SPECIAL FEATURES OF RESTRAINT STRAINS AND THEIR INFLUENCE IN SERVICE STATE DESIGN OF CONCRETE STRUCTURES

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SUMMARY: The nature and special features of restraint effects compared with load effects are discussed. Structures subjected to restraint effects and the cause of these effects are listed. A method for calculating and limiting of crack widths based on the stochastical nature of concrete tensile strenght is shown. Finally the results and their applicability is examined.

INTRODUCTION

Important reinforced concrete structures, and load-bearing structures in particular, are generally designed for certain external load values. In accordance with the method of deterministic structural design the loading values are chosen in such a way that the actual load value can only very seldom surpass the characteristic value of the load in question. In practice the characteristic live load values are in most cases very high as compared with actual mean load values which occur on average. Since the characteristic live loads are a basis for structural design, this usually results in relatively heavy reinforcement in ultimate state, and the crack load of a structure is relatively low with respect to the capacity of a structure.

It is clear that no cracks resulting from actual average load should develop in this kind of structure.

Practice has shown, however, that there are cracks in reinforced concrete structures. The reason for these observed cracks must lie elsewhere than in the misjudgement of the magnitude of design loads or in the inaccurary of structural design. The reason for this is very often that stresses resulting from the restrained deformations or displacements of a structure, and which develop in concrete structures to some extent in all circumstances, have generally been ignored in design.

In most cases these stresses due to restrained deformations were overlooked since they were not sufficiently known, but the structural design was also often knowingly carried out for external design loads only, although it was known that restraint strains developing in structures give rise to cracks. This negligence results not only from a lack of knowledge but in many cases also from the fact that only functional requirements but not methods for treating the stresses in question are presented in the instructions or regulations concerning construction.

In modern construction the significance of quality is becoming increasingly important. The central factor in the pursuit of quality is the quality of visible surfaces, to which the non-formation of cracks or at least the limitation of cracks to an acceptable level is closely related. On the other hand the acceptable level depends on several factors, although factors contributing to durability and outward appearance are generally regarded as the main criterion. As questions pertaining to quality have gained in importance, studies concerning the serviceability limit state of reinforced concrete structures and the properties of previously overlooked so-called restraint strains, together with their effects, have been increasingly undertaken. In the design of reinforced concrete buildings requiring high levels of refinement, in which case more sophisticated methods can be employed, the strains in question have also been taken into consideration.

So far the assessment of restraint effects and their limitation of cracks have been carried out by undeveloped methods or even by theoretically erroneous methods, which does not of course lead to the best result, neither qualitatively nor economically. The calculation method of restraint effects according to the theory of elasticity, based on the uncracked cross-section, leads, among other things, to the impossibility of carrying out a design task. On the other hand the crack width formulae based on the classical sliding bond and on the product of the average crack spacing and mean steel stress underestimate the crack widths which correspond to the actual stress state of restraint strains. Since the development of cracks and stiffness in reinforced concrete structures are dependent on the state of stress, problems arise in describing these phenomena, by means of a mathematically treated model, between the uncracked state and fully cracked state. Since the restraint strains often also depend on environmental conditions, the effect of time should also be considered. The combination of cracking phenomena and microcracking with moisture and temperature variations and with the irreversible or reversible deformations which are dependent on these is an extremely difficult task, which for the time being has not been solved completely.

Because the values of restraint effects depend primarily on the stiffness of the structure, it is obvious that knowing the deformation behaviour of the structure by the best possible means is most important.

This includes

- the essence of the concrete tensile strength
- the interaction between reinforcement and concrete
- cracking phases and crack propagation
- the tension stiffening effect
- mean concrete and steel strains
- the dependence between action effects and displacements
- the dependence between stiffness and stress
- the redistribution of load effects
- the relaxation of release of restraint effects.

Owing to cracking and the dependence of time the estimation of these factors is not always easy.

The main idea of this contribution is based on the fact that all the material properties and also other model parameters are more of less of stochastic nature. This stochastic nature is intentionally taken into consideration only in conjunction with the tensile strength of concrete, for which a simple dependence is assumed, and by means of which the stochastic nature of crack spacing is indirectly presented.

ESSENCE OF RESTRAINT EFFECTS

In the examination of the behaviour of reinforced concrete structures actions directed to the structures should be dealt with by classifying them according to the way in which they were produced.

Direct action can be defined as a population of divided or concentrated forces acting upon a structure which may be static of dynamic in nature. The internal forces resulting from direct actions are regarded as load effects and the loading action as force-controlled.

Indirect action is understood as a population of deformations or displacements within a structure. Indirect actions can in structures produce internal force effects and displacements, or in some cases displacements only as in statically determined structures. Restraint effects are produced in statically indeterminate structures by indirect action. Internal restraint stresses of self-equilibrium stresses may develop both in statically determined and indeterminate structures. The state of stress probably resulting from indirect action is called a strain-controlled or deformation-controlled state. The stress of restraint effects and internal restraint stresses may occur at the same time or separately, depending on the source of restrained strains, and deformation-controlled stress and boundary conditions.

The mode of origin of force-controlled stresses can be specified in the following manner. These stresses are caused by the static self-weight of structures, the acceleration power of a structure and by external static or dynamic loads. According to the laws of structural mechanics the load effects of a structure are in a state of equilibrium with the external forces and the self-weight. Furthermore, the load effects should meet the compatibility requirements at the edges and supports of the structure. When a structure is statically determined only those deformations caused by loading are dependent on the load-deformation behaviour of the structure. In statically indeterminate structures, however, compatibility requirements also render the action effects dependent on the behaviour of the structure. Since the stiffness of the reinforced concrete structures is mainly dependent on the magnitude of a stress, the dependence between actions and stresses is not linear. Non-linearity between actions and stresses means that in statically indeterminate structures there occurs a redistribution of load effects and stresses dependent on the distribution of stiffness in such a way that stress is transferred from flexible to more rigid areas.

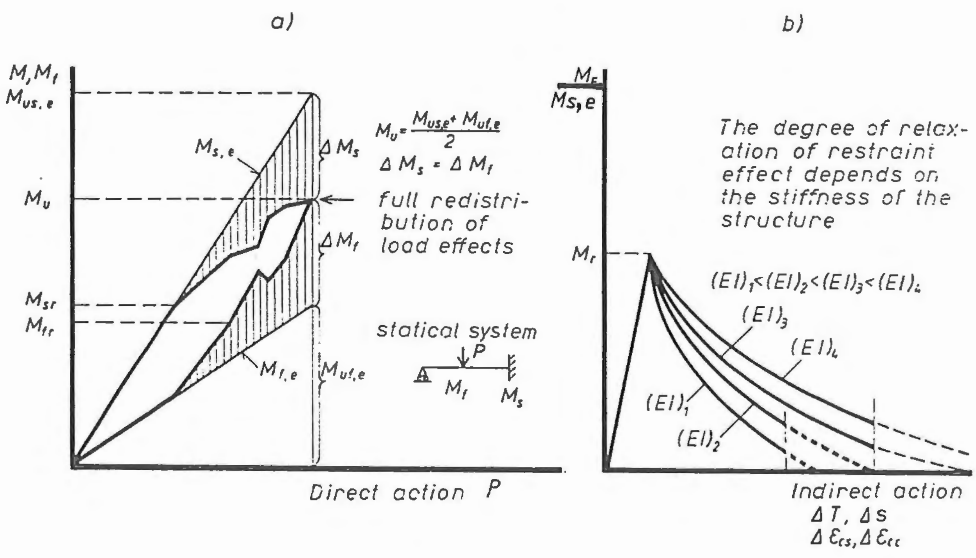


Fig. 1. a) Change in load effects of a reinforced concrete beam, designed according to elastic theory and restrained at one end, as a function of external load (direct action).
b) Change in restraint effect of a corresponding static system as a function of restrained deformation (indirect action).

The mode of origin of the displacement-controlled stresses can be specified in the following way. Stresses bringing about restraint effects are developed in a statically indeterminate structure when deformations resulting from indirect action independently of external load cannot occur freely, but are forced to adjust to the support and continuity conditions. For the accomplishment of this compatibility restraint effects are produced, which give rise to additional deformations and displacements that satisfy the compatibility conditions. Since the restraint effects and the internal stresses result from indirect action or from strain or displacement, the magnitude of these forces is dependent on rigidity alone. The non-linearity of the dependence between the displacement and the restraint effect in the case of reinforced concrete structures thus becomes obvious.

The difference between load effects due to direct action and restraint effects due to indirect action lies in that load effects must satisfy both equilibrium and compatibility conditions, whereas restraint effects are not necessary from the point of view of equilibrium. On the other hand the displacements and deformations resulting from restraint effects must satisfy the compatibility conditions.

Restraint effects can thus occur only in statically indeterminate structures, and are produced from sources of deformations and displacements necessary to satisfy the compatibility conditions of a structure.

In some cases load effects caused e.g. by temperature differences can also develop in statically determinate structures, but then there is a question of second order effects resulting from extra deflections of the structure.

