DEVIATION FROM RIGIDITY IN GYRODYNAMICS

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SUMMARY: In the present paper a near-rigid solid is defined. The concept is then introduced into gyrodynamics. After defining the properties of a suitable floating coordinate system, the latter is used to simplify the equations of gyrodynamics for deformable bodies. The concept of a body of constant configuration in a floating coordinate system is introduced.

INTRODUCTION

The concept of rigidity is the basis of gyrodynamics of solid bodies. Numerous equations, and results, are based on the concept of rigidity, in spite of the fact that it is well known that rigid bodies do not exist.

The present paper sets out to investigate in which fashion deviation from rigidity does affect the equations and the results.

As it turns out, under certain circumstances deformable axisymmetric gyros retain their deformed shape when expressed in an appropriate floating coordinate system, such that equations which are based on the concept of rigidity need not be abandoned.

What is needed is a clearer delimitation of the term "rigid body" and of the new concept of a body of constant configuration in a floating coordinate system, which is introduced in the present paper.

DEFORMATIONS

A typical single solid gyro, such as e.g. a spacecraft, is made of materials that exhibit elastic-hysteretic behaviour. Thus deformations due to gyroscopic forces are inevitable. Such deformations are usually small. We impose the condition that deformations be small and refer to such a solid body from now on as a near-rigid body.

In the present article it is shown in which fashion a typical single solid body gyro deviates from its rigid body configuration.

The single solid body

A typical solid of principal inertia tensor

$$[I]_{\text{undef}} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \tag{1}$$

before deformation, will change to

$$[I]_{\text{def}} = \begin{bmatrix} A + \Delta A & I_{xy} & I_{xz} \\ I_{xy} & B + \Delta B & I_{yz} \\ I_{xz} & I_{yz} & C + \Delta C \end{bmatrix} \tag{2}$$

after deformation, when one and the same C_{xyz} coordinate system is used to describe both tensors. The orientation of the C_{xyz} system is defined by the principal axes of the undeformed solid. Its origin is the body's mass centre C. For very small principal inertia moment changes ΔA, ΔB, ΔC, one may write

$$[I]_{\text{def}} = \begin{bmatrix} A & I_{xy} & I_{xz} \\ I_{xy} & B & I_{yz} \\ I_{xz} & I_{yz} & C \end{bmatrix} \tag{3}$$

Now let the principal coordinate system of the deformed gyro be $Cx'y'z'$, then

$$[I']_{\mathrm{def}} = \begin{bmatrix} A' & 0 & 0 \\ 0 & B' & 0 \\ 0 & 0 & C' \end{bmatrix} \quad (4)$$

Mathematically, these principal inertia moments are the eigenvalues of the matrix of equation (2).

With the principal axes system $Cx'y'z'$ of the deformed gyro assumed known, the inertia tensor of the deformed gyro in the principal axes system $Cxyz$ of the undeformed gyro can be found from

$$[I]_{\mathrm{def}} = [T]^{\mathrm{T}}[I']_{\mathrm{def}}[T] \quad (5)$$

For very small Cardan angles ξ, η, ζ, measured from the principal axes system of the undeformed state to the principal axes system of the deformed state,

$$[T] = \begin{bmatrix} 1 & \zeta & -\eta \\ -\zeta & 1 & \xi \\ \eta & -\xi & 1 \end{bmatrix} \quad (6)$$

Neglecting products and squares of the Cardan angles, and neglecting ΔA, ΔB and ΔC, one obtains

$$[I]_{\mathrm{def}} = \begin{bmatrix} A & (A-B)\zeta & (C-A)\eta \\ (A-B)\zeta & B & (B-C)\xi \\ (C-A)\eta & (B-C)\xi & C \end{bmatrix} \quad (7)$$

Summing up, a near-rigid solid is consequently characterized by

$$\frac{|\Delta A|}{A} \ll 1 \qquad \frac{|\Delta B|}{B} \ll 1 \qquad \frac{|\Delta C|}{C} \ll 1 \tag{8a}$$

$$|\xi| \ll 1 \qquad |\eta| \ll 1 \qquad |\zeta| \ll 1 \tag{8b}$$

A near-rigid solid in its deformed state is characterized by small deformations leading to (a) small inertia moment changes and (b) small inertia products in comparison with the principal inertia moments of the solid in its undeformed state. The small inertia moment changes ΔA, ΔB and ΔC are insignificant (in comparison with the inertia moments A, B, and C) and can be neglected. The small inertia products I_{xy} , I_{xz}, and I_{yz} are significant (in comparison with zero) and must be retained.

