THE PROPAGATION OF NONLINEAR WAVES IN A STRUCTURE CONSISTING OF NET SYSTEM

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SUMMARY: In this paper a motion of a continuous model network is investigated under a transverse impact caused by a point load and by a rigid cone. The nonlinear differential equations obtained are solved analytically and with the finite difference method. The network consists of two sets of flexible filaments, the elements of which are fixed at the intersection points. The equations,describing the front of the plane waves, are derived and solved for two cases: the net a)without and b)with prestressing before shock. Experimental investigations are performed by Moire method. The results confirm the theoretical solution.

INTRODUCTION

The fundamental research of dynamic elastic bindings is by Acad. H.A. Rahmatulin [1]. In this work a motion of endless orthotropic net is investigated under a cross impact by a point load and by a rigid cone.

Nowadays the net structures are often used,but the analysis including large deformations is not yet solved effectively [2].

BASIC EQUATIONS

In derivation of equations of motion we use results by dynamic elastic bindings [1,3].

The equations which describe the net as a continuous structure are presented in the works [4,5,8]and can be written in the following form:

a) the motion of the net on the smooth cone:

$$\begin{aligned}\rho[r\frac{\partial^2\theta}{\partial t^2}+2\frac{\partial\theta}{\partial t}\frac{\partial r}{\partial t}] &= \frac{\partial}{\partial s_1}(\sigma_1\cos\phi_1)+\sigma_1\sin\phi_1\frac{\partial\theta}{\partial s_1}\sin\alpha+\\ &+\frac{\partial}{\partial s_2}(\sigma_2\cos\phi_2)+\sigma_2\sin\phi_2\frac{\partial\theta}{\partial s_2}\sin\alpha\\ \rho[\frac{1}{\sin\alpha}\frac{\partial^2 r}{\partial t^2}-r\sin\alpha(\frac{\partial\theta}{\partial t})^2] &= \frac{\partial}{\partial s_1}(\sigma_1\sin\phi_1)-\sigma_1\cos\phi_1\frac{\partial\theta}{\partial s_1}\sin\alpha+\\ &+\frac{\partial}{\partial s_2}(\sigma_2\sin\phi_2)-\sigma_2\cos\phi_2\frac{\partial\theta}{\partial s_2}\sin\alpha\end{aligned} \tag{1}$$

The reaction on the surface of the cone:

$$R=\cos\alpha[\sigma_1\cos\phi_1\frac{\partial\theta}{\partial s_1}-\sigma_2\cos\phi_2\frac{\partial\theta}{\partial s_2}-\rho r(\frac{\partial\theta}{\partial t})^2]$$

The geometrical conditions:

$$r\frac{\partial\theta}{\partial s_1}=(1+e_1);\qquad \frac{1}{\sin\alpha}\frac{\partial r}{\partial s_1}=(1+e_1)\sin\phi_1$$

$$r\frac{\partial\theta}{\partial s_2}=(1+e_2);\qquad \frac{1}{\sin\alpha}\frac{\partial r}{\partial s_2}=(1+e_2)\sin\phi_2$$

where

σ_1 and σ_2 — stress parameters

e_1 and e_2 — deformation parameters

s_1 and s_2 — Lagrange coordinates

ϕ_1 and ϕ_2 — angle parameters

r — radius of a circle in the point (s_1, s_2)

θ — central angle of this circle

α — angle parameter of the cone

ρ — the density of the net

t — time

b) for the plane motion the system of equations is:

$$\begin{aligned}&\rho\frac{\partial^2 x}{\partial t^2}=\frac{\partial}{\partial s_1}(\sigma_1\cos\gamma_1)+\frac{\partial}{\partial s_2}(\sigma_2\sin\gamma_2)\\ &\rho\frac{\partial^2 y}{\partial t^2}=-\frac{\partial}{\partial s_1}(\sigma_1\sin\gamma_1)+\frac{\partial}{\partial s_2}(\sigma_2\cos\gamma_2)\\ &(1+e_1)\cos\gamma_1=1+\frac{\partial x}{\partial s_1};\qquad (1+e_1)\sin\gamma_1=-\frac{\partial y}{\partial s_1}\\ &(1+e_2)\cos\gamma_2=1+\frac{\partial y}{\partial s_2};\qquad (1+e_2)\sin\gamma_2=\frac{\partial x}{\partial s_2}\end{aligned} \tag{2}$$