Restraint effects can be either unintentional or intentional. Restraint effects in concrete structures are unintentionally produced, for example, as a consequence of

- temperature differences,
- shrinkage differences,
- creep of the structure and
- settlement of supports.

Restraint effects are intentionally produced by prestressing the structure.

Internal stresses are produced when deformations in the cross-section are prevented. These stresses can also be either intentional or unintentional.

Unintentional internal stresses may result, for instance, from

- temperature differences,
- shrinkage differences,
- creep of cross-section and
- plastic deformation.

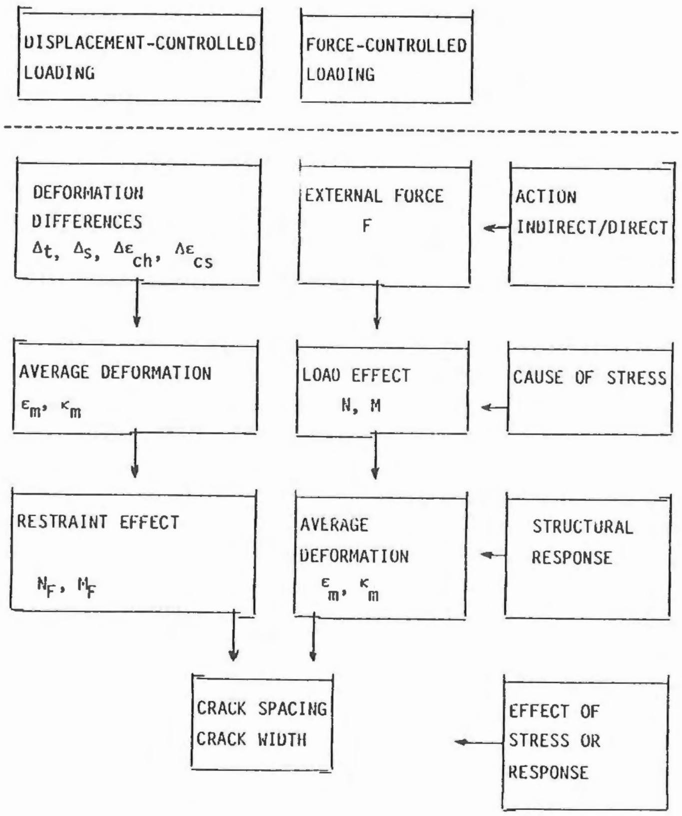


Fig. 2. Contributing factors in force and displacement-controlled loading.

These stresses can develop in statically determined and indeterminate structures, but the sum of these stresses in each cross-section is zero. Internal stresses are also not necessary from the point of view of equilibrium. Where deformations causing restraint effects also require compatibility with the support and boundary conditions of statically indeterminate structures, the deformations causing internal stresses again require compatibility in each cross-section. Internal stresses are produced when deformations independent of external loads tend towards the value defined by the thermodynamic properties of the materials, and the value is incompatible with mechanical behaviour, requiring the principle of a minimum strain condition. As regards their mode of origin these stresses are very similar to those corresponding to restraint effects. If restraint effects are produced as a consequence of temperature difference, internal restraint stresses will also generally occur.

PRACTICAL APPEARANCE OF RESTRAINT EFFECTS

In actual concrete structures strain-controlled loads should be taken into consideration usually in cases such as

- basin structures
- bridge structures
- high chimneys
- water towers
- nuclear power stations
- foundation walls
- massive structures
- balcony structures
- facade elements
- composite structures and
- structures exposed to major temperature differences or fire.

Common features of this kind of structure are

- static indeterminacy
- some degree of restraint displacement
- settlement of foundations
- pre or post-tensioning
- massiveness
- concreting at long intervals
- environmental actions and
- high hydration heat and shrinkage of fresh concrete.

The disadvantages of restraint strains can be reduced by

- exerting an influence on the properties of fresh concrete
- forming movement joints within a structure
- allowing movement of the connecting parts of a structure
- making the connection to adjacent structures more flexible and
- reinforcing a structure considering the restraint stresses.

These measures are not always adequate, since restraint strains no account of which is taken are produced in most structural members, either intentionally or unintentionally. If the structure is not reinforced properly, this leads to a reduction in its serviceability brought about by

- the corrosion of steel and visual defects caused by cracking,
- reduced thermal insulation capacity,

- reduced airtightness and
- reduced soundproofing capacity.

In some cases the capacity of the structure may also decrease.

The temperature of bridge structures, results from solar radiation and changes in outdoor temperature. The wind speed, material properties and the quality of a concrete surface also have an effect on temperature differences.

In prestressed concrete bridges the effect of thermal stresses is dependent mainly on the static system of the structure.

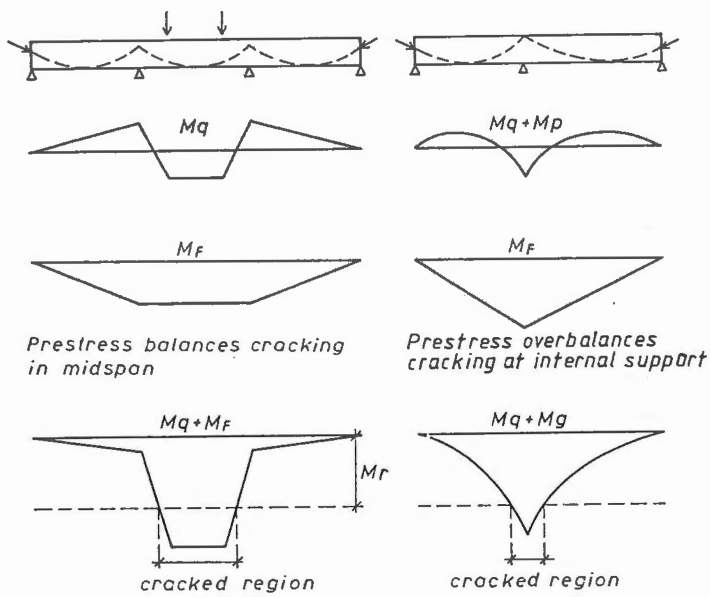


Fig. 3. Effect of prestressing force on cracking in bridges of different static systems.

Partial prestressing is the best way of controlling stresses caused by a temperature gradient and of limiting the crack width.

Other objects subjected to thermal stresses are reinforced concrete facade elements and other structures exposed to outdoor temperatures and solar radiation, the deformations of which are prevented to some extent /5, 8, 10, 13, 20/.

Simultaneously with the temperature stresses, stresses due to shrinkage differences may also act on the facade elements. Deformations parallel to the panel caused by changes in the average temperature of the panel and by shrinkage and prevented by using tie reinforcement and by a bond between the outer panel and heat insulation can be distinguished from deformations which cause the warping of the panel due to temperature and shrinkage differences of the inner and outer surfaces of the panel and which are prevented by using tie bars. For the reasons mentioned restraint stresses can be reduced in most cases by designing the tie reinforcement to be flexible and positioning it so that it prevents as little restraint displacement as possible, but also so that it is suitable for its main purpose: the supporting of the panel. The bond of the insulation to the panel can also be prevented in certain areas, if necessary, or the stiffness of the insulation can be chosen in accordance with the magnitude of movements in such a way that the panel does not crack. The requirement of an uncracked state does not always meet with success, in which case the panel reinforcement must be chosen in such a way that cracks of an acceptable size only are produced. In dimensioning the reinforcement of the outer panel for restraint tension it is not worth starting from the 1st cracking of the panel. It is more realistic and economical to presume that there is already a small crack in the panel, and to restrict the crack width by the amount of steel needed for the purpose.

Reinforced concrete structures, which are exposed to thermal stresses and also very likely to simultaneous shrinkage stresses, include different structures of the process industry, industry chimneys, basin structures and other special structures such as nuclear power plants.

Basin structures exposed to noticeable thermal stresses include containers for liquefied natural gas (LNG), which, as far as the author knows, have not yet been made of concrete in Finland, and the oil tanks of oil-drilling rigs which are anchored to the sea bed and in which the temperature of crude oil and that of sea water outside may be +45 ^{o}C and +5 ^{o}C respectively. Especially in the case of emergency cooling a great temperature difference may develop in condensing water basin structures of nuclear power plants.

There is a substantial amount of literature in which thermal stresses in nuclear power plant structures are discussed. In the main, research has been carried out in those countries where nuclear power has been utilized to generate energy /1, 2, 7, 12, 16, 17/.

Experimental research on restraint effects, restraint stresses and internal stresses in other kinds of structures has also been carried out /3, 4, 6, 15, 21, 24/. Restraint stresses caused by fire are studied in some effects in industrial chimneys. Restraint stresses caused by settlement of supports of foundations are discussed in /14, 22, 23/.

EVALUATION OF CRACK WIDTHS CAUSED BY RESTRAINT EFFECTS

Deterministic variables

Interaction between concrete and reinforcement

The structural elementary unit of a bent beam situated in the vicinity of a crack in the region of bond length is considered below.

