OPTIMIZATION OF STRUCTURES TO RESIST EARTHQUAKES - A FINITE ELEMENT APPROACH

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SUMMARY: The finite element method is used in connection with a direct search algorithm to optimize general structures resisting earthquakes. By mathematical approximation the eigenproblem size is reduced and by using only the first mode a design concept is presented. Examples show the efficiency of the direct search algorithm.

INTRODUCTION

This paper shows how by using a finite element idealization an optimal design for earthquake loading can be found. The advantage of the finite element idealization with its different element types is that not only special types of structures like shear buildings can be optimized efficiently.

Design variables $\underline{x}$ are the element thicknesses of plate elements and the cross-section areas of bars and beams. With beam elements it is assumed that inertia properties and section moduli can be depicted as nonlinear functions of the cross-section area x. Objective $f(\underline{x})$ is the cost of the structure, which has to be a minimum under given constraints $\underline{G}(\underline{x}) \geq 0$. The cost is assumed to depend linearly or nonlinearly only on the design variables and not on any mechanical values like stresses and deformations.

Restrictions on the design variables, called "constructional constraints" and on nodal deformations and element stresses, called "structural constraints" result in a highly nonlinear constrained minimization problem. By the constructional constraints the engineer has a tool to give some useful criterion to the algorithm by contributing his design experience. The structural constraints define upper and lower bounds for stresses and deformations as required by the design codes. By equality constraints the design variables become independent of the finite element idealization, e.g. different elements have the same design variable. Considering this in advance the size of the otpimization problem is reduced.

Especially the structural constraints are highly nonlinear in the design variables. Gradients with respect to the design variables can usually be obtained only by approximation. The sophisticated methods of mathematical programming like quasi Newton and conjugate gradient method are therefore not appropriate.

Direct search algorithms, which do not require the computation of gradients, were investigated for several mathematical test examples by Schwefel /1/. One of the most reliable algorithms presented there, is used here to solve the described optimization problem.

FINITE ELEMENT FORMULATION

The equilibrium condition of a structure under earthquake loading in the displacement method with matrix notation is

$$[\underline{K}(\underline{x}) + \underline{K}_g(\underline{F}_s)]\underline{r}(t) + \underline{M}\ \ddot{\underline{r}}(t) = -\underline{M}\ \underline{R}\ \ddot{r}_a(t) \quad . \qquad (1)$$

The elastic stiffness matrix $\underline{K}(\underline{x})$ depends on the design variables $\underline{x}$. The geometric stiffness matrix $\underline{K}_g(\underline{F}_s)$ is supposed to depend only on the static loading of the structure. A lumped mass matrix $\underline{M}$ is assumed. All matrices are of dimension n by n. The time dependent nodal deformations and accelerations are $\underline{r}(t)$ and $\ddot{\underline{r}}(t)$.

Earthquakes generate by the horizontal component of the ground acceleration $\ddot{r}_a(t)$ mainly lateral forces represented by the time independent loadvector $\underline{R}$ (usually a (0,1) vector) /2/.

By solving the eigenproblem

$$[\underline{K}(\underline{x}) + \underline{K}_g(\underline{F}_s)]\underline{\phi} - \underline{M}\ \underline{\phi}\ \underline{\Omega} = \underline{0} \qquad (2)$$

one obtains the modal matrix $\underline{\phi}$ and the diagonal spectral matrix $\underline{\Omega} = \lceil\omega_i^2\rfloor$. Based on the orthonormality requirement of the eigenvectors $\underline{\phi}^i$ modal analysis is performed.

The response spectrum S_a represents a smoothed maximum value function of response acceleration of a single degree of freedom system with variable natural period T_i. In Figure 1(a) response spectrum for steel with 5 % critical damping and 0,1 g maximal ground acceleration (Mercally intensity VII) is shown /3/. Using the response spectrum S_a and the time independent

