SOME QUESTIONS OF THE DYNAMICAL TASK OF THERMOELASTICITY

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AIM OF THE INVESTIGATION

The experience is that the change of temperature causes deformation and this is true the other way round as well. For this reason in elasticity examinations - strictly speaking - the thermodynamical effects can not be disregarded, the deformation and temperature field has to be dealt with coupled. It is to be noted that this coupling is the most frequent in elasticity but the scientific literature deals with coupling other fields as well e.g.: electric and magnetic fields /33/.

In stationary TE (thermoelasticity) tasks however the differential equation system describing the phenomenon breaks up and the temperature field can be determined separately, and with the knowledge of this the deformation field can be produced. For this reason certain thermodynamical tasks belong to the circle of TE.

In case of quasi-static tasks in order to simplify the solution it is customary to disregard the quantity realizing the coupling. This causes estimable inaccuracy.

The scientific literature mentions the circle of uncoupled dynamical tasks of TE as well, this however - because of its inaccuracy - tells little about the phenomenon.

On the basis of what was mentioned above, the dynamical tasks of TE have to be regarded as coupled as well.

In the technical problems of recent times dynamical tasks are brought up more and more frequently for in the field of reactor technology, space research, supersonic flying, modern weapons, magneto hydrodynamical generators, fast moving internal combustion engines the changes in time are rather fast.

Though the idea of the coupled discussion of the deformation and temperature fields is not new, the intensive examinations started in the 1950's only /44, 45/. The introduction of that form of the heat conduction law which is valid in case of dynamical tasks was suggested by Maxwell in 1867.

The examination of the temperature wave with the application of the Fourier's heat conduction law results infinite wave velocity. For this reason the aim of the modification of the heat conduction law is twofold. On one hand theoretical - there cannot be infinite velocity in the nature, on the other hand - in case of certain tasks to increase precision. Danilovskaya - tho whom the solution of the first dynamical task of TE is related - in the course of her examinations experienced that in certain characteristics changes appear before the disturbance front already. Some scientists e.g.: Parkus, Barenblat etc. think that the reason of this is the structure of the heat conduction law. The explanation for this is that the heat conduction task formulated on the basis of the Fourier's heat conduction law leads to a parabolic-type differential equation, from which the velocity of the temperature wave has an infinite value. The idea is obvious that with the modification of the heat conduction law we get a hyperbolic equation from which the wave velocity has a finite value.

It is not by chance that the question of the deficiency of the heat conduction law occurred exactly within the scope of TE. The reason of this is that the velocity of the elastic wave and the accompanying thermodynamic phenomenon is at least one order of magnitude bigger than that of the temperature disturbance.

On the basis of these it seemed expedient to include the equations of TE in one uniform system, make them applicable in thermodynamical tasks and apply them. On the other hand - just because of the character of the task - at the examinations we have taken into consideration the necessity and possibility of experimental control.

Method of investigation

The equations of TE consist of the connections of elasticity and thermodynamics. The former completely the latter partially is treated by continuumechanics /7, 25, 33/.

Because of the character of the problem extensive mathematical apparatus is needed. For the description differential geometry for the solution theory of partial differential equations /4, 8/ and numerical methods are needed.

The possible temperature ranges make need of more that the basic knowledge of materials customary in elasticity /58/.

The experimental controlling methods need further knowledge of materials and electrodynamics /9/.

What concerns the technic of the examinations we progress from the connections of universal validity towards the special case. Where needed, special attention is paid to the numeric control and the elucidation of the experimental possibilities.

Brief review of the scientific literature

The scientific literature of the TE is rather extensive. Numerous technical books deal with the TE in general and with thermal stresses /1, 2, 3, 5, 26, 27, 29, 31/. Most of this works devote a separate chapter to dynamical problems /1, 2, 5, 27, 29,31/ and many of the books outline first of all dynamical problems /30, 34/.

Though the basic connections of TE have been laid earlier, the first actual dynamical task of TE was - according to our knowledge - solved by Danilovskaya at the beginning of the 1950's /44, 45/. According to her results certain characteristics change before the appearance of the wave frontal ready - which in the opinion of some scientists - e.g.: Parkus and Barenblat - can be explained by the structure of the heat conduction law. So joining Maxwell - who raised the deficiency of Fourier's heat conduction law in 1867 already - numerous scientists e.g.: Braun /10/, Gurtin, Pipkin /16/, Farkas /20, 35/, Gyarmatri /39/, Meixner /36/, Vogel /52, 53, 54, 55/, etc. dealt with the modification of the heat conduction law or the application of the modified heat conduction law in TE e.g.: Puri/14/, Popov /15/, Baltov, Vassilev /40/, Beevers, Engelbrecht /13/, Lukov /23/, Norwood, Warren /38/, Lord, Shuman /37/, etc. The application of that form of the modified heat conduction law which was used by the scientists mentioned above needs the knowledge of the relaxation time. This was determined by Maurer /11/ on the basis of the structure of material.

The coupled equations of the deformation and temperature fields will later on be stated and used relying on Biot /17/, Boley, Weiner /2/, Nowacki /1/, Parkus /5/ and Goldenblat /3/.

