NATURAL FREQUENCIES OF A CIRCULAR SAW BLADE AND THE EFFECT OF COLLARS AND DAMPING MATERIALS IN REDUCING VIBRATION NOISE

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ABSTRACT: The experiment described in this paper proved that the resonant frequencies of a circular saw blade system could be estimated by using the formulae for calculating the natural frequencies of an annular plate if some dimensions were modified. The presence of some irregular modal shapes was noted. The resonant frequencies increased when collars of larger diameter were used. It was found that the use of larger collars could reduce the mechanical vibration noise from the saw blade and also improve the stability of the latter. Suitable damping materials inserted between the circular saw blade and the collars were also found to lower the mechanical vibration noise. The experiment was carried out without a rotating test.

INTRODUCTION

Circular woodcutting saws are widely used in the lumber, plywood, chipboard, fibreboard and furniture industries. The total number of circular woodcutting saws in operation round the world probably runs into millions. The noise radiated from a circular saw is a significant problem, and normally the noise level at the operator's position exceeds 100 dB(A) /1/.

In the discussion on the noise problem of a circular saw a distinction is generally drawn between idling noise and cutting noise. Idling saw noise may be considered under two aspects: a) vibration noise generated by the mechanical transverse vibration of the circular saw blade and b) aerodynamic noise resulting from interaction of the rigid saw blade, including teeth, with the surrounding air. In the case of cutting noise, besides saw blade vibration noise and aerodynamic noise, there is the vibration noise due to workpiece vibration. Usually, cutting noise level is higher than idling noise level. However, when a circular saw is under resonance while idling, the noise level may sharply increase by about 15 dB, and during cutting the workpiece will partly damp the saw blade vibration /4,5/.

There are two different views concerning the force source exciting the transverse vibration of a circular saw blade. One is that the transverse vibration is caused by mechanical factors, such as imbalance or eccentricity of rotor.

The alternative views is that the transverse oscillation of the saw blade is aerodynamic in origin /5/. Considering that both views have been proved experimentally, we can accept each of them, i.e. the transverse vibration of a circular saw blade is both mechanical and aerodynamic in origin.

In addition to causing noise, transverse vibration effects the stability of a circular saw. The displacement on the rim of a circular saw is much greater when it is vibrating at its natural frequencies than when it is far from these. Under resonance a circular saw will lose its stability. As a result, the teeth are more easily worn out, the direction of cut in the workpiece goes out of true, the cut surface of the workpiece is rough, and the cutting kerf in the workpiece is somewhat larger than normal. Concerning this last point, it was estimated that the potential annual saving in material achievable by reducing the kerf from 3,8 mm to 3,3 mm on all circular saws was 40000 m^3 with a current market value of $ 4,21 million /6/.

Therefore, it is worth predicting the natural frequencies of any given circular saw, so as to be able to avoid operating it in their vicinity.

In practice, a circular saw is used together with a pair of collars which clamp the saw blade. When larger collars are used, the stability of a circular saw blade is improved /6,7/. This means that the resonant frequencies are also changed.

Furthermore, if some kind of damping material is inserted between saw blade and collars, it may be possible to reduce to a certain extent the displacement on the rim of the saw blade and the noise level due to the mechanical transverse vibration. It is this idea that prompted the experiment described in this paper.

CALCULATING THE NATURAL FREQUENCIES OF A CIRCULAR SAW BLADE

A circular saw blade is a two-dimensional centric-symmetrical sheet of elastic material lying in a plane with its centric part clamped by a pair of collars. The natural frequencies can be calculated using the formula for the calculation of the natural frequencies of an annular palte /1,2,3/:

$$f_{ij} = \frac{2\lambda_{ij}^2}{\pi D^2} \sqrt{\frac{Eh^3}{12\rho(1-\nu^2)}} \qquad \begin{array}{l} i = 0,\ 1,\ 2,\ 3,\ \ldots \\ j = 0,\ 1,\ 2,\ 3,\ \ldots \end{array}$$

where D is the outer diameter of the plate
h is the thickness of the plate
ρ is mass density of the plate material
E is Young's modulus
ν is Poisson's ratio
λ_{ij} is a dimensionless parameter

The dimensionless parameter λ_{ij} is a function of the boundary conditions on the plate, the plate geometry (the ratio of the inner to the outer diameter), and Poisson's ratio:

$\lambda_{ij} = \lambda$ (boundary conditions, geometry, Poisson's ratio)

For each boundary condition the parameter λ_{ij} has an infinite number of solutions, each of which corresponds to a mode shape of the plate, with the nodal index i being the number of nodal diameters (n), and the nodal index j being the number of nodal circles (s) in the mode shape. For example, the relationship between the nodal index i and j and the mode shape of a simply supported circular plate is shown in Fig. 1 /3/.

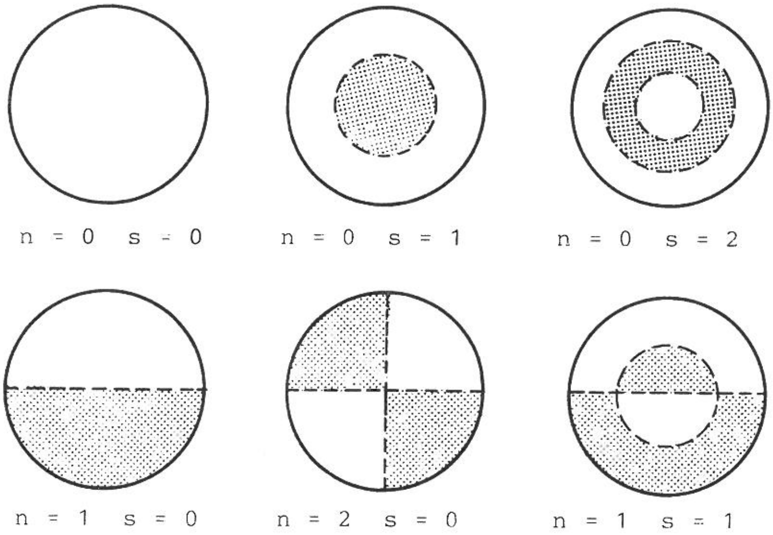


Fig. 1. Relationship between Number of Nodal Diameters (n) and Nodal Circles (s) and Mode Shape of Simply Supported Circular Plate.