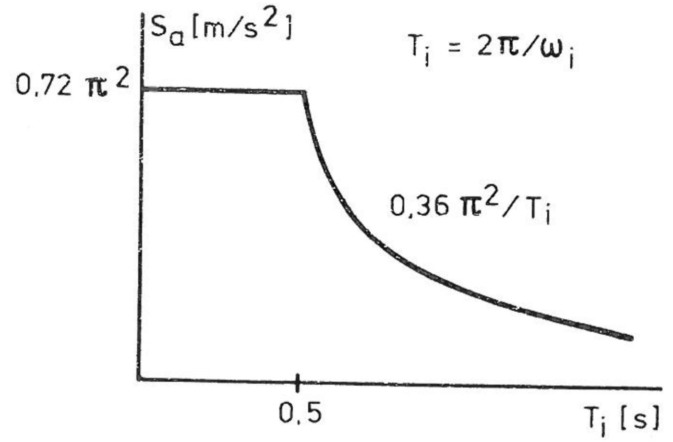


Fig. 1. Response spectrum for steel.

loadvector $\underline{R}$ an upper bound of the normal coordinate q^i of i'th mode is found:

$$q_i = \frac{1}{\omega_i^2} (\underline{\phi}^i)^T \underline{M} \underline{R} S_a(\omega_i) \qquad i = 1,\ldots,n. \tag{3}$$

Therewith nodal deformations $\underline{\bar{r}}^i$ and element stresses $\underline{\bar{\sigma}}^i$ are

$$\underline{\bar{r}}^i = q_i \underline{\phi}^i \qquad i = 1,\ldots,n \tag{4}$$

$$\underline{\bar{\sigma}}^i = \lceil \underline{B}^j(\underline{x}) \; \underline{u}^i \rfloor \qquad j = 1,\ldots,m \tag{5}$$

with the blockdiagonal stress matrix $\lceil \underline{B}^j(\underline{x}) \rfloor$ and the element deformations $\underline{u}^i$; the number of finite elements is m.

It should be noted that often one stress constraint per element is sufficient, because by modal analysis, as performed here, only the value of q^i is specified.

By rms method /2/, /4/ bounds on deformations and stresses caused by all modes are computed:

$$r_j = \sqrt{\sum_{i=1}^{n} (\bar{r}_j^i)^2} \qquad j \in I_e \tag{6}$$

$$\sigma_j = \sqrt{\sum_{i=1}^{n} (\bar{\sigma}_j^i)^2} \qquad j = 1,\ldots,l. \tag{7}$$

The number of stress control points is l. The set of indices of controlled deformations is I_e.

Thus all constraints can be formulated as inequality constraints of the form

$$\underline{G}(\underline{x}) \geqq \underline{0} \; . \tag{8}$$

APPROXIMATIONS

The main problem in the computations described above is the solution of the eigenproblem. Moreover, as in optimization the solution has to be obtained many times.

Earthquake loading is, as mentioned above, a lateral loading. The degrees of freedom of a structure can therefore be parted into primary degrees (horizontal) and secondary degrees (vertical, rotations). Decomposing the eigenproblem according to this and using the assumption that $\underline{M}$ is a lumped mass matrix one obtains /5/

$$\begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} \\ \underline{K}_{21} & \underline{K}_{22} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} - \omega_i^2 \begin{bmatrix} \underline{M}_1 & \underline{0} \\ \underline{0} & M_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{0} \end{bmatrix} . \tag{9}$$

Substituting in this equation the secondary part $\underline{\phi}_2$ by the primary one $\underline{\phi}_1$

$$\underline{\phi}_2 = -(\underline{K}_{22} - \omega_i^2 \underline{M}_2)^{-1} \underline{K}_{21} \underline{\phi}_1 \tag{10}$$

and using series expansion for the inverse with the first eigenvalue, the eigenproblem is reduced to the primary degrees of freedom:

$$(\bar{\underline{K}}_1 - \omega_1^2 \bar{\underline{M}}_1) \underline{\phi}_1 = \underline{0} \tag{11}$$

with

$$\bar{\underline{K}}_1 = \underline{K}_{11} - \underline{K}_{12} \underline{K}_{22}^{-1} \underline{K}_{21} , \tag{12}$$

$$\bar{\underline{M}}_1 = \underline{M}_1 + \underline{K}_{12} \underline{K}_{22}^{-1} \underline{M}_2 \underline{K}_{22}^{-1} \underline{K}_{21}$$

and

$$\underline{\phi}_2 = -(\underline{K}_{22}^{-1} + \omega_1^2 \underline{K}_{22}^{-1} \underline{M}_2 \underline{K}_{22}) \underline{K}_{21} \underline{\phi}_1 . \tag{13}$$

Herewith problem size can be reduced between 50 % and 66 %. This seems to be more advantagous than the usual assumption that a structure behaves like a shear building.

