UNBALANCE EXCITED VIBRATIONS OF A RIGID ROTOR WITH ANISOTROPIC BEARINGS

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SUMMARY: For a rotor, with anisotropic bearings, the response to unbalances can be characterized with elliptical rotor point paths. The magnitude and phase angle of displacements are varying periodically in time. However, the elliptical rotor motion can be considered as the sum of two circular cynchronous motions, one in the spin direction and the other in the opposite direction. By measuring the magnitude and phase of the displacement in two perpendicular directions those circular motion components are obtained. The phenomena of backward precessional critical speed is also treated in this paper. An example of practical application is outlined.

INTRODUCTION

Usually, the vibrations of rotating machinery, e.g. turbines, separators, machine tools /1/, are due to unbalances. Those vibrations are especially crucial when passing through the critical speed of the machinery. Therefore, during the last decades a number of computer programs for predicting unbalance reponse of rotors have been developed /2/. Rather sophisticated mathematical models have been used and a reasonable agreement with reality is generally obtained.

However, these sophisticated models are hardly intelligble for the human mind. By studying very elementary models qualitative understanding of many phenomena is gained. Furthermore in the area of rotor dynamics, many results obtained for single-degree-of-freedom systems, can be applied on multi-degree-of-freedom systems by means of transformation to modal parameters /3/. Tondl /4, 5, 6/ has studied single-degree-of-freedom systems in order to study the stability of rotors.

The eigenfrequencies of a rotor are dependent on the speed of revolution. This is due to the gyral effects. When the spin speed of the rotor coincides with one of the eigenfrequencies a critical speed is obtained. The calculation of critical speeds is of fundamental importance in rotor dynamics. However, also the case when the ratio spin speed/eigenfrequency is an integer may be of some interest. Early investigations of these "second-order critical speeds" have been performed by Fernlund /7/. When the ratio is equal to -1 the speed is denoted critical speed of backward precession /8/.

In this paper it is shown that those critical speeds may be excited due to unbalances when the bearings are anisotropic. A similar example was earlier studied by Gasch & Pfützner /9/.

MATHEMATICAL MODEL

In order to readily understand the effects, analysed in this paper, a very simple mathematical model is studied:

A gyro is mounted on a rigid, massless shaft and symmetrically situated between two identical anisotropic bearings. By means of elementary gyro dynamics the equations of motion are readily derived /9, pp. 93-98/, after assuming small deflections, i.e. linear theory is valid, identical bearings without clearances, the principal axes of bearing stiffness and damping coincide with coordinate axes, viscous damping:

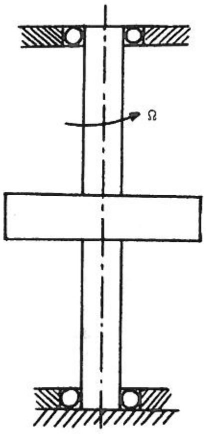

Figure 1. Geometry of rotor-bearing system.

$$m\ddot{u}_x + 2c_x\dot{u}_x + 2k_xu_x = me_o\Omega^2\cos\Omega t\ , \qquad (1)$$

$$m\ddot{u}_y + 2c_y\dot{u}_y + 2k_yu_y = me_o\Omega^2\sin\Omega t\ , \qquad (2)$$

$$J_D\ddot{\Theta}_y - \Omega J_p\dot{\Theta}_x + 2c_xL^2\dot{\Theta}_y + 2k_xL^2\Theta_y = D\Omega^2\cos\Omega t\ , \qquad (3)$$

$$J_D\ddot{\Theta}_x + \Omega J_p\dot{\Theta}_y + 2c_yL^2\dot{\Theta}_x + 2k_yL^2\Theta_x = -D\Omega^2\sin\Omega t\ , \qquad (4)$$

where

c_x, c_y	bearing damping constants in x and y direction respectively,
D	couple unbalance,
J_D	diametral moment of inertia,
J_p	polar moment of inertia,
k_x, k_y	bearing stiffness constants in x and y direction respectively,
$2L$	shaft length,
m	rotor mass,
me_o	static unbalance,
u_x, u_y	radial displacements,
Θ_x, Θ_y	rotations,
Ω	spin (speed) angular velocity.

The definitions, according to ISO-standard, of different unbalances, are given by Schneider /10/ and illustrated in figure 2.

