Rak-11.2107 Sillat ja Perustukset

<u>Esimerkkitehtävät 2009-2011</u>

Rak-11.2107 Sillat ja Perustukset

ESIMERKKITEHTÄVÄT 2009

Esim. 2.1 a) : Maanpaineen laskenta kerroksellisessa maassa (L 2009)

Määritä oheiseen vapaasti seisovaan tukimuuriin kohdistuvan maanpaineen resultantin suuruus ja suunta
a) Kun kysymyksessä on siirtymätön jäykkä tukimuuri (lepopaine)
b) Tukimuurin yläpää siirtyy penkereestä poispäin niin, että aktiivinen maanpaine on täysin kehittynyt
Käytä kuormien ja maaparametrien ominaisarvoja.

$kN := 1000 \cdot N \qquad MN := 1000 \cdot kN$

Alkuarvot:

Käytetään maan kerrosten sis. kitkakulman suunn. arvoina ominaisarvoja eli $\varphi_{di} = \varphi_i$

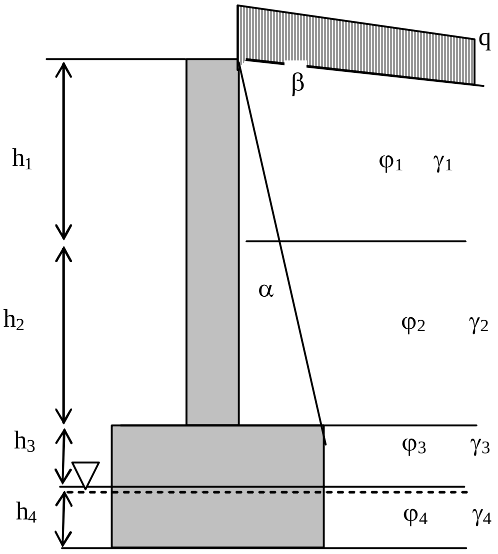
$\gamma_1 := 0.018 \cdot \frac{MN}{m^3} \qquad \varphi_{d1} := 40 \cdot deg \qquad h_1 := 1.5 \cdot m$

$\gamma_2 := 0.020 \cdot \frac{MN}{m^3} \qquad \varphi_{d2} := 38 \cdot deg \qquad h_2 := 1.5 \cdot m$

$\gamma_3 := 0.020 \cdot \frac{MN}{m^3} \qquad \varphi_{d3} := 38 \cdot deg \qquad h_3 := 0.5 \cdot m$

$\gamma_4 := 0.012 \cdot \frac{MN}{m^3} \qquad \varphi_{d4} := 38 \cdot deg \qquad h_4 := 0.5 \cdot m$

$\alpha := -15 \cdot deg \qquad \beta := -12 \cdot deg \qquad q := 0.1 \cdot \frac{MN}{m^2}$

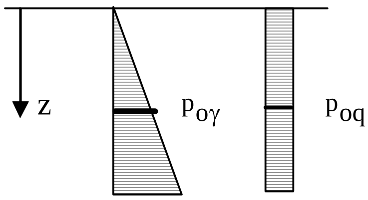


1) Lepopaine: (Otaksutaan vaikuttavan vaakasuoraan suuntaan)

Paineen intensiteetti syvyydellä z:

Maan omasta painosta (γ)

$p_{o\gamma} = K_o \cdot \gamma_s \cdot z$ paine kasvaa lineaarisesti



Tasaisesta pintakuormasta (q):

$p_{oq} = K_o \cdot q$ paine vakio

Lepopainekerroin lasketaan kaavoista:

$K_o = 1 - \sin(\varphi_d)$

$K_{o\beta} = K_o \cdot (1 + \sin(\beta))$ (maapinnan kaltevuus = β)

Kerros 1: 0< z < 1,5 m

$\varphi_d := \varphi_{d1}$

$K_{o\beta1} := (1 - \sin(\varphi_d)) \cdot (1 + \sin(\beta)) \qquad K_{o\beta1} = 0.283 \qquad 1 - \sin(\varphi_d) = 0.357$

$p_{o\gamma1} := K_{o\beta1} \cdot \gamma_1 \cdot h_1 \qquad p_{o\gamma1} = 0.00764 \frac{MN}{m^2}$

$p_{oq1} := K_{o\beta1} \cdot q \qquad p_{oq1} = 0.02829 \frac{MN}{m^2}$

Maanpaine kerroksen alareunassa (z = 1,5 m)

$p_{o1} := p_{o\gamma1} + p_{oq1} \qquad p_{o1} = 0.03593 \frac{MN}{m^2}$

Kerroksen 1 resultantit ja resultanttien etäisyys perustamistasosta:

$P_{o\gamma1} := 0.5 \cdot p_{o\gamma1} \cdot h_1$ $\quad P_{o\gamma1} = 0.00573 \dfrac{MN}{m}$ $\quad d_{\gamma1} := \dfrac{1}{3} \cdot h_1 + h_2 + h_3 + h_4$ $\quad d_{\gamma1} = 3\,m$

$P_{oq1} := p_{oq1} \cdot h_1$ $\quad P_{oq1} = 0.04244\,m \dfrac{MN}{m^2}$ $\quad d_{q1} := \dfrac{1}{2} \cdot h_1 + h_2 + h_3 + h_4$ $\quad d_{q1} = 3.25\,m$

Kerros 2: 1,5 m < z < 3,0 m

$\varphi_d := \varphi_{d2}$

Maakerroksen 1painosta aiheutuva maanpaine lasketaan kuten tasainen pintakuorman q_1

$q_1 := \gamma_1 \cdot h_1$ $\quad q_1 = 0.027 \dfrac{MN}{m^2}$

$K_{o\beta2} := \left(1 - \sin\left(\varphi_d\right)\right) \cdot \left(1 + \sin(\beta)\right)$ $\quad K_{o\beta2} = 0.304$

$p_{o\gamma2} := K_{o\beta2} \cdot \gamma_2 \cdot h_2$ $\quad p_{o\gamma2} = 0.00913 \dfrac{MN}{m^2}$

$p_{oq2} := K_{o\beta2} \cdot \left(q + q_1\right)$ $\quad p_{oq2} = 0.03866 \dfrac{MN}{m^2}$

Maanpaine kerroksen alareunassa (z = 3,0 m)

$p_{o2} := p_{o\gamma2} + p_{oq2}$ $\quad p_{o2} = 0.0478 \dfrac{MN}{m^2}$

Kerroksen 2 resultantit ja niiden etäisyys perustamistasosta:

$P_{o\gamma2} := 0.5 \cdot p_{o\gamma2} \cdot h_2$ $\quad P_{o\gamma2} = 0.00685 \dfrac{MN}{m}$ $\quad d_{\gamma2} := \dfrac{1}{3} \cdot h_2 + h_3 + h_4$ $\quad d_{\gamma2} = 1.5\,m$

$P_{oq2} := p_{oq2} \cdot h_2$ $\quad P_{oq2} = 0.058 \dfrac{MN}{m}$ $\quad d_{q2} := \dfrac{1}{2} \cdot h_2 + h_3 + h_4$ $\quad d_{q2} = 1.75\,m$

Kerros 3: 3,0 m < z < 3,5 m

Maakerrosten 1 ja 2 painosta aiheutuva pintakuorma q_2:

$q_2 := q_1 + \gamma_2 \cdot h_2$ $\quad q_2 = 0.057 \dfrac{MN}{m^2}$

$K_{o\beta3} := K_{o\beta2}$

$p_{o\gamma3} := K_{o\beta3} \cdot \gamma_3 \cdot h_3$ $\quad p_{o\gamma3} = 0.00304 \dfrac{MN}{m^2}$

$p_{oq3} := K_{o\beta3} \cdot \left(q + q_2\right)$ $\quad p_{oq3} = 0.0478 \dfrac{MN}{m^2}$

Maanpaine kerroksen alareunassa (z = 3,5 m)

$p_{o3} := p_{o\gamma3} + p_{oq3}$ $\quad p_{o3} = 0.05084 \dfrac{MN}{m^2}$

Kerroksen 3 resultantit ja niiden etäisyys perustamistasosta:

$P_{o\gamma3} := 0.5 \cdot p_{o\gamma3} \cdot h_3$ $\quad P_{o\gamma3} = 0.00076 \dfrac{MN}{m}$ $\quad d_{\gamma3} := \dfrac{1}{3} \cdot h_3 + h_4$ $\quad d_{\gamma3} = 0.667\,m$

$P_{oq3} := p_{oq3} \cdot h_3$ $\quad P_{oq3} = 0.024 \dfrac{MN}{m}$ $\quad d_{q3} := \dfrac{1}{2} \cdot h_3 + h_4$ $\quad d_{q3} = 0.75\,m$

Kerros 4: 3,5 m < z < 4,0 m

Maakerrosten 1 ja 2 ja 3 painosta aiheutuva pintakuorma q_3:

$$q_3 := q_2 + \gamma_3 \cdot h_3 \qquad q_3 = 0.067\,\frac{MN}{m^2}$$

$$K_{o\beta4} := K_{o\beta3}$$

$$p_{o\gamma4} := K_{o\beta4} \cdot \gamma_4 \cdot h_4 \qquad p_{o\gamma4} = 0.00183\,\frac{MN}{m^2}$$

$$p_{oq4} := K_{o\beta4} \cdot (q + q_3) \qquad p_{oq4} = 0.05084\,\frac{MN}{m^2} \qquad q = 0.1\,\frac{MN}{m^2}$$

Maanpaine kerroksen alareunassa (z = 4,0 m)

$$p_{o4} := p_{o\gamma4} + p_{oq4} \qquad p_{o4} = 0.05267\,\frac{MN}{m^2}$$

Kerroksen 4 resultantit ja niiden etäisyys perustamistasosta:

$$P_{o\gamma4} := 0.5 \cdot p_{o\gamma4} \cdot h_4 \qquad P_{o\gamma4} = 0.00046\,\frac{MN}{m} \qquad d_{\gamma4} := \frac{1}{3} \cdot h_4 \qquad d_{\gamma4} = 0.167\,m$$

$$P_{oq4} := p_{oq4} \cdot h_4 \qquad P_{oq4} = 0.025\,\frac{MN}{m} \qquad d_{q4} := \frac{1}{2} \cdot h_4 \qquad d_{q4} = 0.25\,m$$

Lepopainekuorman resultantti ja sen etäisyys perustamistasosta:

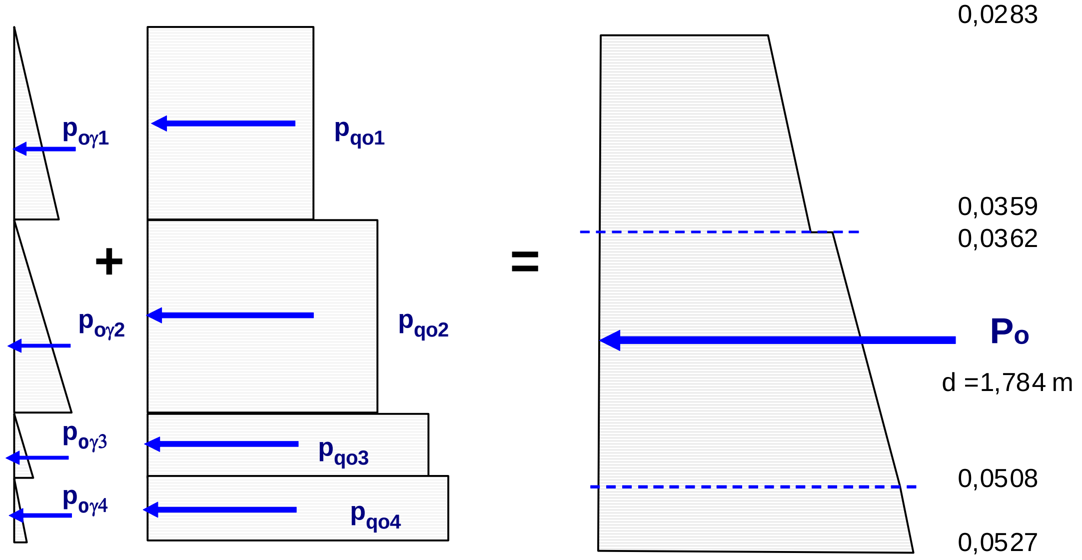
$$P_o := P_{o\gamma1} + P_{o\gamma2} + P_{o\gamma3} + P_{o\gamma4} + P_{oq1} + P_{oq2} + P_{oq3} + P_{oq4}$$

$$M_o := P_{o\gamma1} \cdot d_{\gamma1} + P_{o\gamma2} \cdot d_{\gamma2} + P_{o\gamma3} \cdot d_{\gamma3} + P_{o\gamma4} \cdot d_{\gamma4} + P_{oq1} \cdot d_{q1} + P_{oq2} \cdot d_{q2} + P_{oq3} \cdot d_{q3} + P_{oq4} \cdot d_{q4}$$

$$P_o = 0.164\,\frac{MN}{m} \qquad d_{res} := \frac{M_o}{P_o}$$

$d_{res} = 1.784\,m$ Resultantin etäisyys perustamistasosta

Maanpainekuvio:



2) Aktiivinen maanpaine:

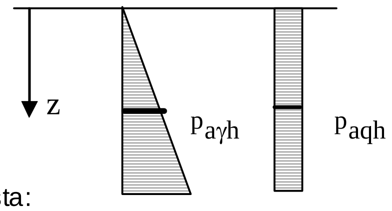
Paineen horisontaalisen komponentin intensiteetti syvyydellä z:

Maan omasta painosta (γ)

$$p_{a\gamma h} = K_{ah} \cdot \gamma_s \cdot z \qquad \text{(kasvaa lineaarisesti)}$$

Tasaisesta pintakuormasta (q):

$$p_{aqh} = K_{ah} \cdot q \qquad \text{(vakio)}$$



Aktiivisen maanpaineen horisontaalikomponentin kerroin kaavasta:

$$K_{ah} = \frac{\cos(\varphi_d + \alpha)^2}{\cos(\alpha)^2 \cdot \left(1 + \sqrt{\dfrac{\sin(\varphi_d + \delta_d) \cdot \sin(\varphi_d - \beta)}{\cos(\alpha - \delta_d) \cdot \cos(\alpha + \beta)}}\right)^2}$$

(α = seinän kaltevuus pystytasosta
β = maanpinnan kaltevuus vaakatasosta
δ = seinäkitkakulma
φ_d = maan sisäisen kitkakulman suun. arvo)

Kerros 1: 0< z < 1,5 m

$\varphi_d := \varphi_{d1}$ (maan sisäinen kitkakulma kerroksessa 1)

$\delta_d := \varphi_{d1}$ (seinäkitkakulma δ = φ koska oletettu liukupinta maan sisässä)

$$K_{ah1} := \frac{\cos(\varphi_d + \alpha)^2}{\cos(\alpha)^2 \cdot \left(1 + \sqrt{\dfrac{\sin(\varphi_d + \delta_d) \cdot \sin(\varphi_d - \beta)}{\cos(\alpha - \delta_d) \cdot \cos(\alpha + \beta)}}\right)^2} \qquad K_{ah1} = 0.177$$

$p_{a\gamma 1} := K_{ah1} \cdot \gamma_1 \cdot h_1 \qquad p_{a\gamma 1} = 0.00477 \dfrac{MN}{m^2}$

$p_{aq1} := K_{ah1} \cdot q \qquad p_{aq1} = 0.01767 \dfrac{MN}{m^2}$

Maanpaine kerroksen alareunassa (z = 1,5 m)

$p_{a1} := p_{a\gamma 1} + p_{aq1} \qquad p_{a1} = 0.02244 \dfrac{MN}{m^2}$

Maanpaineiden horisontaalikomponenttien resultantit ja niiden etäisyys perustamistasosta:

$P_{a\gamma 1} := 0.5 \cdot p_{a\gamma 1} \cdot h_1 \qquad P_{a\gamma 1} = 0.00358 \dfrac{MN}{m} \qquad d_{\gamma 1} = 3\,m$ (ks edellä)

$P_{aq1} := p_{aq1} \cdot h_1 \qquad P_{aq1} = 0.0265 \dfrac{MN}{m} \qquad d_{q1} = 3.25\,m$

Kerros 2: 1,5 m < z < 3,0 m

$\varphi_d := \varphi_{d1}$

$\delta_d := \varphi_{d2}$

$$K_{ah2} := \frac{\cos(\varphi_d + \alpha)^2}{\cos(\alpha)^2 \cdot \left(1 + \sqrt{\frac{\sin(\varphi_d + \delta_d) \cdot \sin(\varphi_{d2} - \beta)}{\cos(\alpha - \delta_d) \cdot \cos(\alpha + \beta)}}\right)^2} \qquad K_{ah2} = 0.185$$

$p_{a\gamma 2} := K_{ah2} \cdot \gamma_2 \cdot h_2 \qquad p_{a\gamma 2} = 0.00555 \frac{MN}{m^2}$

$p_{aq2} := K_{ah2} \cdot (q + q_1) \qquad p_{aq2} = 0.02348 \frac{MN}{m^2}$ ($q_1 = \gamma_1 h_1$, ks kohta 1)

Maanpaine kerroksen alareunassa (z = 3,0 m)

$p_{a2} := p_{a\gamma 2} + p_{aq2} \qquad p_{a2} = 0.02903 \frac{MN}{m^2}$

Maanpaineiden horisontaalikomponenttien resultantit ja niiden etäisyys perustamistasosta:

$P_{a\gamma 2} := 0.5 \cdot p_{a\gamma 2} \cdot h_2 \qquad P_{a\gamma 2} = 0.00416 \frac{MN}{m} \qquad d_{\gamma 2} = 1.5\,m$

$P_{aq2} := p_{aq2} \cdot h_1 \qquad P_{aq2} = 0.03522 \frac{MN}{m} \qquad d_{q2} = 1.75\,m$

Kerros 3: 3,0 m < z < 3,5 m

$\varphi_d := \varphi_{d1}$

$\delta_d := \frac{3}{4} \cdot \varphi_{d3}$ (seinäkitkakulma $\delta = 3/4\ \varphi$; PRO 2004)

$$K_{ah3} := \frac{\cos(\varphi_d + \alpha)^2}{\cos(\alpha)^2 \cdot \left(1 + \sqrt{\frac{\sin(\varphi_d + \delta_d) \cdot \sin(\varphi_{d2} - \beta)}{\cos(\alpha - \delta_d) \cdot \cos(\alpha + \beta)}}\right)^2} \qquad K_{ah3} = 0.209$$

$p_{a\gamma 3} := K_{ah3} \cdot \gamma_3 \cdot h_3 \qquad p_{a\gamma 3} = 0.00209 \frac{MN}{m^2}$

$p_{aq3} := K_{ah3} \cdot (q + q_2) \qquad p_{aq3} = 0.03289 \frac{MN}{m^2}$ ($q_2 = \gamma_2 h_2 + \gamma_1 h_1$)

Maanpaine kerroksen alareunassa (z = 3,5 m)

$p_{a3} := p_{a\gamma 3} + p_{aq3} \qquad p_{a3} = 0.03498 \frac{MN}{m^2}$

Maanpaineiden horisontaalikomponenttien resultantit ja niiden etäisyys perustamistasosta:

$P_{a\gamma 3} := 0.5 \cdot p_{a\gamma 3} \cdot h_3 \qquad P_{a\gamma 3} = 0.00052 \frac{MN}{m} \qquad d_{\gamma 3} = 0.667\,m$

$P_{aq3} := p_{aq3} \cdot h_3 \qquad P_{aq3} = 0.01644 \frac{MN}{m} \qquad d_{q3} = 0.75\,m$

Kerros 4: 3,5 m < z < 4,0 m

$K_{ah4} := K_{ah3}$

$p_{a\gamma4} := K_{ah4} \cdot \gamma_4 \cdot h_4 \qquad p_{a\gamma4} = 0.00126 \dfrac{MN}{m^2}$

$p_{aq4} := K_{ah4} \cdot (q + q_3) \qquad p_{aq4} = 0.03498 \dfrac{MN}{m^2} \qquad (q_3 = \gamma_3 h_3 + \gamma_2 h_2 + \gamma_1 h_1)$

Maanpaine perustamistasossa (z = 4 m)

$p_{a4} := p_{a\gamma4} + p_{aq4} \qquad p_{a4} = 0.03624 \dfrac{MN}{m^2}$

Maanpaineiden horisontaalikomponenttien resultantit ja niiden etäisyys perustamistasosta:

$P_{a\gamma4} := 0.5 \cdot p_{a\gamma4} \cdot h_4 \qquad P_{a\gamma4} = 0.00031 \dfrac{MN}{m} \qquad d_{\gamma4} = 0.167\,m$

$P_{aq4} := p_{aq4} \cdot h_4 \qquad P_{aq4} = 0.01749 \dfrac{MN}{m} \qquad d_{q4} = 0.25\,m$

Akt. maanpainekuvion vaakakomponentin resultantti ja sen paikka:

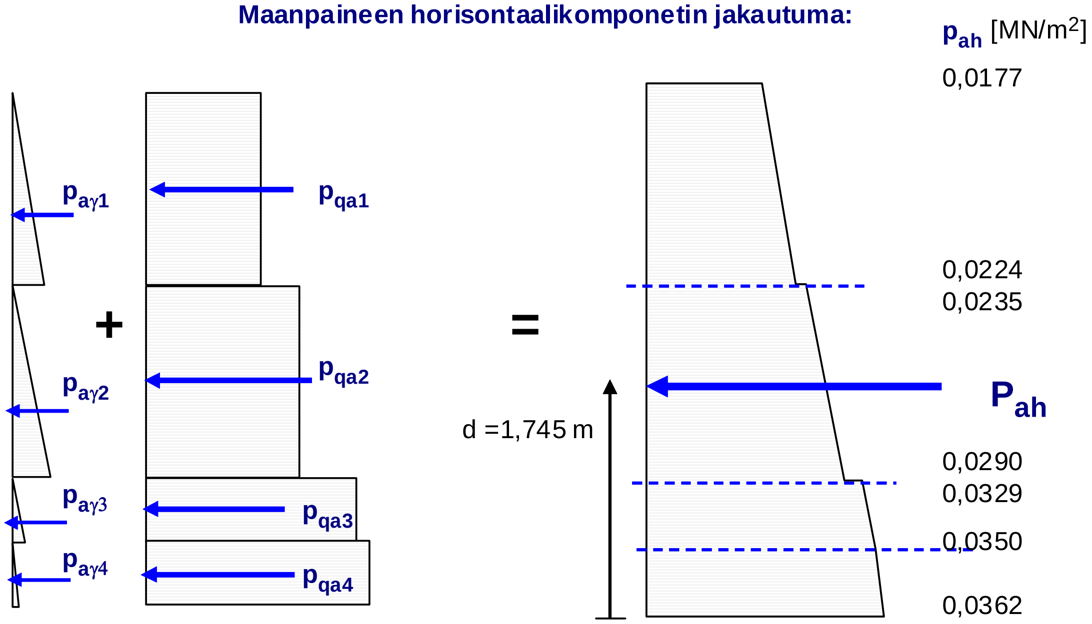
$$P_{ah} := P_{a\gamma1} + P_{a\gamma2} + P_{a\gamma3} + P_{a\gamma4} + P_{aq1} + P_{aq2} + P_{aq3} + P_{aq4}$$

$$M_{ao} := P_{a\gamma1} \cdot d_{\gamma1} + P_{a\gamma2} \cdot d_{\gamma2} + P_{a\gamma3} \cdot d_{\gamma3} + P_{a\gamma4} \cdot d_{\gamma4} + P_{aq1} \cdot d_{q1} + P_{aq2} \cdot d_{q2} + P_{aq3} \cdot d_{q3} + P_{aq4} \cdot d_{q4}$$

$P_{ah} = 0.10423 \dfrac{MN}{m} \qquad d_{res} := \dfrac{M_{ao}}{P_{ah}}$

$d_{res} = 1.745\,m$ Resultantin etäisyys perustamistasosta

Maanpaineen horisontaalikomponetin jakautuma:



Aktiivisen maanpaineen suuruus ja suunta, kun myös vertikaalikomponentti huomioitu :

Maanpaine poikkeaa horisontaalisuunnasta kulman δ - α verran ks. luennot kaavat (12) ja (13):

Kerros 1: $\delta_d := \varphi_{d1}$

$$p_{1y} := \frac{p_{aq1}}{\cos(\delta_d - \alpha)} \qquad p_{1y} = 0.0308\,\frac{MN}{m^2} \qquad \delta_d - \alpha = 55\,deg$$

(= maanpaineen suunnan poikkeama vaakatasosta)

$$p_{1a} := \frac{p_{a1}}{\cos(\delta_d - \alpha)} \qquad p_{1a} = 0.03912\,\frac{MN}{m^2}$$

Kerros 2: $\delta_d := \varphi_{d2}$

$$p_{2y} := \frac{p_{aq2}}{\cos(\delta_d - \alpha)} \qquad p_{2y} = 0.03902\,\frac{MN}{m^2} \qquad \delta_d - \alpha = 53\,deg$$

$$p_{2a} := \frac{p_{a2}}{\cos(\delta_d - \alpha)} \qquad p_{2a} = 0.04823\,\frac{MN}{m^2}$$

Kerros 3: $\delta_d := \frac{3}{4}\varphi_{d3}$

$$p_{3y} := \frac{p_{aq3}}{\cos(\delta_d - \alpha)} \qquad p_{3y} = 0.04534\,\frac{MN}{m^2} \qquad \delta_d - \alpha = 43.5\,deg$$

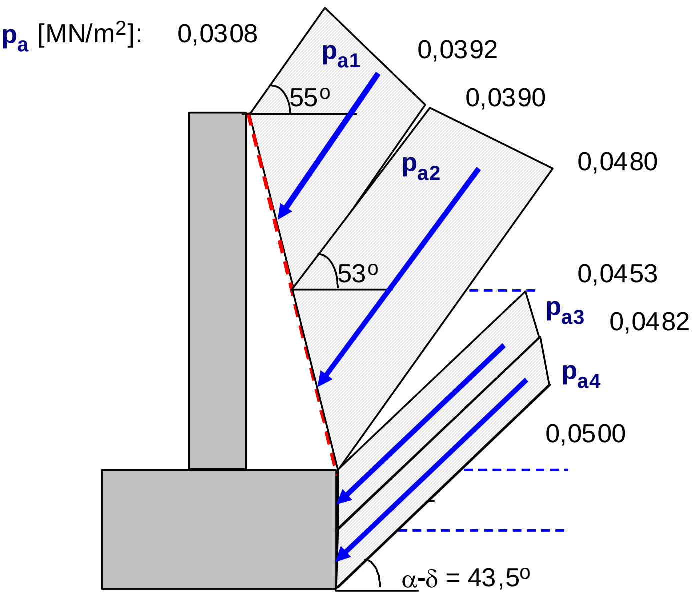
$$p_{3a} := \frac{p_{a3}}{\cos(\delta_d - \alpha)} \qquad p_{3a} = 0.04822\,\frac{MN}{m^2}$$

Kerros 4: $\delta_d := \frac{3}{4}\varphi_{d4}$

$$p_{4y} := \frac{p_{aq4}}{\cos(\delta_d - \alpha)} \qquad p_{4y} = 0.04822\,\frac{MN}{m^2} \qquad \delta_d - \alpha = 43.5\,deg$$

$$p_{4a} := \frac{p_{a4}}{\cos(\delta_d - \alpha)} \qquad p_{4a} = 0.04996\,\frac{MN}{m^2}$$

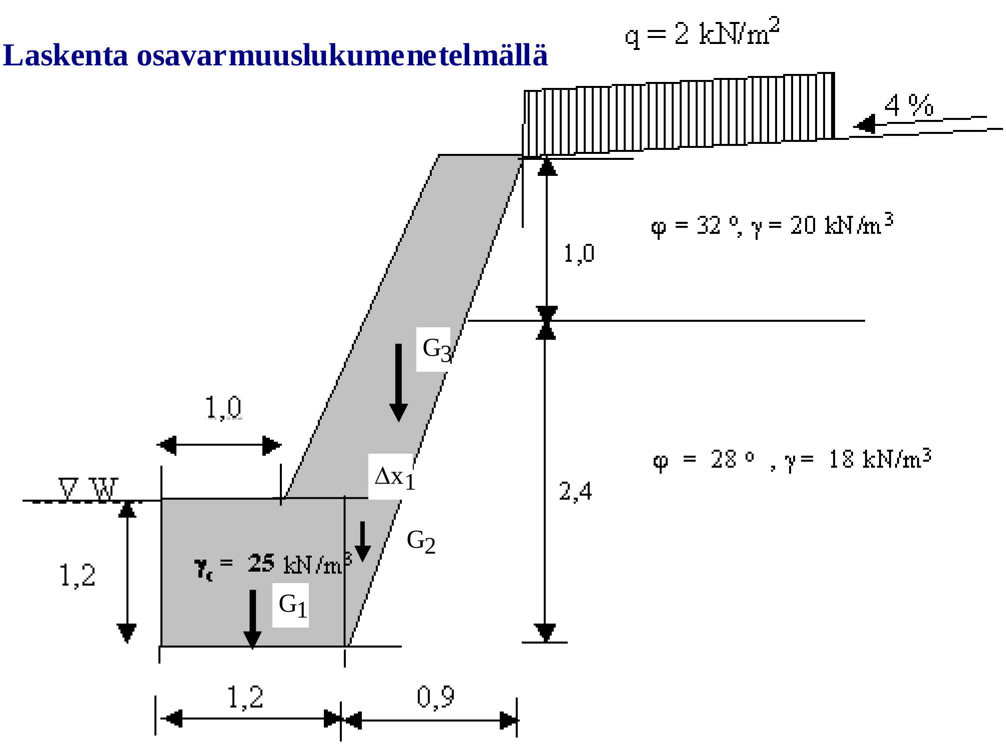
Lopullinen maanpainekuvio:



Esim 2.1: Maanpaineen laskenta kerroksellisessa maassa (vanha)

Määritä kantavuuslaskennassa tarvittavat kuormaresultantin pystysuora komponentti sekä momenttiresultantti oheisen tukimuurin pohjalaatan pohjapinnan piinopisteen suhteen.

Laskenta osavarmuuslukumenetelmällä



Lähtöarvot (kuva)

$h_1 := 1$	$h_2 := 1.2$	$h_3 := 1.2$		m	korkeudet
$\gamma_{s1} := 20$	$\gamma_{s2} := 18$	$\gamma_w := 10$	$\gamma_c := 25$	$\frac{kN}{m^3}$	ominaispainot
$q := 2$				$\frac{kN}{m^2}$	Pintakuorma

Aktiivinen (Coulombin) maanpaine:

Kerros 1:

$\alpha := \operatorname{atan}\left(\frac{0.9}{3.4}\right)$ $\qquad \alpha = 14.82648\,deg$ $\qquad$ tukimuurin kaltevuuskulma

$\beta := \operatorname{atan}\left(\frac{4}{100}\right)$ $\qquad \beta = 2.29061\,deg$ $\qquad$ maanpinnan kaltevuuskulma

$\varphi_{d1} := \operatorname{atan}\left(\frac{\tan(32 \cdot deg)}{1.25}\right)$ $\qquad \varphi_{d1} = 26.56026\,deg$ $\qquad$ kitkakulman suunnitteluarvo, osavarmuuskerroin = 1,25 kantavuutta laskettaesssa, PRO-2004

$\delta_{d1} := \frac{3}{4} \cdot \varphi_{d1}$ $\qquad \delta_{d1} = 19.9202\,deg$ $\qquad$ Huom! seinäkitkakulma ($\delta = 3/4\ \varphi$), PRO-2004

Maanpaineen horisontaaliomponentin kerroin:

$$K_{ah1} := \frac{\cos(\varphi_{d1} + \alpha)^2}{(\cos(\alpha))^2 \cdot \left(1 + \sqrt{\frac{\sin(\varphi_{d1} + \delta_{d1}) \cdot \sin(\varphi_{d1} - \beta)}{\cos(\alpha - \delta_{d1}) \cdot \cos(\alpha + \beta)}}\right)^2} \qquad K_{ah1} = 0.24765$$

Maapaineiden horisontaalikomponentit:

Paine syvyydellä 1 m:

$p_{\gamma 1h} := K_{ah1} \cdot \gamma_{s1} \cdot h_1$ $\quad p_{\gamma 1h} = 4.95298 \frac{kN}{m^2}$ kolmiokuorman intensiteetti syvyydellä h_1

$p_{q1h} := K_{ah1} \cdot q$ $\quad p_{q1h} = 0.4953 \frac{kN}{m^2}$ tasainen kuorma

$p_{1y} := p_{\gamma 1h} + p_{q1h}$ $\quad p_{1y} = 5.44828$ maanpaine syvyydellä h_1

Maanpainekuormien vaakakomponenttien resultantit ja niiden etäisyys peruslaatan alapinnasta:

$P_{\gamma 1h} := 0.5 \cdot K_{ah1} \cdot \gamma_{s1} \cdot h_1^{\,2}$ $\quad P_{\gamma 1h} = 2.47649 \frac{kN}{m}$ $\quad d_{\gamma 1h} := \frac{h_1}{3} + h_2 + h_3$ $\quad d_{\gamma 1h} = 2.73333$ n

$P_{q1h} := K_{ah1} \cdot q \cdot h_1$ $\quad P_{q1h} = 0.4953 \frac{kN}{m}$ $\quad d_{q1h} := \frac{h_1}{2} + h_2 + h_3$ $\quad d_{q1h} = 2.9$ m

Vertikaalikomponentit:

$P_{\gamma 1v} := \tan(\delta_{d1} - \alpha) \cdot P_{\gamma 1h}$ $\quad P_{\gamma 1v} = 0.22075 \frac{kN}{m}$

$P_{q1v} := \tan(\delta_{d1} - \alpha) \cdot P_{q1h}$ $\quad P_{q1v} = 0.04415 \frac{kN}{m}$

Kerros 2:

$\varphi_{d2} := \operatorname{atan}\left(\frac{\tan(28 \cdot deg)}{1.25}\right)$ $\quad \varphi_{d2} = 23.04333\,deg$

$\delta_{d2} := \frac{3}{4} \cdot \varphi_{d2}$ $\quad \delta_{d2} = 17.28249\,deg$

$$K_{ah2} := \frac{\cos(\varphi_{d2} + \alpha)^2}{\cos(\alpha)^2 \cdot \left(1 + \sqrt{\frac{\sin(\varphi_{d2} + \delta_{d2}) \cdot \sin(\varphi_{d2} - \beta)}{\cos(\alpha - \delta_{d2}) \cdot \cos(\alpha + \beta)}}\right)^2} \qquad K_{ah2} = 0.30034$$

maanpaine etäisyydellä 1 m maanpinnasta:

$p_{q2h} := K_{ah2} \cdot (q + \gamma_{s1} \cdot h_1)$ $\quad p_{q2h} = 6.60746$ tasainen kuorma

maanpaine etäisyydellä 2.2 m maanpinnasta:

$p_{\gamma 2h} := K_{ah2} \cdot \gamma_{s2} \cdot h_2$ $\quad p_{\gamma 2h} = 6.48733 \frac{kN}{m^2}$ Kerroksen 2 kolmiokuorman intensiteetti kerroksen alalaidassa (syvyydellä h_2)

$p_{2a} := p_{\gamma 2h} + p_{q2h}$ $\quad p_{2a} = 13.09479 \frac{kN}{m^2}$ maanpaine syvyydellä h_2

Resultantit kerroksessa 2:

$P_{\gamma 2h} := 0.5 \cdot K_{ah2} \cdot \gamma_{s2} \cdot h_2^{\,2}$ $\quad P_{\gamma 2h} = 3.8924 \frac{kN}{m}$ $\quad d_{\gamma 2h} := \frac{h_2}{3} + h_3$ $\quad d_{\gamma 2h} = 1.6$ m

$P_{q2h} := K_{ah2} \cdot (q + \gamma_{s1} \cdot h_1) \cdot h_2$ $\quad P_{q2h} = 7.92895 \frac{kN}{m}$ $\quad d_{q2h} := \frac{h_2}{2} + h_3$ $\quad d_{q2h} = 1.8$ m

Vertikaalikomponentit:

$P_{\gamma 2v} := \tan(\delta_{d2} - \alpha) \cdot P_{\gamma 2h}$ $\quad P_{\gamma 2v} = 0.16695 \frac{kN}{m}$

$P_{q2v} := \tan(\delta_{d2} - \alpha) \cdot P_{q2h}$ $\quad P_{q2v} = 0.34009 \frac{kN}{m}$

Kerros 3: (pohjavedenpinnan alapuolella)

$p_{q3h} := K_{ah2} \cdot (q + \gamma_{s1} \cdot h_1 + \gamma_{s2} \cdot h_2)$ $\quad p_{q3h} = 13.09479$ tasainen kuorma kerroksessa 3 (= p2a)

$p_{\gamma 3h} := K_{ah2} \cdot (\gamma_{s2} - \gamma_w) \cdot h_3$ $\quad p_{\gamma 3h} = 2.88326$ kolmiokuorman intensiteetti syvyydellä h_1

$p_{3a} := p_{q3h} + p_{\gamma 3h}$ $\quad p_{3a} = 15.97804$ $\quad$ maanpaine syvyydellä h_3

Resultantit kerroksessa 3:

$P_{q3h} := K_{ah2} \cdot (q + \gamma_{s1} \cdot h_1 + \gamma_{s2} \cdot h_2) \cdot h_3 \quad P_{q3h} = 15.71375 \, \frac{kN}{m} \qquad d_{q3h} := \frac{h_3}{2} \qquad d_{q3h} = 0.6 \, m$

$P_{\gamma 3h} := 0.5 \cdot K_{ah2} \cdot (\gamma_{s2} - \gamma_w) \cdot h_3^{\,2} \quad P_{\gamma 3h} = 1.72995 \, \frac{kN}{m} \qquad d_{\gamma 3h} := \frac{h_3}{3} \qquad d_{\gamma 3h} = 0.4 \, m$

Vertikaalikomponentit:

$P_{\gamma 3v} := \tan(\delta_{d2} - \alpha) \cdot P_{\gamma 2h} \qquad P_{\gamma 3v} = 0.16695 \, \frac{kN}{m}$

$P_{q3v} := \tan(\delta_{d2} - \alpha) \cdot P_{q3h} \qquad P_{q3v} = 0.67399 \, \frac{kN}{m}$

Omapaino ja epäkeskisyydet pohjapinnan painopisteeseen nähden:

$\Delta x_1 := \frac{1.2 \cdot 0.9}{3.4} \qquad \Delta x_1 = 0.31765 \, m$ $\qquad$ kolmiomaisen osapinnan mitta

$\Delta x_3 := \frac{1.1 \cdot 0.9}{3.4} \qquad \Delta x_3 = 0.29118 \, m$ $\qquad$ etumuurin pp:n epäkeskeisyys (ks kuva)

$G_1 := (\gamma_c - \gamma_w) \cdot 1.2^2 \qquad G_1 = 21.6 \qquad e_1 := 0$ $\qquad$ (Huom! Omalle painolle ei käytetä kantavuutta laskettaessa osavarmuuskertoimia, PRO-2004)

$G_2 := (\gamma_c - \gamma_w) \cdot \frac{\Delta x_1}{2} \cdot 1.2 \qquad G_2 = 2.85882 \qquad e_2 := 0.6 + \frac{\Delta x_1}{3} \qquad e_2 = 0.70588 \, m$

$G_3 := \gamma_c \cdot (\Delta x_1 + 0.2) \cdot 2.2 \qquad G_3 = 28.47059 \qquad e_3 := 0.4 + \frac{(0.2 + \Delta x_1)}{2} + \Delta x_3 \qquad e_3 = 0.95 \, m$

Mitoituskuormien resultantit:

Maanpaineen vaakakomponenttien resultantti:

$P_\gamma := P_{\gamma 1h} + P_{\gamma 2h} + P_{\gamma 3h} \qquad P_\gamma = 8.09884 \, \frac{kN}{m}$ $\qquad$ (Huom! Maanpainekuormille ei käytetä kuormien osavarmuuskertoimia, PRO-2004)

$P_q := P_{q1h} + P_{q2h} + P_{q3h} \qquad P_q = 24.138 \, \frac{kN}{m}$

$H := P_\gamma + P_q \qquad H = 32.23684 \, \frac{kN}{m}$

Omapaino :

$G := G_1 + G_2 + G_3 \qquad G = 52.92941 \, \frac{kN}{m}$

(Maanpaineen pystykomponenttien resultantti:)

(Huom! Jätetään usein huomioimatta laskelmissa

$P_{\gamma v} := P_{\gamma 1v} + P_{\gamma 2v} + P_{\gamma 3v} \qquad P_{\gamma v} = 0.55465 \, \frac{kN}{m}$

$P_{qv} := P_{q1v} + P_{q2v} + P_{q3v} \qquad P_{qv} = 1.05823 \, \frac{kN}{m}$

$P_v := P_{\gamma v} + P_{qv} \qquad P_v = 1.61288 \, \frac{kN}{m}$ $\qquad$ (Pystykomponenttien osuu tässä pieni)

Pystykuormien resultantti:

$V := G + P_v \qquad V = 54.54229 \, \frac{kN}{m}$

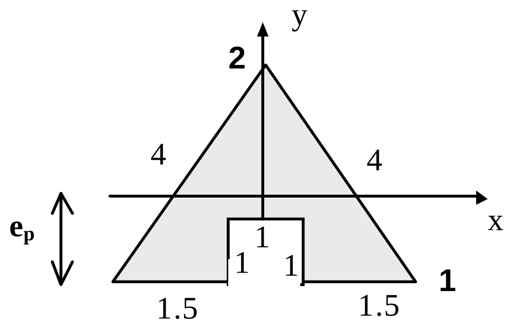
Momentti pohjapinnan painopisteen suhteen:

$M_d := -G_2 \cdot e_2 - G_3 \cdot e_3 + P_{\gamma 1h} \cdot d_{\gamma 1h} + P_{\gamma 2h} \cdot d_{\gamma 2h} + P_{\gamma 3h} \cdot d_{\gamma 3h} + P_{q1h} \cdot d_{q1h} + P_{q2h} \cdot d_{q2h} + P_{q3h} \cdot d_{q3h}$

$M_d = 9.76057 \, \frac{kNm}{m}$

Huom! Oheinen laskutapa maanpaineen määrittämiseksi kerroksellisessa maassa on likimääräinen. Coulombin maanpaineteorian johdossa oletetaan, että maa on homogeenista ts. tasalaatuista.