To avoid unnecessary complications we specifically exclude from consideration the special case $B = A$, $C = 0$ (i.e. rod of zero cross-section), which is of no practical interest.

If the inertia moment changes ΔA , ΔB , and ΔC are looked upon as measures of the dilatation of a gyro body, and the inertia products I_{xy}, I_{yz} , and I_{xz} as measures of distortion, then a near-rigid solid can also be defined as a solid, (a) whose dilatation is small and insignificant and (b) whose distortion is small but significant.

The Axisymmetric Gyro

We shall define as axisymmetric a gyro which in its undeformed state satisfies the condition

$$B = A \tag{9}$$

From equations (7) and (9) we then deduce that the inertia product I_{xy} vanishes, and that in its deformed state an axisymmetric gyro has an inertia tensor of

$$[I] = \begin{bmatrix} A & 0 & (C-A)\eta \\ 0 & A & (A-C)\xi \\ (C-A)\eta & (A-C)\xi & C \end{bmatrix} \tag{10}$$

Observations

We conclude that a near-rigid solid whose inertia tensor in the undeformed state is

$$[I]_{\text{undef}} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \tag{11}$$

will have an inertia tensor in the deformed state of

$$[I]_{\text{def}} = \begin{bmatrix} A & I_{xy} & I_{xz} \\ I_{xy} & B & I_{yz} \\ I_{xz} & I_{yz} & C \end{bmatrix} \tag{12}$$

with

$$I_{xy} = (A - B)\zeta \tag{13a}$$

$$I_{yz} = (B - C)\xi \tag{13b}$$

$$I_{zx} = (C - A)\eta \tag{13c}$$

and all inertia moments and inertia products expressed in terms of the $Cxyz$ principal coordinate system of the near-rigid solid in its undeformed state. It is worthy to note that for the practically important class of axisymmetric gyros, $B = A$, and consequently $I_{xy} = 0$ for their deformed state.

Floating Coordinates

Considerations similar to those used to derive equation (12) can also be followed to derive the inertia tensor of a deformed gyro, which was axisymmetric in its undeformed state, using a floating $Cuvz$

principal coordinate system instead of a body-fixed $Cxyz$ coordinate system (Canavin and Likins /1/). The result is

$$[I] = \begin{bmatrix} A & 0 & I_{uz} \\ 0 & A & I_{vz} \\ I_{uz} & I_{vz} & C \end{bmatrix} \tag{14}$$

with

$$I_{vz} = (A - C)\xi \tag{15a}$$

$$I_{uz} = (C - A)\eta \tag{15b}$$

where this time ξ and η are Cardan angles measured about the u-axis and the (carried) v-axis respectively.

If the inertia products $I_{uz} = (C - A)\eta$ and $I_{vz} = (A - C)\xi$ of equations (15) are constant, then the body retains its configuration expressed in $Cuvz$ coordinates. Such a body shall be referred to as a body of constant configuration in a floating coordinate system.

The floating $Cuvz$ coordinate system has an angular velocity of $\bar{\Omega}$. It represents the angular velocity of the internal (i.e. centrifugal and reversed Coriolis) force system. The $Cxyz$ coordinate system has an angular velocity of ω. It represents the angular velocity of the gyro body.

Elastic Displacements

The amount of deformation occurring depends not only on the elastic properties of the gyro material and on the shape and size of the gyro body, but also on the mounting, such that a general statement is difficult to make from the elastic and installation end. From the dynamics end, however, some observations can be made. If we exclude body dilatation, then we exclude in effect loading due to centrifugal forces, i.e. terms containing ω_v^2 and ω_z^2. The terms remaining are of Coriolis type, i.e. they contain the product $\omega_v\omega_z$, such that one can generally state that (Magnus /2/)

$$I_{vz} = \alpha\omega_v\omega_z \tag{16}$$

where α is an elastic deformation coefficient, measured in Ws^5, which includes all effects other than ω_v and ω_z . It is important to realize that α does not only depend on the gyro body and the material(s) it is made of, but also on the mounting conditions.

Hysteretic Displacements

If an elastic-hysteretic body exhibits an elastic deformation and the body is spinning at an angular velocity $\dot{\sigma}$, then the body masses will be dragged along (Zhang and Ling /3/4/), producing a centrifugal moment

$$I_{uz} = -\beta\omega_v\omega_z\dot{\sigma} \tag{17}$$

where

$$\dot{\sigma} = \omega_z - \Omega_z. \tag{18}$$

The quantity β is an energy dissipation coefficient, measured in Ws^6. It is important to realize that β depends not only on the gyro body's shape,size and material, but also on the mounting conditions.