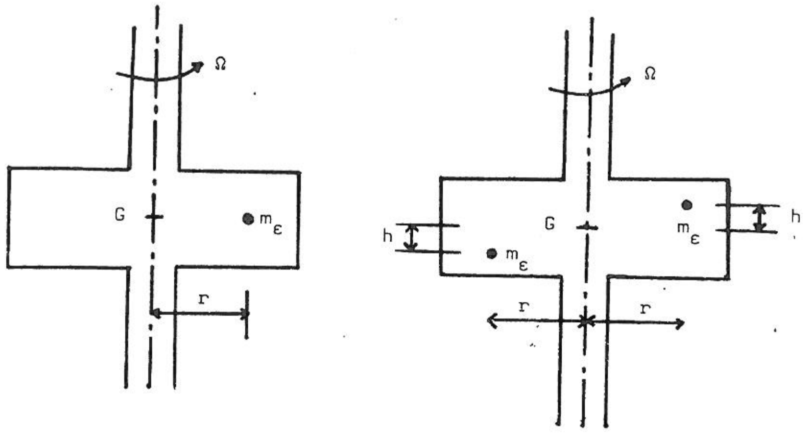

Figure 2. (a) Static unbalance $me_o = m_\varepsilon r$, m_ε unbalance mass, r radial distance to unbalance mass. (b) Couple unbalance $D = 2m_\varepsilon rh$, r radial distance to unbalance masses, h axial distance to unbalance masses.

Due to symmetry the equations constitute two independent systems. We will confine our analysis to equations (3) and (4) where the gyral effects are included. By means of a simple change of notation the results can also be applied on the radial displacement. By introducing the notations

$$\Theta = \Theta_y - i\Theta_x ,$$

$$\Theta^* = \Theta_y + i\Theta_x ,$$

$$c = L^2(c_y+c_x) ,$$

$$k = L^2(k_y+k_x) ,$$

$$\Delta c = L^2(c_y-c_x) ,$$

$$\Delta k = L^2(k_y-k_x) ,$$

where i is the imaginary unit a single complex equation is obtained:

$$J_D\ddot{\Theta} - i\Omega J_p\dot{\Theta} + c\dot{\Theta} + \Delta c\dot{\Theta}^* + k\Theta + \Delta k\Theta^* = D\Omega^2\exp(i\Omega t) . \qquad (5)$$

By means of the trial solution

$$\Theta = \hat{\Theta}_+\exp(i\Omega t) + \hat{\Theta}_-\exp(-i\Omega t)$$

where $\hat{\Theta}_+$ and $\hat{\Theta}_-$ are complex constants, by employing the identities

$$\Theta^* = \hat{\Theta}^*_+\exp(-i\Omega t) + \hat{\Theta}^*_-\exp(i\Omega t) \ ,$$

$$\dot{\Theta}^* = -i\Omega\hat{\Theta}^*\exp(-i\Omega t) + i\Omega\hat{\Theta}^*_-\exp(i\Omega t)$$

and by identification of the coefficients of $\exp(i\Omega t)$ and $\exp(-i\Omega t)$ two complex algebraic equations are obtained

$$(-J_D\Omega^2+J_p\Omega^2+i\Omega c+k)\hat{\Theta}_+ + (i\Omega\Delta c+\Delta k)\hat{\Theta}^*_- = D\Omega^2 \ , \tag{6}$$

$$(-i\Omega\Delta c+\Delta k)\hat{\Theta}^*_+ + (-J_D\Omega^2-J_p\Omega^2-i\Omega c+k)\hat{\Theta}_- = 0 \ . \tag{7}$$

ISOTROPIC CASE

In the isotropic case, i.e. $\Delta c = \Delta k = 0$, $\hat{\Theta}_-$ vanishes and equation (6) is reduced to

$$(-J_D\Omega^2+J_p\Omega^2+i\Omega c+k)\hat{\Theta}_+ = D\Omega^2 \ . \tag{8}$$

The backward precessional rotation component vanishes and the rotor points describe circular orbits,

$$\Theta = \hat{\Theta}_+ \ e^{i\Omega t} \ .$$

Since $\hat{\Theta}_+$ is a complex constant it can be defined by magnitude and constant phase angle,

$$\hat{\Theta}_+ = |\hat{\Theta}_+|e^{-i\varphi} \ ,$$

where φ is phase angle between excitation and response.

For

$$\Omega = \Omega_+ \equiv \sqrt{\frac{k}{J_D - J_p}}$$

the unbalance moment $D\Omega^2$ is merely balanced by the moments due to viscous forces. This speed is called critical speed of forward precession.

