.055					0.149
	0.059				0.200	
	0.063				0.209	
	0.083		0.186		break	0.175 ?
	0.117		0.140			
	0.118			0.149		
7	0.012		0.037		0.055	
	0.013					0.045
	0.026		0.067			0.072
	0.027				0.110	
	0.037			0.117		
	0.038		0.091	0.11?	0.114	0.117
	0.042			0.129 *		
	0.043				0.118	
	0.045			0.11 *		
	0.050		0.116	0.121 c		0.096
	0.051				0.121	
	0.052			0.11 *		
				0.137		
	0.075		0.083			
	0.077			0.097		
8	0.012		?		0.0607	0.045
	0.023			0.082		
	0.024					0.078
	0.025		0.054	(0.06 ?)		
				0.079 *		
	0.026				0.099	
	0.035		0.084	0.086 *		0.112
	0.369			?		
	0.0376				0.084	
	0.0387					0.083