The values of parameter λ for an annular plate with Poisson's ratio 0,3 and with the boundary condition clamped at centre-free along edge have been elaborated in Table 1. When the natural frequencies of a given circular saw blade clamped by a pair of collars are estimated using the formulae and Table 1, it should be considered that the rim of the saw blade contains teeth and some slots, and that the effective diameter of the collars clamping the saw blade is a little less than their actual diameter. However, if the effective diameter of the collars is taken to be, for example, 20 % less than their actual diameter, and the outer diameter of the circular saw blade is taken as it is, the result of the calculation for estimating its natural frequencies is accurate to within 5 %, as will be seen later.

EXPERIMENTAL PROCEDURES

The experiment described in this paper was divided into four procedural steps.

The first step was to measure the natural frequencies of a circular saw

Table 1. Values of Parameter λ_{ij} for Annular Plate Clamped at Centre-Free along Edge, $\nu = 0,3$

No of Nodal Circles s	No of Nodal Diameters n	d/D								
		0,10	0,20	0,30	0,40	0,50	0,60	0,70	0,80	0,90
0	0	4,24	5,18	6,66	9,02	13,02	20,52	36,95	84,50	344,40
0	1	3,48	4,81	6,55	9,12	13,29	20,94	37,50	85,16	345,16
0	2	5,62	6,45	7,96	10,46	14,70	22,50	39,28	87,19	347,45
0	3	12,45	12,61	13,28	14,96	18,56	25,96	42,65	90,72	351,34
0	4	21,84	21,86	22,07	22,98	25,60	32,06	48,07	95,92	356,76
1	0	25,26	32,29	42,61	58,55	85,03	133,91	239,84	543,48	2188,82
1	1	27,67	34,53	44,63	60,38	86,71	135,46	241,28	544,83	2190,09
1	2	36,94	41,96	50,95	65,95	91,74	140,09	245,59	548,87	2193,91
1	3	53,20	55,41	62,05	75,45	100,17	147,77	252,73	555,59	2200,27
1	4	73,56	74,19	78,12	89,07	112,05	158,50	262,68	564,96	2209,19
2	0	73,90	94,08	123,47	168,69	243,69	381,86	680,65	1535,26	6155,58
2	1	77,49	96,95	125,82	170,69	245,44	383,41	682,05	1536,53	6156,76
2	2	90,18	106,13	133,08	176,76	250,68	388,06	686,24	1540,37	6160,30
2	3	113,04	122,62	145,69	187,06	259,49	395,82	693,23	1546,75	6166,21
2	4	142,55	146,64	164,07	201,80	271,94	406,73	703,02	1555,69	6174,47
3	0	146,69	186,43	244,19	333,07	480,45	751,84	1338,43	3015,33	12076,00
3	1	151,27	189,76	246,79	335,20	482,27	753,42	1339,84	3016,60	12077,16
3	2	166,55	200,11	254,68	341,63	487,71	758,17	1344,06	3020,41	12080,65
3	3	194,21	218,32	268,16	352,45	496,83	766,09	1351,10	3026,77	12086,46
3	4	231,60	245,05	287,56	367,78	509,65	777,20	1360,96	3035,67	12094,59
4	0	243,96	309,70	405,27	552,36	796,25	1245,30	2215,72	4989,25	19971,74
4	1	249,27	313,33	408,01	554,56	798,10	1246,90	2217,13	4990,52	19972,89
4	2	266,48	324,47	416,30	561,21	803,67	1251,70	2221,37	4994,31	19976,34
4	3	297,56	343,75	430,33	575,34	812,96	1259,71	2228,43	5000,65	19982,10
4	4	341,25	371,93	450,34	588,04	826,00	1270,93	2238,32	5009,51	19990,15

blade with collars of five different diameters.

The characteristics of the saw blade:

Outer Diameter D = 350 mm
Thickness h = 2,55 mm
Number of Teeth z = 56
Number of Slots = 4
Young's Modulus E = $2{,}1 \cdot 10^{11}$ N/m^2
Poisson's Ratio ν = 0,3

The individual outer diameters of the collars:

50 mm, 70 mm, 100 mm, 140 mm and 200 mm

The circular saw blade was clamped by collars and excited at its rim. Granulated sugar was spread on the surface of the saw blade in order to obtain nodal shapes when the saw blade was vibrated at its natural frequencies. With the smallest collars, the nodal shapes on the surface of the saw blade were obtained when the number of nodal circles (s) changed from 0 to 4, and when the number of nodal diameters (n) changed from 0 to 4. With the largest collars the nodal shapes were obtained when the number of nodal diameters changed from 0 to 4 and the number of nodal circles remained 0. With the other collars, the nodal shapes were obtained when the number of nodal circles changed from 0 to 1.

The second step was to measure the linearly weighted noise level while the circular saw blade, clamped by different collars, was excited using an amplified random noise kept at uniform level as the exciting force.