Esimerkki 3.1: Sydänkuvion määritys



Poikkileikkausarvot:

$A_1 := 0.5 \cdot 4 \cdot 2 \cdot \sqrt{3}$ $\quad m^2$ $\quad$ Tasasiv. kolmio

$A_2 := 1 \cdot 1$ $\quad m^2$ $\quad$ Neliö

$A := 0.5 \cdot 4 \cdot 2 \cdot \sqrt{3} - 1 \cdot 1$ $\quad A = 5.9282$ $\quad m^2$ $\quad$ Kokon. ala

$$e_p := \frac{A_1 \cdot \frac{1}{3} \cdot 2 \cdot \sqrt{3} - A_2 \cdot 0.5}{A} \qquad e_p = 1.26514 \quad m$$

Poikkileikkauksen painopisteen et. kannasta

$$I_{x1} := \frac{4 \cdot \left(2 \cdot \sqrt{3}\right)^3}{36} + A_1 \cdot \left(e_p - \frac{1}{3} \cdot 2 \cdot \sqrt{3}\right)^2 \qquad I_{x1} = 4.7033 \quad m^4$$

Kolmion jäyhyysmomentti pp:n suhteen

$$I_{x2} := \frac{1 \cdot 1^3}{12} + A_2 \cdot \left(e_p - 0.5\right)^2 \qquad I_{x2} = 0.66877 \quad m^4$$

Neliön jäyhyysmomentti pp:n suht

$$I_x := I_{x1} - I_{x2} \qquad I_x = 4.03453 \quad m^4$$

Pinnan jäyhyysmomentti x- aks. suht

$$I_y := \frac{2 \cdot \sqrt{3} \cdot 4^3}{48} - \frac{1^4}{12} \qquad I_y = 4.53547 \quad m^4$$

Pinnan jäyhyysmomentti y- aks. suht

$$i_x := \sqrt{\frac{I_x}{A}} \qquad i_x = 0.82496 \quad m$$

hitaussäde x- aks. suht

$$i_y := \sqrt{\frac{I_y}{A}} \qquad i_y = 0.87468 \quad m$$

hitaussäde y- aks. suht

Sydänkuviota rajoittavien suorien yhtälöt:

$$1 + \frac{x_i \cdot x}{i_y^2} + \frac{y_i \cdot y}{i_x^2} = 0 \qquad (1)$$

neutraaliakseli ($\sigma = 0$)

Lasketaan piirtämistä varten suorien pp- akseleista erot amat osa ξ_i ja η_i:

$$i := 1 .. 2 \qquad \xi_i = -\frac{i_y^2}{x_i} \qquad \eta_i = -\frac{i_x^2}{y_i} \qquad (2)$$

Sijoittamalla kuperien nurkkapisteiden (1, 2) koordinaatit

$x_1 := 2 \qquad y_1 := -e_p$

$x_2 := 0 \qquad y_2 := 2 \cdot \sqrt{3} - e_p$

Leikkauspisteet

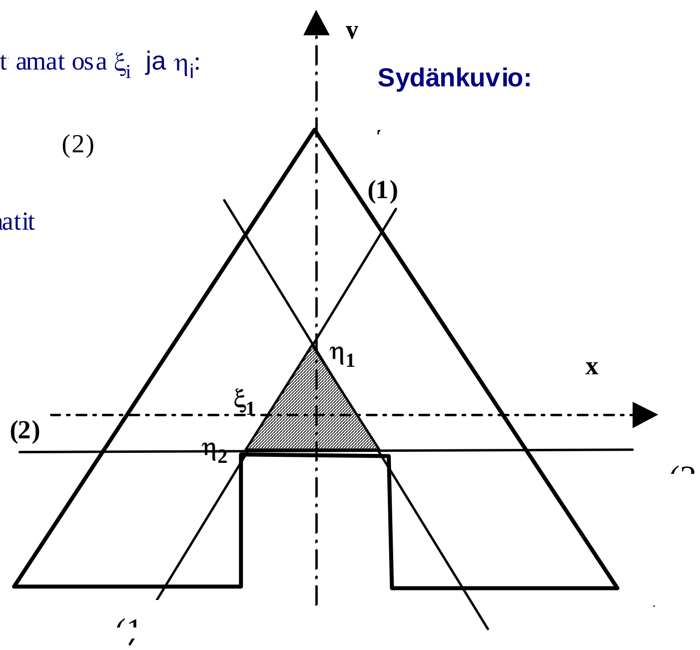
$$\xi_1 := -\frac{i_y^2}{x_1} \qquad \eta_i := -\frac{i_x^2}{y_i}$$

$\xi_1 = -0.38253 \qquad \eta_1 = 0.53794$

$\eta_2 = -0.30949$

(suora (2) x-aks. suunt.)

Sydänkuvio:



Huom! Sydänkuvio symmetrinen y -aks. suhteen

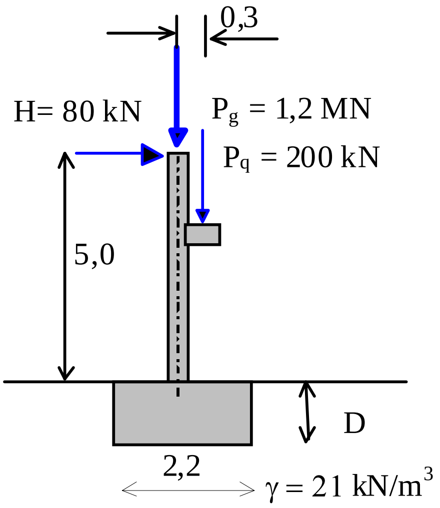
Esim 4.1: Murtovarmuustarkastelu kantavuuskaavalla

Mikä on oheisen hallin pilariperustuksen tarvittava perustamissyvyys maapohjan kantavuuden perusteella arvioiutuna, kun tarkastelu tehdään

a) Kokonaisvarmuuskerroin menetelmällä

b) Rajatilamenettelyllä.

Peruslaatan leveys toisessa suunnassa on 2,5 m. (Omaa painoa ei tarvitse ottaa huomioon.)



Alkuarvot: (kuva)

$\gamma_1 := 21 \cdot 10^{-3} \dfrac{MN}{m^3}$ $\quad \gamma_2 := \gamma_1$ $\quad \varphi := 28 \cdot deg$

$h := 5$ $\quad B := 2.2$ $\quad L := 2.5$ $\quad$ m

$P_g := 1.2$ $\quad P_q := 0.2$ $\quad H_w := 0.08$ $\quad$ MN

Murtotilatarkastelut:

a) Mitoitus kokonaisvarmuuskerroinmenettelyllä:

Pohjapaine: (Tehdään alkuoletus: $D := 1.0$ m)

$M := 0.3 \cdot P_q + (5 + D) \cdot H_w$ $\qquad M = 0.54$ MNm

$N := P_g + P_q$ $\qquad N = 1.4$ MN

$e_N := \dfrac{M}{N}$ $\qquad e_N = 0.38571$ m

$B_t := B - 2 \cdot e_N$ $\qquad B_t = 1.42857$ m

$q_d := \dfrac{N}{B_t \cdot L}$ $\qquad q_d = 0.392$ $\dfrac{MN}{m^2}$

Kantavuus:

$$q_{md} = \gamma_1 \cdot D \cdot N_D \cdot s_D \cdot i_D + \frac{1}{2} \cdot \gamma_2 \cdot B \cdot s_B \cdot i_B \qquad (1)$$

$N_D := \tan\left(45 \cdot deg + \dfrac{\varphi}{2}\right)^2 \cdot e^{\pi \cdot \tan(\varphi)}$ $\qquad N_D = 14.71988$ (PRO 2004 taul 9)

$s_D := 1 + 0.2 \cdot \left(\dfrac{B_t}{L}\right)$ $\qquad s_D = 1.11429$

$i_D := \left(1 - \dfrac{H_w}{N}\right)^2$ $\qquad i_D = 0.88898$

$i_B := (i_D)^2$ $\qquad i_B = 0.79028$

$N_B := 1.5 \cdot (N_D - 1) \cdot \tan(\varphi)$ $\qquad N_B = 10.94249$ (PRO 2004 taul 9)

$s_B := 1 - 0.4 \cdot \left(\dfrac{B_t}{L}\right)$ $\qquad s_B = 0.77143$

Vaadittava min. perustamissyvyys:

(Kokonaisvarmuuskerroin = 2)

(1) =>
$$D_{min} := \frac{2 \cdot q_d - \frac{1}{2} \cdot \gamma_2 \cdot B_t \cdot N_B \cdot s_B \cdot i_B}{\gamma_1 \cdot N_D \cdot s_D \cdot i_D}$$
=> $D_{min} = 2.234$ m > kuin alussa oletettu 1 m

Iteroimalla D:tä päästään 'tarkkaan' ratkaisuun:

$D = 2.70$ m **(OK)**

b) Mitoitus rajatilamenettelyllä:

Kitkakulman laskenta-arvo:

$D := 1.75$ Arvataan perustamissyvyys

$\varphi_d := \operatorname{atan}\left(\frac{\tan(\varphi)}{1.25}\right)$ $\varphi_d = 23.043\,\mathrm{deg}$ (maan kitkak. osavarmuusluku 1.25 kantavuutta laskettaessa)

Pohjapaine: (Käytetään kuormille osavarmuuslukuja 1,2 ja 1,6)

$M_d := 0.3 \cdot 1.6 \cdot P_q + (5 + D) \cdot 1.6 \cdot H_w$	$M_d = 0.96$	MNm
$N_d := 1.2 \cdot P_g + 1.6 \cdot P_q$	$N_d = 1.76$	MN
$H_d := 1.6 \cdot H_w$	$H_d = 0.128$	MN
$e_N := \frac{M_d}{N_d}$	$e_N = 0.54545$	m
$B_t := B - 2 \cdot e_N$	$B_t = 1.10909$	m
$q_d := \frac{N_d}{B_t \cdot L}$	$q_d = 0.63475$	$\frac{MN}{m^2}$

Kantavuus:
$$q_{md} = \gamma_1 \cdot D \cdot N_D \cdot s_D \cdot i_D + \frac{1}{2} \cdot \gamma_2 \cdot B \cdot s_B \cdot i_B \qquad (1)$$

$N_D := \tan\left(45 \cdot \mathrm{deg} + \frac{\varphi_d}{2}\right)^2 \cdot e^{\pi \cdot \tan(\varphi_d)}$	$N_D = 8.6998$	(PRO 2004 taul 9)
$s_D := 1 + 0.2 \cdot \left(\frac{B_t}{L}\right)$	$s_D = 1.08873$	
$i_D := \left(1 - \frac{H_d}{N_d}\right)^2$	$i_D = 0.85983$	
$i_B := (i_D)^2$	$i_B = 0.73932$	
$N_B := 1.5 \cdot (N_D - 1) \cdot \tan(\varphi_d)$	$N_B = 4.91287$	(taul 9)
$s_B := 1 - 0.4 \cdot \left(\frac{B_t}{L}\right)$	$s_B = 0.82255$	

Vaadittava min. perustamissyvyys:

(1) =>
$$D := \frac{q_d - \frac{1}{2} \cdot \gamma_2 \cdot B_t \cdot N_B \cdot s_B \cdot i_B}{\gamma_1 \cdot N_D \cdot s_D \cdot i_D}$$
$D = 3.508$ m => ei järkevä. ei suppene alkup. mitoilla !

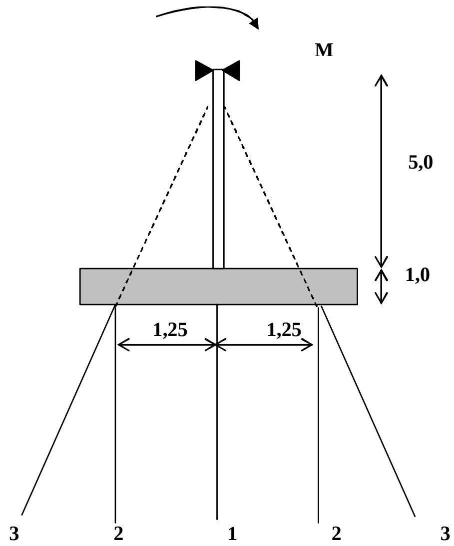
Tarvittava perustamissyvyys tulee näillä anturan mitoilla liian suureksi.
==> kasvatetaan peruslaatan leveyttä:

Peruslaatan leveys:	a) Kokonaisvarmuuslukumenetelmä.: Perustamissyvyys	b) Osavarmuuslukumenetelmä: Perustamissyvyys
B	**Dmin**	**D.min**
2,2	2,70	Ei ratkaisua
2,5	1,85	3,20
2,8	1,39	2,14
3,0	1,17	1,75

Esim 5.1: Paalutus osana st. määräämätöntä rakennetta
(Laskenta voimamenetelmällä)

1) Laskenta kiertokeskiötä hyväksi käyttäen

Oheista paaluille perustettua pilaria (500 x 500) kuormittaa yläpäässä momentti M=1 MNm. Laske pilarin alapäätä rasittavat leikkausvoima **Qz** ja taivutusmomentti **My**. Paalut ovat puupaaluja k = EA/l = 20 MN/m ja vinopaalut kaltevuudessa 4:1. Pilarille Ec = 25000 MN/m2



Lähtoarvot:

$EI_p := 25000\cdot\frac{0.5^4}{12}$ $\quad EI_p = 130.208 \;\; MNm^2$ pilarin t-jäykkyys

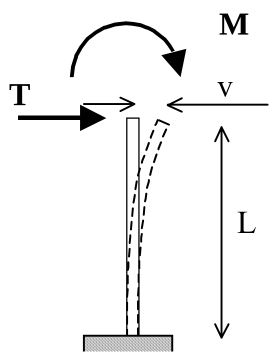
$L := 5$

$k := 20 \quad \frac{MN}{m}$ paalun puristusjäykkyys

$M := 1 \quad$ MN ulkoinen kuorma

Staattisen perusmuodon siirtymien laskenta:

Voimamenetelmää käytettäessä valitaan st, perusmuoto (tässä irroitetaan yläpään kiinnitys) ja lasketaan lasketaan perusmuodon pilarin yläpään vaakasiirtymät. Tuntematon suure on tukireaktio **T,** jolle saadaan ratkaisuyhtälö ehdosta **v = 0.**



1. Pilarin taipuma kuormasta M:

$v_M := \frac{M}{2\cdot EI_p}\cdot L^2$ $\quad v_M = 0.096$

2. Pilarin taipuma kuormasta T:

$v_T = \frac{L^3}{3\cdot EI_p}\cdot T$ $\quad v_T = 0.32\cdot T$

3. Paalutuksen kiertokeskiön siirtymät:

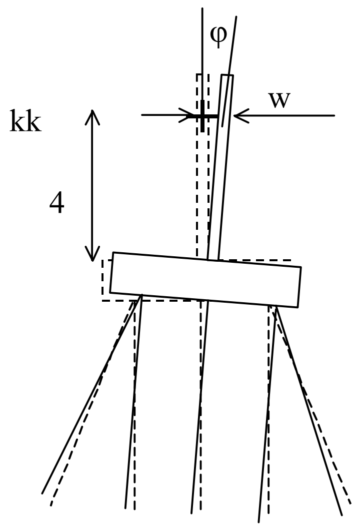
Paalutuksen jäykkyydet:

$p_{x1} := 1 \quad p_{x2} := \frac{4}{\sqrt{17}} \quad p_{z2} := \frac{1}{\sqrt{17}} \quad r_{y1} := 1.25$

$k_{11} := k\cdot\left(5\cdot p_{x1}^2 + 6\cdot p_{x2}^2\right) \quad k_{22} := k\cdot\left(6\cdot p_{z2}^2\right) \quad k_{11} = 212.941$

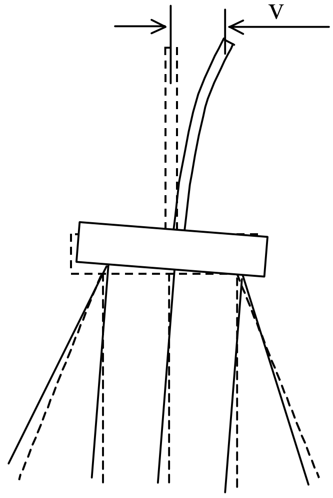
$k_{22} = 7.059$

$k_{33} := k\cdot\left(2\cdot 2\cdot r_{y1}^2\right) \quad k_{33} = 125$



Paalutuksen kiertokeskiön saamat siirtymät kuormista M ja T:

$\Delta h := 1$ (kk :n etäisyys pilarin päästä)



$$w_T = \frac{T}{k_{22}} \qquad \varphi_T = \frac{T\cdot\Delta h}{k_{33}}$$ T:stä

$$\varphi_M := \frac{M}{k_{33}}$$ M:stä

4. Yhteensopivuus pilarin yläpäässä

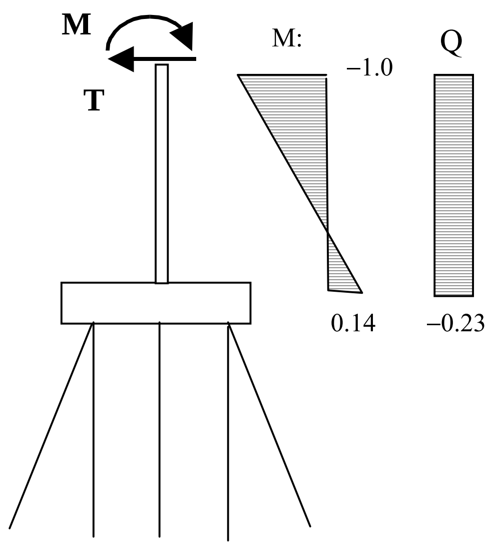
Kokonaissiirtymä pilarin yläpäässä on oikeasti = 0:

$$v = v_M + v_T + w_T + \varphi_T\cdot\Delta h + \varphi_{M.}\cdot\Delta h = 0$$

josta tuntematon tukivoima T voidaan ratkaista:

$$\left(\frac{L^3}{3EI_p} + \frac{1}{k_{22}} + \frac{\Delta h^2}{k_{33}}\right)\cdot T = -\left(\frac{L^2}{2EI.p} + \frac{\Delta h}{k_{33}}\right)M$$

=> $$T := -\frac{\dfrac{L^2}{2\cdot EI_p} + \dfrac{\Delta h^2}{k_{33}}}{\dfrac{L^3}{3\cdot EI_p} + \dfrac{1}{k_{22}} + \dfrac{\Delta h}{k_{33}}}\cdot M \qquad T = -0.221$$ MN



Kysytyt leikkaussuureet pilarin alapäässä:

$Q := T$ $\qquad Q = -0.221$ MN

$M_p := -T\cdot 5 - M$ $\qquad M_p = 0.107$ MNm

2) **Laskenta, kun origo on pilarin juuressa:**

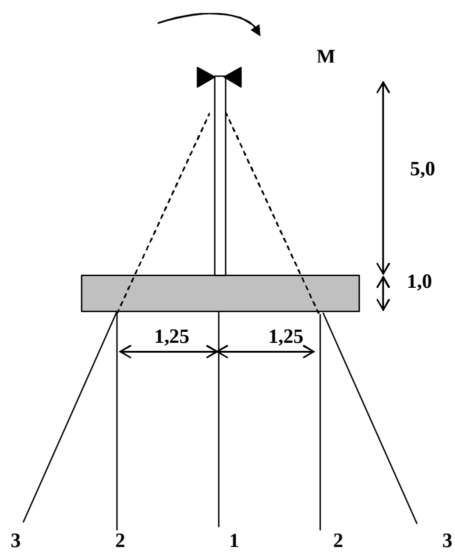
1. Pilarin taipuma kuormasta M:

$$v_M := \frac{M}{2\cdot EI_p}\cdot L^2 \qquad v_M = 0.096$$

2. Pilarin taipuma kuormasta T:

$$v_T = \frac{L^3}{3\cdot EI_p}\cdot T \qquad v_T = 0.32\cdot T$$

3. Paalutuksen kiertokeskiön siirtymät:



Paalutuksen jäykkyydet:

$$p_{x1} := 1 \quad p_{x2} := \frac{4}{\sqrt{17}} \quad p_{z2} := \frac{1}{\sqrt{17}}$$

$$r_{y1} := 1.25 \qquad r_{y2} := -1.25\cdot p_{x2} - 1\cdot(-p_{z2})$$

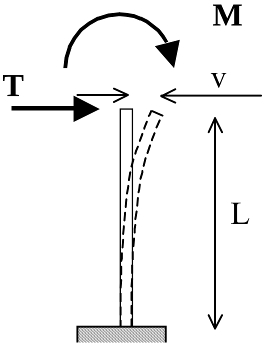
$$r_{y1} = 1.25 \qquad r_{y2} = -0.97$$

$$k_{11} := k\cdot\left(5\cdot p_{x1}^2 + 6\cdot p_{x2}^2\right)$$

$k_{22} := k\cdot\left(6\cdot p_{z2}{}^{2}\right)$

$k_{33} := k\cdot\left(2\cdot 2\cdot r_{y1}{}^{2} + 2\cdot 3\cdot r_{y2}{}^{2}\right)$

$k_{23} := k\cdot\left[3\cdot 2\cdot\left(-p_{z2}\right)\cdot r_{y2}\right]$



Paalutuksen jm.:

$$k_p := \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{23} & k_{33} \end{pmatrix}$$

$k_{11} = 212.941$

$k_{22} = 7.059$

$k_{33} = 237.941$

$k_{23} = 28.235$

Paalutuksen sivusiirtymä- ja kiertymävapausasteisiin (w ja φ) liittyvä jäykkyysmatriisin osa:

$$k_t := \begin{pmatrix} k_{22} & k_{23} \\ k_{23} & k_{33} \end{pmatrix} \qquad \delta = \begin{pmatrix} w \\ \varphi \end{pmatrix} \qquad f = \begin{pmatrix} T \\ M \end{pmatrix}$$

Lasketaan M:n ja T:n aiheuttamat siirtymät origossa ratkaisemalla yhtälöpari:

$$\begin{pmatrix} k_{22} & k_{23} \\ k_{23} & k_{33} \end{pmatrix}\cdot\begin{pmatrix} w \\ \varphi \end{pmatrix} = \begin{pmatrix} T \\ M \end{pmatrix}$$

$$w_T = \frac{k_{33}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot T \qquad \varphi_T := \frac{-k_{23}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot T$$

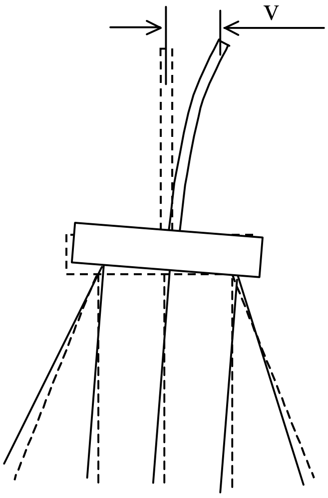
$$w_M = \frac{-k_{23}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot M \qquad \varphi_M = \frac{k_{22}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot M$$

Lasketaan siirtymät pilarin yläpäässä: .