For Ω_+, $\hat{\Theta}_+$ is purely imaginary and the phase angle between excitation and response is $\pi/2$. By studying the phase angle the critical speed can be located after interpolation.

For an isotropic, undamped system with a perfectly balanced rotor equation (5) is reduced to

$$J_D\ddot{\Theta} - i\Omega J_p\dot{\Theta} + k\Theta = 0 \ . \tag{9}$$

By making a trial

$\Theta = \hat{\Theta} e^{i\omega t}$

a characteristic equation of second degree is obtained

$$-\omega^2 J_D + \omega\Omega J_p + k = 0 \; . \qquad (10)$$

For each Ω there are two natural angular frequencies

$$\omega = \frac{1}{2}\frac{J_p}{J_D}\Omega \pm \sqrt{\left(\frac{1}{2}\frac{J_p}{J_D}\Omega\right)^2 + \frac{k}{J_D}} \; . \qquad (11)$$

Due to the gyroscopic effect the natural angular frequencies are dependent on the spin speed. If $\omega = \Omega$, then

$$\Omega = \Omega_+ \equiv \sqrt{\frac{k}{J_D - J_p}} \; . \qquad (12)$$

Ω is denoted as critical speed of forward precession. The case $\Omega = -\omega$ i.e.

$$\Omega_- = \sqrt{\frac{k}{J_D + J_p}} \qquad (13)$$

is also of some interest and Ω_- is denoted as critical speed of backward precession.

The easiest way to understand the situation is by studying the spin-whirl diagram.

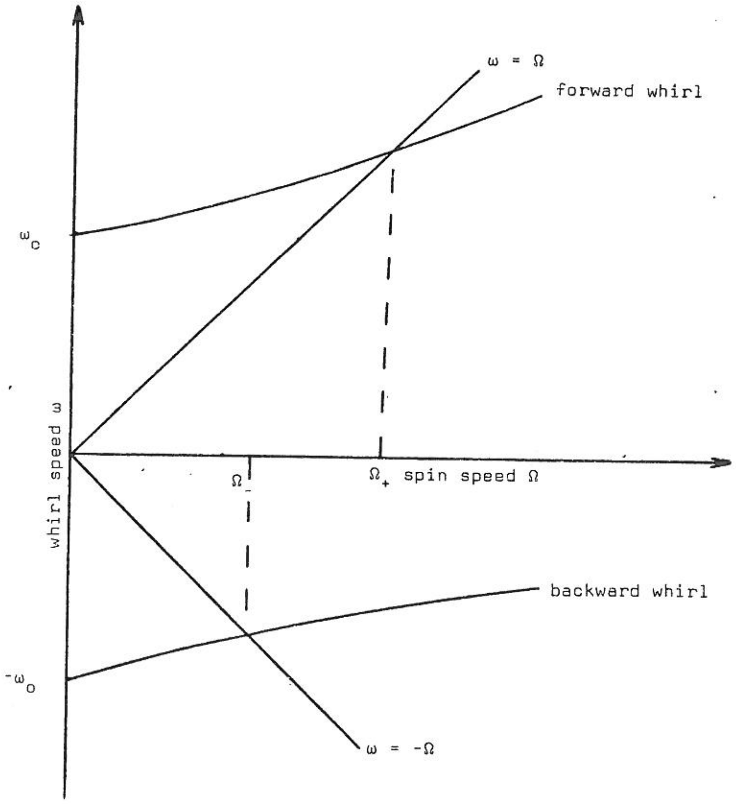

Figure 3. Spin-whirl diagram for $J_p < J_D$, Ω_+ critical speed of forward precession, Ω_- critical speed of backward precession, ω_0 eigenfrequency at zero spin speed.

ANISOTROPIC CASE

In the anisotropic case ($\Delta k \neq 0$ or $\Delta c \neq 0$) there is a coupling between equations (6) and (7). Therefore $\hat{\Theta}_- \neq 0$ and the points of the rotor shaft describe elliptical orbits

$$\Theta = \hat{\Theta}_+ e^{i\Omega t} + \hat{\Theta}_- e^{-i\Omega t} . \tag{14}$$

The deflection/moment ratio Θ' is obtained after division Θ by the unbalance moment $D\Omega^2 \exp(i\Omega t)$, that is

$$\Theta' = \Theta/D\Omega^2 \exp(i\Omega t) = \hat{\Theta}'_+ + \hat{\Theta}'_- e^{-2i\Omega t} . \tag{15}$$

The argument of Θ' is time-dependent when $\hat{\Theta}_- \neq 0$. One phase angle is therefore not sufficient to describe the motion of the rotor. Balda /11/ measured the phase angle in two perpendicular directions. In this paper a, to the author's knowledge, novel method to define the phase angles of an anisotropic rotor system is proposed.