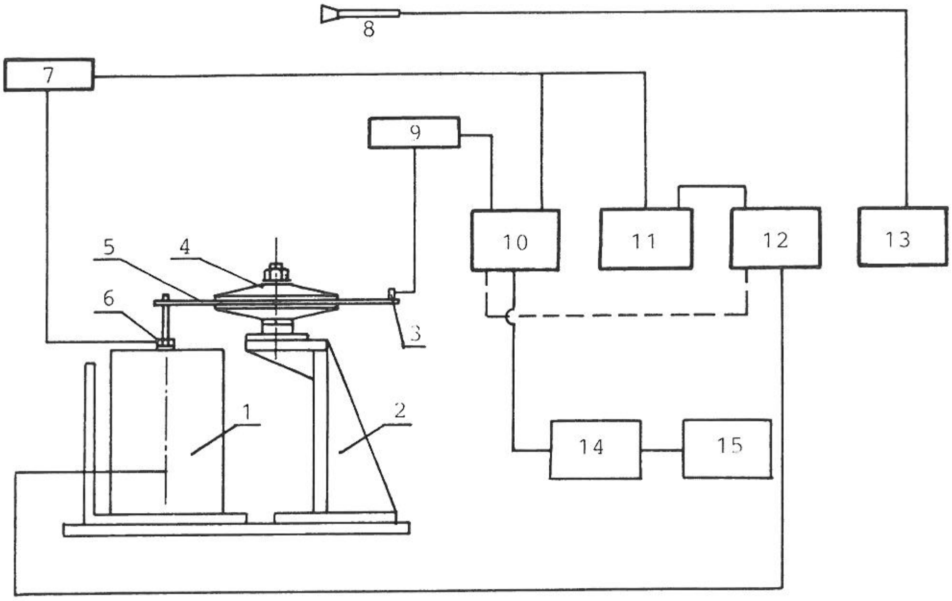
Next, certain damping materials were inserted between saw blade and collars and the same procedure was repeated each time. The damping material used was rubber of thickness 2 mm and paper of thickness 1 mm.

The third step was the frequency analysis of the audible noise from the vibrating saw blade. Three pairs of collars with the diameters 70 mm, 100 mm and 140 mm were used. The saw blade was excited using an amplified random noise kept at uniform level as the exciting force, while the noise level from the saw blade was measured in one third octave. Next, damping material was inserted between saw blade and collars, whose diameter in this case was only 100 mm, and the same procedure was repeated each time.

Finally, the saw blade was excited at different frequencies while the A-weighted noise level (dB) from the saw blade was measured using collars of 70 mm, 100 mm and 140 mm and inserted damping material.

In the course of this experiment, the saw blade was not tested under rotating conditions. The reason behind this was that if the circular saw blade was rotating, the noise level being measured would include mechanical vibration noise and aerodynamic noise, as well as perhaps noise from the electric motor and the rotating machine. Such being the case, the effect of larger collars and inserted damping material in reducing mechanical vibration noise could hardly be distinquished clearly.

The equipment and instruments used in this experiment are shown in Fig. 2.



1 Exciter
2 Saddle
3 Accelerometer
4 Collar
5 Circular Saw Blade
6 Force Transducer
7 Preamplifier
8 Microphone
9 Premplifier
10 Analyser
11 Exciter Control
12 Power Amplifier
13 Frequency Analyser
14 Table Computer
15 Plotter

Fig. 2. Equipment and Instruments Used in the Experiment.

RESULTS AND DISCUSSION

The natural frequencies of the circular saw blade clamped by collars of diameter 50 mm obtained in the experiment are shown in Table 2. The values for the natural frequencies arrived at by calculation alone are also listed in the same table. Note that the diameter of collars was taken as 20 % less than actual diameter, and that the other dimensions were taken as they were. The modal shapes on the surface of the saw blade at resonant frequencies as found in the experiments are shown in Fig. 3.

As can be seen in Table 2, both sets of values for the natural frequencies i.e. those obtained in the experiment and those arrived at by calculation alone, are quite close to each other, the divergence being generally not more

Table 2. Comparison of Calculated and Measured Frequencies.
Collar Diameter 50 mm.

Nodes		Frequency arrived at by calculation alone f_0 H_z	Frequency found by experiment f H_z	Divergence $\delta = \frac{f - f_0}{f_0}$ %
Circle	Diameter			
0	0	90	87	-3,3
0	1	76	77	+1,3
0	2	118	119	+0,8
0	3	258	264	+2,3
0	4	453	431	-4,9
1	0	542	558	+3,0
1	1	591	590	-0,2
1	2	777	769	-1,0
1	3	1106	1052	-4,9
1	4	1526	1480	-3,0
2	0	1584	1586	+0,1
2	1	1656	1657	+0,1
2	2	1906	1891	-0,8
2	3	2358	2290	-2,9
2	4	2960	2853	-3,6
3	0	3144	3110	-1,1
3	1	3234	3248	+0,4
3	2	3521	3463	-1,6
3	3	4068	3962	-2,6
3	4	4816	4584	-4,8
4	0	5228	5189	-0,7
4	1	5331	5284	-0,9
4	2	5662	5558	-1,8
4	3	6257	5953	-4,9
4	4	7113	6748	-5,1