The first eigenvalue is considered only. The sufficiency of this approach is problem dependent and must be checked. Equations (6) and (7) are necessarily not relevant in this case.

Other possibilities to reduce the problem size are

- utilizing symmetry of a structure having in mind that lateral loading is antimetric;
- neglecting axial deformations of beam elements;
- coupling of degrees of freedom;
- neglecting the mass moment of inertia.

OPTIMIZATION ALGORITHM

The optimization problem is

$$\min \{f(\underline{x})\} \qquad \underline{x} \in R^{\alpha}, \quad f(\underline{x}) : R^{\alpha} \to R$$

subject to

$$\underline{G}(\underline{x}) \geq \underline{0} \qquad \underline{G}(\underline{x}) : R^{\alpha} \to R^{\beta}. \tag{14}$$

The number of design variables is α and the total number of constraints is β.

The present algorithm is originated from Rechenberg /6/. It simulates by normal distributed processes an evolution. The change of the design variables is interpreted as a mutation and the decision to use a new vector or not is said to correspond to a selection.

Starting point is supposed to be a feasible design vector $\underline{x}^0$. Iteration (mutation) is counted by ν. With the normal distributed random vector $\underline{z}$ a new design can be found:

$$\underline{y}^\nu = \underline{x}^\nu + \underline{z}^\nu \tag{15}$$

and

$$\underline{x}^{\nu+1} = \begin{cases} \underline{y}^\nu, & \text{if } \underline{G}(y^\nu) \geq \underline{0} \wedge f(\underline{y}^\nu) \leq f(\underline{x}^\nu) \\ \underline{x}^\nu, & \text{otherwise.} \end{cases} \tag{16}$$

The normal distributed random numbers $\underline{z}^\nu$ are calculated from uniformly distributed random numbers $\bar{\underline{z}}$ in the intervall (0,1). (A routine computing uniform random numbers belongs to the standard library of any computer.) A transformation with the stepwidths s_i $(i = 1,\ldots,\alpha)$ results in /7/:

$$\tilde{z}_i^\nu = \sqrt{-2 \ln \bar{z}_i^\nu} \sin (2\pi\bar{z}_{i+1}^\nu) \qquad i: \text{ uneven}$$

$$\tilde{z}_i^\nu = \sqrt{-2 \ln \bar{z}_{i-1}^\nu} \cos (2\pi\bar{z}_i^\nu) \qquad i: \text{ even}$$

$$z_i^\nu = s_i \, \tilde{z}_i^\nu \, . \tag{17}$$

Stepwidth is increased or diminished by up to 15 %, if one success does not occur in average within five iterations. Iteration stops, if accuracy criteria on objective and stepwidth are active or the computing time reaches a given limit. The algorithm solves the constrained problem by accepting only feasible design vectors. This is identical with using an infinite outer penalty function. As the problem is usually not convex only a local minimum can be found in general. This has to be taken into account with applications of the algorithm.

In reference /1/ a FORTRAN code of the algorithm is given. Numerical investigations show that this relatively simple method is characterized by an extreme reliability, which is very important for engineering problems /1/. Rechenberg /6/ proved linear convergence with special problems. In contrary to Schwefel the author changed the algorithm in the respect that instead of controlling the feasibility of a new design the decrease of the objective is controlled first. This results in a 50 % saving of computing time for the following application (see eq. (6)) /8/.

APPLICATION

To solve the design problem by the described direct search algorithm a modular code was written. In a first stage a program for plane structures was taken in which the user can choose the design spectrum and the eigen-problem solver. Objective is the volume of the structure.

Four different routines are available for solving the eigenproblem:

- the complete solution with the computation of all eigenvalues and modes by Householder reduction;
- computation of only the first eigenvalue and mode by von Mises vector iteration without condensation;
- exact condensation for zero mass moment of inertia and computation of all eigenvalues and modes by Householder reduction;
- condensation as described in "Approximations" and computation of only the first eigenvalue and mode by von Mises vector iteration.