From equation (7) we obtain

$$\hat{\Theta}_- = \frac{-(-i\Omega\Delta c + \Delta k)\hat{\Theta}^*_+}{-J_D\Omega^2 - J_p\Omega^2 - i\Omega c + k} . \tag{16}$$

The backward precessional rotational component is thus indirectly excited by the moment

$$-(-i\Omega\Delta c + \Delta k)\hat{\Theta}^*_+ ,$$

which is caused by anisotropy of the rotor bearing system.

For $\Omega = \Omega_-$ the real part of the denominator vanishes, and

$$\arg\left[(1 + \frac{i\Omega_- \Delta c}{\Delta k}) \frac{\hat{\Theta}^*_+}{\hat{\Theta}} \right] = \frac{\pi}{2} .$$

For a lightly damped system i.e. $\Omega_-\Delta c/\Delta k << 1$ one can claim that the argument of $\hat{\Theta}^*_+/\hat{\Theta}_-$ is equal to $-\pi/2$ at the critical speed of backward precession.

One way to describe the angular deflection of a rotor in anisotropic bearings is

$$\Theta_x = \Theta_{xc} \cos \Omega t + \Theta_{xs} \sin \Omega t , \tag{17}$$

$$\Theta_y = \Theta_{yc} \cos \Omega t + \Theta_{ys} \sin \Omega t . \tag{18}$$

Unfortunately the meaning of the coefficients Θ_{xc}, Θ_{xs}, Θ_{yc} and Θ_{ys} is not readily understood. However, by means of complex algebra one can readily rewrite expressions (17) and (18) as (14). Furthermore expression (14) is rewritten as

$$\Theta = \Theta_+ e^{i(\Omega t-\varphi_+)} + \Theta_- e^{-i(\Omega t-\varphi_-)} = e^{-i\varphi_+} (\Theta_+ e^{i\Omega t} + \Theta_- e^{-i\Delta\varphi} e^{-i\Omega t}) . \qquad (19)$$

The rotation is thus characterized with four parameters:

$\Theta_+ = |\hat{\Theta}_+|$ amplitude of forward precessional rotation component,

$\Theta_- = |\hat{\Theta}_-|$ amplitude of backward precessional rotation component,

φ_+ = phase angle between unbalance moment and forward precessional rotation components,

$\Delta\varphi = \varphi_- - \varphi_+$ phase angle between forward and backward rotational components.

For a stationary motion the parameters Θ_+, Θ_-, φ_+ and φ_- are time independent. For a forward precessional critical speed $\varphi_+ = \pi/2$ and for a backward precessional critical speed $\Delta\varphi = \pi/2$.

EXAMPLE OF PRACTICAL APPLICATION

Balda /11/ presented a paper concerning rotor balancing. The deflections and phase angles were measured in two perpendicular directions. Due to anisotropy in the rotor-bearing system different results were obtained in x- and y-direction respectively.

In this paper it is shown how one can use those figures to calculate phase and amplitude of backward and forward rotation components.

The previous results, derived for an angular degree of freedom, can be readily applied on the radial degrees of freedom, by means of simple change of notation.

We will calculate the complex coefficients $\hat{r}_+$, $\hat{r}_-$ defining the dynamic deflection,

$$r = \hat{r}_+ e^{i\Omega t} + \hat{r}_- e^{-i\Omega t} . \qquad (20)$$

The deflections (r_I and r_{II}) and phase angles (φ_I and φ_{II}) in the x- and y-directions have been measured.

When the rotor is deflected in the x-direction the angle between force and displacement is measured to φ_I. Ωt is therefore equal to $\varphi_I + 2\pi n$ (n is an arbitrary integer) and the following equation is obtained

$$\hat{r}_+ e^{i\varphi_I} + \hat{r}_- e^{-i\varphi_I} = r_I . \qquad (21)$$

When the rotor is deflected in the y-direction $\Omega t = \varphi_{II} + \pi/2 + 2\pi n$ and the following equation is obtained after division by i