It will later be shown, that the energy dissipation rate can be expressed as

$$\dot{D} = \beta\omega_v^2\omega_z^2\dot{\sigma}^2 \tag{19}$$

THE ANGULAR MOMENTUM

The angular momentum $\bar{H}$ of a body m is defined by

$$\bar{H} = \int_m \bar{r} \times \dot{\bar{r}}\, dm \tag{20}$$

Any mass element dm that is located at $\bar{\rho}$ in a solid body in its undeformed state, will be displaced by a (small) distance $\bar{\delta}$ in the deformed state, such that

$$\bar{r} = \bar{\rho} + \bar{\delta} \tag{21}$$

The angular velocity of the body (more accurately of the C principal coordinate system of the body in its undeformed state) is $\bar{\omega}$.

Thus

$$\dot{\bar{r}} = \dot{\bar{\rho}} + \dot{\bar{\delta}} = \overset{0}{\bar{\rho}} + \bar{\omega} \times \bar{\rho} + \overset{0}{\bar{\delta}} + \bar{\omega} \times \bar{\delta} \tag{22}$$

with $\overset{0}{\bar{\rho}} = \bar{0}$ because $\bar{\rho}$ refers to the undeformed state.

Entering equation (22) into equation (20) gives

$$\bar{H} = \int_m \bar{r} \times (\bar{\omega} \times \bar{\rho})\, dm + \int_m \bar{r} \times \overset{0}{\bar{\delta}}\, dm + \int_m \bar{r} \times (\bar{\omega} \times \bar{\delta})\, dm \tag{23}$$

The last and the first term on the right hand side can be combined to give

$$\int_m \bar{r} \times (\bar{\omega} \times (\bar{\rho} + \bar{\delta}))\, dm = \int_m \bar{r} \times (\bar{\omega} \times \bar{r})\, dm \tag{24}$$

Now, if the coordinate system employed to describe the motion of the gyro is chosen such that

$$\int_m \bar{r} \times \overset{0}{\bar{\delta}}\, dm = \bar{0} \tag{25}$$

then the angular momentum expression

$$\int_m \bar{r} \times (\bar{\omega} \times \bar{r})\, dm \tag{26}$$

for the body in its deformed state is mathematically similar to the body's angular momentum expression in the undeformed state which would be

$$\int_m \bar{\rho} \times (\bar{\omega} \times \bar{\rho})\, dm \tag{27}$$

The floating $Cuvz$ coordinate system used in the subsequent application satisfies the condition $\overset{0}{\bar{\delta}} = \bar{0}$ exactly when $\dot{\omega}_v = \dot{\omega}_z = 0$, and consequently represents a coordinate system for which equation (25) is exactly satisfied.

Since

$$\int_m \bar{\rho} \times (\bar{\omega} \times \bar{\rho})\ dm = \{\bar{e}\}^T [I]_{undef} \{\bar{\omega}\} \tag{28}$$

it may be concluded that

$$\bar{H} = \int_m \bar{r} \times (\bar{\omega} \times \bar{r})\, dm = \{\bar{e}\}^T [I]_{def} \{\bar{\omega}\} \tag{29}$$

provided condition (25) applies, i.e. the simple $\{\bar{e}\}^T [I]_{def} \{\bar{\omega}\}$ formulation can only be used if the appropriate floating coordinate system is employed to express the components.

ENERGY RATES

The torque $\bar{M}$ applied to any body changes the body's angular momentum according to Euler's law

$$\bar{M} = \dot{\bar{H}} \tag{30}$$

In terms of a coordinate system rotating at $\bar{\Omega}$,

$$\dot{\bar{H}} = \overset{0}{\bar{H}} + \bar{\Omega} \times \bar{H} \tag{31}$$

where $\bar{\Omega}$ is the angular velocity of a suitably chosen floating principal coordinate system.