Applications with up to 15 variables and 90 constraints showed that the method is very reasonable for earthquake design. In all examples a solution was found. To verify the correctness of a solution it is necessary to begin with at least two different start vectors (see section "Optimazation algorithm"). Comparisons resulted in maximal 3.5 % different objective function values.

EXAMPLES

A plane frame of a school building (Fig. 2), which was optimal for static loading, is optimized for the design spectrum of Figure 1.

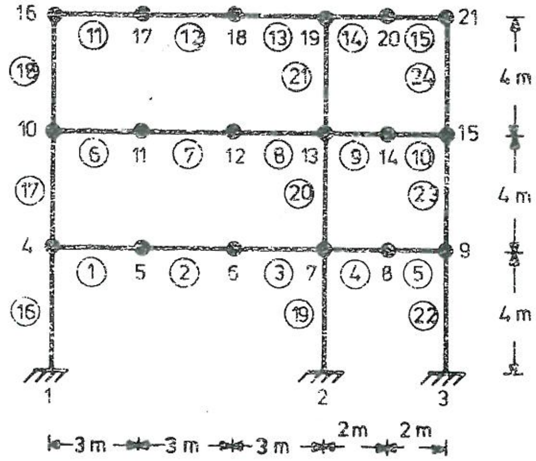


Fig. 2. Plane three storey frame.

The elements have I-cross-sections. The inertia properties J_i and section moduli Z_i are approximated as functions of the cross-section area x_i:

$$J_i = 3.78\, x_i^2$$

and

$$Z_i = 1.58\, x_i^{1.5} .$$

The material is mild steel St37 with a modulus of elasticity of $2.1 \cdot 10^8$ kN/m^2 and a yield stress of $2.4 \cdot 10^5$ kN/m^2. Table 1 shows the assignment of nodes, mass and mass moment of inertia.

Table 1. Node-mass assignment of three storey frame.

node	mass [t]	mass moment of inertia [t m²]
4,10	9.9	0.99
5,6,11,12	13.8	1.38
7,13	19.7	1.97
8,14	12.2	1.22
9,15	9.1	0.91
16	1.95	0.195
17,18	3.84	0,384
19	3.25	0.325
20	2.60	0.260
21	1.30	0.130

The horizontal deformation of node 16 must not exceed 0.05 m ($r_{37} \leq 0.05$). Within all elements normal stress is restricted to the yield stress as required by the design code /4/. From construction it was necessary to have the columns from bottom to top and the girders in each storey of the same size. Table 2 shows the assignment of elements and design variables and the minimal values of design variables as a result of the static loading.

Table 2. Element-design variable assignment.

design variable number	element	lower bound of x_i [m²]
1	1,2,3,4,5	0.0207
2	6,7,8,9,10	0.0208
3	11,12,13,14,15	0.0069
4	16,17,18	0.0130
5	19,20,21,	0.0158
6	22,23,24	0.0094

The first eigensolver routine of section "Application" was used for calculating stresses and deformations. A computation with the minimal cross-sections causes an overstressing of up to 72 %, but feasible deformations. By raising the design variables with the ratios of overstressing a first start vector is found (Table 3). As stepwidths s_i each component is set to 0.01 m^2 at the beginning. To improve the results a second start vector with 0.023 m^2 for each design variable is used.

Table 3. Start and final vectors of design variables.

design variable number	final design I x_i [m^2]	final design II x_i [m^2]
1	0.0219	0.0219
2	0.0208	0.0208
3	0.0069	0.0069
4	0.0159	0.0149
5	0.0258	0.0291
6	0.0155	0.0122

Figure 3 shows the iteration history for both start vectors. The strengthening of the structure is represented by the percentage of yield stress reached in each element. For both start vectors and the related

