A new paradigm for fatigue analysis - evolution equation based continuum approach

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Content

Introduction - fatigue models

Problems in fatigue analyses:

- Multiaxiality
- Damage accumalation rules
- Low-cycle- and high-cycle -fatigue regimes are treated separately
- Mostly based on well defined cycles.

A more fundamental approach for HCF based on *evolution equations* proposed by Ottosen, Stenström and Ristinmaa in IJF 2008.

It provides a well defined and consistent approach for *multiaxial* fatigue analysis which can be "easily" extended to *anisotropic* and *stochastic appraches* and in which the *gradient effects* can be included. In addition, the LCF and HCF regimes can be treated in a uniform manner.

Evolution equation based HCF model

Key ingredients are:

Endurance surface

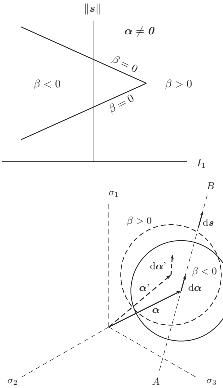
$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0,$$

evolution equations for damage D and the internal variables $\{\boldsymbol{\alpha}\}$

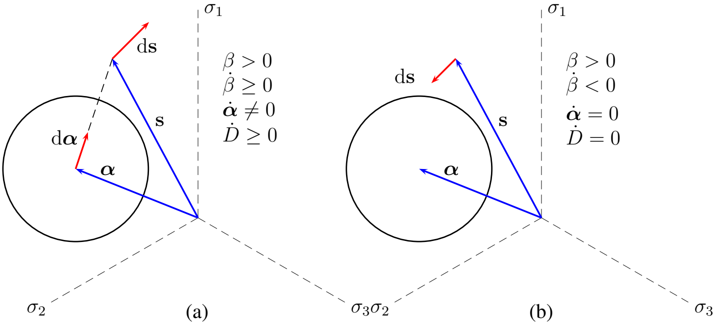
$$\{\dot{\alpha}\} = \{\boldsymbol{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta},$$

and

$$\dot{D} = g(\beta, D)\dot{\beta}.$$



Conditions for evolution



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Endurance surface

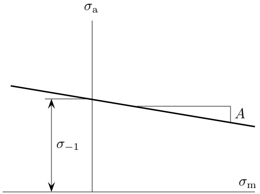
Original formulation by Ottosen et al. for isotropic fatigue

$$\beta = \frac{1}{\sigma_{-1}} \left[\sqrt{3\bar{J}_2} + AI_1 - \sigma_{-1} \right] = 0,$$

where $\bar{J}_2 = \frac{1}{2}\mathrm{tr}\,(\boldsymbol{s} - \boldsymbol{\alpha})^2, \quad I_1 = \mathrm{tr}\,\boldsymbol{\sigma}, \quad A = \sigma_{-1}/\sigma_0 - 1$, and

$$\sigma_{-1} = \sigma_{\mathrm{af},R=-1}$$

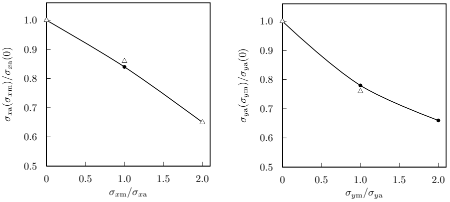
$$\sigma_0 = \sigma_{\mathrm{af},R=0}$$



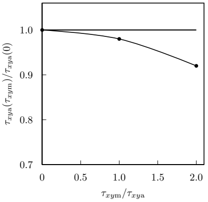
Effect of mean stress

The model describes well the mean stress effect in cyclic tension as well as the non-linear effect on mean shear stress on the fatigue strength.

Transversely isotropic case: forged 34CrMo6 steel.



cyclic normal stress in
longitudinal and transverse directions



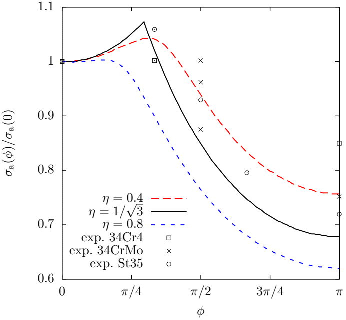
mean shear stress effect
on fatigue stress

Anisotropic case

So far transversely isotropic and orthotropic symmetry has been considered. Formulation based on structural tensors.

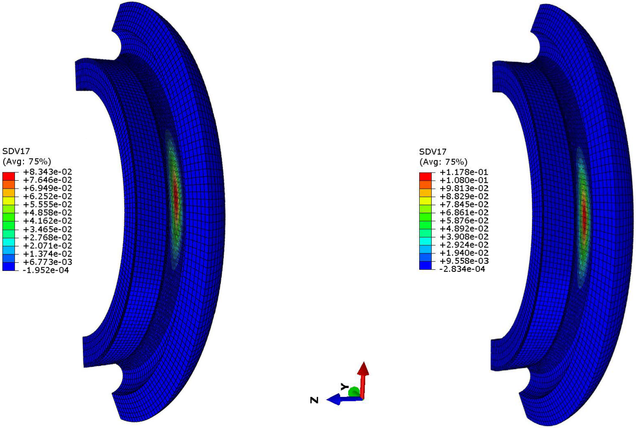
Denoting: $\tau_{-1} = \eta\sigma_{-1}$, consider biaxial loading

$$\sigma_x = \sigma_\mathrm{m} + \sigma_\mathrm{a} \sin \omega t, \quad \sigma_y = \sigma_\mathrm{m} + \sigma_\mathrm{a} \sin(\omega t + \phi)$$



Some results - HCF industrial test case

Transversely isotropc HCF-analysis of a forged 34CrMo6 steel fillet. The fatigue model is implemented in Abaqus FE program using the UMAT subroutine. Colour shows the value of damage D.