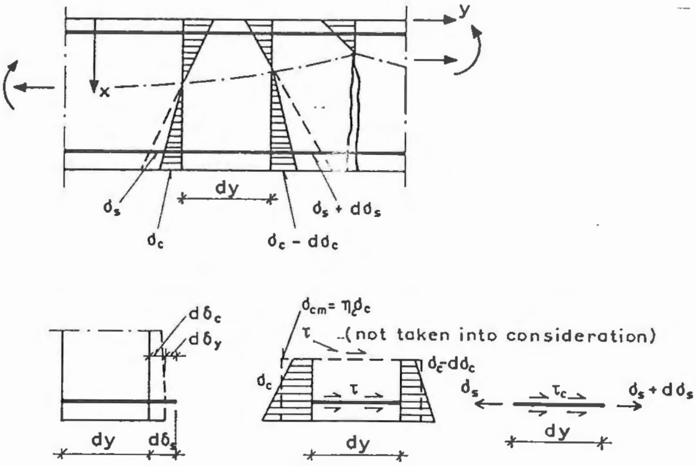


Fig. 4. Basic bond equations.

For displacements of the lower part of the element

$$d\ \delta_s = \frac{\sigma_s}{E_s}\ dy \qquad (1)$$

$$d\ \delta_c = \frac{\sigma_{ct}}{E_c}\ dy \qquad (2)$$

the compatibility condition

$$d\ \delta_y = d\delta_s - d\delta_c = \frac{\sigma_s - n_s \cdot \sigma_{ct}}{E_s}\ dy \qquad (3)$$

is obtained.

For stresses, the equilibrium conditions from the concrete section of the elementary unit are

$$A_{ceff} \cdot \sigma_{ct} \cdot \eta_c - (\sigma_{ct} - d\,\sigma_{ct}) \cdot \eta_c \cdot A_{ceff} - \tau_{cy}\,\pi \cdot \Sigma\,\phi\,dy = 0 \qquad (4)$$

and from the reinforcement proportion

$$-\sigma_s \cdot A_s + (\sigma_s + d\,\sigma_s) \cdot A_s - \tau_{cy} \cdot \pi \cdot \Sigma\,\phi\,dy = 0 \qquad (5)$$

from which we can derive

$$d\,\sigma_{ct} = \frac{\pi \cdot \Sigma\,\phi}{\eta_c \cdot A_{ceff}}\,\tau_{cy}\,dy \qquad (6)$$

and

$$d\,\sigma_s = \frac{\pi \cdot \Sigma\,\phi}{A_s}\,\tau_{cy}\,dy \qquad (7)$$

By substituting expressions (6) and (7) into expression (3) the differential equation

$$\frac{d^2\delta}{dy^2} = \frac{\pi \cdot \Sigma\,\phi}{E_s A_s}\left(1 - \frac{n_s}{\eta_c \cdot A_{ceff}}\right)\tau_{cy} \qquad (8)$$

is obtained.

The deformations of concrete are often ignored, in which case

$$\frac{d^2\delta}{dy^2} = \frac{\pi \cdot \Sigma\,\phi}{E_s A_s}\,\tau_{cy} \quad \text{and} \qquad (9)$$

$$\sigma_s = \frac{d\delta}{dy} \cdot E_s \qquad (10)$$

are obtained.

In order to find a solution for stresses and slip the relation $\tau_{cy} = f(\delta)$ between the local bond stress and the slip should be known.

By integrating equations 3, 6 and 7 and by taking into consideration the boundary conditions that pertain to this case the following equations, based on all bond considerations, are obtained.

$$\varepsilon_{sy} = \varepsilon_{s1} + \frac{\pi \cdot \Sigma \phi}{E_s A_s} \int_0^y \tau_{cy}\, dy \qquad (11)$$

$$\varepsilon_{cy} = \varepsilon_{c1} + \frac{\pi \cdot \Sigma \phi}{E_c \cdot \eta_c \cdot A_{ceff}} \int_0^y \tau_{cy}\, dy \qquad (12)$$

$$\delta_y = \delta_{sy} - \delta_{cy} = \delta_{s1} + \int_0^y \varepsilon_{sy}\, dy - \delta_{c1} - \int_0^y \varepsilon_{cy}\, dy \qquad (13)$$

Depending on the location of the place in the structure to be examined and on the stress level of the structure different boundary conditions are obtained for equations by means of which the values ε_{s1}, ε_{c1}, δ_{c1} and δ_{s1} are determined.

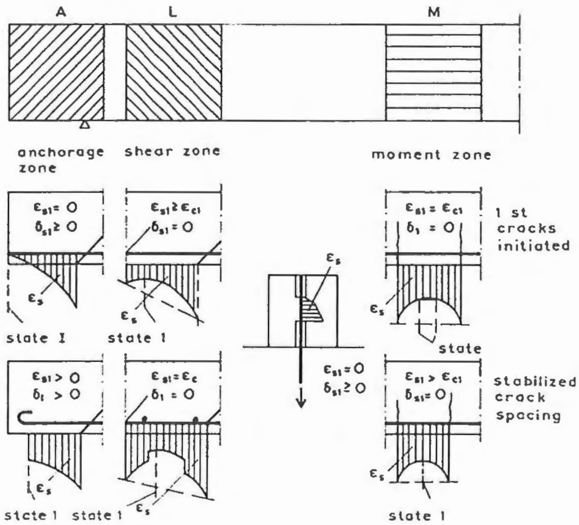


Fig. 5. Common bond stress cases and corresponding boundary conditions.

Equation (8) is the differential equation of the so-called sliding bond, which is based on the fact that there exist slips between the concrete and the reinforcement. In practice, however, it has been found that there are also different kinds of bond mechanism; as a result of this the concrete is detached from the reinforcing bar at a certain distance on both sides of the crack. Since the reinforcing bar (steel) can elongate freely within this distance, it is a factor which increases the crack width and reduces the stiffness. By taking account of the detachment distance in crack considerations means that the sliding and non-sliding bond mechanisms are combined or that the formation mechanism of internal cracks is used in bond considerations. In this study the total detachment lenght was estimated to be 4ϕ.

The solution to equation (8) must generally be made numerically, in which case iteration should also be carried out.

Formation of first crack

A crack is produced in a tensioned reinforced beam when $\sigma_{ct} = \frac{N}{Ai}$ exceeds the value of tensile strength at the weakest point. The increase in stress in the reinforcement is

$$\Delta\sigma_s = \frac{A_{ceff}}{A_s} \cdot \sigma_{ct} = \frac{f_{ctt}}{\rho_{ceff}} \qquad (14)$$

and the stress in the reinforcement at the crack is

$$\sigma_{s2} = \sigma_{s1} + \Delta\sigma_s = \sigma_{s1} + \frac{f_{ctt}}{\rho_{ceff}} \qquad (15)$$

In direct tension, $A_c = A_{ceff}$ and $\rho = \rho_{eff}$ are valid.

Since $\sigma_{ct} \cdot n_s = \sigma_{s1}$ and $\varepsilon_{s1} = \varepsilon_{c1}$ the equations

$$\sigma_{s2} = \sigma_{s1} \left(1 + \frac{1}{n_s \cdot \rho_{ceff}}\right) \qquad (16)$$

and

$$\varepsilon_{s2} = \varepsilon_{s1} \left(\frac{1 + n_s \cdot \rho_{ceff}}{n_s \cdot \rho_{ceff}}\right) \qquad (17)$$

can be derived.

In a bent rectangular beam a crack is formed immediately when $\sigma_{ct} = \frac{M}{W_i}$ exceeds the value of tensile strength at the weakest point.

The increase in steel stress in the reinforcement is

$$\Delta\sigma_s = \frac{\sigma_{ctf} \cdot bh^2}{A_s \cdot 6 \cdot z} = \frac{\sigma_{ctf} \cdot h}{\rho \cdot 6 \cdot z} \approx 0.20 \frac{f_{ctf} \cdot A_c}{\rho_{eff} \cdot A_{ceff}} \qquad (18)$$

The stress in the reinforcement at the crack when $f_{ct} = \sigma_{s1}/n_s$ is

$$\sigma_{s2} = \Delta\sigma_s + \sigma_{s1} = 0.20 \frac{f_{ctf} \cdot A_c}{\rho_{ceff} \cdot A_{ceff}} + \sigma_{s1} \qquad (19)$$

or

$$\sigma_{s2} = \sigma_{s1} \left(1 + 0.20 \frac{A_c}{n_s \ \rho_{ceff} \ A_{ceff}}\right) \qquad (20)$$

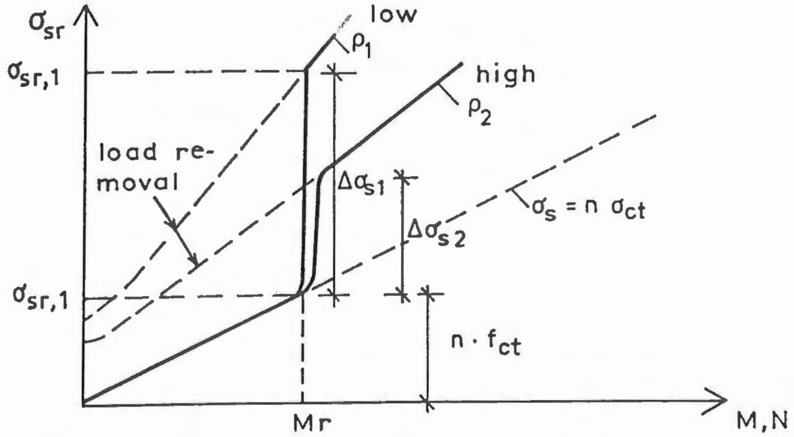


Fig. 6. Dependence of σ_{sr} on the action effect.