Combining equations (30) and (31) leads to

$$\bar{M} = \overset{0}{\bar{H}} + \bar{\Omega} \times \bar{H} \tag{32}$$

Forming an inner product with the angular velocity $\bar{\omega}$ of the near-rigid body (or more accurately of the body-fixed $Cxyz$ principal coordinate system of the near-rigid body in its undeformed state) gives

$$\bar{M} \cdot \omega = \overset{0}{\bar{H}} \cdot \bar{\omega} + (\bar{\Omega} \times \bar{H}) \cdot \bar{\omega} \tag{33}$$

Since the kinetic energy, when condition (25) applies,

$$T = \frac{1}{2} \bar{H} \cdot \bar{\omega} \tag{34}$$

and since $(\bar{\Omega} \times \bar{H}) \cdot \bar{\omega} = -\bar{H} \cdot (\bar{\Omega} \times \bar{\omega})$ one obtains for the kinetic energy rate

$$\dot{T} = \frac{1}{2}(\overset{\circ}{\bar{H}} \cdot \bar{\omega} + \bar{H} \cdot \overset{\circ}{\bar{\omega}}) \qquad (35)$$

We further introduce for the elastic energy rate

$$\dot{E} = \frac{1}{2}(\overset{\circ}{\bar{H}} \cdot \bar{\omega} - \bar{H} \cdot \overset{\circ}{\bar{\omega}}) \qquad (36)$$

and for the energy dissipation rate

$$\dot{D} = (\bar{\Omega} \times \bar{H}) \cdot \bar{\omega} \qquad (37)$$

Because of physical reasons, there is a restriction on the energy dissipation rate. It must always be positive.

The rate of work done by the applied torque is

$$\dot{U} = \bar{M} \cdot \bar{\omega} \qquad (38)$$

Using equations (35), (36), (37) and (38), equation (33) now reads

$$\dot{U} = \dot{T} + \dot{E} + \dot{D} \qquad (39)$$

or more conveniently

$$\dot{U} - \dot{D} = \dot{T} + \dot{E} \qquad (40)$$

In words:

the rate of external work fed into a gyro, minus the internal energy dissipation rate, equals the kinetic energy rate plus the elastic energy rate.

APPLICATION TO A CONSTRAINED SYSTEM

In Figure 1 a flexible disk gyro is shown, mounted on a rigid shaft system driven at constant speeds by motors 1 and 2. Motor 1 develops a (constant) angular speed ω_v, motor 2 a (constant) angular velocity ω_z. The disk is subject to an angular velocity