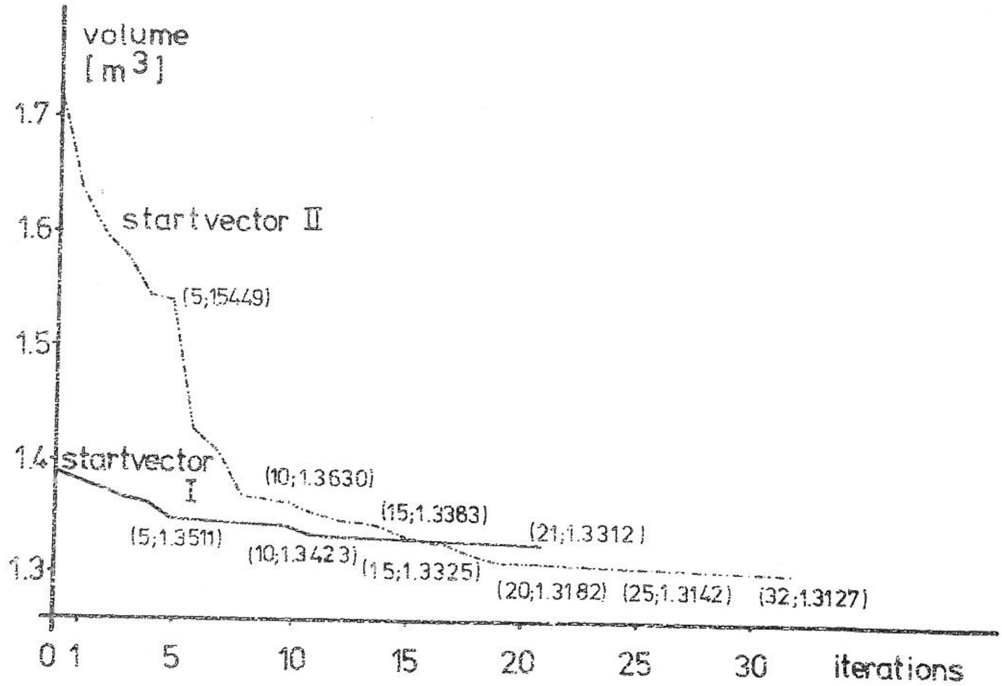


Fig. 3. Iteration history of three storey frame.

final designs the strengthening is depicted in Figure 4.

The difference between both solutions is 1.4 %. The final designs represent both local minima, which are caused by the activation of at least two constraints. The mechanical behaviour of both designs is different: whereas the column in the middle of both designs is relatively strong, only in the final design II the outer columns are according to the structure of different size. The symmetry of design I is contradictory to the unsymmetric structure.

As a second example a six storey plane frame (Fig. 5) is optimized for the design spectrum of Figure 1.