Tukivoiman T aiheuttama vaakasiitymä pilarin yläpäässä

$$w_T = \frac{k_{33} - k_{23}\cdot L}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot T \qquad w_{T2} = \varphi_T\cdot L = \frac{-k_{23} + k_{22}\cdot L}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot T\cdot L$$

$$w_M = \frac{-k_{23}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot M \qquad w_{M2} = \varphi_M\cdot L = \frac{k_{22}}{k_{22}\cdot k_{33} - k_{23}{}^{2}}\cdot M\cdot L$$



4. Yhteensopivuus pilarin yläpäässä

Kokonaissiirtymä pilarin yläpäässä on oikeasti = 0:

$$v = v_M + v_T + w_T + w_M + \varphi_T\cdot L + \varphi_{M.}\cdot L = 0$$

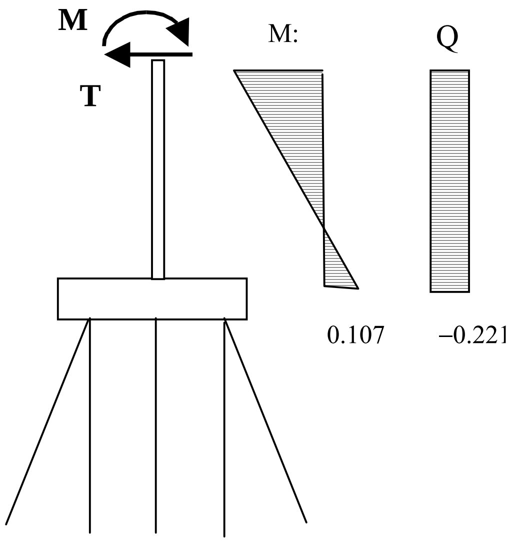
josta tuntematon tukivoima T voidaan ratkaista:

$$\left[\frac{L^3}{3EI_p}+\frac{k_{33}-k_{23}\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}+\frac{\left(-k_{23}+k_{22}\cdot L\right)\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}\right]\cdot T = -\left(\frac{L^2}{2EI.p}+\frac{-k_{23}}{k_{22}\cdot k_{33}-k_{23}^{\,2}}+\frac{k_{22}\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}\right)M$$

$$T := -\frac{\dfrac{L^2}{2\cdot EI_p}+\dfrac{-k_{23}}{k_{22}\cdot k_{33}-k_{23}^{\,2}}+\dfrac{k_{22}\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}}{\dfrac{L^3}{3EI_p}+\dfrac{k_{33}-k_{23}\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}+\dfrac{\left(-k_{23}+k_{22}\cdot L\right)\cdot L}{k_{22}\cdot k_{33}-k_{23}^{\,2}}}$$

$T = -0.221$ MN **OK**

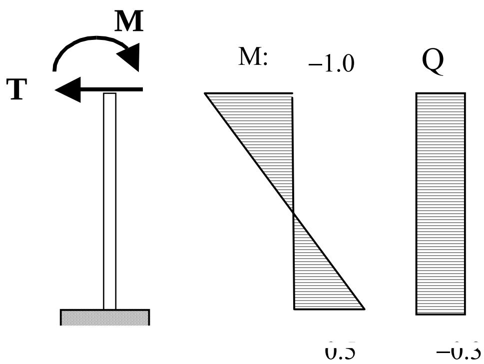
$M_p := -T\cdot 5 - M$ $M_p = 0.107$



Jos paalutus olisi täysin jäykkä ts. pilarin alapää jäykästi kiinnitetty

$T := \frac{-3}{2}\cdot\frac{M}{L}$ $T = -0.3$ MN

$M_p := -T\cdot 5 - M$ $M_p = 0.5$ MNm



Paalutuksen joustavuus siis pienentää pilarin leikkaussuureita alapään kiinnityskohdassa.

Esim 5.2: Paalutus osana st. määräämätöntä rakennetta
(Laskenta siirtymämenetelmällä)

$EA := 25000 \cdot 0.5^2$ $\quad EA = 6250.000$ MN $\quad L := 5$ m

$EI := 130.208$ MN·m $\quad k := 20 \ \frac{MN}{m}$

Paalutuksen jäykkyysmatriisi

$p_{x1} := 1 \quad p_{z1} := 0 \quad r_{y1} := -1.25$ $\quad$ Pystypaalujen suuntakosinit ja momettivarret

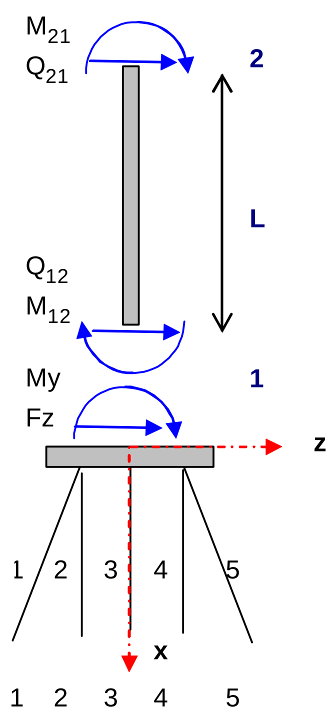
$p_{x2} := \frac{4}{\sqrt{17}} \quad p_{z2} := \frac{1}{\sqrt{17}} \quad r_{y2} := -1.25 \cdot p_{x2} - 1 \cdot (-p_{z2})$ $\quad$ Vinopaalujen suuntakosinit ja momettivarret

$k_{11} := k \cdot \left(5 \cdot p_{x1}{}^2 + 6 \cdot p_{x2}{}^2\right) \qquad k_{11} = 212.941 \ \frac{MN}{m}$

$k_{22} := k \cdot \left(6 \cdot p_{z2}{}^2\right) \qquad k_{22} = 7.059 \ \frac{MN}{m}$

$k_{33} := k \cdot \left(2 \cdot 2 \cdot r_{y1}{}^2 + 2 \cdot 3 \cdot r_{y2}{}^2\right) \qquad k_{33} = 237.941$ MN·m

$k_{23} := k \cdot \left[3 \cdot 2 \cdot (-p_{z2}) \cdot r_{y2}\right] \qquad k_{23} = 28.235$ MN



$w_2 := 0$

$$K_1 := \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{23} & k_{33} \end{pmatrix} \qquad K_1 = \begin{pmatrix} 212.941 & 0.000 & 0.000 \\ 0.000 & 7.059 & 28.235 \\ 0.000 & 28.235 & 237.941 \end{pmatrix}$$

Pilarin jäykkyysmatriisi taivutusvapausasteille:

$$K_2 := \begin{pmatrix} 12 \cdot \frac{EI}{L^3} & 6 \cdot \frac{EI}{L^2} & -12 \cdot \frac{EI}{L^3} & 6 \cdot \frac{EI}{L^2} \\ 6 \cdot \frac{EI}{L^2} & 4 \cdot \frac{EI}{L} & -6 \cdot \frac{EI}{L^2} & 2 \cdot \frac{EI}{L} \\ -12 \cdot \frac{EI}{L^3} & -6 \cdot \frac{EI}{L^2} & 12 \cdot \frac{EI}{L^3} & -6 \cdot \frac{EI}{L^2} \\ 6 \cdot \frac{EI}{L^2} & 2 \cdot \frac{EI}{L} & -6 \cdot \frac{EI}{L^2} & 4 \cdot \frac{EI}{L} \end{pmatrix} \qquad K_2 = \begin{pmatrix} 12.500 & 31.250 & -12.500 & 31.250 \\ 31.250 & 104.166 & -31.250 & 52.083 \\ -12.500 & -31.250 & 12.500 & -31.250 \\ 31.250 & 52.083 & -31.250 & 104.166 \end{pmatrix}$$

Kootaan rakenteen jäykkyysmatriisi nurkan 1 ja 2 tasapainoyhtälöistä (luennot kaava (88)):

$$\left(12 \cdot \frac{EI}{L^3} + k_{22}\right) \cdot w_1 + \left(6 \cdot \frac{EI}{L^2} + k_{23}\right) \cdot \varphi_1 + 6 \cdot \frac{EI}{L^2} \cdot \varphi_2. = 0$$

$$\left(6 \cdot \frac{EI}{L^2} + k_{23}\right) \cdot w_1 + \left(4 \cdot \frac{EI}{L} + k_{33}\right) \cdot \varphi_1 + 2 \cdot \frac{EI}{L} \cdot \varphi_2 = 0$$

$$6\cdot\frac{EI}{L^2}\cdot w_1 + 2\cdot\frac{EI}{L}\cdot\varphi_1 + 4\cdot\frac{EI}{L}\cdot\varphi_1 = M$$

Yhtälöryhmä matriisimuodossa:

$$K\cdot\delta = f$$

missä K = rakenteen jäykkyysmatriisi, f = kuormavektori ja δ = ratkaistavat siirtymät

$$K := \begin{pmatrix} 12\cdot\frac{EI}{L^3} + k_{22} & 6\cdot\frac{EI}{L^2} + k_{23} & 6\cdot\frac{EI}{L^2} \\ 6\cdot\frac{EI}{L^2} + k_{23} & 4\cdot\frac{EI}{L} + k_{33} & 2\cdot\frac{EI}{L} \\ 6\cdot\frac{EI}{L^2} & 2\cdot\frac{EI}{L} & 4\cdot\frac{EI}{L} \end{pmatrix} \begin{matrix} w_1 \\ \varphi_1 \\ \varphi_2 \end{matrix} \qquad K = \begin{pmatrix} 19.559 & 59.485 & 31.250 \\ 59.485 & 342.108 & 52.083 \\ 31.250 & 52.083 & 104.166 \end{pmatrix}$$

Kuormitusvektori $M := 1$ MNm

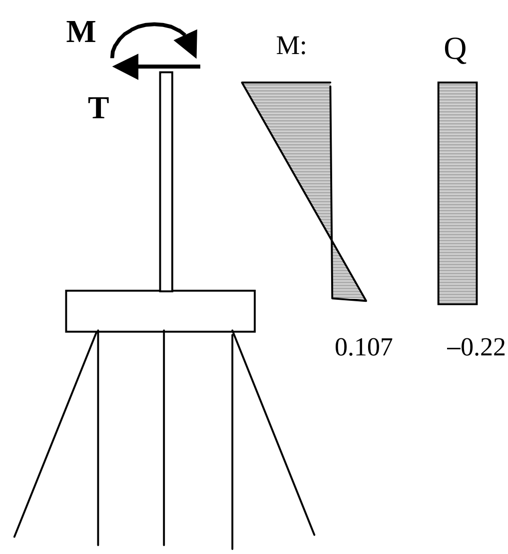
$$f := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Siirtymien ratkaisu:

$$\delta := K^{-1}\cdot f \qquad \begin{pmatrix} w_1 \\ \varphi_1 \\ w_2 \\ \varphi_2 \end{pmatrix} := \begin{pmatrix} \delta_0 \\ \delta_1 \\ 0 \\ \delta_2 \end{pmatrix} \qquad \begin{pmatrix} w_1 \\ \varphi_1 \\ w_2 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} -0.05628 \\ 0.00623 \\ 0.00000 \\ 0.02337 \end{pmatrix}$$

Pilarin voimasuureet:

$$\begin{pmatrix} Q_{12} \\ M_{12} \\ Q_{21} \\ M_{21} \end{pmatrix} := K_2\cdot\begin{pmatrix} w_1 \\ \varphi_1 \\ w_2 \\ \varphi_2 \end{pmatrix} \qquad \begin{pmatrix} Q_{12} \\ M_{12} \\ Q_{21} \\ M_{21} \end{pmatrix} = \begin{pmatrix} 0.221 \\ 0.107 \\ -0.221 \\ 1.000 \end{pmatrix}$$



Taivutusmomentti ja leikkausvoima

$M_{12} := 0.107$ MNm taivutusmomentti pilarin juuressa

$Q_{12} := 0.221$ MN leikkausvoima pilarin juuressa

Paaluvoimat:

$N_1 := k\cdot(-p_{z2})\cdot w_1 + r_{y2}\cdot\varphi_1$ $\quad N_1 = 0.267$ MN Vinopaalu

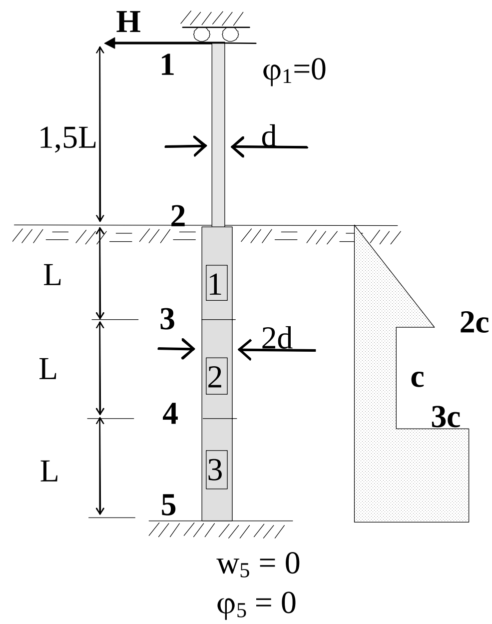
$N_2 := k\cdot(-1.25)\cdot\varphi_1$ $\quad N_2 = -0.156$ MN Pystysuora paalu

$N_3 := 0$

$N_4 := -N_2$

$N_5 := -N_1$

Esim 6.1: Suurpaaluperustuksen laskenta elementtimenetelmällä



Määritä oheiselle poikkileikkaukseltaan pyöreästä pilarista ja suurpaalusta koostuvalle perustuksen siirtymät, sivupaine sekä taivutus- ja leikkausvoimakuviot elementtimenetelmän mukaisella ratkaisulla pisteissä 1...5 käyttämällä paalulle kuvan mukaista elementtijakoa. Sivupaine muuttuu oheisen kuvion mukaisesti c = 100 MN/m³ ja kuormituksena on vaakavoima H 1 MN.
L = 4 m ja d = 0.9 m. Paalu ja pilari ovat betonia K40.

$kN := 1000 \cdot N \qquad MN := 1000 \cdot kN$

Lähtöarvot :

$d := 0.9 \cdot m \qquad L := 4 \cdot m \qquad c := 100 \cdot \frac{MN}{m^3}$

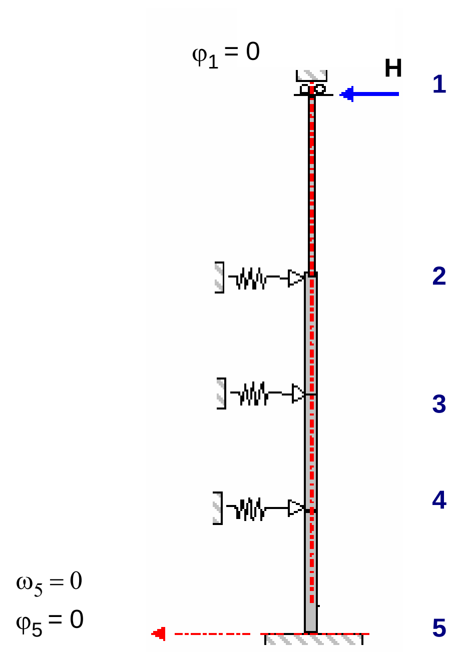
$K_c := 45 \cdot \frac{MN}{m^2} \qquad K_0 := 25 \cdot 10^6 \cdot \frac{MN}{m^2}$

$E_c := \sqrt{K_0 \cdot K_c} \qquad E_c = 33541 \frac{MN}{m^2}$

$I := \frac{\pi \cdot d^4}{64} \qquad I_2 := \frac{\pi \cdot (2 \cdot d)^4}{64}$

$EI_1 := E_c \cdot I \quad EI_1 = 1080.2\, MN \cdot m^2$ pilari

$EI_2 := E_c \cdot I_2 \quad EI_2 = 17283.7\, MN \cdot m^2$ paalu



Maan sivuvastusta kuvaavat jousivakiot:

$k_2 := \frac{L}{2} \cdot d \cdot c \qquad k_2 = 180.0 \frac{MN}{m}$

$k_3 := \frac{L}{2} \cdot d \cdot \left[\left(\frac{c + 2 \cdot c}{2} \right) + c \right] \qquad k_3 = 450.0 \frac{MN}{m}$

$k_4 := \frac{L}{2} \cdot d \cdot (c + 3 \cdot c) \qquad k_4 = 720.0 \frac{MN}{m}$

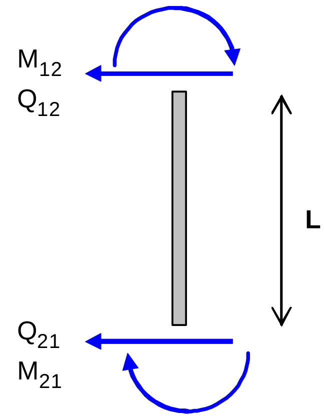
Muutetaan matriiseissa käytettävät suureet dimensioimattomiksi (MathCadin takia) :

$EI_1 := \frac{EI_1}{\mathrm{UnitsOf}(EI_1)} \cdot 10^{-6} \qquad EI_2 := \frac{EI_2}{\mathrm{UnitsOf}(EI_2)} \cdot 10^{-6} \qquad L := \frac{L}{\mathrm{UnitsOf}(L)}$

$k_2 := \frac{k_2}{\mathrm{UnitsOf}(k_2)} \cdot 10^{-6} \qquad k_3 := \frac{k_3}{\mathrm{UnitsOf}(k_3)} \cdot 10^{-6} \qquad k_4 := \frac{k_4}{\mathrm{UnitsOf}(k_4)} \cdot 10^{-6}$