With the equations determined by them numerous authors have made numerical calculations in the circle of dynamical problems of TE. So among others Boley, Hetnarski /42/, Nickell, Sackmann /43/, Takeuti /49/, Kiss Lajos /46/, Ignaczak, Hatnarski.

Bargman /2/, Nigul, Engelbrecht /32/, Takeuti /51/ etc. dealt with including the equations serving to examine the dynamical problems of TE into one system.

Vogel and his colleagues completed calculations and measurements for the examination of the modified/parabolic/heat conduction equation /52, 53, 54, 55/.

The effort to find information about the experiments and experimental methods belonging to the circle of dynamical problems of TE was unsuccessful.

Short summary of the examinations

Basic equations of thermoelasticity

Let us sum up basic equations of thermoelasticity on the basis of the scientific literature. Let us apply the additive and differentiating convention and do the examinations in a coordinate system of Descartes. Let us presume small deformations and reversible processes.

The equations can be stated according to the basic equations of continuummecahnics /7, 25/, TE /1, 2, 3, 5,29/, and thermodynamics /18, 19, 47, 23/.

The kinematic equation expresses the coupling of the ε_{ij} deformation and u_i displacement fields:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

Here ε_{ij} and u_i are the function of place and time. The equation of motion is used in the form of

$$\frac{1}{\rho}\,\sigma_{ij,j} + g_i = \ddot{u}_i \tag{2}$$

where $\sigma_{ij} = \sigma_{ji}$, respectively σ_{ij}, g_i are the function of place and time, whereas ρ is the function of place and temperature. The constitutive equation in case of linearly elastic continuum is:

$$\varepsilon_{ij} = a_{ijkl}\,\sigma_{kl} + \int_0^{\theta} \alpha_{ij}\, d\theta\ , \tag{3}$$

where the θ temperature difference is the function of place and time, a_{ijkl} and α_{ij} are the functions of place and temperature. The first law of thermodynamics has the form

$$\rho\dot{u} = \sigma_{kl}\, v_{kl} - h_{k,k} + \rho\, r \tag{4'}$$

where u, v_{kl}, h_k and r are the function of place and time. Mention has to be made that from (4') with disregarding the density of the internal source of heat and making use of the connection between the heat flux intensity and the quantity of heat after settling we get

$$\rho du = \sigma_{ij}\, d\varepsilon_{ij} + dq. \tag{4}$$

The relation (4) will be used later on. The form of the second law of thermodynamics

$$dq = Tds. \tag{5}$$

Here the q quantity of heat, the s entropy and the temperature T are the function of place and time. Finally the heat conduction equation on the basis of Fourier's heat conduction law

$$\dot{q} = r + (\kappa_{ij}\,\theta_{,j})_{,i} \tag{6}$$

where κ_{ij} is the function of place and time.

With the equation system (1-6) arbitrary problem of TE can be examined. The unknown ε_{ij}, u_i, σ_{ij}, T, q,u and s can be determined as there is further connection between them. however the solution of this partial differential equation system with the appropriate additional conditions is not known yet for the general case.

The researcher dealing with mechanics is from the above unknowns first of all interested in σ_{ij} stress, ε_{ij} strain, u_i displacement and T temperature. According to this it is customary to trace the equation system back to a system with two unknowns which consists of the so called general equation of motion and the general heat conduction equation.

The modified heat conduction law

The one dimensional dynamical task of heat conduction - in case the temperature dependence of the material is disregarded - with paying no attention to the mechanical characteristics interaction is described on the basis of Fourier's heat conduction law by the

$$a\,T_{xx} - T_t = 0 \tag{7}$$

differential equation. Taking into consideration the temperature dependence of the material characteristics in the equation (7) would result further parts. Neglecting these parts would decrease the precision, does not change the statements, however, made on the basis of the qualitative examinations of the equation.

The analysis of the differential equation (7) shows that the temperature wave propagates with infinite speed, which is in contradiction with the experience. To solve this contradiction instead of (7) the

$$a\,T_{xx} - T_t - \tau T_{tt} = 0 \tag{8}$$

connection is suggested /10, 13, 14, 15, 16, 31, 52, 53, 54, 55/. According to this the velocity of the temperature is

$$v_T = \sqrt{\frac{a}{\tau}} \tag{9}$$

which in case of $\tau \neq 0$ is a finite value. Experiments have been made to determine the numerical value of τ with the starting point of material structural characteristics /11/.

The production of the equation (8) is basically possible in two ways /10/: we modify either the heat conduction law of the equation of state. On the basis of the above introduction hereupon we will display the generalized method of modification.

let us examine a one dimensional dynamical task of heat conduction. Let us neglect the mechanical interactions and the temperature dependence of the material characteristics and let us presume that the material is homogenous,isotropic. According to the first law of thermodynamics

$$\rho u_t = h_x. \tag{10}$$

The equation of state adequately to the experience should be adopted in

$$\varphi(u,T,T_x,T_t) = 0 \tag{11}$$

form. The heat conduction law is looked for adequately to the experience too in

$$\Psi(h,h_t,R,T_x,T_t) = 0 \tag{12}$$

form. It is worth noting that (11) and (12) can be stated in a more general form as well, these are considered to be the first approach of the task.