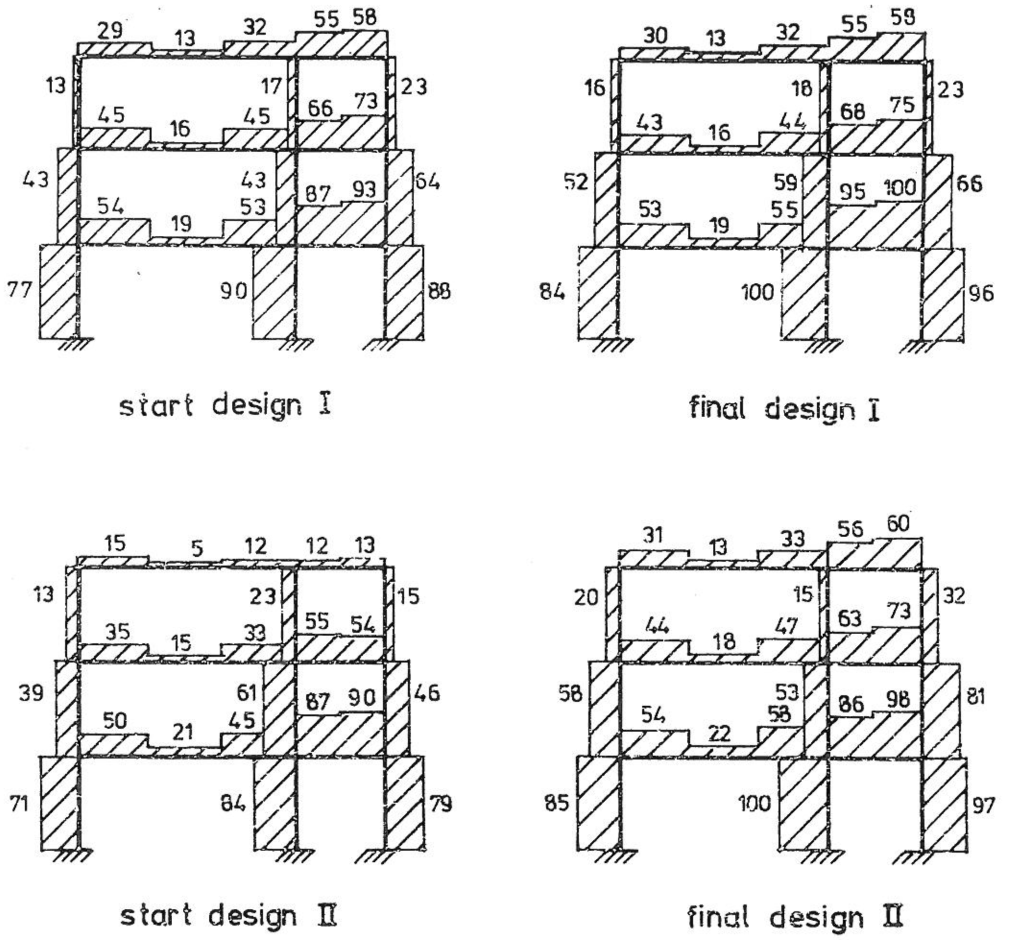
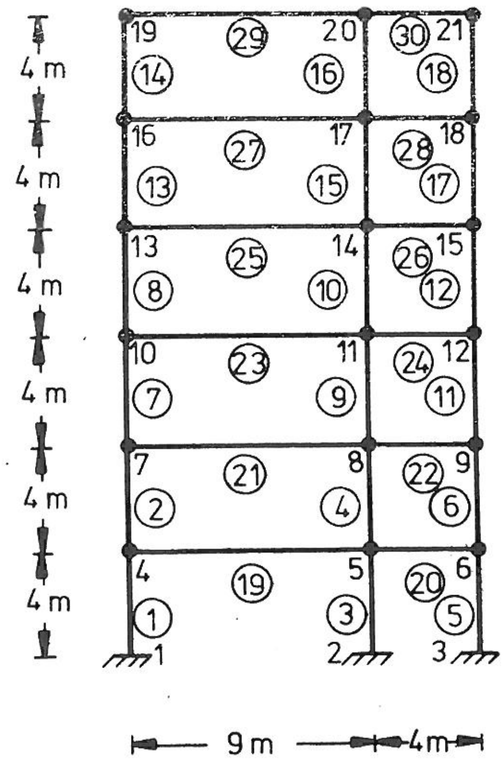


Fig. 4. Stresses in start and final designs as percentage of yield stress.



node	mass [t]	mass moment of inertia [tm^2]
4, 7, 10 13, 16	23.7	2.37
5, 8, 11 14, 17	39.5	3.95
6, 9, 12 15, 18	15.2	1.52
19	5.79	0.579
20	8.39	0.836
21	2.60	0.260

Fig. 5. Plane six storey frame.

The data of material, cross-section properties and step-width are the same as in the first example. Stresses in all elements and the horizontal deformation of node 19 are restricted ($r_{46} \leq 0.05$ m). To solve the eigenproblem the fourth solver of section "Application" is used. Table 4 represents the assignment of elements and design variables and minimal and starting values of the design variables.

Table 4. Assignment of elements and design variables.

design variable number	element	lower bound of x_i [m^2]	start vector I [m^2]	start vector II [m^2]
1	1,2	0.015	0.05	0.06
2	3,4	0.020	0.05	0.06
3	5,6	0.015	0.05	0.06
4	7,8	0.015	0.05	0.06
5	9,10	0.020	0.05	0.06
6	11,12	0.015	0.05	0.06
7	13,14	0.015	0.05	0.06
8	15,16	0.020	0.05	0.06
9	17,18	0.015	0.05	0.06
10	19,20	0.020	0.05	0.06
11	21,22	0.020	0.05	0.06
12	23,24	0.020	0.05	0.06
13	25,26	0.020	0.05	0.06
14	27,28	0.020	0.05	0.06
15	29,30	0.020	0.05	0.06

Figure 6 shows the iteration history for both start vectors. The number of iterations is 45 and 68, respectively. These curves show that one should