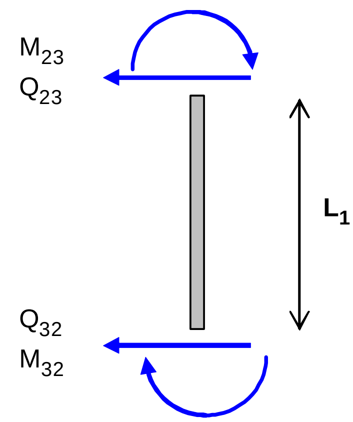
Paaluelementin jäykkyysmatriisi

$$K_2 := \begin{pmatrix} \frac{12\cdot EI_2}{L^3} & \frac{6\cdot EI_2}{L^2} & -\frac{12\cdot EI_2}{L^3} & \frac{6\cdot EI_2}{L^2} \\ \frac{6\cdot EI_2}{L^2} & \frac{4\cdot EI_2}{L} & -\frac{6\cdot EI_2}{L^2} & \frac{2\cdot EI_2}{L} \\ -\frac{12\cdot EI_2}{L^3} & -\frac{6\cdot EI_2}{L^2} & \frac{12\cdot EI_2}{L^3} & -\frac{6\cdot EI_2}{L^2} \\ \frac{6\cdot EI_2}{L^2} & \frac{2\cdot EI_2}{L} & -\frac{6\cdot EI_2}{L^2} & \frac{4\cdot EI_2}{L} \end{pmatrix} \quad K_2 = \begin{pmatrix} 3240.7 & 6481.4 & -3240.7 & 6481.4 \\ 6481.4 & 17283.7 & -6481.4 & 8641.8 \\ -3240.7 & -6481.4 & 3240.7 & -6481.4 \\ 6481.4 & 8641.8 & -6481.4 & 17283.7 \end{pmatrix}$$



Pilari : $L_1 := 1.5\cdot L$

$$K_1 := \begin{pmatrix} \frac{12\cdot EI_1}{L_1^3} & \frac{6\cdot EI_1}{L_1^2} & -\frac{12\cdot EI_1}{L_1^3} & \frac{6\cdot EI_1}{L_1^2} \\ \frac{6\cdot EI_1}{L_1^2} & \frac{4\cdot EI_1}{L_1} & -\frac{6\cdot EI_1}{L_1^2} & \frac{2\cdot EI_1}{L_1} \\ -\frac{12\cdot EI_1}{L_1^3} & -\frac{6\cdot EI_1}{L_1^2} & \frac{12\cdot EI_1}{L_1^3} & -\frac{6\cdot EI_1}{L_1^2} \\ \frac{6\cdot EI_1}{L_1^2} & \frac{2\cdot EI_1}{L_1} & -\frac{6\cdot EI_1}{L_1^2} & \frac{4\cdot EI_1}{L_1} \end{pmatrix} \quad K_1 = \begin{pmatrix} 60 & 180 & -60 & 180 \\ 180 & 720.2 & -180 & 360.1 \\ -60 & -180 & 60 & -180 \\ 180 & 360.1 & -180 & 720.2 \end{pmatrix}$$



Kootaan koko rakenteen jäykkyysmatriisi pilarin ja paaluelementtien jäykkyysmatriiseista:

$$C = \begin{bmatrix} \frac{12EI_1}{L_1^3} & \frac{6EI_1}{L_1^2} & \frac{-12EI_1}{L_1^3} & \frac{6EI_1}{L_1^2} & & & & & & \\ \frac{6EI_1}{L_1^2} & \frac{4EI_1}{L_1} & \frac{-6EI_1}{L_1^2} & \frac{2EI_1}{L_1} & & & & & & \\ \frac{-12EI_1}{L_1^3} & \frac{-6EI_1}{L_1^2} & \frac{12EI_1}{L_1^3}+\frac{12EI_2}{L_2^3} & \frac{-6EI_1}{L_1^2}+\frac{6EI_2}{L_2^2} & \frac{-12EI_2}{L_2^3} & \frac{6EI_2}{L_2^2} & & & & \\ \frac{6EI_1}{L_1^2} & \frac{2EI_1}{L_1} & \frac{-6EI_1}{L_1^2}+\frac{6EI_2}{L_2^2} & \frac{4EI_1}{L_1}+\frac{4EI_2}{L_2} & \frac{-6EI_2}{L_2^3} & \frac{2EI_2}{L_2} & & & & \\ & & \frac{-12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2} & \frac{12EI_2}{L_2^3}+\frac{12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2}+\frac{6EI_2}{L_2^2} & \frac{-12EI_2}{L_2^3} & \frac{6EI_2}{L_2^2} & & \\ & & \frac{6EI_2}{L_2^2} & \frac{2EI_2}{L_2} & \frac{-6EI_2}{L_2^2}+\frac{6EI_2}{L_2^2} & \frac{4EI_2}{L_2}+\frac{4EI_2}{L_2} & \frac{-6EI_2}{L_2^2} & \frac{2EI_2}{L_2} & & \\ & & & & \frac{-12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2} & \frac{12EI_2}{L_2^3}+\frac{12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2}+\frac{6EI_2}{L_2^2} & \frac{-12EI_2}{L_2^3} & \frac{6EI_2}{L_2^2} \\ & & & & \frac{6EI_2}{L_2^2} & \frac{2EI_2}{L_2} & \frac{-6EI_2}{L_2^2}+\frac{6EI_2}{L_2^2} & \frac{4EI_2}{L_2}+\frac{4EI_2}{L_2} & \frac{-6EI_2}{L_2^2} & \frac{2EI_2}{L_2} \\ & & & & & & \frac{-12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2} & \frac{12EI_2}{L_2^3} & \frac{-6EI_2}{L_2^2} \\ & & & & & & \frac{6EI_2}{L_2^2} & \frac{2EI_2}{L_2} & \frac{-6EI_2}{L_2^2} & \frac{4EI_2}{L_2} \end{bmatrix}$$

Lisätään sivuvastuksen jousivakiot rivien 2, 4 6 lävistäjäalkioon (Huom! MathCadin numerointi alkaaa 0:sta)

$$C_{2,2} := C_{2,2} + k_2 \qquad C_{4,4} := C_{4,4} + k_3 \qquad C_{6,6} := C_{6,6} + k_4$$

Rakenteen jäykkyysmatriisi ennen reunaehtojen soveltamista: $K_0 := C$

$K_0 =$

	0	1	2	3	4	5	6	7	8	9	
0	60	180	-60	180	0	0	0	0	0	0	w_1
1	180	720.2	-180	360.1	0	0	0	0	0	0	φ_1
2	-60	-180	3480.7	6301.3	-3240.7	6481.4	0	0	0	0	w_2
3	180	360.1	6301.3	18003.8	-6481.4	8641.8	0	0	0	0	φ_2
4	0	0	-3240.7	-6481.4	6931.4	0	-3240.7	6481.4	0	0	w_3
5	0	0	6481.4	8641.8	0	34567.4	-6481.4	8641.8	0	0	φ_3
6	0	0	0	0	-3240.7	-6481.4	7201.4	0	-3240.7	6481.4	w_4
7	0	0	0	0	6481.4	8641.8	0	34567.4	-6481.4	8641.8	φ_4
8	0	0	0	0	0	0	-3240.7	-6481.4	3240.7	-6481.4	w_5
9	0	0	0	0	0	0	6481.4	8641.8	-6481.4	17283.7	φ_5

Muodostetaan kuormitusvektori:

$i := 0, 1 .. 6 \qquad f_i := 0 \qquad f_0 := 1$ (Pilarin yläpäässä on vaakavoima H = 1)

Rreunaehdot $\varphi_1 = w_5 = \varphi_5 = 0$ otetaan huomioon pyyhkäisemällä matriisiyhtälöistä vastaavat rivit ja sarakkeet pois (nrot 1, 8 ja 9) (Huom! numerointi MathCadissä alkaa 0:sta)

Lopullinen ratkaisuyhtälö nurkkasiirtymien δ ratkaisemiseksi:

$K \cdot \delta = f$ missä

$$K := \begin{pmatrix} 60.013 & -60.013 & 180.038 & 0.000 & 0.000 & 0.000 & 0.000 \\ -60.013 & 3480.702 & 6301.341 & -3240.690 & 6481.379 & 0.000 & 0.000 \\ 180.038 & 6301.341 & 18003.832 & -6481.379 & 8641.839 & 0.000 & 0.000 \\ 0.000 & -3240.690 & -6481.379 & 6931.379 & 0.000 & -3240.690 & 6481.379 \\ 0.000 & 6481.379 & 8641.839 & 0.000 & 34567.357 & -6481.379 & 8641.839 \\ 0.000 & 0.000 & 0.000 & -3240.690 & -6481.379 & 7201.379 & 0.000 \\ 0.000 & 0.000 & 0.000 & 6481.379 & 8641.839 & 0.000 & 34567.357 \end{pmatrix} \qquad f = \begin{array}{cc} \begin{pmatrix} 1.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} & \begin{array}{c} w_1 \\ w_2 \\ \varphi_2 \\ w_3 \\ \varphi_3 \\ w_4 \\ \varphi_4 \end{array} \end{array}$$

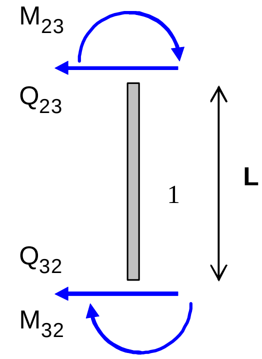
Yhtälöstä (1) saadaan ratkaisuna siirtymävektori :

$$\delta := K^{-1} \cdot f \qquad \delta^T = (0.025538 \quad 0.005027 \quad -0.001283 \quad 0.001236 \quad -0.000598 \quad 0.000018 \quad -0.000082)$$

Pilarin voimasuureet ratkaistaan pilarin jäykkyysmatriisin avulla:

$$\begin{pmatrix} Q_{12} \\ M_{12} \\ Q_{21} \\ M_{21} \end{pmatrix} := K_1 \cdot \begin{pmatrix} \delta_0 \\ 0 \\ \delta_1 \\ \delta_2 \end{pmatrix} \qquad \begin{pmatrix} Q_{12} \\ M_{12} \\ Q_{21} \\ M_{21} \end{pmatrix} = \begin{pmatrix} 1.000 \\ 3.231 \\ -1.000 \\ 2.769 \end{pmatrix}$$

Suurpaalun saadaan voimasuureet vastaavasti elementeittäin:



$$\begin{pmatrix} Q_{23} \\ M_{23} \\ Q_{32} \\ M_{32} \end{pmatrix} := K_2 \cdot \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} \qquad \begin{pmatrix} Q_{23} \\ M_{23} \\ Q_{32} \\ M_{32} \end{pmatrix} = \begin{pmatrix} 0.1 \\ -2.8 \\ -0.1 \\ 3.1 \end{pmatrix}$$

Elementti 1

$$\begin{pmatrix} Q_{34} \\ M_{34} \\ Q_{43} \\ M_{43} \end{pmatrix} := K_2 \cdot \begin{pmatrix} \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{pmatrix} \qquad \begin{pmatrix} Q_{34} \\ M_{34} \\ Q_{43} \\ M_{43} \end{pmatrix} = \begin{pmatrix} -0.5 \\ -3.1 \\ 0.5 \\ 1.3 \end{pmatrix}$$

Elementti 2

$$\begin{pmatrix} Q_{45} \\ M_{45} \\ Q_{54} \\ M_{54} \end{pmatrix} := K_2 \cdot \begin{pmatrix} \delta_5 \\ \delta_6 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} Q_{45} \\ M_{45} \\ Q_{54} \\ M_{54} \end{pmatrix} = \begin{pmatrix} -0.5 \\ -1.3 \\ 0.5 \\ -0.6 \end{pmatrix}$$

Elementti 3

Kuvaajat:

Paalun ja pilarin sivusiirtymä w : (poimitaan ratkaisuvektorista tarvittavat siirtymät)

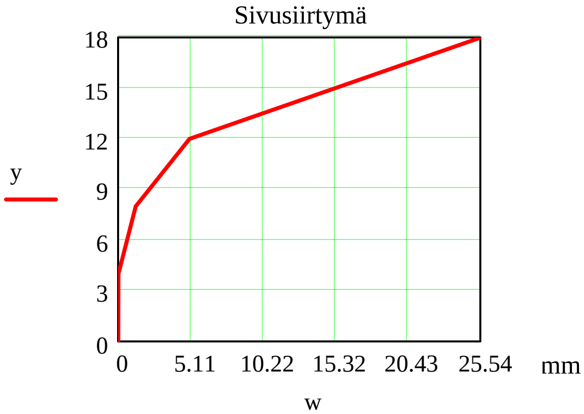
$$y := \begin{pmatrix} 4.5 \cdot L \\ 3 \cdot L \\ 2 \cdot L \\ L \\ 0 \end{pmatrix} \qquad w := \begin{pmatrix} \delta_0 \\ \delta_1 \\ \delta_3 \\ \delta_5 \\ 0 \end{pmatrix} \cdot 1000 \qquad y = \begin{pmatrix} 18.0 \\ 12.0 \\ 8.0 \\ 4.0 \\ 0.0 \end{pmatrix} \qquad w = \begin{pmatrix} 25.5 \\ 5.0 \\ 1.2 \\ 0.0 \\ 0.0 \end{pmatrix} \; \mathrm{mm}$$

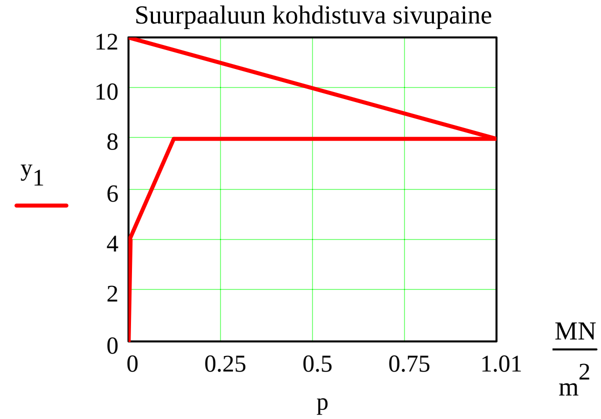
(y -koordinaatti paalun juuresta ylöspäin)

Suurpaalun kohdistuva sivupaine p : (sivupaine $p_i = c_i w_i$)

$$p = c_i \cdot w_i \qquad c := 100 \ \frac{\mathrm{MN}}{\mathrm{m}^3}$$

$$y_1 := \begin{pmatrix} 3 \cdot L \\ 2 \cdot L \\ 2 \cdot L \\ L \\ L \\ 0 \end{pmatrix} \qquad p := \begin{pmatrix} 0 \\ 2 \cdot c \cdot \delta_1 \\ c \cdot \delta_3 \\ c \cdot \delta_5 \\ 3 \cdot c \cdot \delta_5 \\ 0 \end{pmatrix} \qquad y_1 = \begin{pmatrix} 12.0 \\ 8.0 \\ 8.0 \\ 4.0 \\ 4.0 \\ 0.0 \end{pmatrix} \qquad p = \begin{pmatrix} 0.00000 \\ 1.00538 \\ 0.12364 \\ 0.00183 \\ 0.00550 \\ 0.00000 \end{pmatrix} \frac{\mathrm{MN}}{\mathrm{m}^2}$$



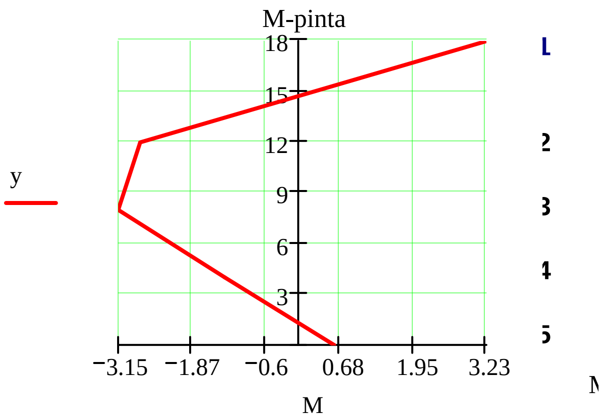


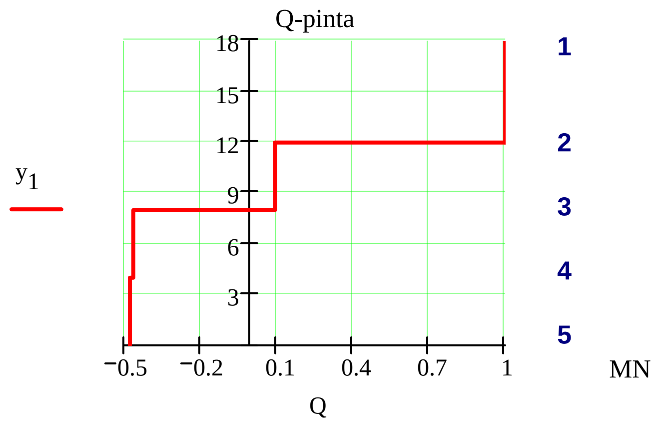
Paalun ja pilarin momenttipinta :

$$y := \begin{pmatrix} 4.5 \cdot L \\ 3 \cdot L \\ 2 \cdot L \\ L \\ 0 \end{pmatrix} \quad M := \begin{pmatrix} M_{12} \\ M_{23} \\ M_{34} \\ M_{45} \\ -M_{54} \end{pmatrix} \quad y = \begin{pmatrix} 18.0 \\ 12.0 \\ 8.0 \\ 4.0 \\ 0.0 \end{pmatrix} \quad M = \begin{pmatrix} 3.23 \\ -2.77 \\ -3.15 \\ -1.30 \\ 0.59 \end{pmatrix} \quad \mathrm{MN \cdot m}$$

Paalun ja pilarin leikkausvoimajakautuma Q :

$$y_1 := \begin{pmatrix} 4.5 \cdot L \\ 3 \cdot L \\ 3 \cdot L \\ 2 \cdot L \\ 2 \cdot L \\ L \\ L \\ 0 \end{pmatrix} \quad Q := \begin{pmatrix} Q_{12} \\ -Q_{21} \\ Q_{23} \\ -Q_{32} \\ Q_{34} \\ -Q_{43} \\ Q_{45} \\ -Q_{54} \end{pmatrix} \quad y_1 = \begin{pmatrix} 18.0 \\ 12.0 \\ 12.0 \\ 8.0 \\ 8.0 \\ 4.0 \\ 4.0 \\ 0.0 \end{pmatrix} \quad Q = \begin{pmatrix} 1.000 \\ 1.000 \\ 0.095 \\ 0.095 \\ -0.461 \\ -0.461 \\ -0.474 \\ -0.474 \end{pmatrix} \quad \mathrm{MN}$$



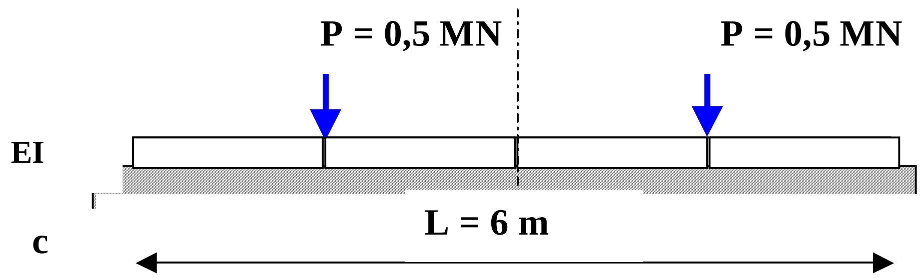


Tarkastetaan vaakasuorien voimien tsp: $k_2 \cdot \delta_1 + k_3 \cdot \delta_3 + k_4 \cdot \delta_5 - Q_{54} = 1.0$ **= H** **(OK)**

(Huom! Tässä esimerkissä paalun jako vain 3:een elementtiin on liian karkea ja tulokset siksi likimääräisiä).