Stochastical variabes of concrete

In spite of the apparent homogeneity of concrete its tensile strength is distributed stochastically within a structure. In consequence of this random nature the formation of cracks also occurs in random places in those areas where the smallest local value of the tensile strength has been exceeded.

The stochastical nature of cracking consists of the different factors of uncertainty, which generally consist of

- the random nature of the physical properties of materials
- the uncertainty of the crack models and
- the uncertainty factors of statistical models.

The nearer to the beginning the initiation of crack formation, the greater is the proportion of stochastic cracking. In this case the so- called bond- free zones between the cracks are at their largest, whereas near the stabilized cracking phase the bond-free zones, where new cracks are to be produced, are at their smallest. Expressed in statistical mathematics the stochastic nature of cracking means that the tensile strength of concrete has a certain standard deviation. If there were no deviation, i.e. if the tensile strength were constant, all cracks would be produced at the same moment at which the tensile strength is exceeded in the weakest point.

The determination of the location of a crack is impossible beforehand, but it is necessary that the random nature of the tensile strength of concrete and thereby the stochastic situation of cracks are known no matter how simply, for example, in the calculations of crack widths and average deformations.

A better picture of the random nature of tensile strength can be given by examining the tension zone of a reinforced concrete beam, for example. In principle, the tensile strength can be unequal in size in each cross-section of the tension zone, thus including all the values of the continuous density function of the hypothetical statistical distribution of the tensile strength. In practice, however, the density function is not continuous, since the tensile strength is regionally almost equally high and the regions in question are always limited in number. Thus the tension zone of a reinforced concrete bar or a beam can be assumed to consist of small pieces in the area of which the tensile strength is constant. If there were no reinforcing steels, the beam would fail after the first crack is produced or in a place where the tensile strength of concrete is locally at its lowest. Since the tensile force carried by the concrete is transmitted, however, to the reinforcement, the whole crack propagation corresponding to the tensile strength region can be described by increasing the stress uniformly over the whole area of the structure.

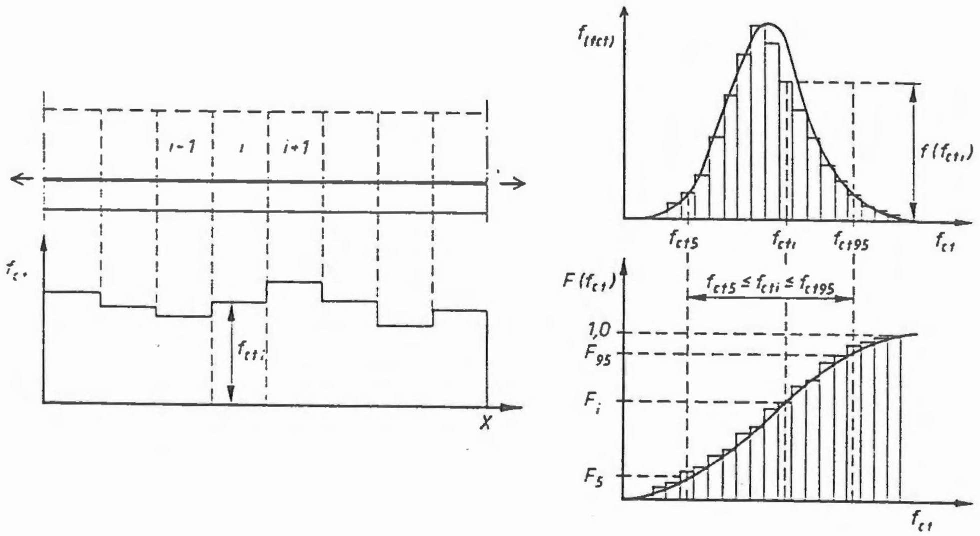


Fig. 7. Idealized tensile strength variations of concrete in the tension zone of a beam /9/.

If the tensile strength were constant, all possible cracks would be produced, once the cracking force has been exceeded, in the area where the tensile strength of concrete is exceeded. Owing to the random nature of the tensile strength, the cracks develop gradually in any of the strength zones including the probability area, these zones lying between the tensile strength values corresponding to the first crack formation and the stabilized cracking phase.

Distributions of random variables are estimated in statistical mathematics using the functions of the derivatives of the continuous cumulative distribution and probability functions or, in other words, by means of density functions. It is very commonly assumed that the strength values of materials are normally distributed. The same applies here to the tensile strengths. Depending on the purpose, it is true that by selecting distribution models which are clearly incompatible with the facts, very satisfactory results can also be achieved.

The said strength zone of the tensile strength, where the development of a crack pattern occurs, can be expressed mathematically, if it is assumed that the steel stress σ_{sr} corresponds to the tensile strength which corresponds to the first crack formation, and that the steel stress f_{sy} corresponds to the tensile strength which corresponds to the last crack formation and that the strength interval is obtained by linear interpolation.

When the tension zone is exposed to the stress $\sigma_i = f_{cti}$, all the parts are stressed in turn at the values $f_{ctlow} < f_{ct} < f_{cti}$. When the stress is $\sigma_i = f_{cti}$, the cracking probability of the elementary unit in question is obtained as the integral of the density function of the tensile strength

$$P_{ir} = P(f_{cti}) = \int_{-\infty}^{f_{cti}} f(f_{ct})\, df \qquad (21)$$

Correspondingly, the probability of noncracking is $1 - P_{ir}$. Since the stress values $\sigma_o < f_{ctlow}$ and $\sigma_i > f_{ctupp}$ do not produce cracks as presumed, the probability that no crack is produced in the elementary unit can be calculated, instead of the value $\int_{-\infty}^{-\infty} f(f_{ct})\, df = 1$, in regard to any strength zone which better corresponds to actual physical facts.

By using normal distribution and the area defined above as a strength zone and by setting the lower limit to 5 % of the proportion falling below the corresponding value with a 50 % probability, and the upper limit respectively to 5 % of the proportion falling beyond the upper characteristic value, the probability of crack formation can be expressed by means of conditional probability as follows.

The probability that in the area of constant stress no new crack will occur between the two existing cracks when the stress is $\sigma_i = f_{cti}$ is

$$P_{nr} = \frac{1 - P_{ir}}{1 - P_{10}} \qquad (22)$$

Fig. 8. shows values of the probability P_{nr} as a function of the ratio $\varepsilon_{s2}/\varepsilon_{sr1}$, when the strength zone is $f_{ct5} < f_{cti} < f_{ct95}$. The probability of crack formation thus increases, with an increase in f_{cti}. At a certain value of f_{cti} the probability of crack formation decreases conversely to an increase in variation coefficient for tensile strength.

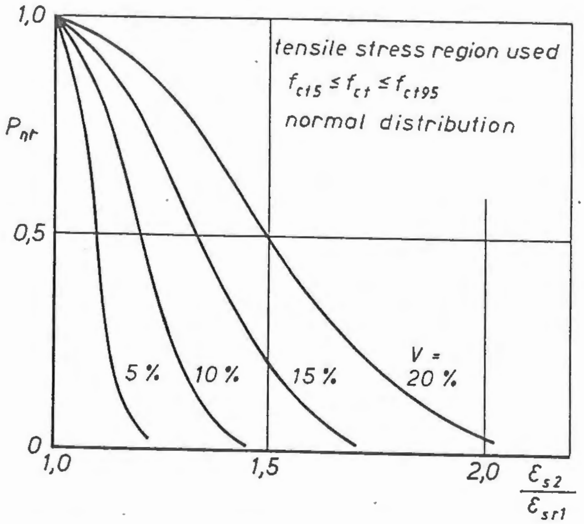


Fig. 8. Probability of non-formation of cracks as a function of stress and the variation coefficient of tensile strength.

Crack propagation

Following the formation of the first crack, a subsequent crack cannot be produced before the tensile force necessary for the new crack to form has been transmitted through the bond from the reinforcement to the concrete. The distance in question, which theoretically is needed for the transmission of force, is the anchorage length of the bond. It is also the smallest possible crack spacing if the tensile strength at that particular point is exceeded.

When the degree of stress increases, new cracks are randomly produced between the already existing cracks or outside at points between or external to the anchorage lengths starting from the cracks and corresponding to the states of stress in question, and where the tensile strength of concrete is at its weakest.