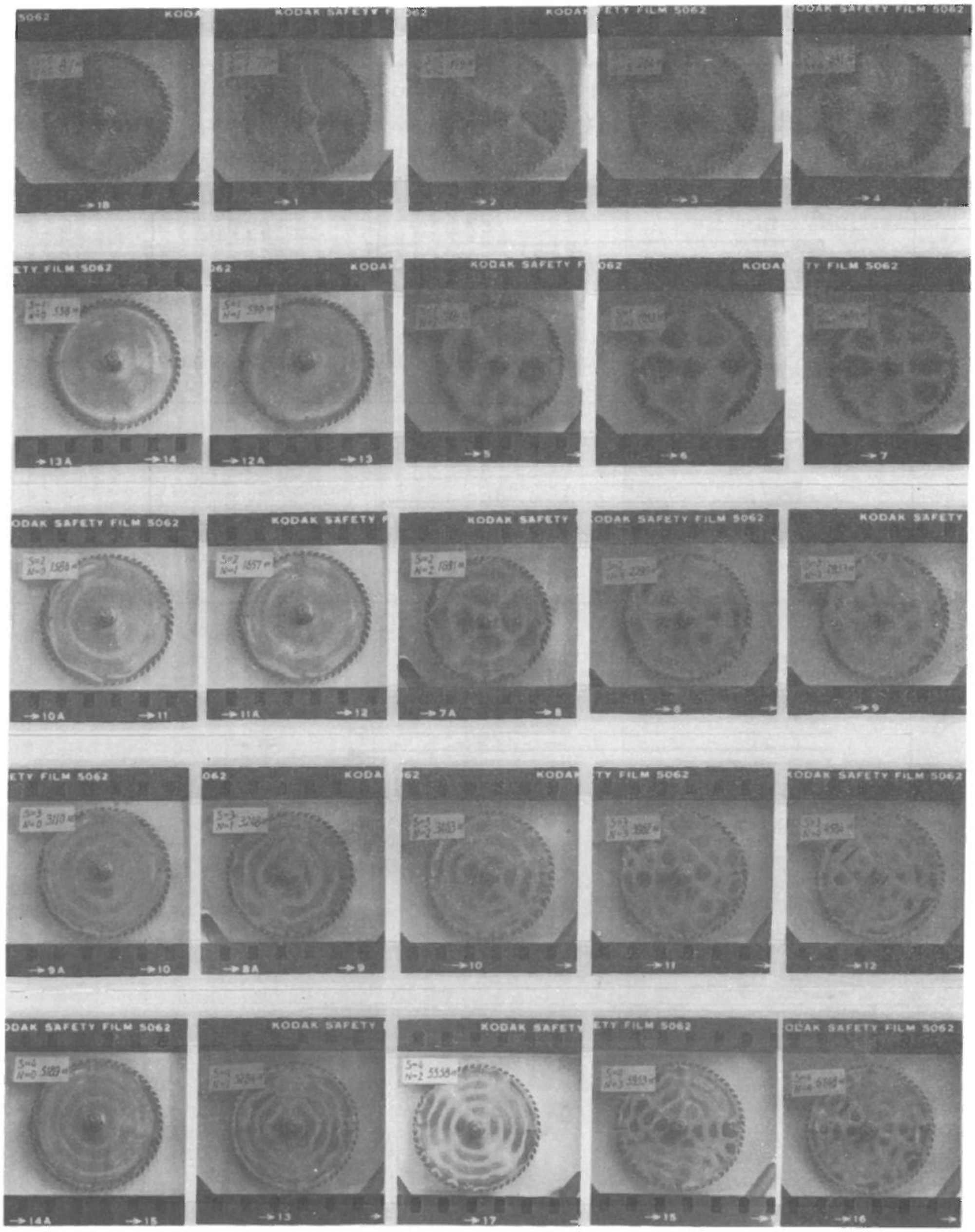


Fig. 3. Modal Shapes at Resonant Frequencies.

than 5 %. In the case of most of the frequencies measured in the experiment, the value was less than that arrived at by calculation alone. The reasons for this may perhaps partly be explained by: a) the effect of teeth and slots (mainly slots); b) the influence of the point of excitation, which was tantamount to adding a limitation on the saw blade; c) the collar diameter used for the calculations being taken as 20 % less, which was perhaps not the optimum. In spite of this, it seems possible to use the formulae given earlier to estimate the natural frequencies of a circular saw, and especially the lower frequencies.

A circular saw blade has a countless number of degrees of freedom and therefore a countless number of resonant frequencies also. For example, the number of circles of modal shapes (s) could in theory be 0, 1, 2, ... to inifinity, and the number of diameters of modal shapes (n) also could be 0, 1, 2, ... to inifinity. In the experiment, of course, it was not possible to obtain all of them. However, besides these regular modal shapes having circles and diameters, there were also some irregular modal shapes, some examples of which are shown in Fig. 4. Curiously enough, some of them also had circles and diameters whose numbers were the same as those seen in Fig. 3. A typical example was the modal shape having 1 circle and 2 diameters and a frequency of 732 Hz, while in Table 2 and Fig. 3 the modal shape with the same number of circles and diameters has a frequency of 769 Hz. One possible explanation for this is symmetry in the vibration system. As could be seen, there were some symmetrical axes on the circular saw blade: one pair were the lines connecting the ends of four slots; one pair was between the slots, and another was the line connecting the point of excitation and the centre of the saw blade, and in this experiment was the same as one in the first pair. When the circular saw blade was being excited at its resonant frequencies, the diameter lines of modal shape generally lay on the symmetrical location relative to those symmetrical lines, or just on those lines. Wherever they