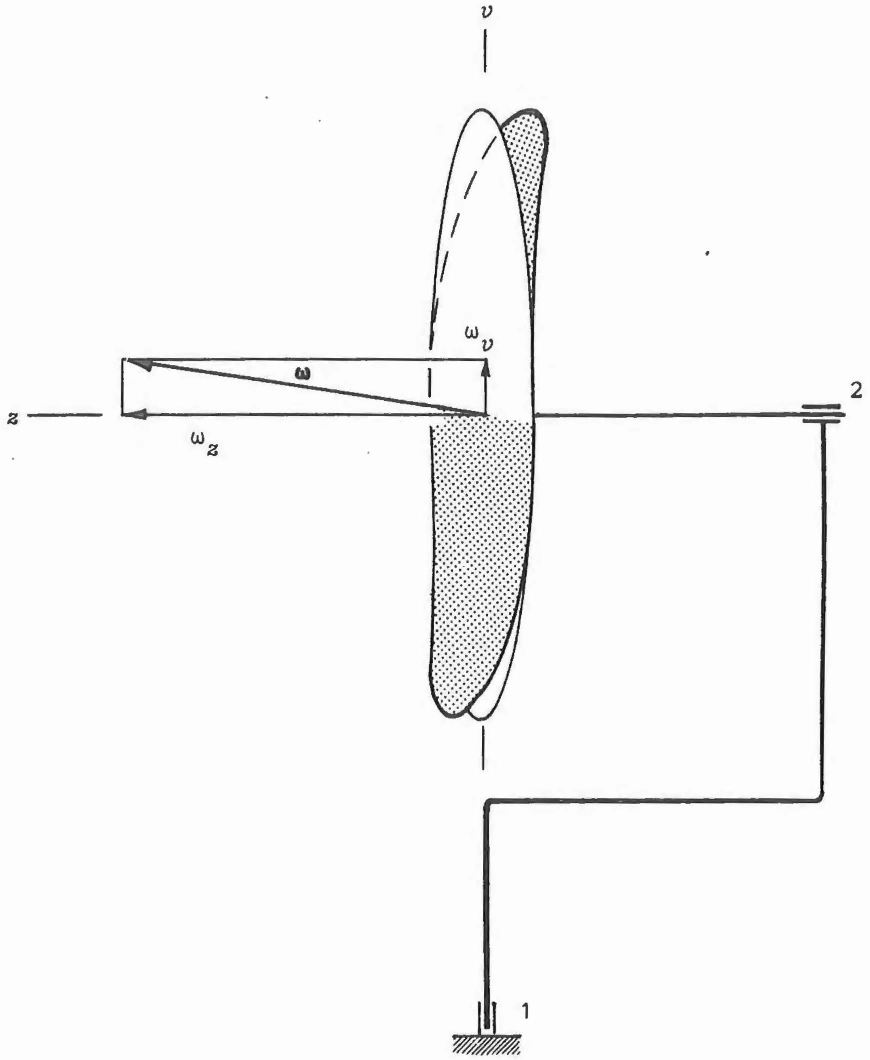

Figure 1 Disk Gyro

$$\overline{\omega} = [\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z] \begin{bmatrix} 0 \\ \omega_v \\ \omega_z \end{bmatrix} \tag{41}$$

The floating $Cuvz$ coordinate system used to describe the motion has an angular velocity

$$\overline{\Omega} = [\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z] \begin{bmatrix} 0 \\ \omega_v \\ 0 \end{bmatrix} \tag{42}$$

Any real solid disk will be made of an elastic-hysteretic material. The elastic properties alone will lead to a small I_{vz} ; the hysteretic properties to an even smaller I_{uz} .

With

$$[I] = \begin{bmatrix} A & 0 & I_{uz} \\ 0 & A & I_{vz} \\ I_{uz} & I_{vz} & C \end{bmatrix} \tag{43}$$

the gyro's angular momentum becomes

$$\overline{H} = [\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z] \begin{bmatrix} A & 0 & I_{uz} \\ 0 & A & I_{vz} \\ I_{uz} & I_{vz} & C \end{bmatrix} \begin{bmatrix} 0 \\ \omega_v \\ \omega_z \end{bmatrix} \tag{44}$$

$$= [\,\bar{e}_u \quad \bar{e}_v \quad \bar{e}_z\,] \begin{bmatrix} I_{uz}\,\omega_z \\ A\omega_v + I_{vz}\,\omega_z \\ C\omega_z + I_{vz}\,\omega_v \end{bmatrix} \tag{45}$$

The torque transmitted to the gyro is obtained from Euler's law

$$\bar{M} = \dot{\bar{H}} \tag{46}$$

Since $\dot{\delta}_u = \dot{\delta}_v = \dot{\delta}_z = 0$, condition (25) is satisfied. With

$$\dot{\bar{H}} = \overset{0}{\bar{H}} + \bar{\Omega} \times \bar{H} \tag{47}$$

and $$\overset{0}{\bar{H}} = \bar{0} \tag{48}$$

and

$$\bar{\Omega} \times \bar{H} = \begin{bmatrix} \bar{e}_u & \bar{e}_v & \bar{e}_z \\ 0 & \omega_v & 0 \\ I_{uz}\omega_z & A\omega_v + I_{vz}\omega_z & C\omega_z + I_{vz}\omega_v \end{bmatrix} \tag{49}$$

we obtain

$$\bar{M} = [\,\bar{e}_u \quad \bar{e}_v \quad \bar{e}_z\,] \begin{bmatrix} C\omega_v\omega_z + I_{vz}\omega_v^2 \\ 0 \\ -\,I_{uz}\omega_v\omega_z \end{bmatrix} \tag{50}$$

The condition $I_{uz} = I_{vz} = 0$ represents the case of a rigid body. The condition $I_{vz} \neq 0$ and $I_{uz} = 0$ represents the case of a purely elastic body. The case $I_{vz} = 0$ and $I_{uz} \neq 0$ represents a purely hysteretic body. The case $I_{vz} \neq 0$ and $I_{uz} \neq 0$ represents an elastic-hysteretic, i.e. a real solid, body.

The power $\dot{U}$ supplied to the gyro is

$$\dot{U} = \bar{\mathrm{M}} \cdot \bar{\omega} \tag{51}$$

and amounts to

$$\dot{U} = - I_{uz} \omega_v \omega_z^2 \tag{52}$$

Since in the present case all power fed into the system is used to feed the energy dissipation we may express the energy dissipation rate (which can only be positive) as

$$\dot{D} = - I_{uz} \omega_v \omega_z^2 \tag{53}$$

The kinetic energy stored in the gyro is

$$T = \frac{1}{2} \bar{\mathrm{H}} \cdot \bar{\omega} = \frac{1}{2}(A\omega_v^2 + C\omega_z^2 + 2I_{vz}\omega_v\omega_z) \tag{54}$$