The solution of the set task can be received through the solution of the partial differential equation system (10-12). For the system the following specifications can be made. Neither the equation of state nor the heat conduction law can be the functions of place and time so the equations (11) and (112) can not contain explicitly x and t.

(These conditions have already been taken into consideration.) Because of the wave character of the tsk the equation system in the whole range concerning the examined body, in every possible initial and boundary condition has to lead to secondary, hyperbolic equation. On the basis of experiments of universal validity further conditions can be laid down, these at the present examinations are neglected.

Differentiated (11) according to t and (12) according to x and substituting them into (10) then arranging

$$\frac{\rho}{\varphi_u}[\Psi_h(\varphi_T T_t + \varphi_{T_x} T_{xt} + \varphi_{T_t} T_{tt}) +$$

$$+ \Psi_{ht}(\varphi_T T_{tt} + \varphi_{T_x} T_{xtt} + \varphi_{T_t} T_{ttt}) = \qquad (13)$$

$$= \Psi_T T_x + \Psi_{T_x} T_{xx} + \Psi_{T_t} T_{tx}.$$

can be written. In order to simplify the examination - in accordance with the customs - let us write that

$$\varphi_u = 1 \text{ es} \qquad \Psi_h = 1. \qquad (14)$$

The received differential equation concerning T is secondary hyperbolic if the next conditions are fulfilled /4/:

$$\Psi_{ht}\varphi_{Tt} = 0 ,$$

$$\Psi_{ht}\varphi_{Tx} = 0, \qquad (15)$$

$$(\varphi_{Tx} - \frac{\Psi_{Tt}}{\rho})^2 + 4(\varphi_{Tt} + \Psi_{ht}\varphi_T)\frac{\Psi_{Tx}}{\rho} > 0.$$

On the basis of these we can examine a few definite cases (chart No 1). It can be seen that the equation of state and heat conduction law of the usual form can not be valid at the same time. The variation however in agreement with the line No 2 or 3 in the chart is possible.

The analysis of the equation system of thermoelasticity

With the utilization of the basic equations introduced in chapter Basic equations of thermoelasticity and the

$$T = \sum_{i=o}^{\infty} \alpha_i s^i \tag{16}$$

connection in case of big temperature change and coupled task after arrangement we get the general equation of heat conduction:

$$\begin{aligned} &\dot{T}[\rho c(\ln \frac{T}{T_o} + 1) - \beta_{kl}\varepsilon_{kl}] - T\beta_{kl}\dot{\varepsilon}_{kl} - \\ &- (\rho c \frac{\dot{T}}{T} - \beta_{kl}\dot{\varepsilon}_{kl}) \sum_{i=o}^{\infty} i\alpha_i (c\ln \frac{T}{T_o} - \frac{1}{\rho}\beta_{kl}\varepsilon_{kl})^i = (\kappa_{kl}T_{,k})_{,l}. \end{aligned} \tag{17}$$

Let us make a few remarks in connection with this. If from series (16) we only take into consideration the part of zero order, then $\alpha_o = T_o$. This is the condition of small temperature change with which (17) results

$$c\rho\dot{T} - T\beta_{kl}\dot{\varepsilon}_{kl} - \beta_{kl}\varepsilon_{kl}\dot{T} = (\kappa_{kl}T_{,k})_{,l}. \tag{18}$$

Comparing this with the general equation of heat conduction concerning small temperature change customary in the scientific literature the difference in one member can be seen.

With the help of the equations introduced above any problem of TE can be examined. The solution of the system, however, effective for the general case is not known because mathematical difficulties. Therefore any simplification leading to a special task is significant. It is customary to write the equations referring to a homogenous, isotropic continuum that has material characteristics which do not depend on temperature. Applying the connections to quasistatic or stationary problems means further simplification.

In case of isobar, isochor, isotherm and adiabatic processes the equations can be written in an easier - mathematically better treatable - form. So we get the next equations for the strain and stress:

$$\sigma_{ij} = -(3\lambda + 2\mu)\alpha_t \, \theta\delta_{ij} ,$$

$$\varepsilon_{ij} = \alpha_t \, \theta\delta_{ij} , \tag{19}$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} ,$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + [\lambda + \frac{(3\lambda + 2\mu)^2\alpha_t^2}{\rho c} T_o]\varepsilon_{kk}\delta_{ij} .$$

According to this the equations describing the isothermic (19)' and adiabatic $(19)_4$ processes have the same structure, the difference is only in the quality of the constant. The significance of this is that the dynamical processes with approximation can always be regarded as adiabatic.

Application for the thermal shock of long bars

Our calculations are made for linearly elastic, isotropic continuum that has material characteristics which do not depend on temperature - for bars. The body force and the intensity of the heat source can be disregarded.