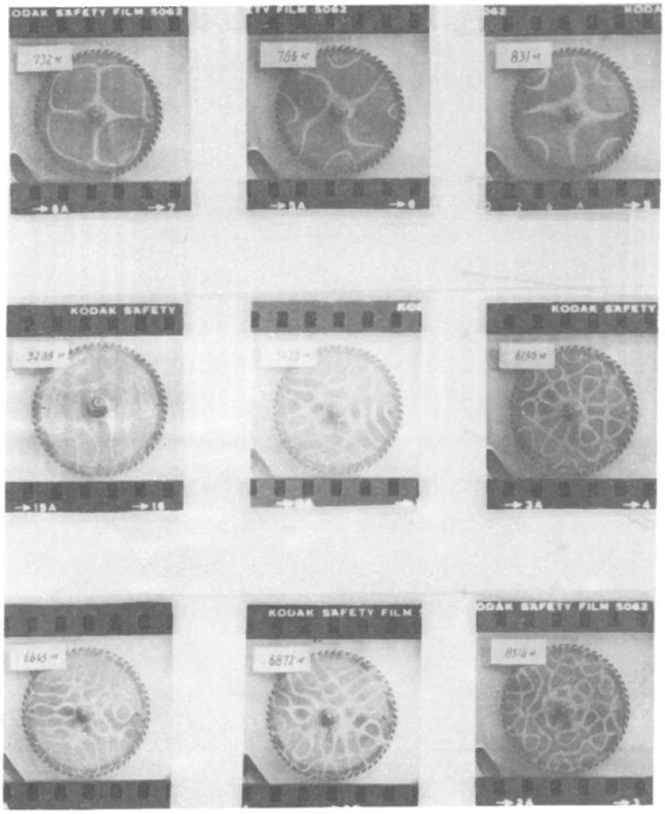


Fig. 4. Some Irregular Modal Shapes.

do in fact lie depends on actual condition. It seems possible that some irregular modal shapes can be combined in some way by two, or even more, normal modal shapes or are probably caused by non-linearity of the saw blade due to the presence of teeth, slots and so forth. But that is going beyond the scope of this paper.

When a circular saw blade was equipped with collars of different diameters, its resonance frequencies changed. The resonant frequencies of the saw blade clamped by collars of five different diameters are given in Table 3. Fig. 5 was plotted on the basis of the values in Table 3. As could be expected, the resonant frequencies increased when collars of larger diameter were used.

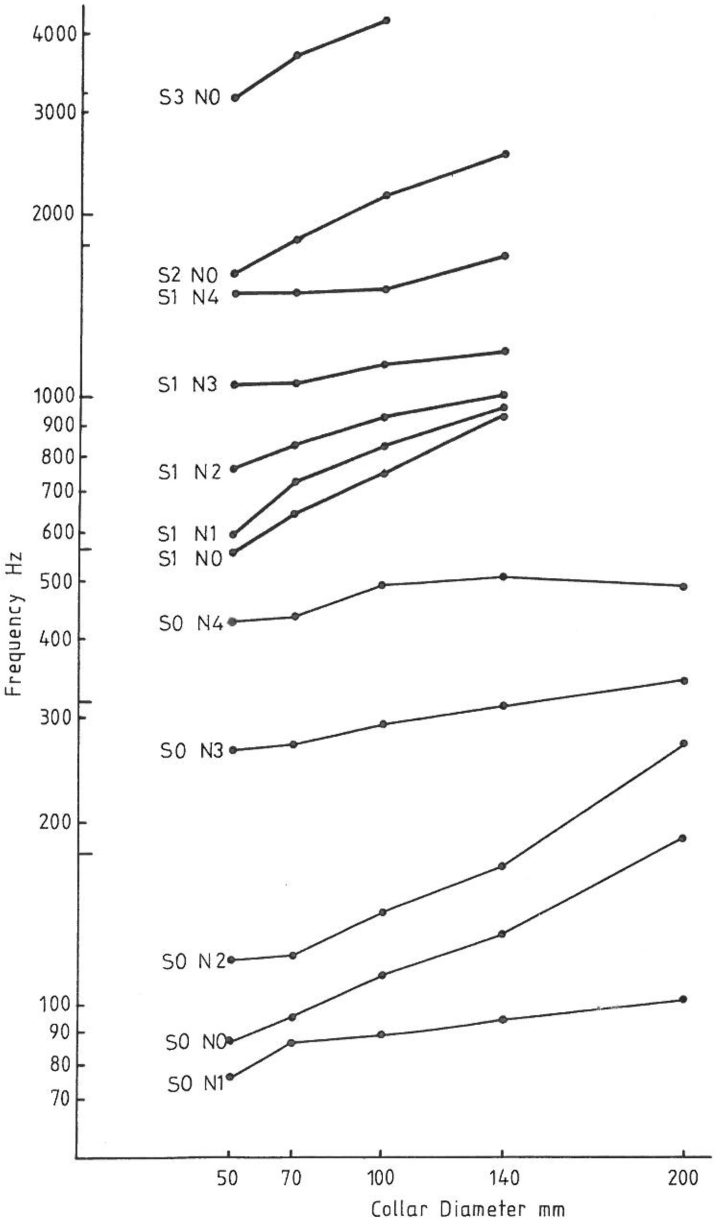


Fig. 5. Relation between Resonant Frequencies and Collar Diameter.

Table 3. Measured Resonant Frequencies of Clamped Blade Using Different Collars.

Nodes		I (ø 50 mm)	II (ø 70 mm)	III (ø 100 mm)	IV (ø 140 mm)	V (ø 200 mm)
Circle	Diameter	f_1 Hz	f_2 Hz	f_3 Hz	f_4 Hz	f_5 Hz
0	0	87	96	112	131	189
0	1	77	87	89	95	103
0	2	119	122	142	169	368
0	3	264	269	290	309	344
0	4	431	439	490	504	486
1	0	558	640	750	943	
1	1	590	724	831	960	
1	2	769	831	921	1004	
1	3	1052	1076	1126	1200	
1	4	1480	1493	1516	1714	
2	0	1586	1832	2166	2512	
3	0	3110	3665	4216		

While the circular saw blade clamped by collars of different diameters was excited using an amplified random noise signal as exciting force, the noise level of the saw blade was measured at a position one metre above the saw blade. The same procedure was repeated when the damping materials, rubber and paper, were inserted between the saw blade and the collars of different diameters. The measured results are shown in Fig. 6. The curves show that the noise level can be somewhat reduced by using larger collars and by inserting damping material, that the use of larger collars produces a more beneficial effect than using damping material, and that if both methods are employed the result is even better again.