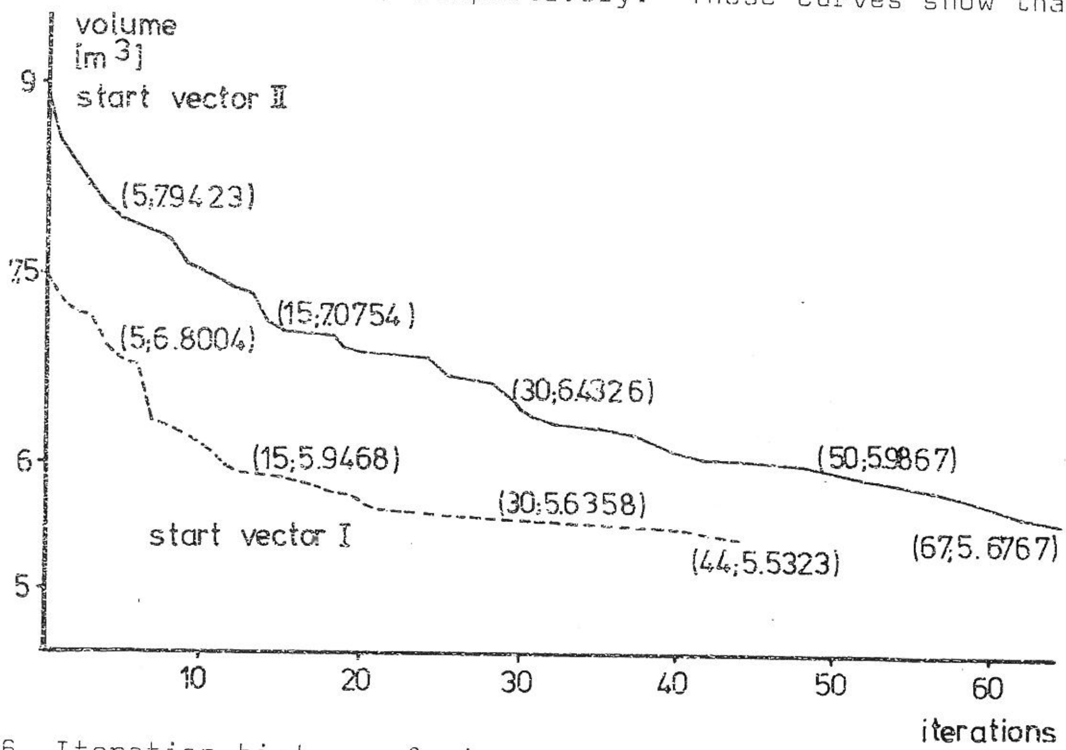


Fig. 6. Iteration history of six storey frame.

start with a feasible design as close as possible to some constraints. With both start vectors the deformation constraint is active at the local minima. The difference of the objective function between the two final designs is 2.5 %. Improvement is 26 % and 37 % respectively.

The final stress distribution and cross-section areas can be seen in Figure 7. Stresses are again percentages of yield stress, and cross-section areas are of dimension $10^{-4} m^2$; in brackets are the minimal allowable cross-section areas for comparison. From Figure 7 it is evident that by the structure