Let us use the equations for the bar shown in Fig. 1 and examine our dimensional task. After arrangement we get the

$$\frac{E}{\rho} u_{xx} - u_{tt} - \frac{E\alpha_t}{\rho} \theta_x = \frac{(E\alpha_t)_x}{\rho} \theta - \frac{E_x}{\rho} u_x .$$

$$\theta_{xx} - \left(\frac{c\rho}{\kappa} + \frac{\gamma}{\kappa} u_x \right)\theta_t - \frac{\gamma}{\kappa} T u_{xt} = - \frac{\kappa_x}{\kappa} \theta_x \tag{20}$$

connections. We get the additional conditions through the following consideration (Fig. 1). In accordance with a given regularity we transfer heat at the free end of the cylindrical, heat insulated at its mantle, fixed bar of "l" length, the temperature and the strain so will change accordingly to this. At the fixed end of the bar the displacement and the temperature change - because the

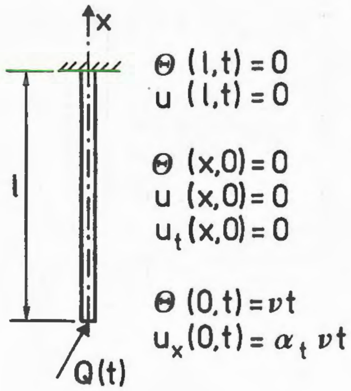

Fig. 1

fixing can be regarded as a heat container with thermodynamically infinite capacity - are zero. Let us suppose in addition that at the beginning of the examination the bar is at rest, its displacement is zero and the temperature is T_o. The solution was completed with the method of finite differences and with the help of digital computer.

TABLE 1.

	State equation φ	Heat conduction law ψ	φT_x	φT_t	φT	ψT_t	ψh_t	ψT_x	$\psi h_t\ \varphi T_t$	$\psi h_t\ \varphi T_x$	Inequality	Possible?
1.	$u - cT = 0$	$h - \kappa T_x = 0$	0	0	$-c$	0	0	$-\kappa$	0	0	0	no
2.	$u - cT - c\tau T_t = 0$	$h - \kappa T_x = 0$	0	$-c\tau$	$-c$	0	0	$-\kappa$	0	0	$4c\tau \frac{\kappa}{\rho} > 0$	yes
3.	$u - cT = 0$	$h + \tau h_t - \kappa T_x = 0$	0	0	$-c$	0	τ	$-\kappa$	0	0	$4\tau c \frac{\kappa}{\rho} > 0$	yes

TABLE 2.

E	$2 \cdot 10^{6}$	kp/cm^{2}
κ	$14{,}15 \cdot 10^{-5}$	$kcal/cmsC^{o}$
α	$1{,}17 \cdot 10^{-5}$	$1/C^{o}$
T_0	300	K^{o}
h	1	cm

ρ	$8 \cdot 10^{-6}$	kps^{2}/cm^{4}
γ	58,4	$kp/cm^{2}C^{o}$
c	0,114	$kcal/kpC^{o}$
l	100	cm
k	$1 \cdot 10^{-6}$	s

$\tilde{E}$	$-1 \cdot 10^{4}$	kp/cm^{3}
$\tilde{\rho}$	0	kps^{2}/cm^{5}
$\tilde{\alpha}$	$2{,}34 \cdot 10^{-8}$	$1/C^{o}cm$
$\tilde{\kappa}$	$-8{,}49 \cdot 10^{-7}$	$kcal/cm^{2}sC^{o}$
$\tilde{c}$	$0{,}114 \cdot 10^{-2}$	$kcal/kpC^{o}cm$
$\tilde{\gamma}$	$-0{,}234$	$kp/cm^{3}C^{o}$

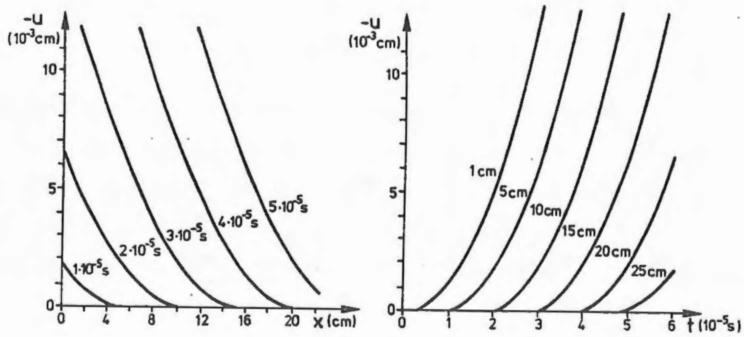

Fig. 2

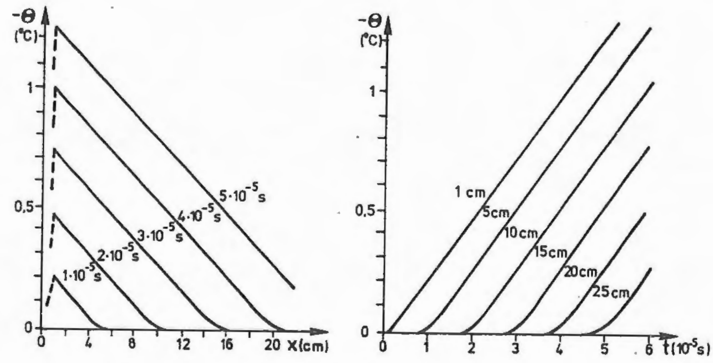

Fig. 3

In case of thermal shock of homogenous bar of isotropic material the starting data are to be found in Table 2. Fig. 2 and 3 show the functions u(x,t) and θ(x,t) at ν=5.10^6 K/s value. Fig. 4 and 5 show the functions u(x,t) and θ(x,t) at different values of ν with parametrically. Figures 6 and 7 also show u(x,t) and θ(x,t) in case of different temperatures at the and of the bar in accordance with Fig. 8.