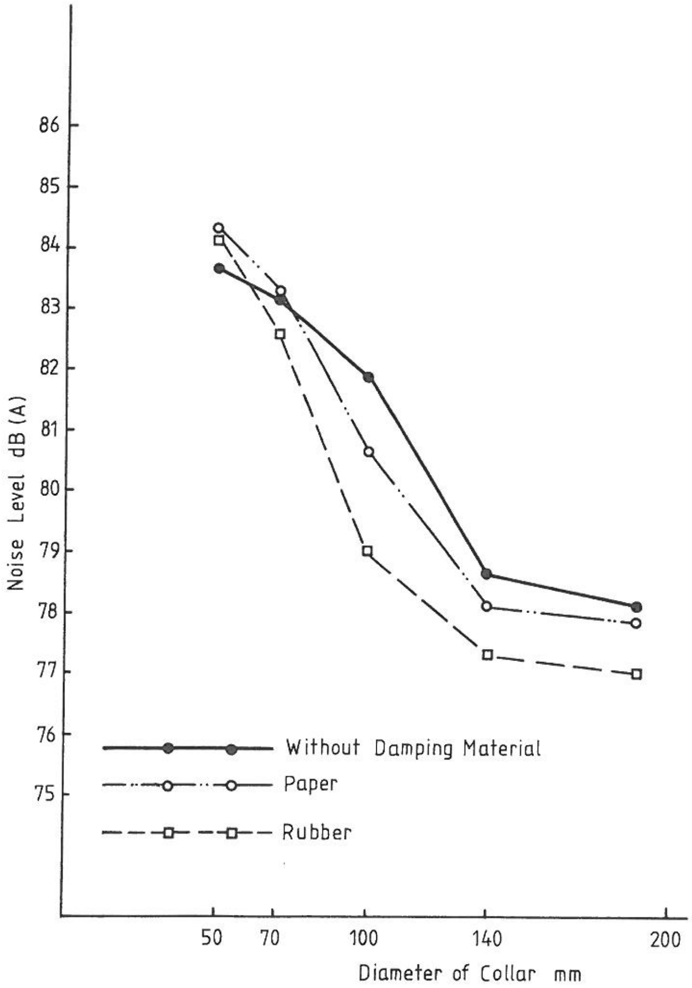


Fig. 6. Noise Level Using Different Collars with and without Damping Materials.

During this step in the procedure, the velocity on the rim of the saw blade was measured also. Some results are shown in Fig. 7 which show that the velocity on the rim of the circular saw blade decreased with the increase in collar diameter. This means that the stability of the saw blade was somewhat improved.

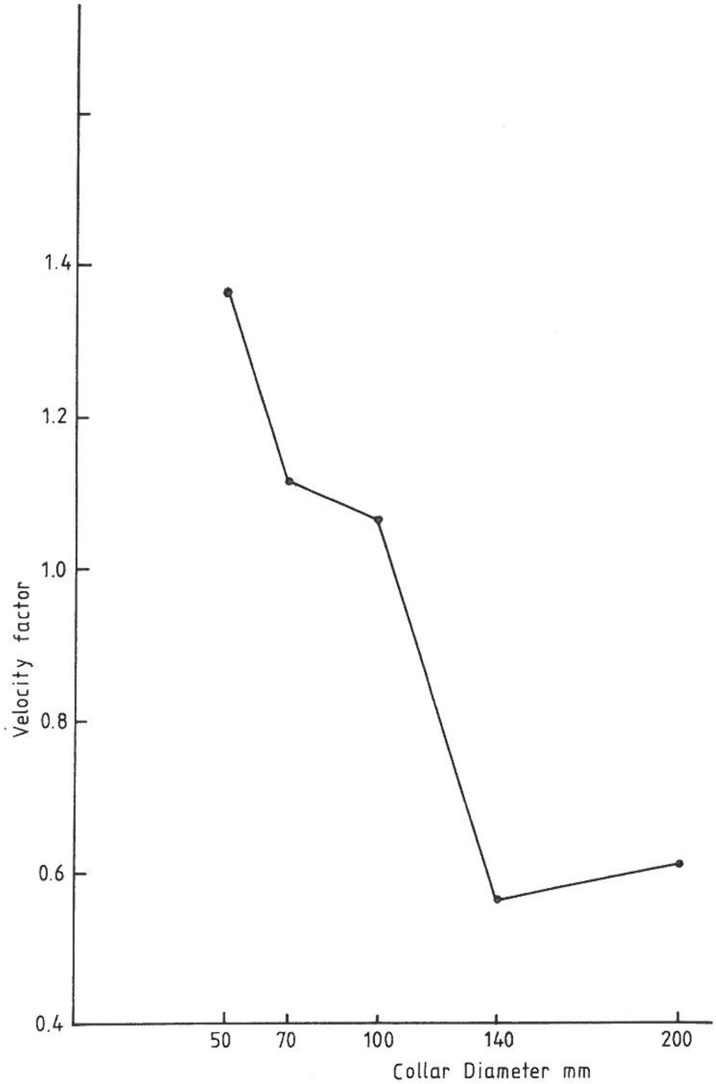


Fig. 7. Relation of Rim Velocity and Collar Diameter.

The analysis of the noise from the saw blade excited by an amplified random noise signal was carried out with a one-third octave band. Fig. 8 shows the noise spectrum curves of the saw blade with the individual collar diameters of 70 mm, 100 mm and 140 mm.

It is evident that when larger collars are used, the noise level at higher frequencies is reduced, while at lower frequencies it increases slightly. The results obtained when damping materials were used are shown in Fig. 9. It seems clear that in the lower frequency range the noise levels with and without the insertion of damping material were almost the same in magnitude, but in the higher frequency range damping material had a clearly lowering effect on the noise level.