where

x, y – the projection of velocity vectors

γ_1, γ_2 – angle parameters

CHARACTERISTIC PROPERTIES OF BASIC EQUATIONS

a) For the case of motion on cone system, differential equations (1) can be written in the form [1]:

$$A\frac{\partial W}{\partial t} + B_1\frac{\partial W}{\partial s_1} + B_2\frac{\partial W}{\partial s_2} + C = 0 \tag{3}$$

$$W = (w \quad u \quad r \quad e_1 \quad e_2 \quad \phi_1 \quad \phi_2)^T$$

$$C = \begin{pmatrix} \frac{2}{r}uw + \frac{\sigma_1}{\rho}(1+e_1)\sin\phi_1\frac{\cos\phi_1}{r^2}\sin\alpha + \frac{\sigma_2}{\rho}(1+e_2)\sin\phi_2\frac{\cos\phi_2}{r^2}\sin\alpha \\ -rw^2\sin\alpha + \frac{\sigma_1}{\rho}(1+e_1)\frac{\cos^2\phi_1}{r}\sin^2\alpha + \frac{\sigma_2}{\rho}(1+e_2)\frac{\cos^2\phi 2}{r}\sin^2\alpha \\ -u \\ -u(1+e_1)\frac{\cos^2\phi_1}{r} \\ -u(1+e_2)\frac{\cos^2\phi_2}{r} \\ u(1+e_1)\sin\phi_1\frac{\cos\phi_1}{r} \\ u(1+e_2)\sin\phi_2\frac{\cos\phi_2}{r} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+e_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+e_2 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & 0 & 0 & -a_1^2\frac{\cos\phi_1}{r} & 0 & \frac{\sigma_1}{\rho}\frac{\sin\phi_1}{r} & 0 \\ 0 & 0 & 0 & -a_1^2\sin\phi_1\sin\alpha & 0 & \frac{-\sigma_1}{\rho}\sin\alpha\cos\phi_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r\cos\phi_1 & \frac{-\sin\phi_1}{\sin\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r\sin\phi_1 & \frac{-\cos\phi_1}{\sin\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0 & 0 & 0 & -a_2^2 \frac{\cos\phi_2}{r} & 0 & \frac{\sigma_2}{\rho}\frac{\sin\phi_2}{r} & 0 \\ 0 & 0 & 0 & -a_2^2 \sin\phi_2 \sin\alpha & 0 & \frac{-\sigma_2}{\rho}\sin\alpha\cos\phi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -r\cos\phi_2 & \frac{-\sin\phi_2}{\sin\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r\sin\phi_2 & \frac{-\cos\phi_2}{\sin\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Following notation is used:

$$u = \frac{\partial r}{\partial t} \qquad w = \frac{\partial \theta}{\partial t}$$
$$a_1^2 = \frac{1}{\rho}\frac{d\sigma_1}{de_1} \qquad a_2^2 = \frac{1}{\rho}\frac{d\sigma_2}{de_2}$$

The condition that a surface $\Phi(s_1, s_2, t) =$ const, be a characteristic surface of Eq.(3) is equivalent to the condition that the determinant of the characteristic matrix be zero [6]:

$$Det(A\frac{\partial \Phi}{\partial t} + B_1\frac{\partial \Phi}{\partial s_1} + B_2\frac{\partial \Phi}{\partial s_2}) = 0 \tag{4}$$