$$r_+ e^{i\varphi_{II}} - r_- e^{-i\varphi_{II}} = r_{II} . \qquad (22)$$

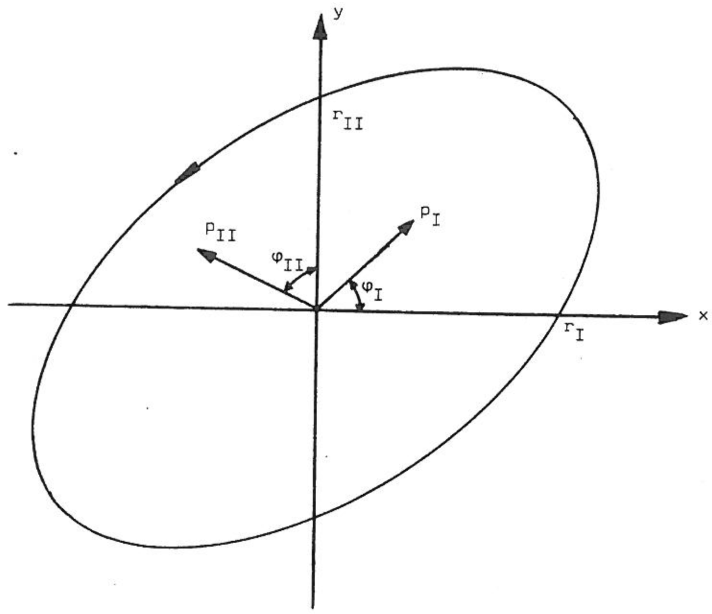

Figure 4. Deflection of rotor during one revolution, φ_I phase angle when rotor deflected in x-direction, φ_{II} phase angle when rotor deflected in y-direction.

By means of elementary algebra expressions for $\hat{r}_+$ and $\hat{r}_-$ are readily derived from (21) and (22):

$$\hat{r}_+ = \frac{r_I\, e^{-i\varphi_{II}} + r_{II}\, e^{-i\varphi_I}}{2\cos(\varphi_I-\varphi_{II})} = r_+\, e^{-i\varphi_+} , \tag{23}$$

$$\hat{r}_- = \frac{r_I\, e^{i\varphi_{II}} - r_{II}\, e^{i\varphi_I}}{2\cos(\varphi_I-\varphi_{II})} = r_-\, e^{+i\varphi_-} . \tag{24}$$

When $r_I = r_{II}$ and $\varphi_I = \varphi_{II}$ then $\hat{r}_- = 0$, thus the backward rotation component is non-existant.

By means of formula (23) and (24) one can calculate amplitude and phase of forward and backward rotational components respectively.

CONCLUSIONS

By studying a very simple mathematical model one can understand the influence of anisotropy of bearings on a rotor-bearing system. It was e.g. shown, that critical speeds of backward precession may be excited due to unbalances, when the bearings have anisotropic stiffness or damping properties. By means of simple change of notation the mathematical procedure described in this paper can be readily extended to a multi-degree-of-freedom system. Equation (5) is then generalized to

$$[M]\ddot{\bar{r}} + [\Delta M]\ddot{\bar{r}}^* - i\Omega[D]\dot{\bar{r}} + [C]\dot{\bar{r}} + [\Delta C]\dot{\bar{r}}^* + [K]\bar{r} + [\Delta K]\bar{r}^* =$$

$$= \bar{F}_+ \exp(i\Omega t) + \bar{F}_- \exp(-i\Omega t) \ , \qquad (25)$$

where

$[C]$, $[\Delta C]$ damping matrices,
$[K]$, $[\Delta K]$ stiffness matrices,
$[M]$, $[\Delta M]$ mass matrices,
$\bar{r}$ complex displacement vector,
$\bar{F}_+$, $\bar{F}_-$ force vectors.

The coefficient matrices $[\Delta C]$, $[\Delta K]$ and $[\Delta M]$ are due to non-symmetrical damping, stiffness and mass properties of bearings, frame or frame feet. Anisotropic properties of the rotor are not included in equation (25). In that case time-dependent coefficient matrices are obtained /12/, equation (25) can be solved by means of the trial

$$\bar{r} = \bar{r}_- \exp(i\Omega t) + \bar{r}_- \exp(-i\Omega t) \ . \qquad (26)$$

If the exciting forces are due to unbalances, $\bar{F}_- = \bar{0}$. If $\bar{F}_+ = \pm \bar{F}_-$, an exciting force with constant direction in a non-rotating coordinate system is described. In the latter case, one can realize that critical speeds of backward precession will be excited even in the entirely isotropic case, i.e. $[\Delta C] = [\Delta K] = [\Delta M] = [0]$.

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