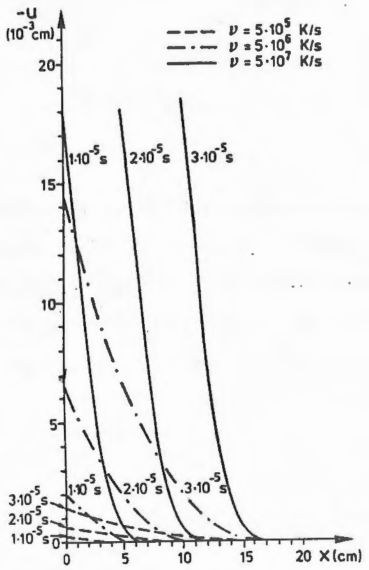

Fig 4.

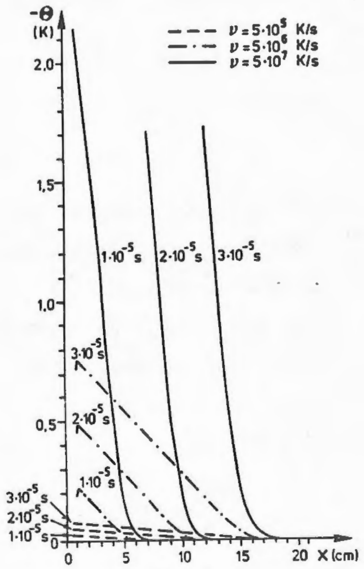

Fig 5.

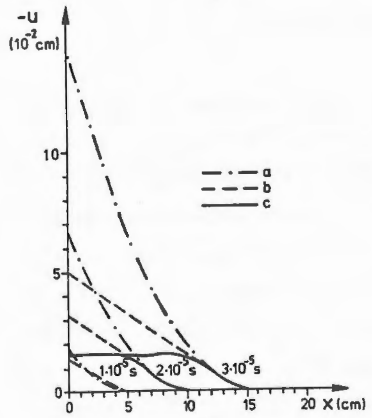

Fig 6.

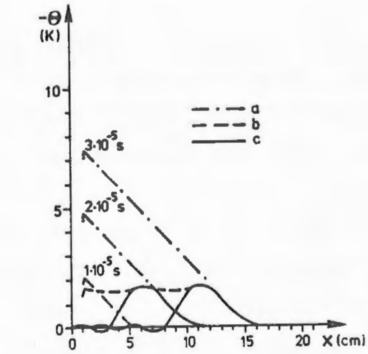

Fig 7.

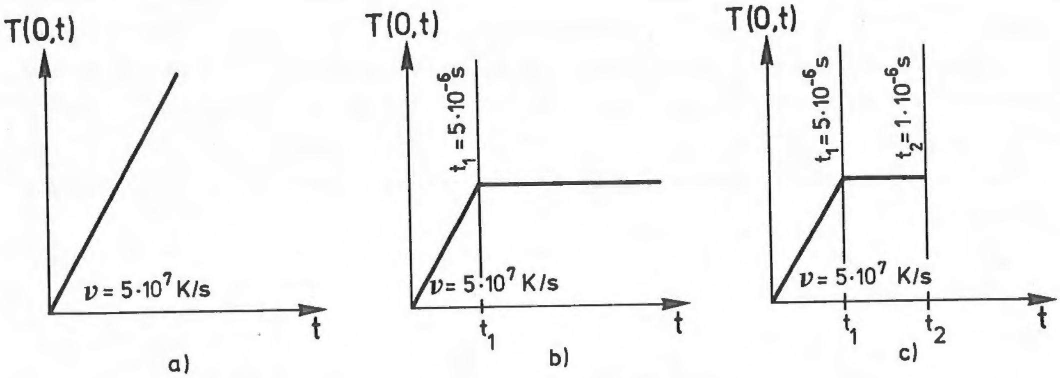

Fig 8.

In case of thermal shock of inhomogeneous bar of isotropic material the starting data can be found in Table 2. The measure of inhomogeneity /58/ corresponds to the imperfect manufacturing that occurs in the routine. The changes of material characteristics are considered to be linear e.g. coefficient of elasticity:

$$E(x) = E + \tilde{E}x.$$

The results do not differ significantly from the homogenous case.

Possibility of analogy in thermoelasticity

Numerical results show that the deformation wave generated by the thermal shock propagates with the velocity of the elastic wave(in case of steel apx. 5.10^3 m/s) respectively accordingly to the coupled task the deformation wave is accompanied by temperature wave the velocity of which equals that of the previous one. This temperature wave is however not to be confused with the propagation of the introduced heat and the temperature change corresponding to that the speed of which is several orders of magnitude smaller. On the basis of these the following statements can be done.