Loading and mesh by Dr. Juho Könnö, Wätrsilä Finland Oy

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LCF-HCF approach

Evolution equation for the $\boldsymbol{\alpha}$-tensor

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}$$

and for damage

$$\dot{D} = K \exp[L\exp(-\xi\bar{\varepsilon}_{\mathrm{p}})\beta + M\langle \mathrm{sgn}(f)\rangle \bar{\varepsilon}_{\mathrm{p}}]\dot{\beta}$$

Plasticity model based on Armstrong-Frederick model

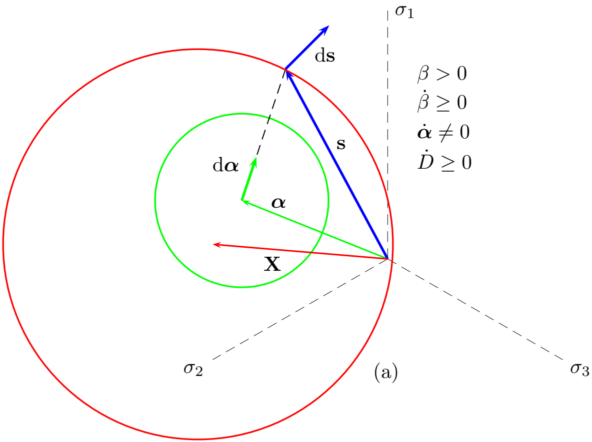
$$f(\boldsymbol{\sigma}, \boldsymbol{X}, R) = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{X}) : (\boldsymbol{s} - \boldsymbol{X})} - (\sigma_{\mathrm{y}} + R) = 0$$

$$\dot{R} = \gamma R_{\infty}\left(1 - R/R_{\infty}\right)\dot{\bar{\varepsilon}}_{\mathrm{p}}$$

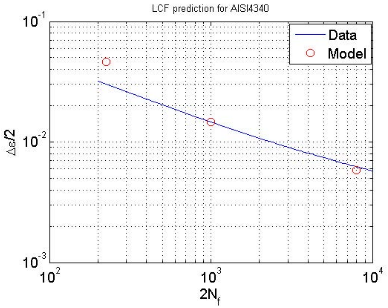
$$\dot{\boldsymbol{X}} = \tfrac{2}{3}X_{\infty}\dot{\boldsymbol{\varepsilon}}_{\mathrm{p}} - \gamma\dot{\bar{\varepsilon}}_{\mathrm{p}}\boldsymbol{X}$$

$$\dot{\boldsymbol{\varepsilon}}_{\mathrm{p}} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$

Illustration in deviatoric plane



$\Delta\varepsilon$-N curve in LCF-regime - AISI 4340



ASTM Handbool (Coffin-Manson + Basquin):

$$\frac{\Delta\varepsilon}{2} = 0.58(2N_{\mathrm{f}})^{-0.57} + 0.0062(2N_{\mathrm{f}})^{-0.09}$$

Stochastic analysis

We have considered stress processes as Ornstein-Uhlenbeck process (a stationary Gauss-Markov process depending on parameters $\boldsymbol{\lambda}, \boldsymbol{\mu}$ and $\boldsymbol{\eta}$)

$$\mathrm{d}\boldsymbol{\sigma}(t) = \boldsymbol{\lambda}(\boldsymbol{\mu} - \boldsymbol{\sigma}(t))\mathrm{d}t + \boldsymbol{\eta}\mathrm{d}\boldsymbol{W}(t)$$

Process $\boldsymbol{W}(t)$ is a Wiener process (Brownian motion)

It is a stochastic differential equation, solution can be found as

$$\boldsymbol{\sigma}(t) = \boldsymbol{\mu} + (\boldsymbol{\sigma}_0 - \boldsymbol{\mu}) \exp(-\boldsymbol{\lambda} t) + \boldsymbol{\eta} \int_0^t \exp(-\boldsymbol{\lambda}(t-s))\mathrm{dW}(s),$$

where the integral is the so called Itô integral wrt the Wiener process.

Gradient effects

Simply substitute

$$\sigma_{-1,\mathrm{corr}} = \sigma_{-1}(1 + \sqrt{As}),$$

where

$$s = \frac{\nabla\sigma_{\mathrm{eff}} \cdot \nabla\sigma_{\mathrm{eff}}}{\sigma_{\mathrm{eff}}}$$

where σ_{eff} is the standard von Mises stress.

A is the only additional material parameter (the Neuber parameter)

Concluding remarks and future work

- Consistent unified approach
- Can be “easilily” extended to anisotropy, stochastic analysis, gradient effect can be included
- *Parameter estimation*
- *Micromechanical motivation of the evolution equations.*

Watercolor by
Pia Erlandsson

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Thank you for your attention!