The Eq.(4) can be written after mathematical operations in the form:

$$\begin{aligned} \Delta \; &= \frac{1}{2}\frac{1}{a_1^2 a_2^2}(\frac{\partial \Phi}{\partial s_1})^2(\frac{\partial \Phi}{\partial s_2})^2 \sin 2\phi_1(\sin 2\phi_1 - \cos 2\phi_2) + \\ &+ \frac{1}{a_1^2 b_1^2}(\frac{\partial \Phi}{\partial s_2})^2[(\frac{\partial \Phi}{\partial s_2})^2 - (\frac{\partial \Phi}{\partial s_1})\cos\phi_1\cos\phi_2\cos(\phi_1 - \phi_2)] + \\ &+ \frac{1}{a_2^2 b_1^2}(\frac{\partial \Phi}{\partial s_2})^2[(\frac{\partial \Phi}{\partial s_1})\cos^2(\phi_1 - \phi_2) - \frac{1}{a_1^2}(\frac{\partial \Phi}{\partial t})^2] + \\ &+ \frac{1}{a_1^2 b_2^2}(\frac{\partial \Phi}{\partial s_1})^2(\frac{\partial \Phi}{\partial s_2})^2 \sin\phi_1 \sin\phi_2\cos(\phi_1 - \phi_2) - \\ &- [(\frac{\partial \Phi}{\partial s_1})^2 - \frac{1}{a_1^2}(\frac{\partial \Phi}{\partial t})][\frac{1}{a_2^2 b_2^2}(\frac{\partial \Phi}{\partial s_1})^2\cos 2\phi_1 + \frac{1}{a_2^2 b_1^2 b_2^2}(\frac{\partial \Phi}{\partial t})^2] + \\ &+ \frac{1}{b_1^2 b_2^2}(\frac{\partial \Phi}{\partial s_2})^2[(\frac{\partial \Phi}{\partial s_1})^2\sin^2(\phi_2 - \phi_1) - \frac{1}{a_1^2}(\frac{\partial \Phi}{\partial t})^2] \end{aligned} \tag{5}$$

where

$$b_1 = \frac{1}{\rho}\frac{\sigma_1}{1 + e_1} \qquad b_2 = \frac{1}{\rho}\frac{\sigma_2}{1 + e_2}$$

The wave equation (5) shows (see [5]) that the waves in the set under a transverse shock by cone can not be separated. Thus, the family of waves create other families.

b) The characteristic equation for system (2) of automodel motion have the following form [6]:

$$
\begin{aligned}
&[(\xi_1^2 - a_1^2\cos^2\gamma_1 - b_1^2\sin^2\gamma_1)(\frac{\partial\omega}{\partial\xi_1})^2 + 2\xi_1\xi_2\frac{\partial\omega}{\partial\xi_1}\frac{\partial\omega}{\partial\xi_2} + \\
&+ (\xi_2^2 - a_2^2\sin^2\gamma_2 - b_2^2\cos^2\gamma_2)(\frac{\partial\omega}{\partial\xi_2})^2][(\xi_1^2 - a_1^2\sin^2\gamma_1 - b_1^2\cos^2\gamma_1)(\frac{\partial\omega}{\partial\xi_1})^2 + \\
&+ 2\xi_1\xi_2\frac{\partial\omega}{\partial\xi_1}\frac{\partial\omega}{\partial\xi_2} + (\xi_2^2 - a_2^2\cos^2\gamma_2 - b_2^2\sin^2\gamma_2(\frac{\partial\omega}{\partial\xi_2})^2] - \\
&- [(a_1^2 - b_1^2)\sin\gamma_1\cos\gamma_1(\frac{\partial\omega}{\partial\xi_1})^2 - (a_2^2 - b_2^2)\sin\gamma_2\cos\gamma_2(\frac{\partial\omega}{\partial\xi_2})^2]^2 = 0
\end{aligned}
\qquad (6)
$$

where

$\xi_1 = \frac{s_1}{v_0 t}$ and $\xi_2 = \frac{s_2}{v_0 t}$;v_0 - const.