The amplitude of the temperature wave accompanying the deformation wave - in spite of the rather fast temperature change adequate to the shock at the end of the bar - is small and its velocity - as it has already been mentioned is rather big. Furthermore this temperature change is negative and this fact needs further explanation.

The experimental justification of the results seems so expedient. This involves however twofold difficulty: rather small temperature difference has to be measured by rather fast change.

only such a measurement arrangement comes into question at which the inertia of the measuring element is small.

We also examine the possibility of the creation of an electric analogy because the similarity of the telegraph equations /9, 41/ and the equation system (20) allows us to hope for promising experimental results.

According to the telegraph equations /9/:

$$- e_x = Ri + Li_t, \qquad - i_x = Ge + Ce_t. \tag{21}$$

These connections can be produced if we apply the generalized Kirchhoff and Ohm laws for the circuit made after Fig. 9.

After differentiating (21) and arranging we get

$$\frac{1}{LC} i_{xx} - i_{tt} + \frac{G}{LC} e_x = \frac{R}{L} i_t , \qquad e_{xx} - RCe_t + Li_{xt} = RGe. \tag{22}$$

Let us write the general equation of motion and the general equation of heat conduction describing the thermal shock of the long bar in a slightly modified form according to the followings:

$$\frac{E}{\rho} u_{xx} - u_{tt} - \frac{E\alpha_t}{\rho} T_x = 0 , \qquad T_{xx} - f_T T_t - g(T) u_{xt} = \frac{\gamma}{\kappa} u_x T_t. \tag{23}$$

Corresponding the i current to u displacement and e voltage to T temperature on the basis of the comparison of the equations the analogy can be seen for which the fulfillment of the following conditions is necessary

$$\frac{1}{LC} = \frac{E}{\rho} \; ; \; \frac{G}{LC} = -\frac{E\alpha_t}{\rho} \; ; \; \frac{R}{L} \to 0 \; ; \; RC = f_T \; ;$$
$$L = g(e) \; ; \; RG \to 0 \; ; \; \frac{\gamma}{\kappa} \to 0 \; . \qquad (24)$$

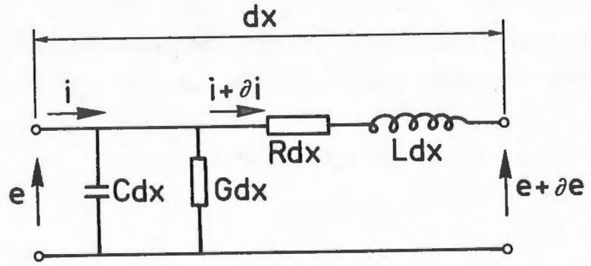

Fig.9.

Here the exact structure of the g(e) function can be determined on the basis of $(20)_2$.

The electric model of the long bar can be produced by the adequate number of repetition of the basic substituting circuits made according to Fig. 9 with regard to the conditions (24). Having for the first circle voltage adequate to the temperature change the approximation with given precision of functions i(x,t) and e(x,t) can be measured at discrete places - continuously in time.

Summary, results

In accordance with the set aim relying on the scientific literature I wrote down the equation system of TE with the help of the basic equations of continuummechanics and thermodynamics. I analyzed the equation system, investigated the possibility of modification of the heat conduction law, made calculations for the thermal shock of long bars and examined the possibility of an electric analogy.

The results of the examinations can be summed up as follows

I. The equations of TE included in the system are suitable for describing an arbitrary coupled dynamical task. On the basis of the examination of these the followings can be stated.

I.a) In case of a coupled task the equation (8) is valid which differs from the equation customary in the scientific literature in its $-\beta_{kl}\varepsilon_{kl}\dot{T}$ member.

I.b) The equations describing adiabatic and isothermic processes have the same structure, the difference is in the quality of the constants. The significance of this is that the dynamical processes with good approximation can be regarded adiabatic.

II. I elaborated a method for the separation of possible and impossible heat conduction laws and equations of state. With the help of this I determined that

II.a) The heat conduction law and the equation of state of the customary form cannot be valid at the same time. The ones according to lines 2 and 3 of Table 1 are possible however.

II.b) in dynamical - particularly rapid dynamical - tasks the Fourier's heat conduction law is not appropriate, modified equation has to be applied.

III. With the utilization of the equation system I made calculations for the examination of the thermal shock of long bars, which show the followings:

III.a In spite of the large quantity of heat conducted at the end of the bar with the exception of the place x 0 = the bar turned cooler everywhere. The reason of this is the difference between the velocity of the elastic wave and temperature wave.

III.b Technically reasoned inhomogeneity in the displacement and temperature fields does not cause significant difference compared to the homogenous case.

IV. Examining the possibility of analogy I determined that

IV.a with electric circuit built up on the basis of the telegraph equations the thermal shock of long bars can be modelled.