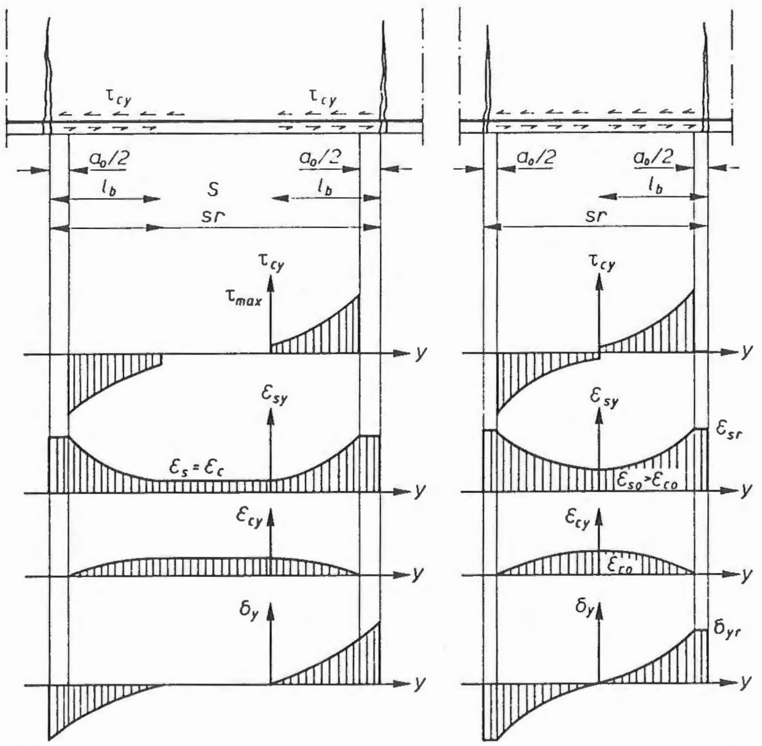
When stress or displacement increase, the number of cracks also increases and the average crack width shortens and roughly approaches the stabilized value. Following this there will be very few new cracks but the crack widths will enlarge with an increase in stress.

The cracking phases can be determined as follows: /9/

a) First, i.e. initial cracking phase when $S_r > 2\ l_b$

b) Crack propagation phase, i.e. primary cracking phase when $S_r \leq 2\ l_b$

c) Stabilized cracking phase, i.e. minimum crack spacing phase when $s_r = s_{rm}$



First cracking phase

Crack propagation phase
Stabilized cracking phase

Fig. 9. Phases of cracking and corresponding stresses and deformations between two cracks.

The final value of the crack spacing is determined by the condition that the increased force causing the cracks must be balanced by the sum total of the tensile stresses of concrete.

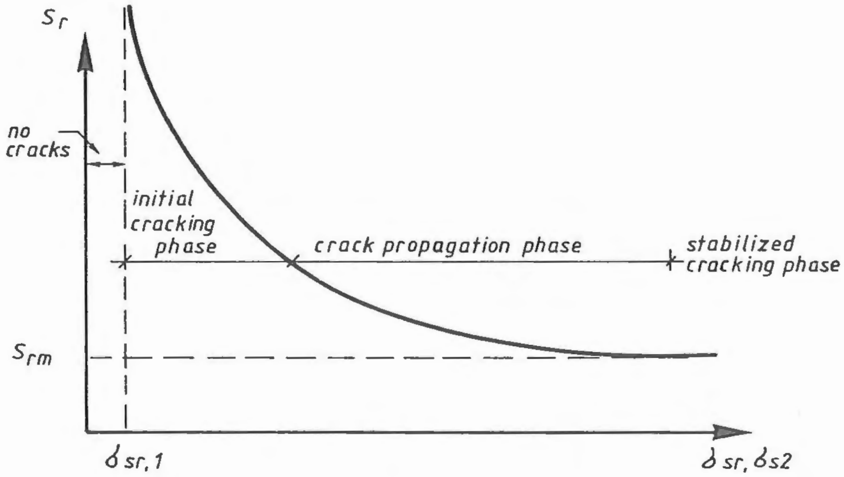


Fig. 10. Interdependence of crack spacing and the degree of stress.

Crack spacing

The crack spacing is dependent on the degree of stress and on the random character of crack formation. It is also a random variable, as are all factors dependent on material properties and structural dimensions. Exact calculational determination of crack spacing, depending on the degree of stress, may be impossible. However, since the smallest and largest approximate numbers of cracks which stress can produce in a structure are known, the mean value of crack spacing in proportion to the minimum crack spacing corresponding to the desired probability can be calculated by making certain simplifications and statistical assumptions based on the properties of the tensile strength of concrete. Corresponding to the analogy of stochastic nature of the concrete tensile strength we can write:

$$P(f_{ct} < f_{cti}) = F(f_{cti}) = \frac{\text{number of cracked sections}}{\text{total number of sections}}$$

$$\text{or} \quad F(f_{cti}) = \frac{\text{number of cracks}}{\text{largest possible number of cracks}} = \frac{n}{m}$$

Where the crack spacing in the crack formation phase is dependent on stress, the variable $s_r = \ell/n$ is at its minimum in the stabilized cracking phase and the quantity $s_{rm} = \ell/m$ a constant. Thus $s_r/s_{rm} = m/n$ and when $\sigma_{ct} = f_{cti}$,

$$\frac{s_r}{s_{rm}} = \frac{1}{F(f_{cti})} \tag{23}$$

The equation (23) describes the development of cracks in the crack formation phase, taking into account the random nature of the tensile strength of concrete.

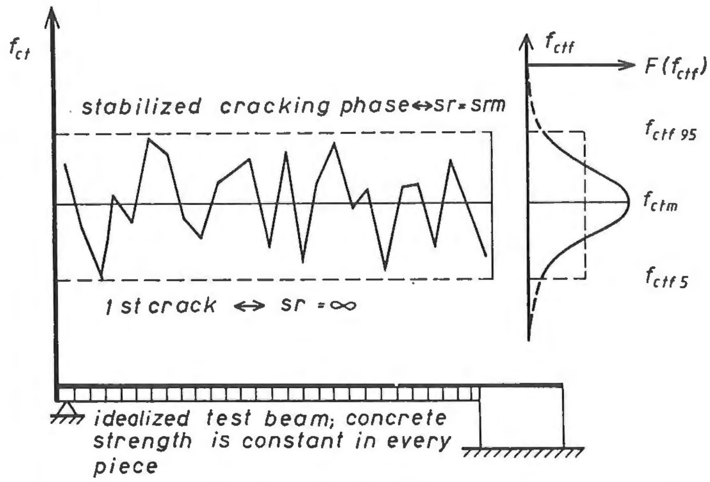


Fig. 11. Modelling of concrete tensile strength in the tension zone of a beam using a probability density function of normal distribution.

If the distribution is restricted to the lower and upper parts and if the distance $\Delta f_{ct} = f_{ctupp} - f_{ctlow}$ is assumed as the useful region of tensile strength the relation s_r/s_{rm} can be roughly expressed as the relation between the strain in steel at the crack and the strain in steel at the first crack, using different values of the coefficient of variation of tensile strength, since the value of strain in steel at the crack under tensile load, when the tensile strength has a value of f_{cti} is approximately

$$\varepsilon_{s2} = \frac{f_{cti}}{\rho_{ceff} \cdot E_s}(1 + \rho_{ceff} \cdot n_s) \approx \frac{f_{cti}}{\rho_{ceff} \cdot E_s} \tag{24}$$

since $\varepsilon_{c1} \approx 0$

and under bending load approximately

$$\varepsilon_{s2} = n_s \cdot \frac{f_{cti}}{E_s} + 0.2 \frac{A_c \cdot f_{cti}}{\rho_{eff} \cdot A_{ceff} \cdot E_s} \approx 0.2 \frac{A_c \cdot f_{cti}}{\rho_{ceff} \cdot A_{ceff} \cdot E_s}$$

$$\approx 0.2 \frac{f_{cti}}{\rho \cdot E_s} , \qquad (25)$$

since $\varepsilon_{c1} \approx 0$.