Esim 7.1 Kimmoisella alustalla oleva palkki differenssi-menetelmällä

Laske taipuma, pohjapainejakautuma ja taivutusmomenttipinta oheisen kimmoisalla alustalla (alustaluku c= 10 MN/m3) olevan palkin alla differenssimenetelmällä jakamalla palkki neljään tasaväliseen osaan otaksumalla pohjapainen muuttuvan lineaarisesti diff.pisteiden välillä.
Palkin poikkileikkaus; h x b = 0,3 x 0,4 m. ja kimmokerroin E = 14 000 MN/m2



a) Taipuisa palkki differenssimenetelmällä:

Lähtöarvot:

$P := 0.5$ MN — Ulkoinen kuorma

$c := 10 \ \frac{MN}{m^3}$ — Alustaluku

$b := 0.4$ m $\quad h := 0.3$ m — palkin poikkileikkausmitat

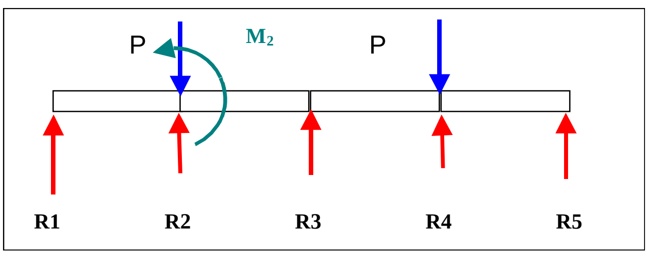
$L := 6.0$ m $\quad \Delta x := 1.5$ m — Palkin pituus, diff.pisteiden väli

Palkin jäykkyys:

$E := 14000 \ \frac{MN}{m^2} \qquad I := \frac{b \cdot h^3}{12} \qquad I = 9 \cdot 10^{-4} \ m^4 \qquad EI := E \cdot I \qquad EI = 12.6 \ \frac{MN}{m^2}$

Reaktiovoimat eli "tukireaktiot" taipumien avulla:

(ol. lineaarinen pohjapainejakautuma ks luennt)



$$R_1 = \frac{\Delta x \cdot b \cdot c}{8} \cdot (3 \cdot w_1 + w_2)$$

$$R_2 = \frac{\Delta x \cdot b \cdot c}{8} \cdot (w_1 + 6 \cdot w_2 + w_3) \qquad (1)$$

$$R_3 = \frac{\Delta x \cdot b \cdot c}{8} \cdot (2 \cdot w_2 + 6 \cdot w3)$$

Symmetria:=> $w_2 = w_4$ ja $w_1 = w_5$

Ulkoiset tasapainoyhtälöt (ks kuva)

$$M_2 = R_1 \cdot \Delta x$$

$$M_3 = R_1 \cdot 2 \cdot \Delta x + R_2 \cdot \Delta x - P \cdot \Delta x \qquad (2)$$

$$2 \cdot R_1 + 2 \cdot R_2 + R_3 = 2 \cdot P$$

(= palkin "globaali" voimatasapainoyhtälö)

Toisaalta palkin taivutusmomentti sisäpisteessä i voidaan lausua differenssiyhtälön avulla (l.(97)):

$$M_i = -\frac{EI}{\Delta x}\cdot\left(w_{i-1} - 2\cdot w_i + w_{i+1}\right)$$

Taivutusmomentit pisteissä 2 ja 3::

$$M_2 = -\frac{EI}{\Delta x^2}\cdot\left(w_1 - 2\cdot w_2 + w_3\right) \qquad (3)$$

$$M_3 = -\frac{EI}{\Delta x^2}\cdot\left(2\cdot w_2 - 2\cdot w_3\right)$$

Kaavoista (2) ja (3) saadaan ratkaisuyhtälöt taipumille, kun sijoitetaan R_i:t yhtälöistä (1)

$$\frac{\Delta x\cdot b\cdot c}{8}\cdot\left(3\cdot w_1 + w_2\right)\cdot\Delta x = -\frac{EI}{\Delta x^2}\cdot\left(w_1 - 2\cdot w_2 + w_3\right)$$

$$\frac{\Delta x\cdot b\cdot c}{8}\cdot\left(3\cdot w_1 + w_2\right)\cdot 2\cdot\Delta x + \frac{\Delta x\cdot b\cdot c}{8}\cdot\left(w_1 + 6\cdot w_2 + w_3\right)\cdot\Delta x - P\cdot\Delta x = -\frac{EI}{\Delta x^2}\cdot\left(2\cdot w_2 - 2\cdot w_3\right)$$

$$2\cdot\left[\frac{\Delta x\cdot b\cdot c}{8}\cdot\left(3\cdot w_1 + w_2\right)\right] + 2\cdot\left[\frac{\Delta x\cdot b\cdot c}{8}\cdot\left(w_1 + 6\cdot w_2 + w_3\right)\right] + \frac{\Delta x\cdot b\cdot c}{8}\cdot\left(2\cdot w_2 + 6\cdot w3\right) = 2\cdot P$$

Taipumien ratkaisuyhtälöryhmä on muotoa:

$$K\cdot w = f$$

Käytetään vakiotermeille merkintöjä

$$c_1 := \frac{\Delta x^2\cdot b\cdot c}{8} \qquad c_2 := \frac{EI}{\Delta x^2} \qquad c_1 = 1.125 \qquad c_2 = 5.6$$

Saadaan kerroinmatriisiksi K:

$$K := \begin{pmatrix} 3\cdot c_1 + c_2 & c_1 - 2\cdot c_2 & c_2 \\ 7\cdot c_1 & 8\cdot c_1 + 2\cdot c_2 & c_1 - 2\cdot c_2 \\ 8\cdot c_1 & 16\cdot c_1 & 8\cdot c_1 \end{pmatrix} \qquad K = \begin{pmatrix} 8.975 & -10.075 & 5.6 \\ 7.875 & 20.2 & -10.075 \\ 9 & 18 & 9 \end{pmatrix}$$

(= "kerroinmatriisi")

Oikean puolen kuormavektori:

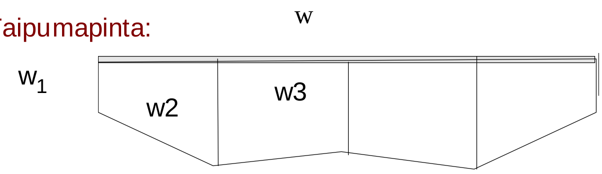
$$f := \begin{pmatrix} 0 \\ P\cdot\Delta x \\ 2\cdot P\cdot\Delta x \end{pmatrix} \qquad f = \begin{pmatrix} 0 \\ 0.75 \\ 1.5 \end{pmatrix}$$

(= "kuormavektori")

Taipumat:

$$w := K^{-1}\cdot f \qquad w = \begin{pmatrix} 0.0271 \\ 0.0482 \\ 0.0433 \end{pmatrix} mm$$

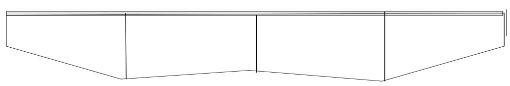
Taipumapinta:



Pohjapaine:

$$\sigma := c\cdot w \qquad \sigma = \begin{pmatrix} 0.271 \\ 0.482 \\ 0.433 \end{pmatrix} \frac{MN}{m^2}$$

Pohjapaine:



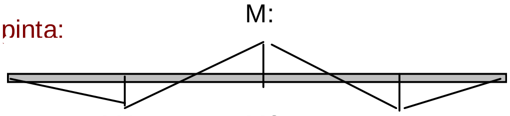
Taivutusmomentit:

$M_1 := 0$

$M_2 := -\frac{EI}{\Delta x^2}\cdot\left(w_0 - 2\cdot w_1 + w_2\right)$ $\quad M_2 = 0.146$ MNm

$M_3 := -\frac{EI}{\Delta x^2}\cdot\left(2\cdot w_1 - 2\cdot w_2\right)$ $\quad M_3 = -0.055$ MNm

Momenttipinta:



b) Vastaava täysin jäykkä palkki (EI = oo): (vertailun vuoksi)

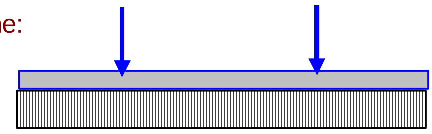
Pohjapaine:

$\sigma := \frac{2\cdot P}{L\cdot b}$ $\quad \sigma = 0.417 \ \frac{MN}{m^2}$ (= tasainen)

Taipuma:

$w := \frac{\sigma}{c}$ $\quad w = 0.0417$ m (= vakio)

Pohjapaine:



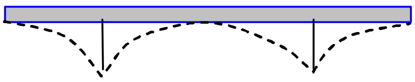
Taivutusmomentit:

$M_1 := 0$

$M_2 := \sigma\cdot b\cdot\frac{\Delta x^2}{2}$ $\quad M_2 = 0.188$ MNm

$M_3 := \sigma\cdot b\cdot\frac{(2\cdot\Delta x)^2}{2} - P\cdot\Delta x$ $\quad M_3 = 0$ MNm

Taivutusmomenttikuvio:



Rak-11.2107 Sillat ja Perustukset

ESIMERKKITEHTÄVÄT

Aalto-yliopisto
Rakennustekniikan laitos

Sillat ja perustukset

Esimerkkitehtävät

Tekijät	**Jutila, A., Syrjä, R.**
Julkaisija	**Aalto-yliopisto**
	Insinööritieteiden korkeakoulu
	Rakennustekniikan laitos
Luokka	**66**
Paikka	**Espoo**
Vuosi	**2011**

ALKUSANAT

Tämä opetusmoniste sisältää Sillat ja perustukset -nimisen kurssin (Rak-11.2107) esimerkkitehtävät. Kurssi sisältyy Aalto-yliopiston rakenne- ja rakennustuotantotekniikan tutkinto-ohjelmaan.

Tehtävät on ryhmitelty aihepiireittäin. Kukin tehtävä sisältää tehtävänannon ja malliratkaisun.

Monistetta on työstetty 1990-luvun lopulta lähtien. Esimerkkitehtäviin sisältyy sekä Eurokoodien että B-sarjan ohjeiden mukaisia laskentatapoja.

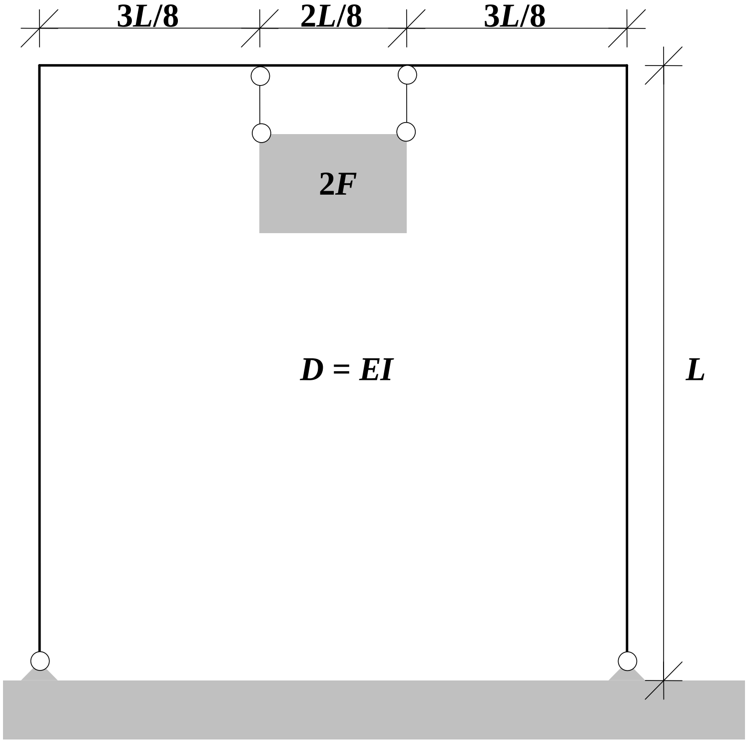
Tätä monistetta on saatavana myös englanninkielisenä (*Bridges and Foundation Structures - Problems*).

Otaniemessä 20.12.2011
Tekijät

SISÄLLYS

1. TEHTÄVÄ

Ratkaise kuvan 1 mukaisen nosturikehän perustuksiin syntyvä vaakavoima yksikkövoima-, kulmanmuutos- ja momenttimenetelmällä!

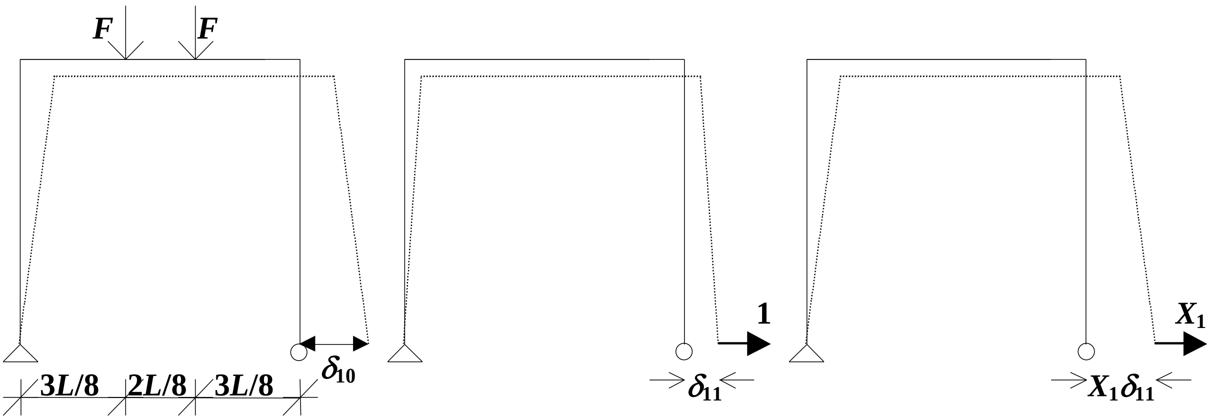


Kuva 1.

YKSIKKÖVOIMAMENETELMÄ

Staattisesti määrättyyn perusmuotoon ulkoisesta kuormasta ($2F$) aiheutuvan siirtymän (δ_{10}) ja tuen tuntemattoman vaakavoiman (X_1) aiheuttaman siirtymän ($X_1\delta_{11}$) summa on nolla. Tässä δ_{11} on yksikkövoiman aiheuttama siirtymä. Ks. kuva 2.

$$\delta_{10}+X_1\delta_{11} = 0 \tag{1}$$



Kuva 2.

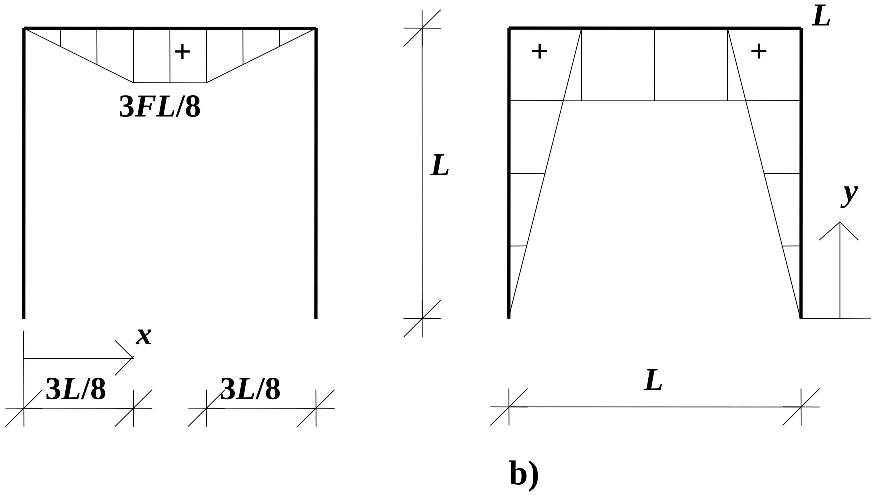
Momentti vaakapalkilla ulkoisesta kuormasta (kuva 3a)

$$M_0 = \begin{cases} Fx, & x = \left[0, \ \frac{3L}{8}\right] \\ \frac{3FL}{8}, & x = \left[\frac{3L}{8}, \ \frac{5L}{8}\right] \\ F(L-x), & x = \left[\frac{5L}{8}, \ L\right] \end{cases} \tag{2a...c}$$

Pilareille ei synny momenttia ulkoisesta kuormasta.

Momentti yksikkövoimasta (kuva 3b)

$$M_1 = \begin{cases} L, & \text{palkki} \\ y, & \text{pilari} \end{cases} \tag{3a, b}$$



Kuva 3.

Siirtymät

$$\begin{cases} \delta_{10} = \int \frac{M_1 M_0}{D} ds \\ \delta_{11} = \int \frac{M_1 M_1}{D} ds \end{cases} \qquad (4a, b)$$

Sijoittamalla näihin momentit saadaan

$$\begin{cases} \delta_{10} = \frac{15FL^3}{64D} \\ \delta_{11} = \frac{5L^3}{3D} \end{cases} \qquad (5a, b)$$

<u>VASTAUS</u>:

Vaakavoima

$$X_1 = \frac{-\delta_{10}}{\delta_{11}} \qquad (6)$$

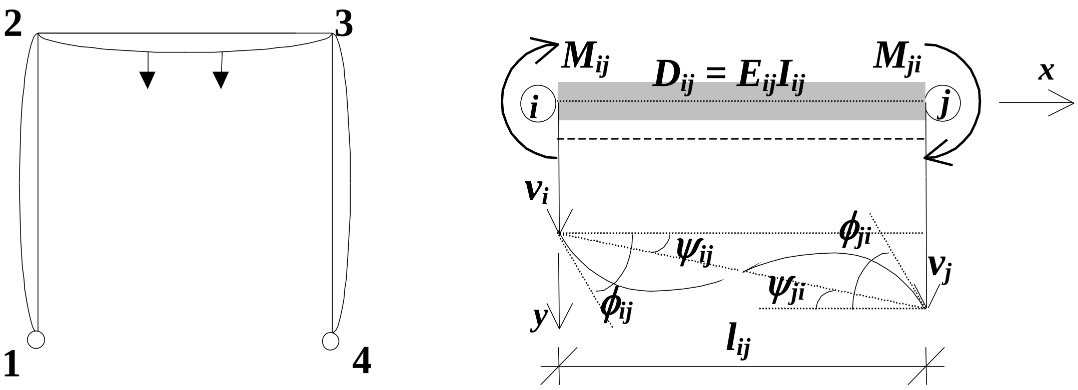
$$= \frac{-9F}{64} \qquad (7)$$

Negatiivinen merkki tarkoittaa, että suunta on päinvastainen kuin alussa oletettiin.

SIIRTYMÄMENETELMÄ (erityisesti kulmanmuutosmenetelmä)

Nurkkamomentit (kuva 4)

$$\begin{cases} M_{12} = 0 \\ M_{21} = a^{o}_{21}\phi_{21} - c^{o}_{21}\psi_{21} + M^{o}_{K21} \\ M_{23} = a_{23}\phi_{23} + b_{23}\phi_{32} - c_{23}\psi_{23} + M_{K23} \\ M_{32} = a_{32}\phi_{32} + b_{32}\phi_{23} - c_{32}\psi_{32} + M_{K32} \\ M_{34} = a^{o}_{34}\phi_{34} - c^{o}_{34}\psi_{34} + M^{o}_{K34} \\ M_{43} = 0 \end{cases} \qquad \text{(8a…f)}$$



a) b)

Kuva 4.

Yhteensopivuusehdot

$$\phi_{21} = \phi_{23} = \phi_2 \qquad (9)$$

$$\phi_{32} = \phi_{34} = \phi_3 = -\phi_2 \qquad (10)$$

Tasapainoehdot

$$\begin{cases} 0 = M_{21} + M_{23} \Rightarrow M_{23} = -M_{21} = M_2 = M \\ 0 = M_{32} + M_{34} \Rightarrow M_{34} = -M_{32} = M_3 = M_2 = M \end{cases} \qquad \text{(11a, b)}$$

Siirtymästä aiheutuva kiertymä perusmuodossa (*Kaavakokoelma*)

$$\psi = \frac{v_2 - v_1}{l} \tag{12}$$

Sauvavakiot perusmuodossa

$$a = \frac{4EI}{l} = \frac{4D}{l} \tag{13}$$

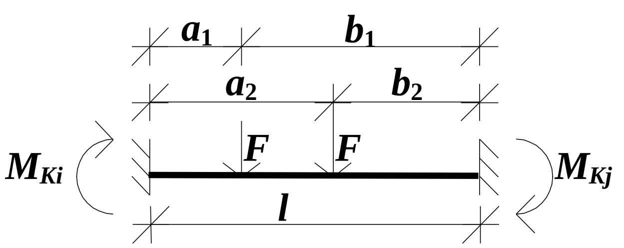
$$b = \frac{2EI}{l} = \frac{2D}{l} \tag{14}$$

$$c = \frac{6EI}{l} = \frac{6D}{l} \tag{15}$$

$$a^o = c^o = \frac{3EI}{l} = \frac{3D}{l} \tag{16}$$

Kuormituksesta aiheutuvat sauvanpäämomentit perusmuodossa

$$M_{Ki} = -M_{Kj} = \frac{-Fa_1b_1^2}{l^2} + \frac{-Fa_2b_2^2}{l^2} \tag{17}$$



Kuva 5.

Siirtymästä aiheutuvat kiertymät

$$\psi_{21} = \psi_{34} = \psi \tag{18}$$

$$\psi_{23} = \psi_{32} = 0 \tag{19}$$

Sauvavakiot

$$a_{21} = a_{23} = a_{32} = a_{34} = \frac{4EI}{L} = \frac{4D}{L} = a \tag{20}$$

$$b_{23} = b_{32} = \frac{2EI}{L} = \frac{2D}{L} = b \tag{21}$$

$$c_{23} = c_{32} = \frac{6EI}{L} = \frac{6D}{L} = c \tag{22}$$

$$a_{21}^{o} = a_{34}^{o} = c_{21}^{o} = c_{34}^{o} = \frac{3EI}{L} = \frac{3D}{L} = a^{o} \tag{23}$$

Kuormituksesta aiheutuvat sauvanpäämomentit

$$M_{K23} = -M_{K32} = \frac{-15FL}{64} \tag{24}$$

Nurkkamomentit sievennyksen jälkeen

$$\begin{cases} -M_2 = a^o\,\phi - a^o\,\psi \\ M_2 = a\,\phi - b\,\phi + M_K \\ -M_3 = -a\,\phi + b\,\phi - M_K \\ M_3 = -a^o\,\phi - a^o\,\psi \end{cases} \tag{25a…d}$$

Yhtälöistä 11

$$\begin{cases} -M_K = +(a^o + a - b)\phi - a^o\psi \\ +M_K = -(a^o + a - b)\phi - a^o\psi \end{cases} \tag{26a, b}$$

Siirtymästä aiheutuva kiertymä

$$\psi = 0 \tag{27}$$

Kaavoista 26a ja 27 saadaan

$$\phi = \frac{-M_K}{a^o + a - b} \tag{28}$$

$$\Rightarrow \phi = \frac{3FL^2}{64D} \tag{29}$$

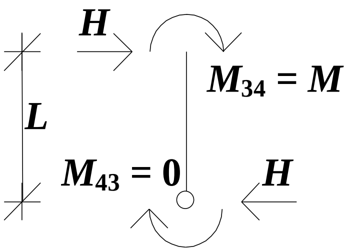
Yhtälöstä 25a

$$M = -a^o \phi \tag{30}$$

$$\Rightarrow M = \frac{-9FL}{64} \tag{31}$$

Tuen vaakavoima (kuva 6)

$$HL = -M_{34} \tag{32}$$



Kuva 6.