$\omega(\xi_1, \xi_2) = 0$ - characteristic curves

$$a_i = \frac{1}{v_0^2\rho}\frac{d\sigma_i}{de_i} \quad \text{and} \quad b_i = \frac{1}{v_0^2\rho}\frac{\sigma_i}{1+e_i}$$

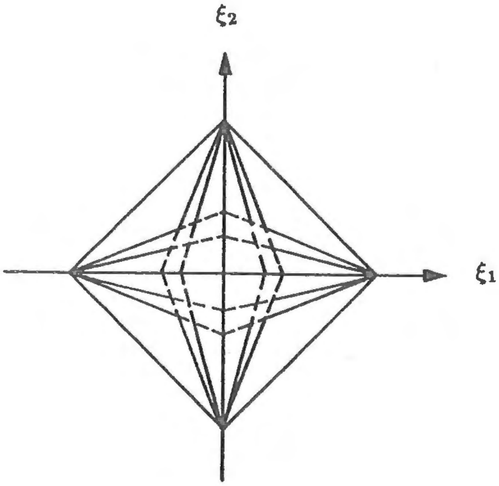

Fig. 1.

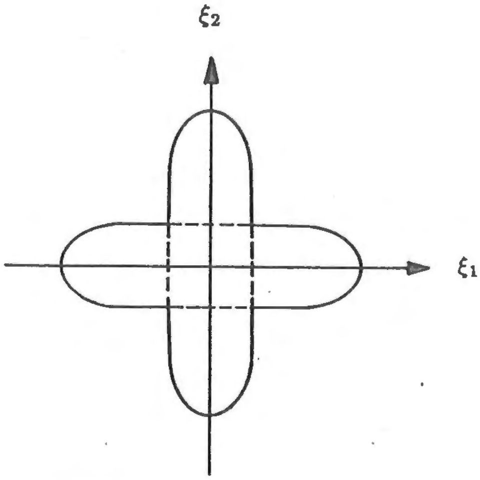

Fig. 2.

Eq.(6) have after mathematical operations and after substituting $\frac{\partial\omega}{\partial\xi_2} = -\frac{\partial\xi_1}{\partial\xi_2}\frac{\partial\omega}{\partial\xi_1}$ the form[7]:

$$
\begin{aligned}
\frac{d\xi_1}{d\xi_2} &= \frac{\xi_1\xi_2 \pm \sqrt{\xi_1^2\xi_2^2 - (\xi_1^2 - a^2)(\xi_2^2 - b^2)}}{(\xi_2^2 - b^2)} \\
\frac{d\xi_1}{d\xi_2} &= \frac{\xi_1\xi_2 \pm \sqrt{\xi_1^2\xi_2^2 - (\xi_1^2 - b^2)(\xi_2^2 - a^2)}}{(\xi_2^2 - a^2)}
\end{aligned}
\qquad (7)
$$

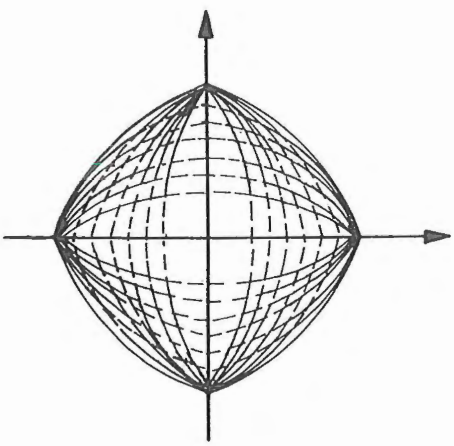

Fig. 3.