The value of strain in the steel at the first crack under tensile stress is correspondingly

$$\varepsilon_{sr,1} = \frac{A_{ceff}}{A_c \cdot E_s} \cdot f_{ct5} = \frac{f_{ct5}}{\rho_{ceff} \cdot E_s} \qquad (26)$$

and under bending stress

$$\varepsilon_{sr,1} = 0.2 \frac{A_c \; f_{ct5}}{\rho_{ceff} \cdot A_{ceff}} \qquad (27)$$

or in other words, in both cases

$$\frac{f_{cti}}{f_{ct5}} \approx \frac{\varepsilon_{s2}}{\varepsilon_{sr,1}} \qquad (28)$$

For calculating the function $1/F\ (f_{cti})$, the values $f_{cti} = (1 \pm k_i\ v)\ \ f_{ctm}$ and $f_{ct5} = (1 - 1.64\ v)\ f_{ctm}$ must be determined, from which the values of the variable $\varepsilon_{s2}/\varepsilon_{sr,1}$ can be obtained. The coefficients k_i of standard deviation corresponding to the cumulative distribution function $F(f_{cti})$ can be obtained from the table of normal distribution. The dependence of crack spacing on the degree of tension in different phases of cracking can be determined from the formula (29) when the tensile strengths of concrete in different sections of the tension area of the structure are normally distributed.

$$\frac{1}{F(f_{cti})} = \frac{s_r}{s_{rm}} \qquad (29)$$

The distribution of tensile strengths can also be simplified by assuming it to be e.g. rectangular distribution. If the same strength area is still used the following can be obtained:

$$f(f_{ct}) = \frac{1}{f_{ct95} - f_{ct5}} \qquad (f_{ct5} \leq f_{ct} \leq f_{ct95}) \tag{30}$$

and

$$F(f_{ct}) = \int_{f_{ct}} f(f_{ct})\, d\, f_{ct} = \frac{f_{ct}}{f_{ct95} - f_{ct5}} + A \tag{31}$$

From the boundary condition $f_{ct} = f_{ct5}$: $f(f_{ct}) = 0 \rightarrow A = \dfrac{-f_{ct5}}{f_{ct95} - f_{ct5}}$

$$\text{or} \quad F(f_{ct}) = \frac{f_{ct} - f_{ct5}}{f_{ct95} - f_{ct5}} \tag{32}$$

$$\text{so} \quad \frac{s_r}{s_{rm}} = \frac{\dfrac{f_{ct95}}{f_{ct5}} - 1}{\dfrac{f_{cti}}{f_{ct5}} - 1} \geq 1 \tag{33}$$

Fig. 12 shows different crack spacings in relation to the distribution of tensile strength. The normal distribution can approximately be compensated by rectangular distribution, if the cracking phase is not yet stabilised.

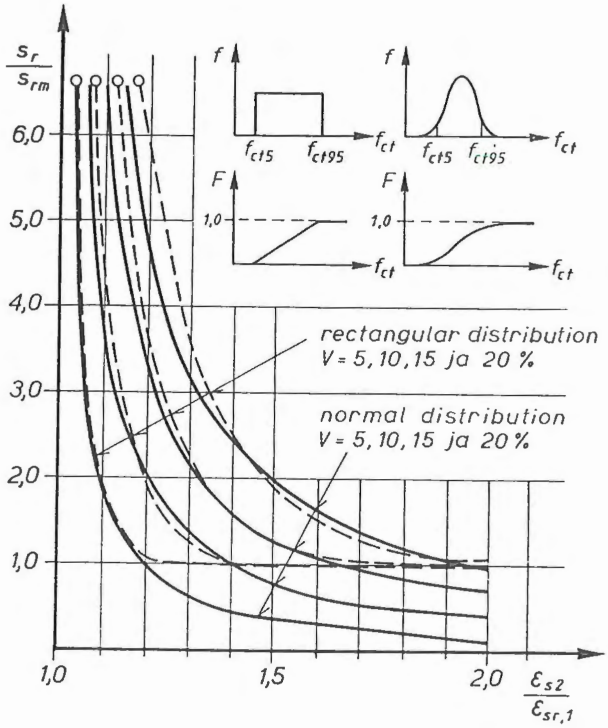


Fig. 12. Dependence of crack spacing on the degree of stress when the tensile strength is normally and evenly distributed.

RESULTS

Mean steel strains

The general distributions and values of stresses and strains are obtained as a numerical solution to the differential equation derived from observations of basic bond equations of the elementary structural unit. Through these distributions and values and crack spacing dependent on the degree of stress determined on a probability basis the average value of strain in steel and the crack widths can be determined. The formation of new cracks is taken into account in the form of the continuously diminishing crack spacing.

The deformations of the entire structure are calculated from the deformations of an elementary structural unit.

A section of beam, equivalent to the length of an average crack spacing corresponding to the degree of stress in question, comprises the elementary structural unit. It is therefore assumed that the average deformation in the region of the elementary unit is equal to the mean value of deformations of crack spacings of varying length.

Because of different boundary conditions in different cracking phases, the strain equations corresponding to each phase must be considered separately.

Approximate formulae can be derived as follows:

Initial cracking phase $s_r \geq 2\, l_b$

$$\varepsilon_{sm} = \frac{\frac{\rho \cdot n_s}{1 + \rho \cdot n_s}\, \varepsilon_{sr}\, (s_r - \frac{2}{3}\, l_0) + w_m}{s_r} \qquad (34)$$

In the formulae, $l_0 = l_b + 2\, \phi$

Crack propagation phase $s_r < 2\, l_b$

Average strain is

$$\varepsilon_{sm} = \frac{w_m}{s_r} \qquad (35)$$

Stabilized cracking phase

$$\varepsilon_{sm} = \frac{w_m}{s_{rm}} \qquad (36)$$

Figure 13 shows curves of average stress calculated using a numerical method, for different strengths of concrete and different variation coefficient values of concrete tensile strength. The curves are based on the values of tensile strength of concrete presented in /10/. The limits corresponding to the different phases of cracking are marked by a broken line. It is noticeable that e.g. in the case of concrete K 30, the crack spacing can not be stabilised prior to the yield of reinforcement, if the amount of steel ρ_{eff} < 0.8...1.0 % depending on the dispersion of tensile strength. If the concrete is harder it is only possible to reach the stabilized cracking phase prior to the yield of reinforcement when $\rho_{eff} \geq$ 1.0...1.5 % depending on the dispersion of tensile strength.

In the service state the stabilized cracking phase can be produced as a result of the steel stresses (approx. 250 N/mm^2), if the amount of steel ρ_{eff} is at least 1.1...1.7 % in concrete K 30 and 1.7...2.5 % in concrete K 50. The variation is due to the dispersion of tensile strength. It is therefore to be expected that in the reinforcement no stress resulting in a stabilized cracking phase according to the standards can be produced, even by load effects, not to mention the restraint effects.

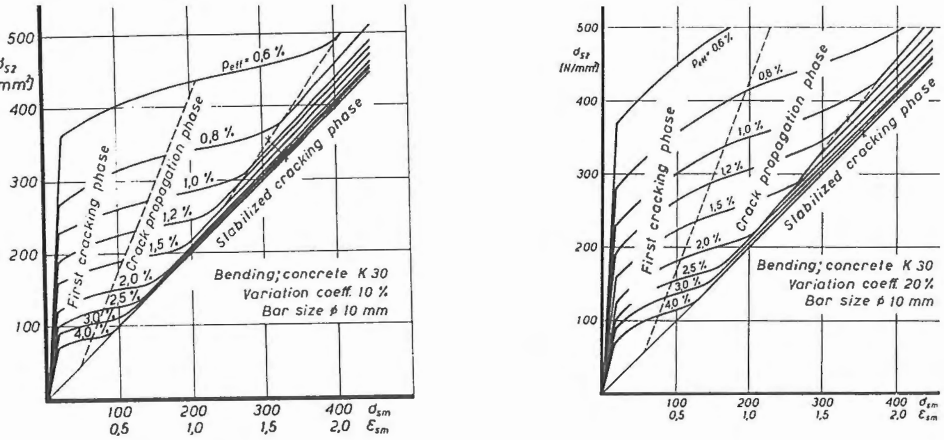


Fig. 13. σ_{s2} - σ_{sm} curves for concrete K 30 and K 50, reinforcement A400H and bar diameter ø 10 mm /9/.

Crack width

Calculating the crack widths is based on the fact that the difference between relative strains in the reinforcement and the concrete is integrated over the crack spacing in each state of stress. The crack spacing is thus a quantity that changes according to stress. Its length is determined by the interaction of concrete and reinforcement.

The interaction is based partly on the sliding bond and partly on the non-sliding bond, which causes a disturbance in the interaction near the cracks. Crack widths in different phases of cracking are determined as follows:

Initial cracking phase

$$w_m = 2 \cdot \int_{y=0}^{y=l_b} (\varepsilon_{sy} - \varepsilon_{cy})\, dy + 2 \int_{l_b}^{l_b + \frac{a_0}{2}} \varepsilon_{sy}\, dy \qquad (37)$$

where

$$l_0 = l_b + 2\,\phi \tag{38}$$

Crack propagation phase

$$w_m = 2 \int_{y=0}^{\frac{s_r}{2} - \frac{a_0}{2}} (\varepsilon_{sy} - \varepsilon_{cy})\, dy + 2 \int_{s_r - \frac{a_0}{2}}^{s_r + \frac{a_0}{2}} \varepsilon_{sy}\, dy \tag{39}$$

Stabilized cracking phase

$$w_m = s_{rm} \cdot \varepsilon_{sm} + a_0 \cdot (\varepsilon_{s2} - \varepsilon_{sm}) \tag{40}$$

where

$s_{rm} \approx l_b + 2\,\phi$

$a_0 = 4\,\phi$

Calculated design curves are presented in Figs. 14 and 15. The first can be used when pure restraint effect is acting. The second is suitable when load effects and restraint effects are acting simultaneously.