<u>VASTAUS</u>: Vaakavoima

$$H = \frac{9F}{64} \tag{33}$$

VOIMAMENETELMÄ (erityisesti momenttimenetelmä)

Sauvanpääkiertymät (kuva 4)

$$\begin{cases} \phi_{21} = \alpha_{21} M_{21} - \beta_{21} M_{12} + \psi_{21} + \alpha_{21}^{o} \\ \phi_{23} = \alpha_{23} M_{23} - \beta_{23} M_{32} + \psi_{23} + \alpha_{23}^{o} \\ \phi_{32} = \alpha_{32} M_{32} - \beta_{32} M_{23} + \psi_{32} + \alpha_{32}^{o} \\ \phi_{34} = \alpha_{34} M_{34} - \beta_{34} M_{43} + \psi_{34} + \alpha_{34}^{o} \end{cases} \tag{34}$$

Yhteensopivuusehdot

$$\phi_{21} = \phi_{23} \tag{35}$$

$$\phi_{32} = \phi_{34} \tag{36}$$

Nurkkamomentit

$$\begin{cases} 0 = M_{21} + M_{23} \Rightarrow M_{23} = -M_{21} = M_2 = M \\ 0 = M_{32} + M_{34} \Rightarrow M_{34} = -M_{32} = M_3 = M_2 = M \\ 0 = M_{12} \\ 0 = M_{43} \end{cases} \tag{37a…d}$$

Siirtymästä aiheutuva kiertymä perusmuodossa

$$\psi = \frac{v_2 - v_1}{l} \tag{38}$$

Sauvavakiot perusmuodossa

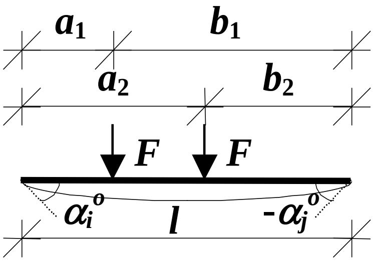
$$\alpha = \frac{l}{3EI} = \frac{l}{3D} \tag{39}$$

$$\beta = \frac{l}{6EI} = \frac{l}{6D} \tag{40}$$

Kuormituksesta aiheutuvat sauvanpääkiertymät perusmuodossa (kuva 7)

$$\alpha_i^o = \frac{Fa_1b_1}{6Dl}(b_1+l)+\frac{Fa_2b_2}{6Dl}(b_2+l) \tag{41}$$

$$\alpha_j^o = \frac{-Fa_1b_1}{6Dl}(a_1+l)+\frac{-Fa_2b_2}{6Dl}(a_2+l) \tag{42}$$



Kuva 7.

Siirtymästä aiheutuvat kiertymät

$$\psi_{21}=\psi_{34}=\psi \tag{43}$$

$$\psi_{23}=\psi_{32}=0 \tag{44}$$

Sauvavakiot

$$\alpha_{21}=\alpha_{23}=\alpha_{32}=\alpha_{34}=\frac{L}{3D}=\alpha \tag{45}$$

$$\beta_{21}=\beta_{23}=\beta_{32}=\beta_{34}=\frac{L}{6D}=\beta \tag{46}$$

Kuormituksesta aiheutuvat sauvanpääkiertymät

$$\alpha_{21}^o=\alpha_{34}^o=0 \tag{47}$$

$$\alpha_{23}^o=-\alpha_{32}^o=\frac{15FL^2}{128D}=\alpha^o \tag{48}$$

Sauvanpääkiertymät sievennyksen jälkeen

$$\begin{cases} \phi_{21} = -\alpha M + \psi \\ \phi_{23} = (\alpha + \beta) M + \alpha^o \\ \phi_{32} = -(\alpha + \beta) M - \alpha^o \\ \phi_{34} = \alpha M + \psi \end{cases} \qquad \textbf{(49a…d)}$$

Yhtälöistä 35 ja 36

$$+\alpha^o = -(2\alpha + \beta) M + \psi \qquad \textbf{(50)}$$

$$-\alpha^o = +(2\alpha + \beta) M + \psi \qquad \textbf{(51)}$$

Siirtymästä aiheutuva kiertymä

$$\psi = 0 \qquad \textbf{(52)}$$

Yhtälöistä 50 ja 52

$$M = \frac{-\alpha^o}{2\alpha + \beta} \qquad \textbf{(53)}$$

Sijoittamalla sauvavakiot saadaan

$$M = \frac{-9FL}{64} \qquad \textbf{(54)}$$

Tuen vaakavoima (kuva 6)

$$HL = -M_{34} \qquad \textbf{(55)}$$

<u>VASTAUS</u>: Vaakavoima

$$H = \frac{9F}{64} \qquad \textbf{(56)}$$

2. TEHTÄVÄ

Määritä kuvan 1 tasakattoisen, teräsrunkoisen asuinrakennuksen reunapilarin peruslaattaa (kuvassa harmaalla) rasittavien voimasuureiden kaikki mahdolliset kuormitusyhdistelmät murtorajatilassa (STR/GEO) Eurokoodin ja sen Suomen kansallisen liitteen mukaan siten, että kaikkien kuormitustapausten kertoimet käyvät ilmi [*NA SFS-EN 1990*, taulukko A1.2(B)]; voimasuureiden arvoja ei tarvitse laskea. Mikä on suurimman mahdollisen kaatavan momentin arvo peruslaatan päällä, kun tuuli on rakennuksen lyhemmän sivun suuntainen? Tuulikuorman oletetaan jakaantuvan tasan peräkkäisille pilareille. Rakennus sijaitsee Kemijärvellä.

Pilareiden määrä

$n_1 =$ 3 kpl

$n_2 =$ 5 kpl

Pilareiden välin puolikas

$a =$ 5 m

Kerrosmäärä ja -korkeus

$n_k =$ 4 kpl

$h_i =$ 3 m

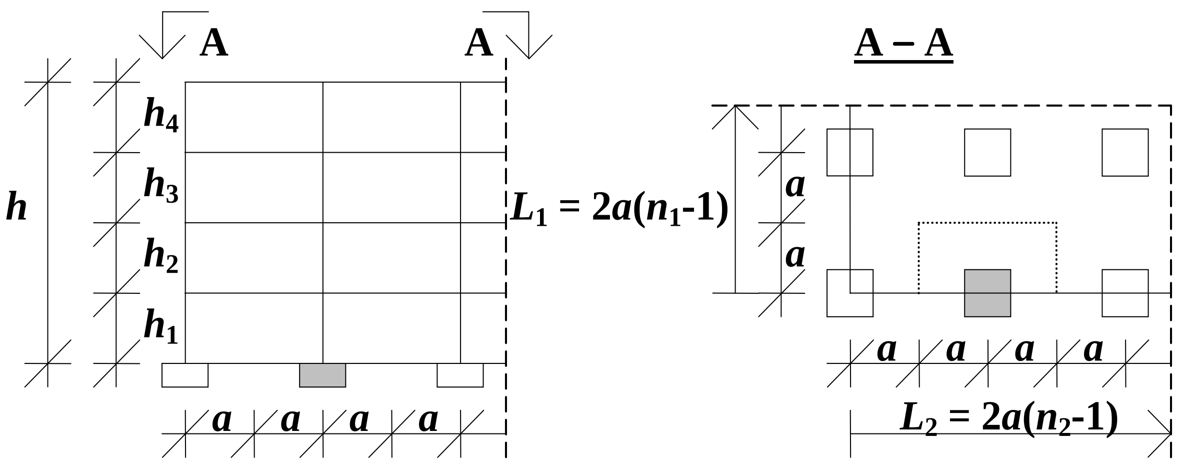
Pystykuormat kertyvät pisteviivalla merkityltä alueelta.

Rakennuksen katon ja välipohjan omapaino

$p =$ 0,005 MN/m^2

Kantavien seinien ja pilarin omapaino ulkoseinän kohdalla

$v =$ 0,020 MN/m



Kuva 1.

KUORMIEN YHDISTELY

Mitoituskaava (*NA* SFS-EN 1990*, taulukko A1.2(B))

$$F_d = \left.\begin{matrix}\xi\gamma_{Gj,\mathrm{sup}} \\ \gamma_{Gj,\mathrm{sup}}\end{matrix}\right\} K_{FI} G_{kj,\mathrm{sup}} + \gamma_{Gj,\mathrm{inf}} G_{kj,\mathrm{inf}} + \left.\begin{matrix}\gamma_{Q1} \\ 0\end{matrix}\right\} K_{FI} Q_{k1} + \left.\begin{matrix}\gamma_{Qi} \\ 0\end{matrix}\right\} K_{FI} \sum \psi_{0i} Q_{ki} \qquad (1)$$

jossa $G_{kj,\,\mathrm{sup}}$ on pysyvän kuorman ominaisarvon yläraja ja vastaava osavarmuusluku on

$$\gamma_{Gj,\,\mathrm{sup}} = 1{,}35$$

$G_{kj,\,\mathrm{inf}}$ on pysyvän kuorman ominaisarvon alaraja ja vastaava osavarmuusluku on

$$\gamma_{Gj,\,\mathrm{inf}} = 0{,}9$$

$Q_{k\,1}$ on määräävän muuttuvan kuorman ominaisarvo ja vastaava osavarmuusluku on

$$\gamma_{Q\,1} = 1{,}5$$

Q_{ki} on muuttuvan kuorman ominaisarvo ja vastaava osavarmuusluku on

$$\gamma_{Qi} = 1{,}5$$

pienennyskerroin on

$$\xi = 0{,}85$$

Nelikerroksinen talo on luotettavuusluokassa RC2 (*NA SFS-EN 1990*, taulukko B1), jolloin kuormakerroin on (*SFS-EN 1990*, taulukko B3)

$$K_{FI} = 1{,}0$$

Yhdistelyarvo hyöty-, lumi- ja tuulikuormalle (*NA SFS-EN 1990*, taulukko A1.1, luokka A)

$$\psi_{0,imposed} = 0{,}7$$
$$\psi_{0,snow} = 0{,}7$$
$$\psi_{0,wind} = 0{,}6$$

Tulot

$$\xi\gamma_{Gj,\,\mathrm{sup}} \approx 1{,}15$$
$$\gamma_{Qi} K_{FI}\, \psi_{0,imposed} = 1{,}05$$
$$\gamma_{Qi} K_{FI}\, \psi_{0,snow} = 1{,}05$$
$$\gamma_{Qi} K_{FI}\, \psi_{0,wind} = 0{,}90$$

***) NA = National Annex, kansallinen liite.**

LISÄVAAKAVOIMA

Vinouden aiheuttaman epäkeskisyyden perusarvo (*SFS-EN 1993-1-1*, kohta 5.3.2)

$$\phi_0 = 0{,}005$$

Rakenteen korkeudesta johtuva pienennyskerroin

$$\alpha_h = \min\begin{cases} \max\begin{cases} \dfrac{2}{\sqrt{h_0 h}} \\ \dfrac{2}{3} \end{cases} \\ 1 \end{cases} \qquad (2)$$

missä

$$h_0 = 1 \text{ m}^{-1}$$

Saadaan

$$\alpha_h = 0{,}667$$

Pilarien lukumäärä rakennuksen lyhemmässä suunnassa

$$m = n_1 \qquad (3)$$

$$= 3$$

Rakenteen leveydestä johtuva pienennyskerroin

$$\alpha_m = \sqrt{\frac{1}{2}\left(1+\frac{1}{m}\right)} \qquad (4)$$

$$= 0{,}816$$

Vinouden aiheuttama kokonaisepäkeskisyys (*SFS-EN 1993-1-1*, kaava 5.5)

$$\phi = \phi_0 \alpha_h \alpha_m \qquad (5)$$

$$= 0{,}002722$$

Lisävaakavoima vaikuttaa kerroksen lattian tasolla ja on

$$H = \phi N \qquad (6)$$

jossa N on vastaava pystyvoima (*SFS-EN 1993-1-1*, kuva 5.4).

RAKENTEEN OMAPAINO

Rakennuksen katto tai välipohja

$$G_{pi} = 2a^2 p \qquad (7)$$

$$= \quad 0{,}250 \ \text{MN}$$

Seinä ja pilari

$$G_{vi} = 2a\,v \qquad (8)$$

$$= \quad 0{,}200 \ \text{MN}$$

Pystyvoiman ominaisarvo yhden kerroksen osalta

$$G_i = G_{pi} + G_{vi} \qquad (9)$$

$$= \quad 0{,}450 \ \text{MN}$$

Rakennuksen kokonaiskorkeus

$$h = n_k h_i \qquad (10)$$

$$= \quad 12{,}000 \ \text{m}$$

Momentti katon aiheuttamasta lisävaakavoimasta

$$M_{G1} = \phi G_{pi} h$$

$$= \quad 0{,}008 \ \text{MNm} \qquad (11)$$

Momentti kolmen ylimmän kerroksen aiheuttamasta lisävaakavoimasta

$$M_{G2} = 3\phi G_i h/2 \qquad (12)$$

$$= \quad 0{,}022 \ \text{MNm}$$

Momentti yhteensä

$$M_G = M_{G1} + M_{G2} \qquad (13)$$

$$= \quad 0{,}030 \ \text{MNm}$$

HYÖTYKUORMA

Rakennus on käyttöluokassa A (*NA SFS-EN 1990*, taulukko A1.1).

Pintakuorma (*NA SFS-EN 1991-1-1*, taulukko 6.2)

$$q_{ki} = \quad 0{,}002 \ \text{MN/m}^2$$

Mitoittava pystyvoima kerroksessa

$$Q_{ii} = 2a^2 q_{ki} \qquad (14)$$

$$= \quad 0{,}100 \ \text{MN}$$

Toisen kerroksen lattian yläpuolella olevien kerrosten määrä

$$n = n_k - 1 \qquad (15)$$

$$= 3$$

Pienennystekijä (*NA SFS-EN 1991-1-1*, kohta 6.3.1.2(11))

$$\alpha_n = \frac{2+(n-2)\psi_{0,imposed}}{n} \qquad (16)$$

$$= 0{,}900$$

Momentti kerroksien hyötykuormien aiheuttamasta lisävaakavoimasta

$$M_i = \phi(\alpha_n h_i + 2h_i + 3h_i)Q_{ii} \qquad (17)$$

$$= 0{,}005 \text{ MNm}$$

LUMIKUORMA

Ominaislumikuorma Kemijärvellä (*NA SFS-EN 1991-1-3*, kuva 4.1)

$$s_k = 0{,}002750 \text{ MN/m}^2$$

Tasakaton muotokerroin (*SFS-EN 1991-1-3*, taulukko 5.2)

$$\mu_i = 0{,}8$$

Tuulensuojaisuuskerroin C_e (normaali maasto) ja lämpökerroin C_t (*SFS-EN 1991-1-3*, kohta 5.2(7) ja 5.2(8))

$$C_e = 1{,}0$$

$$C_t = 1{,}0$$

Lumikuorma pinta-alayksikköä kohti (*SFS-EN 1991-1-3*, kaava 5.1)

$$s = \mu_i C_e C_t s_k \qquad (18)$$

$$= 0{,}002200 \text{ MN/m}^2$$

Mitoittava pystyvoima

$$Q_{ks} = 2a^2 s \qquad (19)$$

$$= 0{,}110 \text{ MN}$$

Momentti lumen aiheuttamasta lisävaakavoimasta

$$M_s = \phi Q_{ks} h \qquad (20)$$

$$= 0{,}004 \text{ MNm}$$

TUULIKUORMA

Lyhemmän sivun pituus

$$L_1 = 2a(n_1 - 1) \quad (21)$$

$$= 20 \text{ m}$$

Pidemmän sivun pituus

$$L_2 = 2a(n_2 - 1) \quad (22)$$

$$= 40 \text{ m}$$

Vaikutuspinta

$$A_{ref} = hL_2 \quad (23)$$

$$= 480 \text{ m}^2$$

Tuulennopeuden modifioimaton perusarvo (*NA SFS-EN 1994-1-4*, kohta 4.2)

$$v_{b,0} = 21 \text{ m/s}$$

Suuntakerroin (*SFS-EN 1991-1-4*, kohta 4.2)

$$c_{dir} = 1{,}0$$

Vuodenaikakerroin (*SFS-EN 1991-1-4*, kohta 4.2)

$$c_{season} = 1{,}0$$

Tuulennopeuden perusarvo (*SFS-EN 1991-1-4*, kaava 4.1)

$$v_b = c_{dir}\, c_{season}\, v_{b,0} \quad (24)$$

$$= 21 \text{ m/s}$$

Ilman tiheys (*SFS-EN 1991-1-4*, kohta 4.5)

$$\rho = 1{,}250 \text{ kg/m}^3$$

Nopeuspaineen perusarvo (*SFS-EN 1991-1-4*, kaava 4.10)

$$q_b = \frac{1}{2}\rho v_b^2 \quad (25)$$

$$= 276 \text{ N/m}^2$$

Maastoluokaksi valitaan II (*SFS-EN 1991-1-4*, liite A).

Altistuskerroin rakennuksen yläosassa (*SFS-EN 1991-1-4*, kuva 4.2)

$$C_{e,wind} = 2{,}5$$

Puuskanopeuspaine (*SFS-EN 1991-1-4*, kaava 4.8)

$$q_p = C_{e,wind}\, q_b \quad (26)$$

$$= 689 \text{ N/m}^2$$

Koska rakenteen korkeus on pienempi kuin leveys, tarkastellaan nopeuspaine yhtenä kaistana (*SFS-EN 1991-1-4*, kohta 7.2.2, kuva 7.4).

Suhde (*SFS-EN 1991-1-4*, kuva 7.5)

$$\kappa = h/d = L_1/h \tag{27}$$

$$= 0{,}600$$

Painekerroin vyöhykkeille D ja E, kun pinta-ala on suurempi kuin 10 m^2 saadaan interpoloimalla (*SFS-EN 1991-1-4*, taulukko 7.1).

$$C_{pe,10,D} = \frac{0{,}8-0{,}7}{1-0{,}25}(\kappa-0{,}25)+0{,}7 \tag{28}$$

$$= 0{,}747$$

$$C_{pe,10,E} = -\left[\frac{0{,}5-0{,}3}{1-0{,}25}(\kappa-0{,}25)+0{,}3\right] \tag{29}$$

$$= -0{,}393$$

Ulkopintoihin vaikuttava tuulenpaine (*SFS-EN 1991-1-4*, kaava 5.1)

$$w_{e,i} = q_p C_{pe,\,10,\,i} \tag{30}$$

$$w_{e,D} = 515 \text{ N/m}^2$$

$$w_{e,E} = -271 \text{ N/m}^2$$

Rakennekerroin (*SFS-EN 1991-1-4*, kohta 6.2)

$$c_s c_d = 1$$

Tuulikuorma (*SFS-EN 1991-1-4*, kaava 5.5)

$$F_{w,e} = c_s c_d (w_{e,D} - w_{e,E}) A_{ref} \tag{31}$$

$$= 0{,}377 \text{ MN}$$

Pilaria kohden

$$F_w = \frac{F_{w,e}}{n_1 n_2} \tag{32}$$

$$= 0{,}025 \text{ MN}$$

Momentti

$$M_w = F_w h/2 \tag{33}$$

$$= 0{,}151 \text{ MNm}$$

<u>VASTAUS</u>:

Kuormitustapaukset

$$
F_d = \left.\begin{array}{l}1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}35\end{array}\right\} G_{k,\mathrm{sup}} + 0{,}9G_{k,\mathrm{inf}} + \left.\begin{array}{l}1{,}5\\1{,}5\\1{,}5\\1{,}5\\1{,}05\\1{,}05\\0\\0\\1{,}05\\1{,}05\\0\\0\\0\end{array}\right\} Q_{ki} + \left.\begin{array}{l}1{,}05\\1{,}05\\0\\0\\1{,}5\\1{,}5\\1{,}5\\1{,}5\\1{,}05\\0\\1{,}05\\0\\0\end{array}\right\} Q_{ks} + \left.\begin{array}{l}0{,}9\\0\\0{,}9\\0\\0{,}9\\0\\0{,}9\\0\\1{,}5\\1{,}5\\1{,}5\\1{,}5\\0\end{array}\right\} Q_{kw} \qquad \begin{array}{r}(34a)\\(34b)\\(34c)\\(34d)\\(34e)\\(34f)\\(34g)\\(34h)\\(34i)\\(34j)\\(34k)\\(34l)\\(34m)\end{array}
$$

Tuuli aiheuttaa momentin suhteen määräävän kuormituksen, joten vaarallisin tapaus on i.