The elastic energy stored in the gyro can be shown to be

$$E = I_{vz}\omega_v\omega_z \tag{55}$$

such that the total mechanical energy stored in the gyro is

$$T + E = \frac{1}{2}(A\omega_v^2 + C\omega_z^2 + 4I_{vz}\omega_v\omega_z) \tag{56}$$

Numerical Example

For an example let us pick a 2D dumbbell gyro consisting of four elasto-hysteretic massless arms each of 0.5 m length with four point masses of 0.25 kg each (Figure 2). Also let $\omega_v = 2$ rad/s and

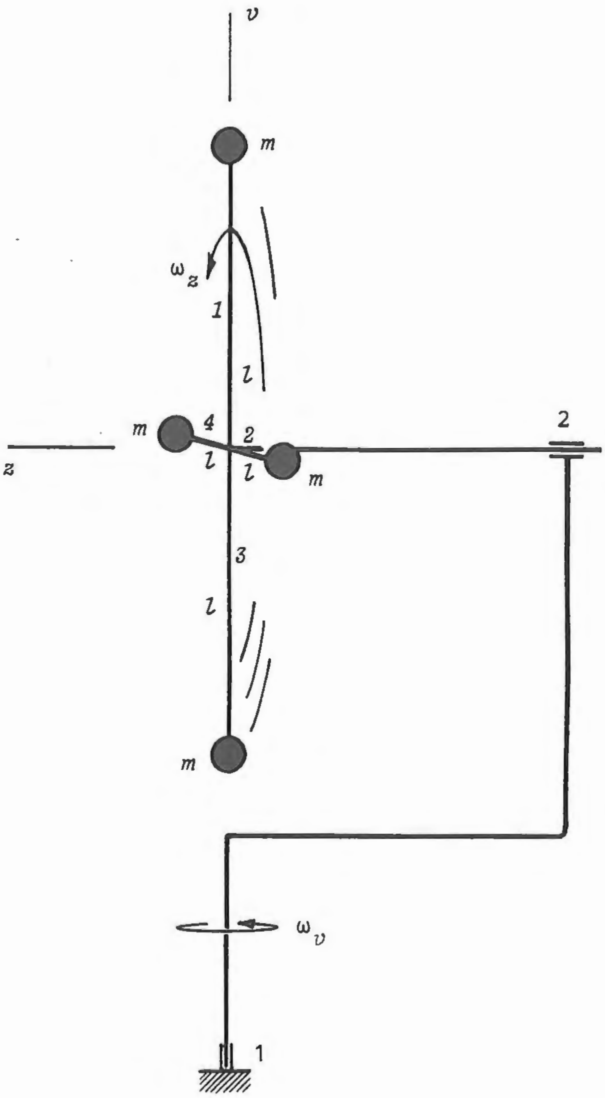

Figure 2 Model in its Undeformed State, used for Numerical Example

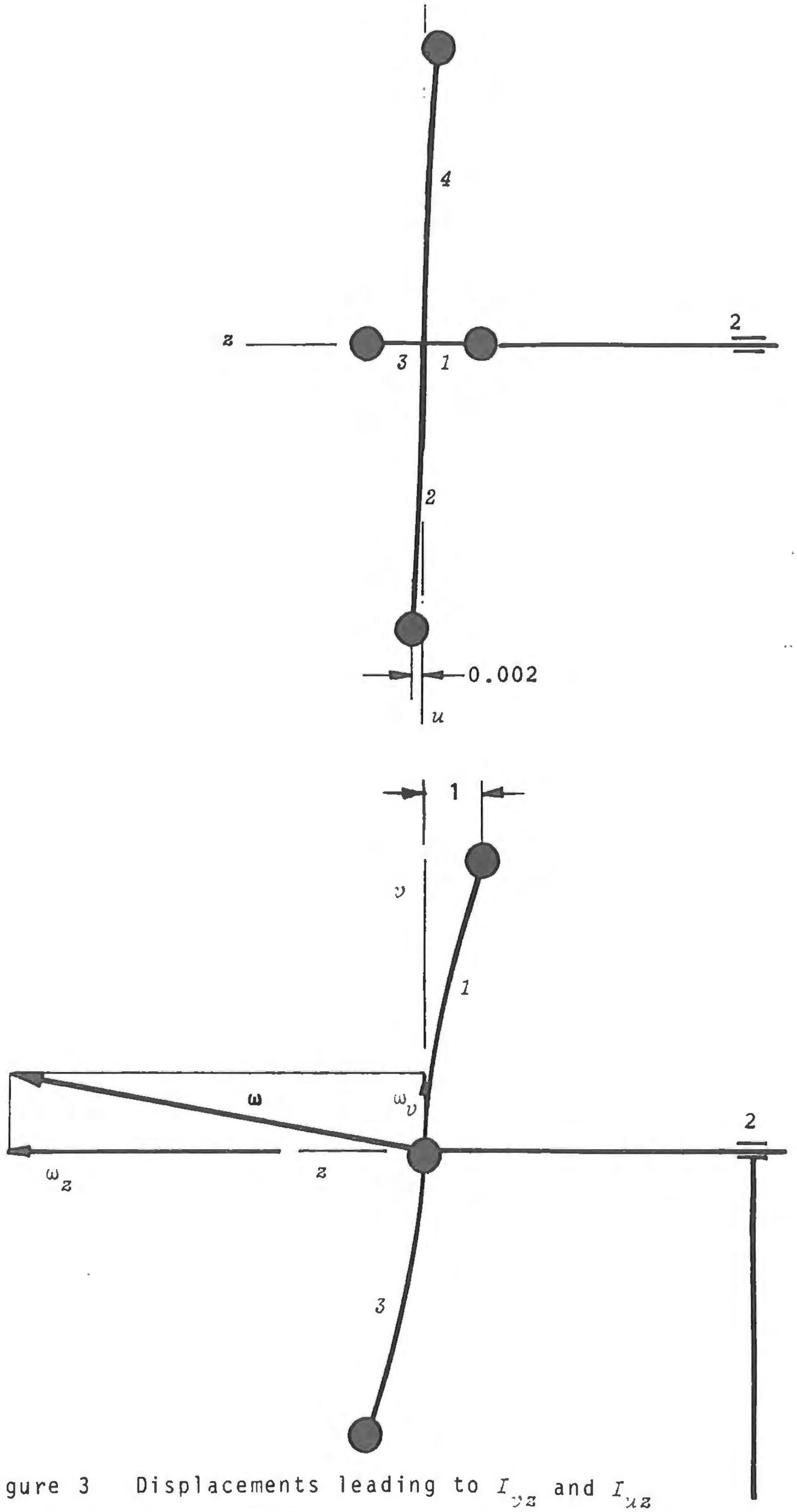

Figure 3 Displacements leading to I_{yz} and I_{xz}

ω_z = 500 rad/s. The (elastic) deformation (of arm *1*) shall be e = - 1 mm and the (hysteretic) deformation (of arm *2*) shall be e_1 = 0.002 mm (Figure 3), both at the moment when $\sigma = 0^\circ$.

We then obtain

$$B = A = 2ml^2 = 2(0.25)0.5^2 = 0.125 \text{ Ws}^3$$

$$C = 0.25\, ml^2 = 0.25 \text{ Ws}^3$$

$$I_{vz} = -2mel = -2(0.25)(-0.001)(0.5) = 0.00025 \text{ Ws}^3$$

$$I_{uz} = -2me_1 l = -2(0.25)\ 0.000002(0.5) = -0.000\ 0005 \text{ Ws}^3$$