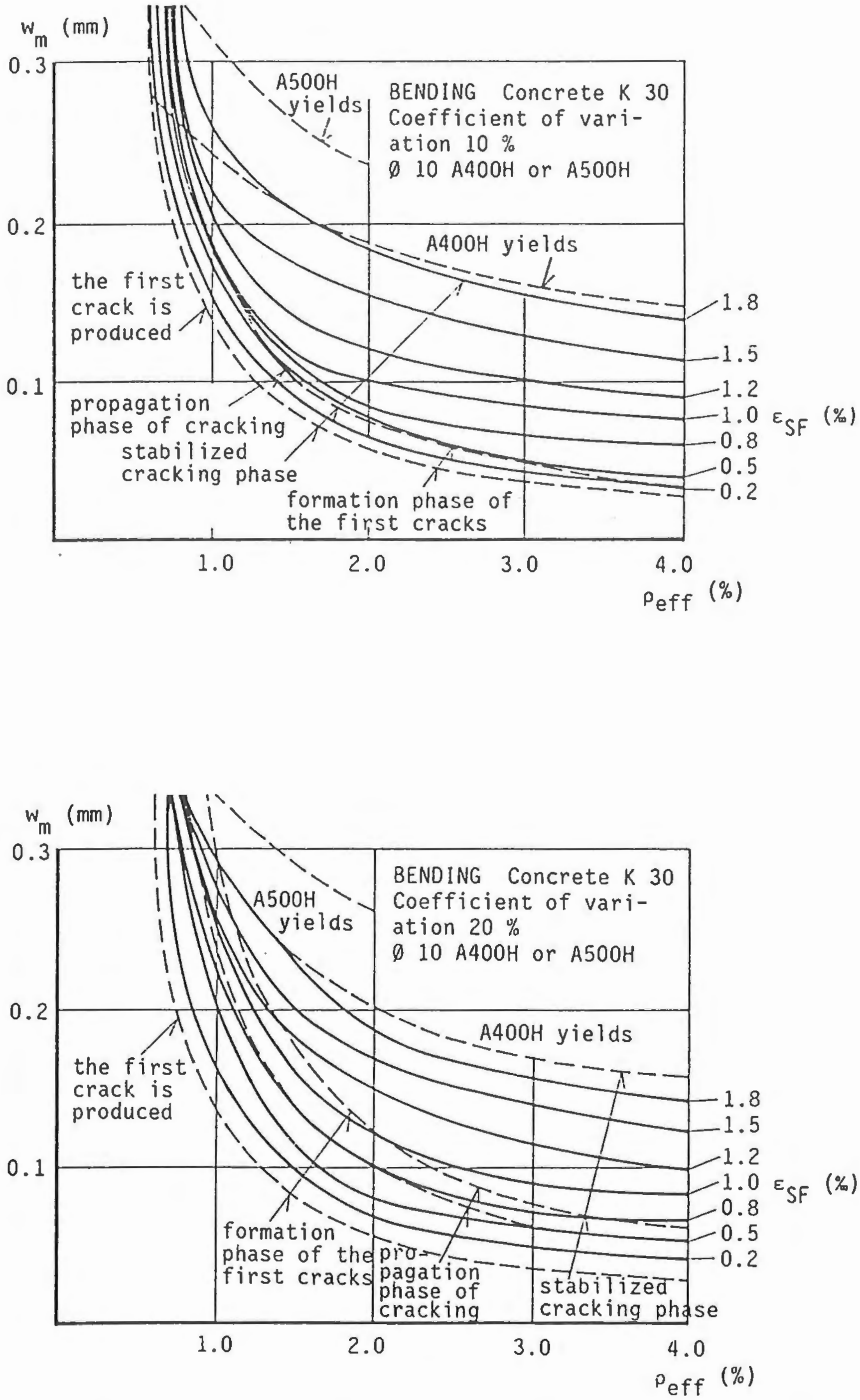


Fig. 14. Design diagram for crack width caused by restraint bending /9/.

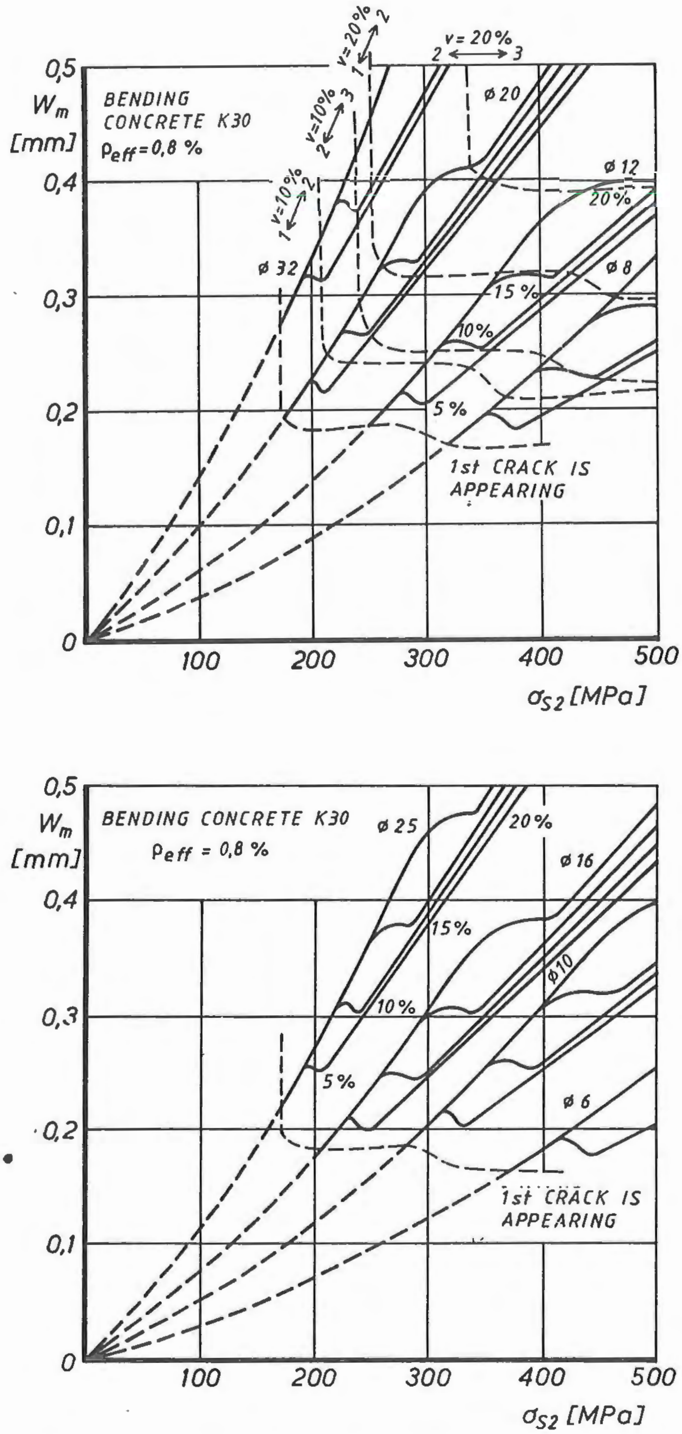


Fig. 15. Crack limitation diagram for bending caused by load effect or combined load and restraint effect /9/.

EVALUATION OF RESULTS

Average stresses in steel

The calculated σ_{s2} - σ_{sm} curves shown in Fig. 13, indicate that the deformation behaviour of a reinforcing bar in a cracked state greatly depends on whether cracking has reached its stabilized phase or not.

It can thus be stated that in such cases, which often occur in practice, for example

- in lightly reinforced structures,
- in a mere restraint strain,
- in prestressed structures and
- under combined stress of a compressive normal force and a moment,

neither is the cracking pattern stabilized nor does classical methods give a sufficiently accurate result to the calculation of crack widths.

On the other hand, when using the method presented here the crack width is determined by the steel strains corresponding to each stress level and by the transfer lengths of bond, as well as by the crack spacing model describing the stochastic nature of cracking, also with regard to the effect of possible damage to bond in the close vicinity of cracks. Since the determination of the calculational values is based on actual bond properties, all the parameters that affect these properties can be changed, if necessary, and a result can thus be obtained which corresponds as closely as possible to physical behaviour.