SYMBOLS

u_i displacement vector

ε_{ij} strain tensor

x locus

t time

ρ mass density

σ_{ij} stress tensor

g_i body force

a_{ijkl} matrix of elasticity

α_{ij} matrix of linear expansion coefficients

α_t coefficient of linear expansion

T temperature

T_o temperature in natural state

$\theta = T - T_o$

$(\ )_x = \frac{\partial(\ )}{\partial x}$

κ_{ij} matrix of heat conduction coefficients

κ coefficient of heat conduction

a coefficient of temperature conduction

v_T velocity of heat propagation

c specific heat

λ, μ Lamé constants

$\gamma = /3\lambda + 2\mu/\alpha_t$

δ_{ij} Kronecker delta

$\beta_{ij} = -\gamma\delta_{ij} = -\ /3\lambda + 2\mu/\alpha_t$

E Young modulus

ν temperature change rate

e voltage

u	internal energy density	i	current intensity
v_{kl}	strain rate tensor	R	resistance
h_k	intensity of heat flux	L	inductivity
r	heat source intensity	C	capacitance
q	heat	G	conductance
s	entropy density	τ	relaxation time

$$(\ )_{,j} = \frac{\partial(\ )}{\partial x_j}$$

REFERENCES

/1/ Nowacki, W., Thermoelasticity. Pergamon Press. 1962.

/2/ Boley, B.A., Weiner, J.H., Theory of Thermal Stresses. John Wiley and Sons. 1960.

/3/ Гольенблат, И.И., Расчеты на прочность, устойчивость и колебания в условиях высоких температур. /Hungarian translation/ Müszaki Könyvkiadó, Budapest 1969.

/4/ Петровский, И.Г., Лекции об уравнениях с частными производными. /Hungarian translation/ Akadémiai Kiadó, Budapest 1955.

/5/ Parkus, H., Thermoelasticity. Springer-Verlag. Wien/New York. 1976.

/6/ Kovács, I., On the Mechanical Properties of Solid Bodies. (In Hungarian) Fizikai Szemle, Vol. XXII. No 11.

/7/ Ландау, Л.Д., Лифшиц, Е.М., Теоретическая физика VII. Теория упругости. /Hungarian translation/ Tankönyvkiadó, Budapest 1974.

/8/ Тихонов, А.Н., Самарский, А.А., Уравнения математической физики. /Hungarian translation/ Akademiai Kiadó, Budapest 1956.

/9/ Simonyi, K., Theoretical Electricity. (In Hungarian) Tankönyvkiadó. Budapest 1973.

/10/ Braun, M., Zur Aurbreitung von Unstetigkeitsflächen in thermoelastischen Stoffen. Doktor Thesis. Stuttgart 1974.

/11/ Maurer, M.J., Relaxation Model for Heat Conduction in Metals. Journal of Applied Physics. Vol. 40 No 13. 1969.

/12/ Béda, Gy., Method to Investigate the Plastic Wave. (In Hungarian) NME Közleményei, Vol. XIV. 1967.

/13/ Beevers, C.E., Engelbrecht, J., Constitutive Rate-Dependent Theory of Termoviscoelasticity.
Известия Академии Наук Эстонской СССР. Том 24. Физика, Математика. 1975. № 4.

/14/ Puri, P., Plane Waves in Generalized Thermoelasticity. Int. Journal of Eng. Sci. Vol. 11.

/15/ Попов, Е.Б., Динамическая связная задача термоупругост для полупространсва с учетом конечности скорости распределения тепла. Прикл. Мат. и Мех. Вып. 2. 1967.

/16/ Gurtin, M.E., Pipkin, A.C., A General Theory of Heat Conduction with Finite Wave Speeds. Arch. Rat. Mech. An. Vol. 31 No 2 1968.

/17/ Biot, M.A., Thermoelasticity and Irreversible Thermodynamics Journal of Applied Physics, Vol. 27 No 3 19.

/18/ Truesdell, C., Rational Thermodynamics. McGraw-Hill. New York. 1969.

/19/ Gyarmati, I., Non-Equilibrium Thermodynamics. (In Hungarian) Müszaki Könyvkiadó. Budapest 1976.

/20/ Farkas, H., On the Phenomenological Theory of Heat Conduction. Int. J. Eng. Sci. 1975. Vol. 13, pp. 1035-1053.

/21/ Bargmann, H., Über den Trägheitseffekt in der Thermoelastizität. ZAMM 60, /1980/ 98-99.

/22/ Ziegler, H., Discussion of Some Objections to Thermomechanical Orthogonality. Ing. Arch. 50 /1981/ pp. 149-164.

/23/ Лыков, А.В., Теория теплопроводности. Издательство "Высшая школа", Москва, 1967.

/24/ Bargmann, H., Dynamic Thermal Shock Resistance. Topics in Applied Cont. Mech. Springer, 1974, pp. 174-181.

/25/ Béda, Gy., Continuummechanics I. (In Hungarian) Lecture note, Tankönyvkiadó, Budapest 1980.

/26/ Stojanovic, R., Nonlinear Thermoleasticity. Udine 1972. Springer-Verlag, Wien-New York.

/27/ Sneddon, I.N., The Linear Theory of Thermoelasticity. Udine 1974. Springer-Verlag. Wien-New York.

/28/ Thermoelasticity. Ed. B.A. Boley. IUTAM Symp. 1968. Springer-Verlag. Wien-New York.

/29/ Коваленко, А.Д., Основы термоупруглсти. Изд. Наукова Думка. Киев, 1970.