The inertia tensor in the deformed state is thus

$$I = \begin{bmatrix} 0.125 & 0 & -0.000\ 0005 \\ 0 & 0.125 & 0.00025 \\ -0.000\ 0005 & 0.00025 & 0.25 \end{bmatrix} \text{ Ws}^3$$

The angular velocities are

$$\overline{\omega} = [\,\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z\,] \begin{bmatrix} 0 \\ 2 \\ 500 \end{bmatrix} \text{ rad/s}$$

$$\overline{\Omega} = [\,\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z\,] \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ rad/s}$$

The angular momentum is

$$\overline{H} = [\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z] \begin{bmatrix} 0 & - \; 0.000\ 25 \\ 0.25 & + \; 0.125 \\ 125 & + \; 0.0005 \end{bmatrix} \quad Ws^2$$

The torque applied to the gyro is

$$\overline{M} = [\overline{e}_u \quad \overline{e}_v \quad \overline{e}_z] \begin{bmatrix} 250 & + \; 0.001 \\ 0 & \\ 0 & + \; 0.0005 \end{bmatrix} \quad Ws$$

the power supplied to feed the hysteresis loss is

$$\dot{U} = 0.000\ 0005(2)500^2 = 0.25 \quad W$$

The Cardan angle

$$\xi = \frac{I_{vz}}{A - C} = \frac{0.00025}{-\ 0.125} = -0.002 \text{ rad} = -0.11459^{\circ}$$