Express mention must here be made that, as was previously pointed out, the saw blade used in this experiment was not tested under rotating conditions and that therefore the noise spectrum curves given here are not the real rotating noise spectrum curves. They merely show the results obtained when

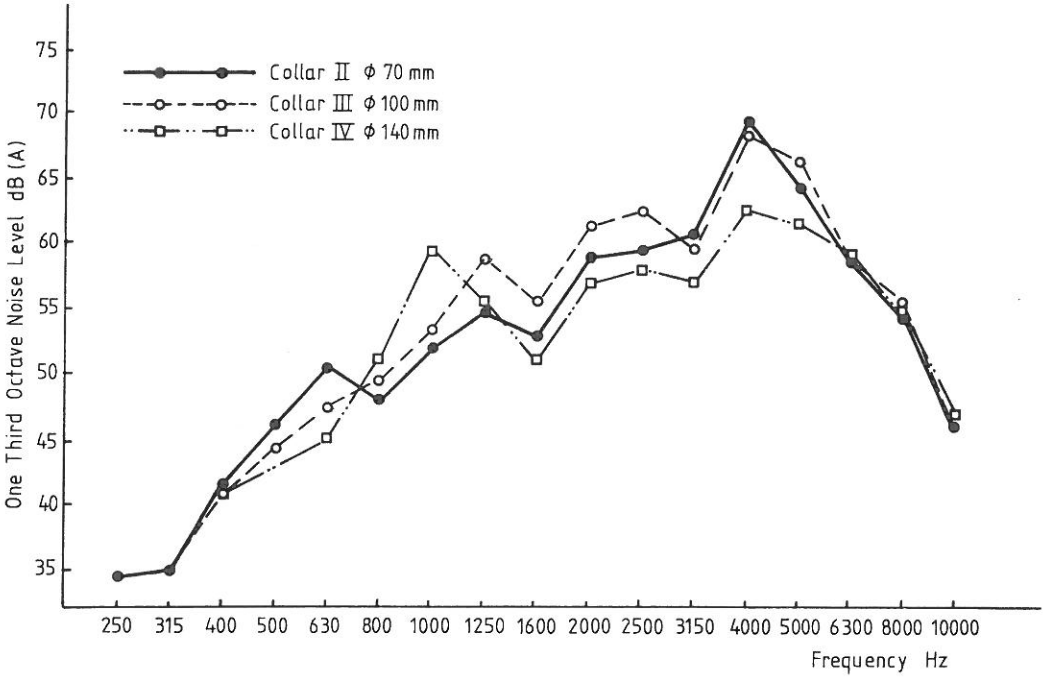


Fig. 8. Noise Spectrum Curves of Saw Blade Using Different Collar Diameters.

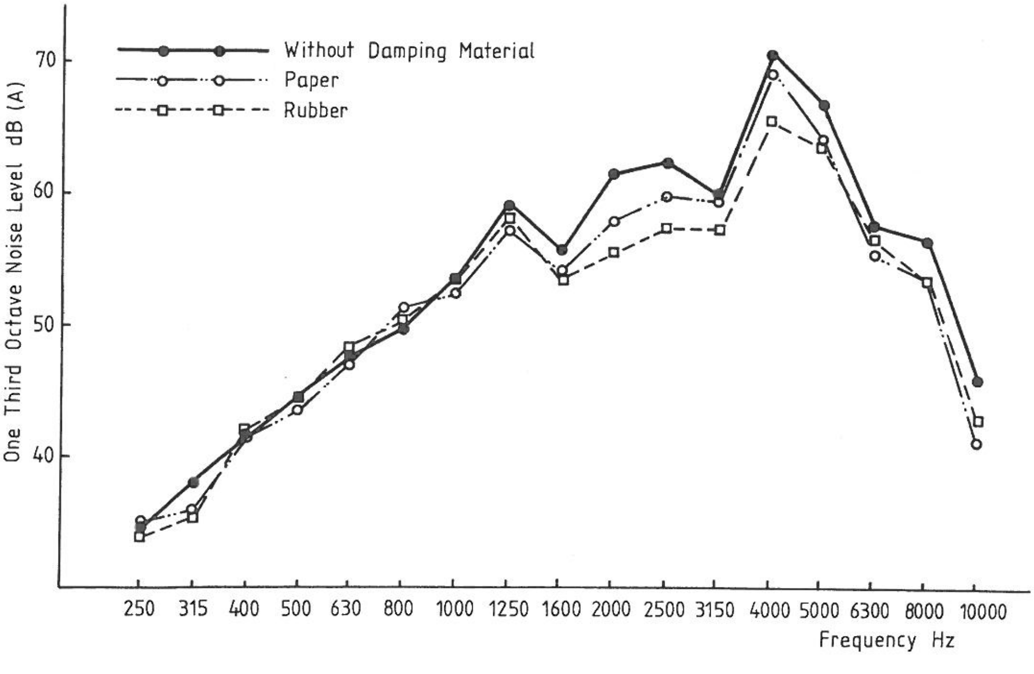


Fig. 9. Noise Spectrum Curves of Saw Blade Using 100 mm Collars with and without Damping Materials.