/30/ Новацкий, В., Динамические задачи термоупругости. Изд. Мир. Москва, 1970.

/31/ Подстригач, Я.С., Коляно, Ю.М., Обобщенная термомеханика. Изд. Наукова Думка, Киев, 1976.

/32/ Нигул, У.К., Энгелбрехт, Ю.К., Нелинейные и линейные перезодные волновые процессы деформации термоупругих и упругих тел. Академия Наук ЭССР. Институт Кибернетики, Таллин. 1972.

/33/ Electromagnetic Interactions in Elastic Solids. Ed. by H. Parkus. CISM. Springer-Verlag. Wien - New York.

/34/ Parkus, H., Instationäre Wärmespannungen. Wien Springer Verlag. 1959.

/35/ Farkas, H., Generalization of the Fourier Law. Periodica Polytechnica, Vol. 24 /1980/ No 4. pp. 291 - 308.

/36/ Meixner, J., On the Linear Theory of Heat Conduction. Arch. Rat. Mech. Anal. 39 /1970/ pp. 108 - 130.

/37/ Lord, H.W., Shulman, Y., A Generalized Dynamical Theory of Thermoelasticity. J. MEch. Phys. Solids Vol. 15 /1967/ pp. 299 - 309.

/38/ Norwood, F.R., Warren, W.E., Wave Propagation in the Generalized Dynamical Theory of Thermoelasticity. Quart. Journ. Mech. and Applied Math. Vol XXII /1969/ pp. 283 - 290.

/39/ Gyarmati, I., On the Wave Approach of Thermodynamics and Some Problems of Non-Linear Theories. J. Non-Equilib. Thermodyn. Vol. 2 /1977/ No. 4 pp. 233 - 261.

/40/ Baltov, A., Vallilev, K., Shock Thermo-Mechanical Wave Propagation in Infinite Medium. Bulgarian Academy of Sciences, Theoretical and Applied Mechanics. Sofia 1977. Year VIII, No 3 pp. 23 - 26.

/41/ Smith, D.R., Palmer, J.T., On the Behavior of the Solution of the Telegraphist's Equation for Large Absorption Arch. Rat. Mech. An. 39 /1970/ pp. 146 - 157.

/42/ Boley, B.A., Hetnarski, R.B., Propagation of Discontinuities in Coupled Thermoelastic Problems. Journal of Applied Mechanics. September 1968 pp. 489 - 494.

/43/ Nickell, R.E., Sackman, J.L., Approximate Solutions in Linear, Coupled Thermoelasticity. Journal of Applied Mechanics. June 1968 pp. 255 - 266.

/44/ Даниловская, В.И., Температурные напряжерия в упругом полупространстве, вознткающие вследствие внезапного нагрева его границы. Пиклaная математика. Том 14. 1950. стр. 316-318.

/45/ Даниловская, В.И., Об одной динамической задаче термоупругости. Прикладная математика и механика. Том. 16. 1952. Стр. 341-344.

/46/ Kiss Lajos, Hegesztés okozta mechanikai folyamatok vizsgálata a varrat környezetében. Kandidátusi értekezés. Miskolc 1969.

/47/ Carslaw, H.S., Jaeger, J.C., Conduction of Heats in Solids. Oxford at the Clarendon Press. 1959.

/48/ Tautz, H., Wärmeleitung und Temperaturausgleich. Akademie-Verlag. Berlin. 1971.

/49/ Takeuti, Y., Furukava, T., Some Considerations on Thermal Shock Problems in a Plate. Journal of Applied Mechanics. Vol. 48, March 1981. pp. 113 - 118.

/50/ Takeuti, Y., Komori, S., Noda, N., Nyuko, H., Thermal-Stress Problems in Industry. 3: Temperature Dependency of Elastic Moduli for Several Metals at Temperatures from -196 to 1000 °C. Journal of Thermal Stresses, 2 /1979/ pp. 233 - 250.

/51/ Takeuti, Y., Foundations for Coupled Thermoelasticity. Journal of Thermal Stresses, 2 /1979/ pp. 233 - 250.

/52/ Vogel, J., Vedeni tepla a konecna rychlost jeho sireni. Acta Polytechnica-Práce CVUT v. Praze, IV.1. 1977.

/53/ Habetinová, L., Neustupa, J., Reseni rovnice vedeni tepla pri konene rychlosti jeho sireni Fourierova metodou. Acta polytechnica-Práce CVUT v. Praze, 21 /II.2/ 1979.

/54/ Vogel, J., Kebralolvá, J., Reseni rovnice vedeni tepla pri konecné rychlosti jeho sireni metodou siti. Acta Polytechnica-Pracé CVUT v. Praze, 9 /II.2/ 1981.

/55/ Vogel, J., Zitny, R., Fusek, I., Habetinova, L., Kebrdlová, J., Zkusenosti s resenin hyperbolické rovnice vedeni tepla numerickymi metodami. Acta Polytechnica-Práce CVUT v. Praze, vedecká Konference, 1982.

/56/ Simmonds, J.G., A Brief on Tensor Analysis. Springer-Verlag. 1982.

/57/ Obádovics, J.Gy., Gyakorlati számitási eljárások. Gondolat. Budapest. 1972.

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