The Cardan angle

$$\eta = \frac{I_{uz}}{C - A} = \frac{-\ 0.000\ 0005}{0.125} = -\ 0.000\ 004 \text{ rad} = -\ 0.000\ 229^{\circ}$$

The elastic deformation coefficient is

$$\alpha = \frac{I_{vz}}{\omega_v \omega_z} = \frac{0.00025}{2(500)} \ Ws^5 = 0.25 \quad \mu Ws^5$$

The hysteretic energy dissipation coefficient, with
$\dot{\sigma} = \omega_z - \Omega_z = 500 - 0 = 500$ rad/s, is

$$\beta = -\frac{I_{uz}}{\omega_v \omega_z \dot{\sigma}} = \frac{0.000\ 0005}{2(500)500} \ Ws^6 = 1 \quad pWs^6$$

Using $\dot{D} = \beta\omega_v^2\omega_z^2\dot{\sigma}^2$ the energy dissipation rate becomes

$$\dot{D} = 1(10^{-12})2^2(500^2)500^2 = \mathit{0.25} \text{ W}$$

and is seen to equal $\dot{U}$.

The gyro's kinetic energy is

$$T = \frac{1}{2}\bar{H}\cdot\bar{\omega} = \frac{1}{2}(A\omega_v^2 + C\omega_z^2 + 2I_{vz}\omega_v\omega_z)$$

$$= \frac{1}{2}(0.125(2^2) + 0.25(500^2) + 2(0.00025)2(500))$$

$$= 31250.5 \text{ Ws}$$

The gyro's angular momentum has a magnitude of

$$H = \sqrt{0.00025^2 + 0.375^2 + 125.0005^2} = 125.0010625 \text{ Ws}^2$$

The elastic energy E stored in the gyro is

$$E = I_{vz}\omega_v\omega_z = 0.00025(2)500 = 0.25 \text{ Ws}$$

such that the total energy stored in the gyro is

$$T + E = \frac{1}{2}(A\omega_v^2 + C\omega_z^2 + 4I_{vz}\omega_v\omega_z) = 31250.75 \text{ Ws}$$

The numerical values shown in italics are those due to the presence of energy dissipation, i.e. hysteresis.

Summing up, the energy rates equation

$$\dot{U} - \dot{D} = \dot{T} + \dot{E}$$

expressed in numbers would read

$$0.\text{[illegible]} - \text{[illegible]} = 0 + 0 \quad \mathrm{W}$$

If the elastic deformation would have been ignored, i.e. if $I_{vz} = 0$ had been used throughout, there would have been no effect on the energy rates (Rimrott /5/).

CONCLUSIONS

From a study of the constrained system, and in particular from a perusal of the numerical example, the reader will conclude that the elastic deformations (as expressed by I_{vz}) lead only to very minor adjustments of the results, compared to results obtained by assuming absolute rigidity. This is not the case for the deformations due to hysteresis (as expressed by I_{uz}) which lead to a new component in the angular momentum, a new component in the expression for the applied torque, and eventually to the conclusion that mechanical energy to compensate for the energy dissipation must continually be fed to the gyro.

The longstanding practice of ignoring elastic deformations in gyrodynamic calculations appears consequently justified.

Deformations due to hysteresis (in spite of the fact that they are typically much smaller than elastic deformations), however, lead to new terms and can therefore not be ignored.

NOTATION

A, B, C	principal inertia moments [Ws^3]
C	mass centre
D	dissipated energy [Ws]
E	elastic energy [Ws]
$\overline{\mathrm{H}}$	angular momentum [Ws^2]

T	kinetic energy [Ws]
U	work [Ws]
$\bar{e}$	standard basis vector
e	elastic deformation [m]
e_1	displacement due to hysteresis [m]
l	length [m]
$\bar{r}$	position vector [m]
u, v, z	floating principal coordinate system for undeformed state
x, y, z	principal coordinate system for undeformed state
$\bar{\Omega}$	angular velocity of $Cuvz$ coordinate system [rad/s]
α	elastic deformation coefficient [Ws^5]
β	energy dissipation coefficient [Ws^6]
$\bar{\delta}$	displacement vector of mass element [m]
ξ, η, ζ	Cardan angles [rad]
$\bar{\rho}$	position vector of mass element in body prior to deformation [m]
$\dot{\sigma}$	Euler spin rate [rad/s]
$\bar{\omega}$	angular velocity of $Cxyz$ coordinate system [rad/s]
$\cdot$	derivative with respect to time, in absolute space
$\circ$	derivative with respect to time, in $Cuvz$ space

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