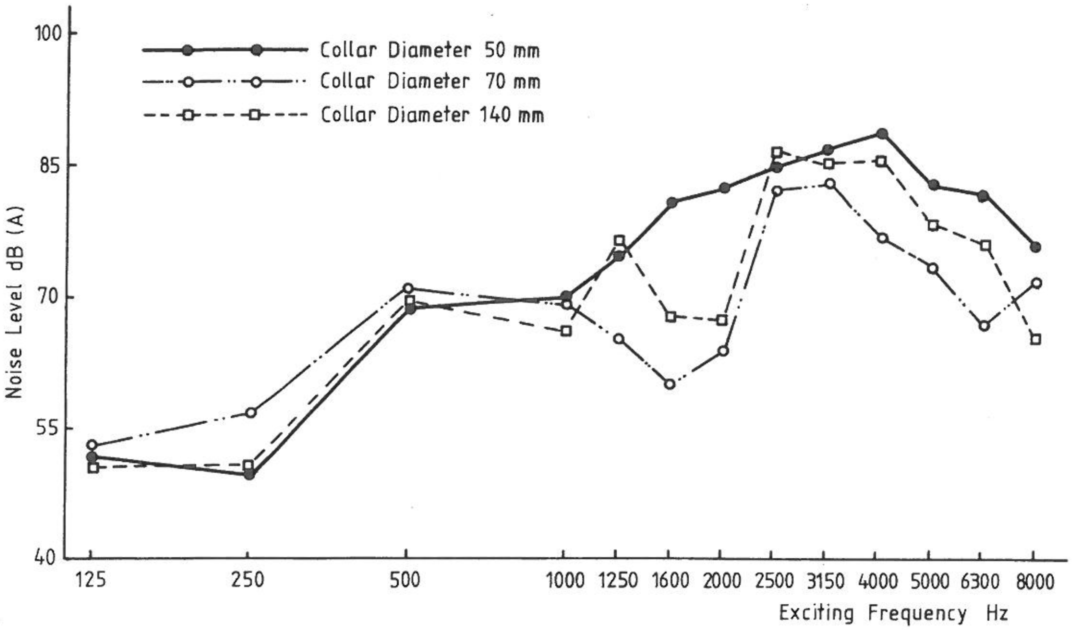


Fig. 10. Noise Level of Saw Blade Excited at Different Frequencies Using Different Collar Diameters.

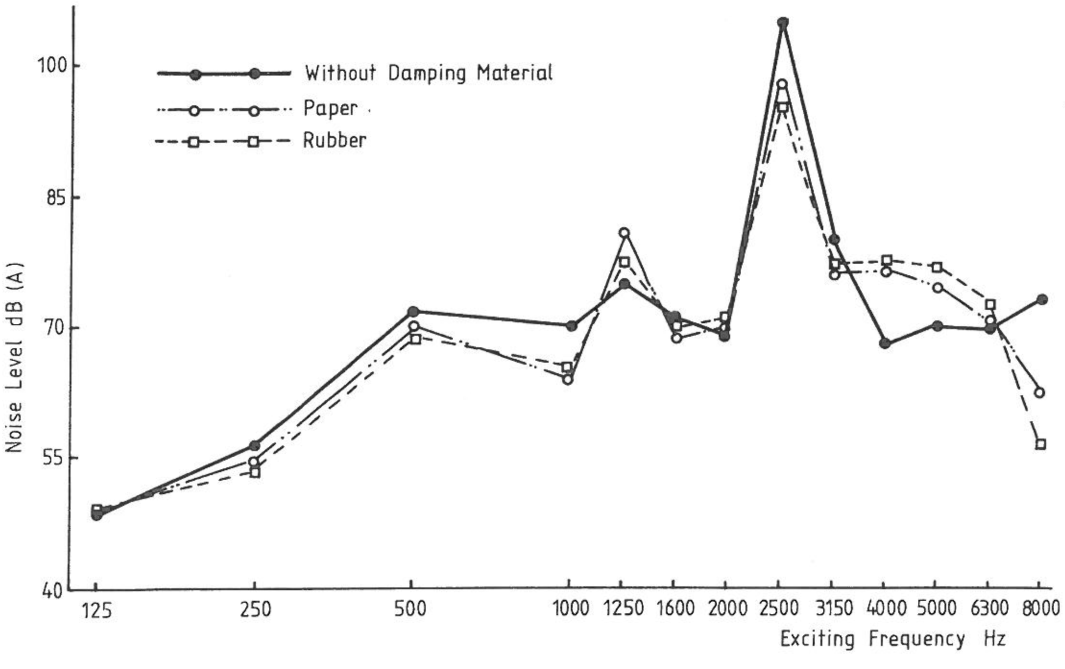


Fig. 11. Noise Level of Saw Blade Excited at Different Frequencies with and without Damping Materials.

the saw blade was excited by an amplified random noise signal. Clearly the level and the position of noise peaks could not be the same as the real ones. However, the main purpose of this experiment was to examine the influence of collars of different diameters as well as certain damping materials on the noise level of a saw blade, and the method of procedure adopted in the experiment is justified with this purpose in view.

In order to estimate the effect of larger collars and damping material in practice, without testing the saw blade in rotation, it was excited at different frequencies and the noise level measured at the same time. The results are shown in Fig. 10 and Fig. 11. As can be seen in Fig. 10, the noise level in the higher frequency area was reduced when larger collars were used, and in the lower frequency area it increased slightly.

When damping materials were used, as is seen in Fig. 11, the noise level was reduced on the whole, but in some ranges of exciting frequencies it somewhat increased, even if the difference was not so clear.

CONCLUSIONS

On the basis of the results and discussion above, the following general conclusions may be drawn:

1. The natural frequencies of a circular saw blade clamped by a pair of collars can be predicted using the formulae for calculating the natural frequencies of an annular plate. However, certain parameters on the saw blade system must be modified in some way, and the results of calculations do not contain the frequencies of irregular modal shapes.
2. The use of collars of larger diameter for clamping a saw blade increases the resonant frequencies of the saw blade system and improves its stability.
3. The mechanical vibration noise level of a saw blade can be reduced using collars of larger diameter. The diameter should be as large as possible.
4. Suitably chosen damping material, inserted between a saw blade and the collars, lowers the mechanical vibration noise level.

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