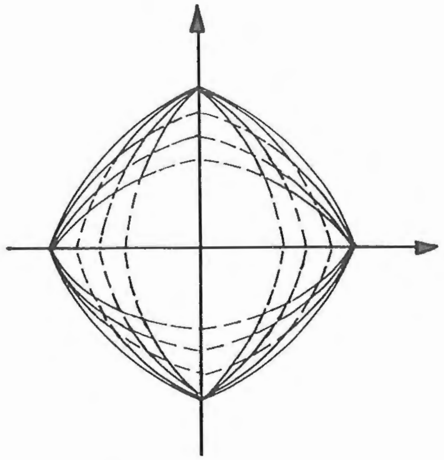

Fig. 4.

where $a_1 = a_2 = a$ and $b_1 = b_2 = b$ (see[7]). Eqs.(7) describe the front of plane waves. The solution of Eq.(7) for the case $b = 0$ (the net is not prestressed) is:

$$|\xi_1 \pm a| = |C\xi_2| \text{ and } |\xi_2 \pm a| = |C\xi_1| \tag{8}$$

Figure 1 shows this case.

Eqs.(6) in the case when b $\neq 0$ (the net is prestressed) have solution in the domain

$$\frac{{\xi_1}^2}{a^2} + \frac{{\xi_2}^2}{b^2} \geq 1 \; and \; \frac{{\xi_1}^2}{b^2} + \frac{{\xi_2}^2}{a^2} \geq 1. \tag{9}$$

System of Eq.(6) is solved by the finite difference method [7]. Figures 2-4 shows the other cases , when the net is prestressed.

EXPERIMENTAL INVESTIGATION USING MOIRE EFFECT

The impact on elastic net was executed with a pneumatic gun Y-13. The motion of the net was registrated by a high-speed camera CKC-I. (speed 3.010^3 photo s^{-1}) . The speed of the motion of a cone was 40-70 ms^{-1} and the speed of propagation of the waves in the net was 200 ms^{-1} . In experiments [9]the Moire-method was used [8]. Figures 5 and 6 show the motion of the waves in plane domain when the net is without and with prestressing,respectively.

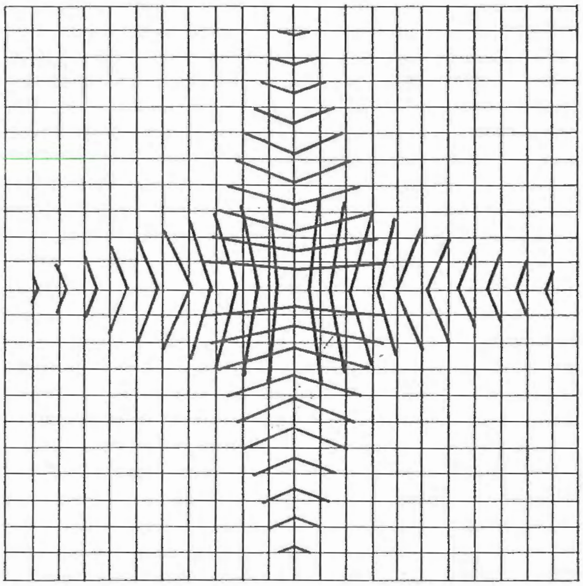

Fig. 5.

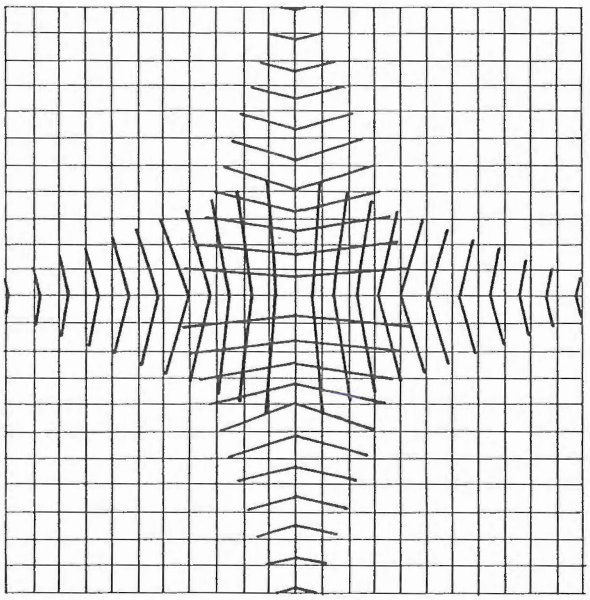

Fig. 6.

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