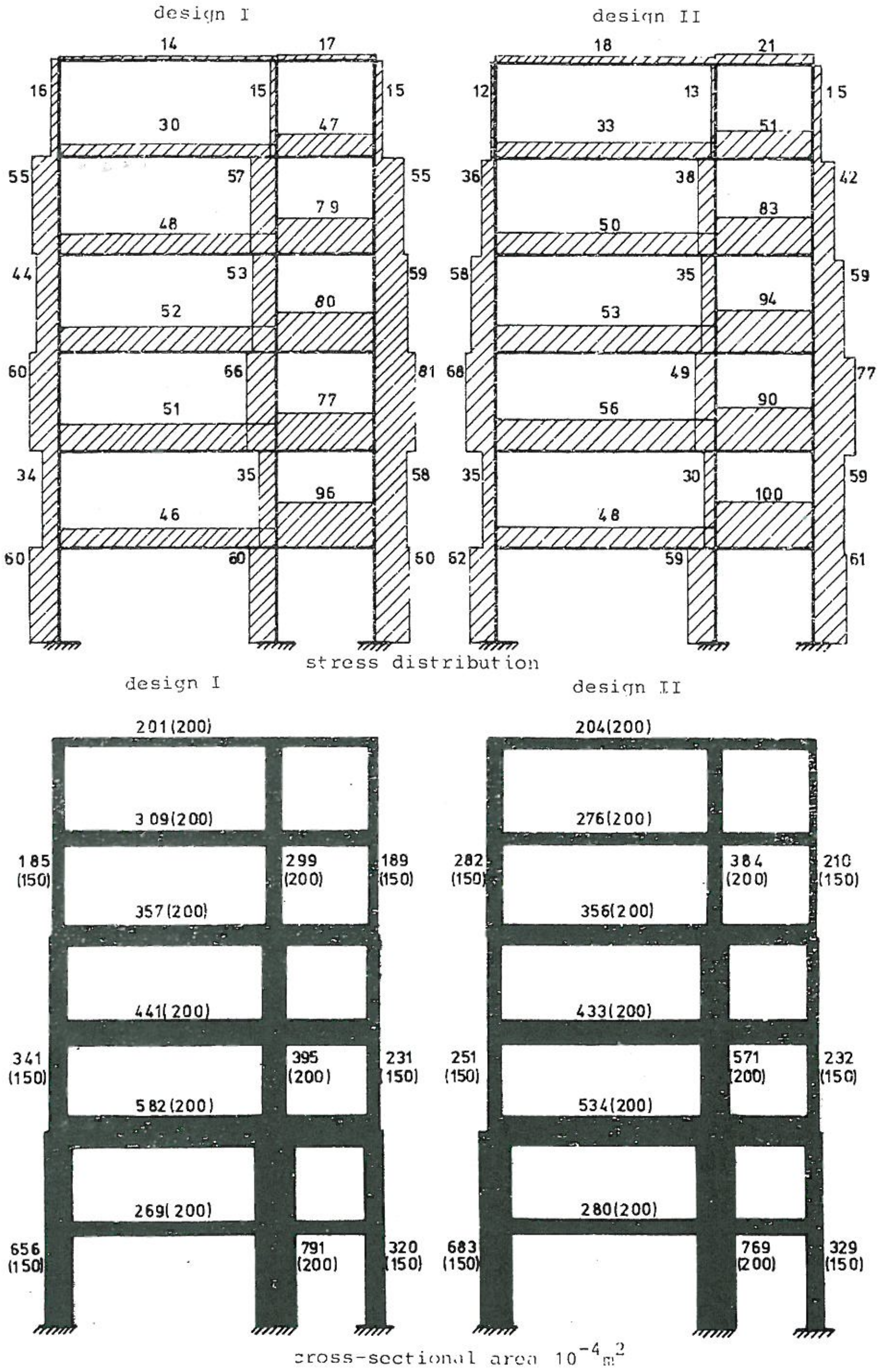


Fig. 7. Stresses and cross-section areas in final designs.

a special mode of action is emphasized and both final designs produce a different layout of it, although the volume of both is nearly the same.

CONCLUSIONS

This paper describes an application of a direct search algorithm to earthquake resistant design. The displacement method and a finite element model are applied. Results of two examples illustrate the procedure.

NOTATION

α	number of design variables
β	number of constraints
$f(\underline{x})$	objective function
$\underline{F}_s$	vector of inner axial forces caused by static loading
g	acceleration due to gravity (9.81 m/s^2)
$\underline{G}(\underline{x})$	vector of constraints
$J_i(x_i)$	moment of inertia
$\underline{K}(\underline{x})$	elastic stiffness matrix
$\underline{K}_g(\underline{F}_s)$	geometric stiffness matrix
l	number of stress control points
m	number of elements
$\underline{M}$	lumped mass matrix
n	number of degrees of freedom
ω_i	natural frequency of i'th mode
$\underline{\Omega}$	spectral matrix
$\underline{\Phi}$	modal matrix, matrix of eigenvectors
q_i	normal coordinate
$\underline{r}(t), \ddot{\underline{r}}(t)$	vectors of nodal deformations and accelerations
$\ddot{r}_a(t)$	horizontal component of ground acceleration during an earthquake
s_i	stepwidth of variable x_i
$S_a(\omega_i)$	response spectrum
$\underline{\sigma}$	stress vector
t	time
T_i	natural period
$\underline{x}$	vector of design variables
$\bar{z}_i, \tilde{z}_i, z_i$	uniformly normal distributed, weighted random numbers
$Z_i(x_i)$	section modulus
Superscript T	transpose of a matrix

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