In comparing the validity region $\sigma_{sm} \geq 0.4\ \sigma_{s2}$ of the formula for average steel strain, in compliance with the CEB Model Code, with the boundary of the initial cracking phase conforming to the calculations presented here, it is interesting to note that the boundary is always in the region of the initial cracking phase. This means that the boundary lies in the stress region where the crack width will be underestimated when using the formula of the CEB Model Code. According to the statistical method the boundary should be approximately

$$\sigma_{sm} \geq 0.4 \cdot \sigma_{s2} + \sigma_{s1} \qquad (41)$$

Tension stiffening effect of concrete

The factor which greatly contributes to the deformation behaviour of a reinforced concrete structure is the tension stiffening effect of the concrete between the cracks, which can be described mathematically by the formula

$$\Delta\varepsilon_s = \varepsilon_{s2} - \varepsilon_{sm}. \qquad (42)$$

In order to describe the stiffening effect the empirical Rao model is perhaps used most frequently /19/. In Fig. 16 the values of the stiffening effect calculated by the used calculation method are compared to the values compatible with the Rao model as a function of the stress level. It is noted that when the variation coefficient for the tensile strength is 20 % the Rao model underestimates the stiffening effect in almost the entire region of the service state. When the coefficient of variation is 10 % the difference at the immediate onset of crack formation is not significant as compared with the Rao model; however, immediately after the initial cracking phase the Rao model overestimates the stiffening effect. Underestimation of the stiffening effect means that deformations and cracks are larger, whereas overestimation is a sign of smaller deformations and crack widths. A similar doubt concerning the validity of the Rao model is also expressed in references /18/ and /23/.

Fig. 17 shows a relative decrease in the tension stiffening effect in comparison with its greatest value corresponding to the first crack and also in comparison with the instantaneous increase in steel stress following crack initiation, when the bar size and the coefficient of variation change.

It is noted that, proportional to a more rapid decrease in the stiffening effect,

- the steel amount is larger,
- the variation coefficient for concrete is lower,
- the bar size is larger (bond weakens).

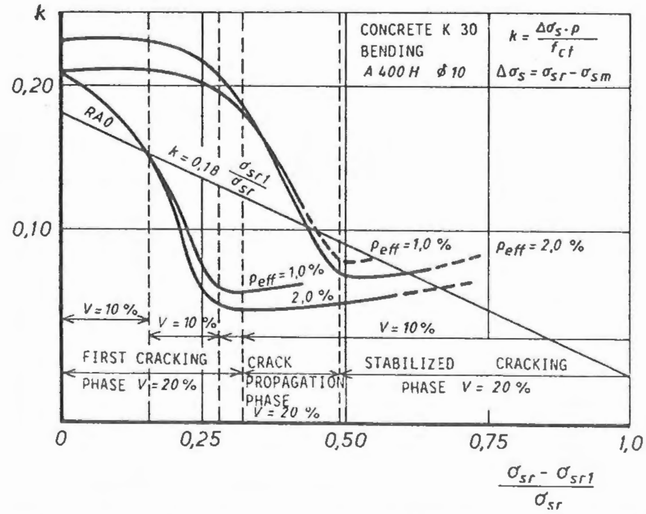


Fig. 16. Tension stiffening according to the calculation method used, compared with Rao:s formula.

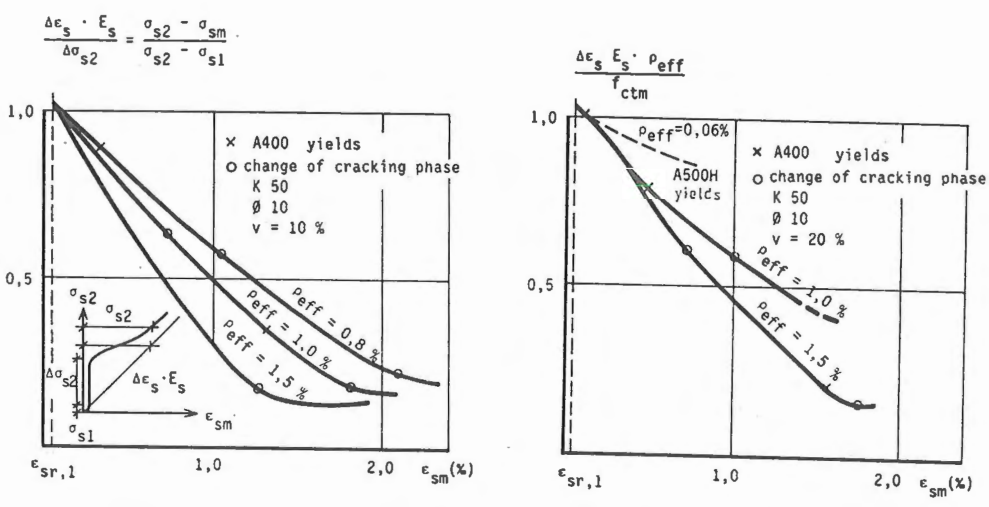


Fig. 17. Relative degree of tension stiffening according to the calculation method used.

Crack widths

From the curves describing average calculated crack widths as a function of the steel stress it appears that the crack width corresponding to the initiation of the first crack increases with a decrease in the effective reinforcement area, with increasing bar size and increasing strength of concrete. The dispersion of the tensile strength of concrete does not affect the initiation of the first crack, but the greater the dispersion, the smaller is the almost linearly increasing portion of the upper curve of each bar size describing initial cracking. In the initial cracking phase the crack width thus increases almost linearly as far as the steel stress σ_{s2} is concerned.

In the crack propagation phase, on the other hand, an increase in crack width is retarded or the widths in a certain stress zone do not increase at all, since there are then no non-sliding bond zones and the formation of cracks takes place in such rapid succession that the average widths of the previous cracks have no time to increase. In this research use is made of a model which estimates the crack spacing and in which the crack formation density is at its maximum at the point where the stress state corresponds to the average tensile strength of concrete (mode of the density function). If the dispersion is small, the formation rate of cracks becomes so great that a point of discontinuity is developed on the model curve describing the crack spacing as a function of the stress, and the crack widths corresponding to this stress zone may even slightly decrease as a result of calculation. This phenomenon is perceptible especially when the dispersion (variation coefficient about 5 %) is small and the bar size increases. An absolute decrease in crack width is at its maximum of the order of 0.01...0.2 mm only. In using the dimensioning curves the crack widths corresponding to this small stress zone can be assumed to be constant without committing a major error. In this cracking phase the crack width thus clearly depends on the crack spacing and the steel amount as well as on the dispersion of tensile strength.

In the stabilized cracking phase the crack spacing is at its minimum and constant, and the average steel strain increases almost linearly.

Consequently the crack width also increases linearly with the steel stress in steel. The increase is not as sharp as in the initial cracking phase.

Considering the effects of different parameters on the crack width the following can be stated: Since the average steel strain decreases while the crack spacing remains unchanged, the average crack spacing decreases with increasing concrete strength. In the weak bond zone crack widths obtained at the same average strain value are larger than in the good bond zone. Regardless of the concrete and the degree of reinforcement an increase in the crack width is 30 to 40 %, if the dimensioning diagrams have been plotted on the good bond zone. A crack width obtained with a bar having weak bond is larger than that obtained with one having good bond properties, since the crack spacing increases while the bond properties weaken.

In the crack diagrams where the average steel strain (restrained strain) serves as a parameter, the lower broken line represents the initiation of the first crack. As is shown, the crack propagation phase is reduced to almost one line, since the crack width in this phase does not increase much with an increasing average steel strain, particularly when the dispersion of the tensile strength is small. Thus the initial cracking and stabilized cracking phases are more pronounced on the diagram.

NOTATION

Capital Latin letters

A_s	area of reinforcement cross-section
$A_{c,eff}$	effective area of concrete cross-section
E_c	modulus of elasticity of concrete

$(EI)_{eff}$	effective stiffness
E_s	modulus of elasticity of steel
F(fctf)	cumulative distribution function or probability mass function of flexural tensile strength
K	nominal strength of concrete, flexural stiffness of a cross-section
M	bending moment
M_F	restraint moment
M_q	moment of variable external load
N	normal force
P	tendon force, probability
T	temperature

Small Latin letters

a_0	length of whole region around crack where the bond is damaged
f_{ct}	tensile strength of concrete
f_{ctf}	flexural tensile strength of concrete
k	coefficient, relative value
l	length
l_b	anchorage length
m	relative moment, number
n	relative normal force, coefficient, number
n_s	elastic modulus ratio of steel to concrete
s_r	average crack spacing
s_{rm}	average crack spacing in the stabilized cracking phase
v	coefficient of variation
w	crack width
w_m	mean crack width
w_k	characteristic value of crack width
x	distance of neutral axis from the top surface of a structure
y	co-ordinate

Capital Greek letters

Δ	difference, increase, settlement
ϕ	diameter of steel bar
Σ	sum (total)

Small Greek letters

ε_c	relative deformation of concrete
ε_m	relative mean strain
ε_s	relative strain of reinforcement
ε_{ch}	creep of concrete
ε_{cs}	shrinkage of concrete
ρ, ρ'	geometrical reinforcement ratio (lower part, upper part)
ρ_{eff}	relative effective reinforcement area
σ_s	stress in reinforcement
$\sigma_{sr,1}$	stress in reinforcement at initation of the 1st crack
τ_{cy}	shear stress in concrete in direction y

Subscripts

F	quantity describing restraint
e	elastic
m	mean
p	prestressed
q	variable load
r	cracking
0, 1, 2, 3...	specific values of quantities
1	uncracked state
2	cracked state

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