Suurin mahdollinen momentti

$$
\begin{aligned} M &= 1{,}15M_G + 1{,}05M_i + 1{,}05M_s + 1{,}5M_w \\ &= 0{,}270\ \mathrm{MNm} \end{aligned} \tag{35}
$$

3. TEHTÄVÄ

Määritä kuvan 1 nelikerroksisen rakennuksen reunapilarin peruslaatalle (kuvassa harmaalla) tulevien pystykuormien ja lyhyen sivun suuntaisten vaakavoimien suurimmat ja pienimmät laskenta-arvot! Kyseessä on rakenteellinen mitoitus. Kuormitusyhdistelmät muodostetaan murtorajatilassa julkaisun *Rakenteiden kuormitusohjeet, RIL 144-2002* mukaan. Tuulikuorman oletetaan jakaantuvan tasan peräkkäisille pilareille. Rakennuksessa on tasakatto ja se sijaitsee Kemijärvellä.

Pilareiden määrä

n_1 = 3 kpl

n_2 = 5 kpl

Pilareiden välin puolikas

a = 5 m

Kerrosmäärä ja -korkeus

n_k = 4 kpl

h_i = 3 m

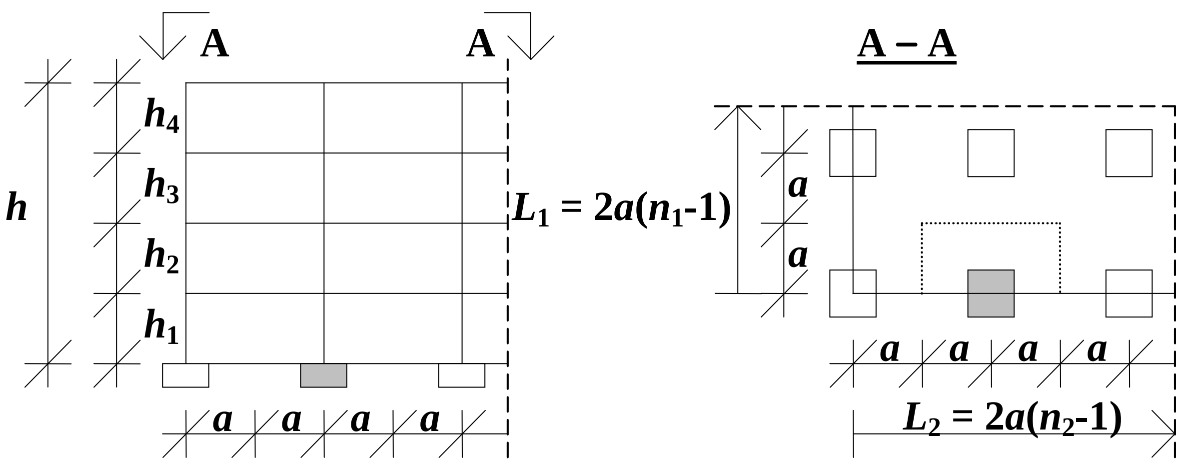
Pystykuormat kertyvät pisteviivalla merkityltä alueelta.

Rakennuksen katon ja välipohjan omapaino

p = 0,005 MN/m^2

Kantavien seinien ja pilarin omapaino ulkoseinän kohdalla

v = 0,020 MN/m



Kuva 1.

RAKENTEEN OMAPAINO

Rakennuksen katto tai välipohja

$$G_{pi} = 2a^2 p \tag{1}$$

$$= \quad 0{,}250 \text{ MN}$$

Seinä ja pilari

$$G_{vi} = 2a\,v \tag{2}$$

$$= \quad 0{,}200 \text{ MN}$$

Pystyvoiman ominaisarvo yhden kerroksen osalta

$$G_i = G_{pi} + G_{vi} \tag{3}$$

$$= \quad 0{,}450 \text{ MN}$$

Summa: rakennuksen katto ja kerrokset

$$G = G_{pi} + \sum_{n=1}^{4} G_i \tag{4}$$

$$= \quad 2{,}050 \text{ MN}$$

LUMIKUORMA

Peruslumikuorma Kemijärvellä (*RIL 144-2002*, kuva 4.121, s. 24)

$$s_k = \quad 0{,}002 \text{ MN/m}^2$$

Tasakaton muotokerroin (*RIL 144-2002*, kuva 4.122d, s. 27)

$$\mu = \quad 1{,}0$$

Mitoittavan pystyvoiman ominaisarvo (*RIL 144-2002*, kohta 4.1, s. 23)

$$Q_k = 2a^2 \mu s_k \tag{5}$$

$$= \quad 0{,}100 \text{ MN}$$

HYÖTYKUORMA

Pintakuorma (*RIL 144-2002*, taulukko 5.12, s. 78)

$$q_{ik} = 0{,}001500 \text{ MN/m}^2$$

Mitoittava pystyvoima kerroksessa

$$Q_{ii} = 2a^2 q_{ik} \tag{6}$$

$$= 0{,}075 \text{ MN}$$

Oleskelukuorman pienennyskerroin (*RIL 144-2002*, taulukko 8.21b, s. 150)

$$k = 0{,}75$$

Mitoittavan pystyvoiman ominaisarvo

$$Q_i = k\sum_{i=1}^{4} Q_{ii} \tag{7}$$

$$= 0{,}225 \text{ MN}$$

LISÄVAAKAVOIMA

Rakennuksen lyhemmän sivun pituus

$$L_1 = 2a(n_1\text{-}1) \tag{8}$$

$$= 20 \text{ m}$$

Rakennuksen pidemmän sivun pituus

$$L_2 = 2a(n_2\text{-}1) \tag{9}$$

$$= 40 \text{ m}$$

Lisävaakavoima vaikuttaa kerroksen lattian tasolla ja on

$$H = \phi N \tag{10}$$

jossa ϕ on rakennuksen lyhemmässä suunnassa (*RIL 144-2002*, kohta 6.5, s. 134)

$$\phi = 1/150$$

$$= 0{,}006667$$

ja N on vastaava pystyvoima.

TUULIKUORMA

Rakennuksen kokonaiskorkeus

$$h = \sum_{n=1}^{4} h_i \tag{11}$$

$$= 12 \text{ m}$$

Oletetaan maastoluokaksi II.

Tuulenpaine (*RIL 144-2002*, taulukko 4.22a, s. 31)

$$q_{wk} = q_0 \left(\frac{h}{h_0} \right)^{0,24} \tag{12}$$

jossa

$$q_0 = 0,000650 \text{ MN/m}^2$$

$$h_0 = 10 \text{ m}$$

Täten

$$q_{wk} = 0,000679 \text{ MN/m}^2$$

Tuulenpuoleisen seinän painekerroin (*RIL 144-2002*, taulukko 4.231a, s. 38)

$$C_{p,t} = 0,7$$

Seinien pituusmittojen suhde, kun tuuli on lyhyen sivun suuntainen

$$L_1/L_2 = 0,500$$

Suojanpuoleisen seinän painekerroin

$$C_{p,s} = -0,5$$

Pilarin kokonaistuulikuorma pitkälle sivulle (*RIL 144-2002*, kaava 4.212b, s. 30)

$$Q_w = \frac{hL_2(C_{p,t} - C_{p,s})q_{wk}}{n_1 n_2} \tag{13}$$

$$= 0,026 \text{ MN}$$

KUORMIEN YHDISTELY

Kuormien yhdistely murtorajatilassa (*RIL 144-2002*, kohta 8.2, s. 149)

$$q_d = \sum_{i=1}^{m} \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} + \sum_{i=3}^{n} \gamma_{qi} q_{ki} \tag{14}$$

voidaan kirjoittaa muotoon

$$q_d = \left.\begin{matrix} 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{matrix}\right\} g + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right\} q_i + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 0{,}8 \\ 0 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0{,}8 \\ 0 \\ 0 \end{matrix}\right\} q_k + \left.\begin{matrix} 0{,}8 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0{,}8 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \end{matrix}\right\} q_w \qquad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \\ (9) \\ (10) \end{matrix} \tag{15a…j}$$

Alaindeksi *d* viittaa laskenta-arvoon (design value).
Kertoimet vaakariveittäin vastaavat samaa tapausta. Jatkossa alaindeksien numerot viittaavat näihin rivinumeroihin. (*RIL 144-2002*, taulukko 8.21a, s. 150)
(Kuormitustapauksia on kaksinkertainen määrä, jos $\gamma_{q\,1} = 0$, kun $\gamma_g = 1{,}2$, ja $\gamma_{q\,1} = 1{,}6$, kun $\gamma_g = 0{,}9$.)

Peruslaatalle tuleva pystyvoiman suurin ja pienin laskenta-arvo

Lumi täysillä (tuuli puolella tai ei tuulta)

$$V_{d,1,2} = 1{,}2G + 1{,}6Q_i + 1{,}6Q_k \tag{16}$$
$$= 2{,}980 \text{ MN}$$

Lumi puolella (tuuli täysillä)

$$V_{d,3} = 1{,}2G + 1{,}6Q_i + 0{,}8Q_k \tag{17}$$
$$= 2{,}900 \text{ MN}$$

Omapaino minimissä (lumi ja tuuli nolla)

$$V_{d,10} = 0{,}9G \tag{18}$$
$$= 1{,}845 \text{ MN}$$

Peruslaatalle tulevan lyhyen sivun suuntaisen vaakavoiman suurin ja pienin laskenta-arvo ja sen vaikutuskorkeus peruslaatasta

Lumi täysillä (tuuli puolella)

$$\pm H_1 = 1{,}2\phi(G_{p5} + 4G_i) + 1{,}6\phi \cdot 4Q_{ii} + 1{,}6\phi Q_k + 0{,}8Q_w \tag{19}$$

$= \quad 0{,}042$ MN

$$e_1 = \frac{1{,}2\phi\left(G_{p5}h + 4G_i\frac{3}{8}h\right) + 1{,}6\phi \cdot 4Q_{ii}\frac{3}{8}h + 1{,}6\phi Q_k h + 0{,}8Q_w\frac{h}{2}}{H_1} \tag{20}$$

$= \quad 5{,}807$ m

Lumi puolella (tuuli täysillä)

$$\pm H_3 = 1{,}2\phi(G_{p5} + 4G_i) + 1{,}6\phi \cdot 4Q_{ii} + 0{,}8\phi Q_k + 1{,}6Q_w \tag{21}$$

$= \quad 0{,}062$ MN

$$e_3 = \frac{1{,}2\phi\left(G_{p5}h + 4G_i\frac{3}{8}h\right) + 1{,}6\phi \cdot 4Q_{ii}\frac{3}{8}h + 0{,}8\phi Q_k h + 1{,}6Q_w\frac{h}{2}}{H_3} \tag{22}$$

$= \quad 5{,}819$ m

Ei vaakakuormia

$H_{10} = \quad 0$ MN

<u>VASTAUS</u>: Suurimmat ja pienimmät laskenta-arvot ja näiden sijainti

Pystyvoima

$V_{d,max} = \quad 2{,}980$ MN

$V_{d,min} = \quad 1{,}845$ MN

Vaakavoima kohtisuorassa pitkää sivua vastaan

$H_{d,max} = \quad 0{,}062$ MN

$e_{d,max} = \quad 5{,}819$ m

$H_{d,min} = \quad 0{,}000$ MN

4. TEHTÄVÄ

Määritä tarkastuskäytävällä varustettuun massiiviseen betonipatoon (kuva 1) vaikuttavien vedenpaineiden ja nosteen resultantit ja vaikutuskohdat *x* -*y* -koordinaatistossa, kun alaveden korkeus on

$h_{a,1}$ = **3,000 m**

Tutki toisena tapauksena kaatumis- ja liukuvarmuus, kun alaveden korkeus on

$h_{a,2}$ = **0 m**

Laskelmat suoritetaan julkaisun *Vesirakenteiden suunnittelu RIL 123-1979* mukaan.

Kitkakerroin padon ja kallion välissä (lievästi rakoillut ja lustoinen kallio, *RIL 123-1979*, s. 64)

μ = **0,700**

Yläveden korkeus

h_y = **9,000 m**

Padon mitat

h = **9,500 m**

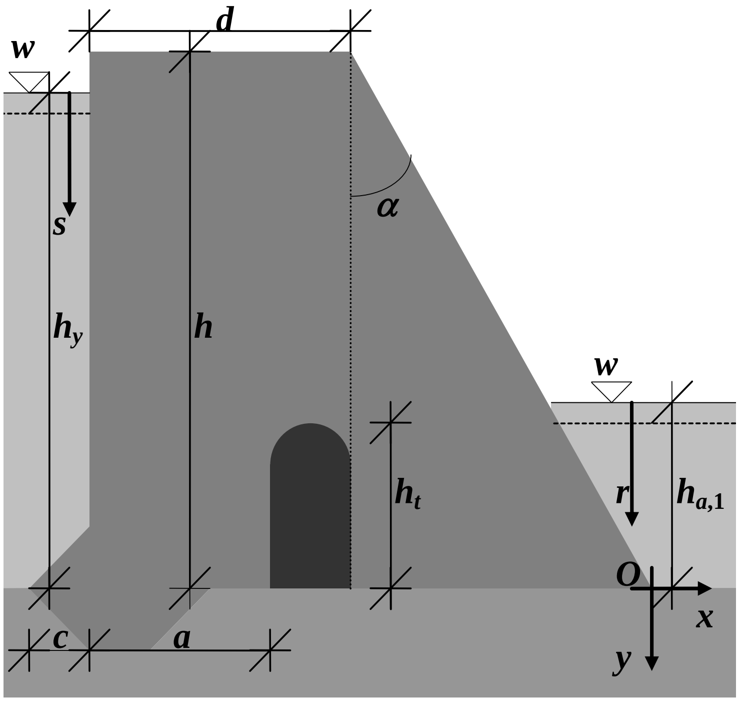
h_t = **2,000 m**

a = **1,000 m**

d = **2,500 m**

c = **0,500 m**

α = **30 °**



Kuva 1.

Teräsbetonin tilavuuspaino

γ_c = 25 000 N/m^3

Makean veden tilavuuspaino

γ_w = 9 810 N/m^3

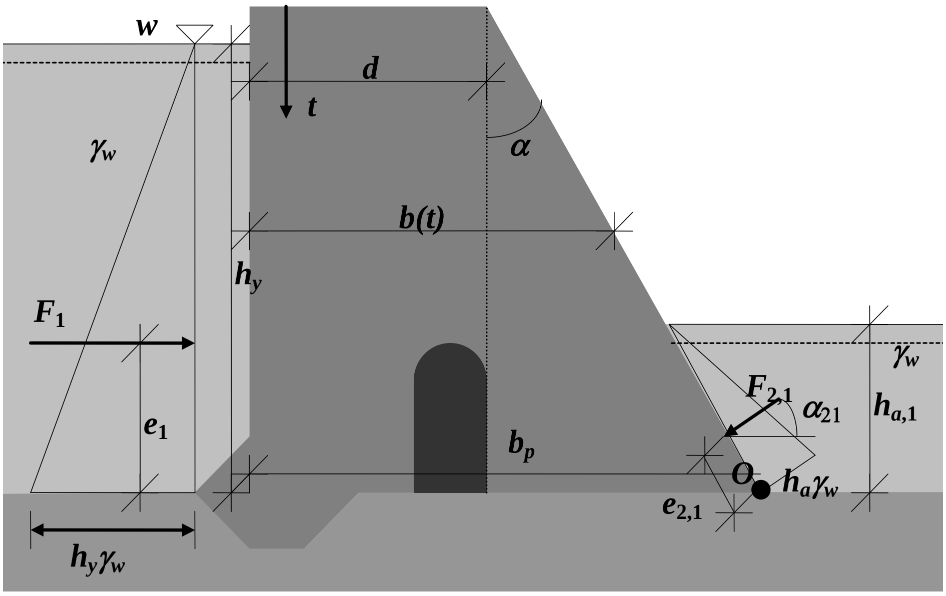
Padon leveys sen "syvyyden" t funktiona (kuva 2)

$$b(t) = d + \frac{t}{\sqrt{3}} \tag{1}$$

Perustamistasossa

$$b_p = b(t = h) \tag{2}$$

$= 7{,}985$ m



Kuva 2.

YLÄVEDEN VEDENPAINE

Resultantin arvo (kuva 2)

$$F_1 = \frac{1}{2}\gamma_w h_y^2 \tag{3}$$

$= 397\ 305$ N/m

ja sen etäisyys perustamistasosta

$$e_1 = h_y/3 \qquad (4)$$
$$= \quad 3{,}000 \text{ m}$$

ALAVEDEN VEDENPAINE

Resultantin horisontaalikomponentin arvo (kuva 2)

$$F_{2,1h} = \gamma_w h_{a,1}^{\ 2} \cos\alpha/2 \qquad (5)$$
$$= \quad 38\,231 \text{ N/m}$$

ja sen etäisyys pisteestä *O*

$$e_{2,1h} = h_{a,1}/3 \qquad (6)$$
$$= \quad 1{,}000 \text{ m}$$

Resultantin vertikaalikomponentin arvo

$$F_{2,1v} = \gamma_w h_{a,1}^{\ 2} \sin\alpha/2 \qquad (7)$$
$$= \quad 22\,073 \text{ N/m}$$

ja sen etäisyys pisteestä *O*

$$e_{2,1v} = e_{2,1h} \tan\alpha \qquad (8)$$
$$= \quad 0{,}577 \text{ m}$$

YLÄVEDEN PUOLEINEN NOSTE

Resultantin arvo (*RIL 123-1979.* Kohta 2. Kuva 7.) (kuva 3)

$$F_{3,i} = \left[0{,}4\left(h_y - h_{a,i}\right) + \frac{1}{2}\left(h_y - 0{,}4\left(h_y - h_{a,i}\right)\right)\right]\gamma_w\left(a + c\right) \qquad (9)$$
$$F_{3,1} = \quad 83\,876 \text{ N/m}$$
$$F_{3,2} = \quad 92\,705 \text{ N/m}$$

Vaikutuskohdan etäisyys padon ylävedenpuoleisesta pystysuorasta seinästä

$$e_{3,i} = \frac{0{,}4\left(h_y - h_{a,i}\right)\frac{a+c}{2} + \frac{1}{2}\left[h_y - 0{,}4\left(h_y - h_{a,i}\right)\right]\frac{a+c}{3}}{0{,}4\left(h_y - h_{a,i}\right) + \frac{1}{2}\left[h_y - 0{,}4\left(h_y - h_{a,i}\right)\right]} - c \tag{10}$$

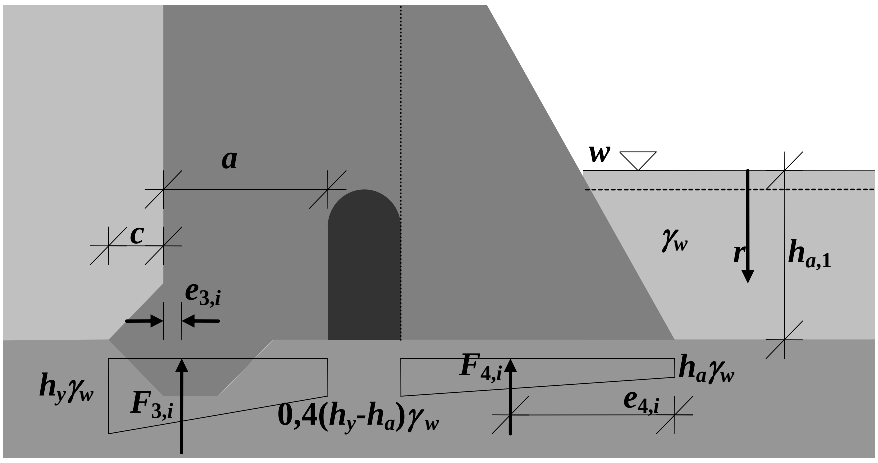
$e_{3,1}$ = 0,105 m

$e_{3,2}$ = 0,143 m

x -koordinaatin arvo

$$x_{3,i} = e_{3,I} - b_p \tag{11}$$

$x_{3,1}$ = -7,880 m



Kuva 3.

ALAVEDEN PUOLEINEN NOSTE

Resultantin arvo (*RIL 123-1979.* Kohta 2. Kuva 7.) (kuva 3)

$$F_{4,i} = \left\{h_{a,i} + \frac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]\right\}\gamma_w\left(b_p - d\right) \tag{12}$$

$F_{4,1}$ = 145 277 N/m

$F_{4,2}$ = 96 851 N/m

Vaikutuskohdan etäisyys padon alanurkasta (origosta)

$$e_{4,i} = \frac{h_{a,i}\dfrac{b_p - d}{2} + \dfrac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]\dfrac{2}{3}\left(b_p - d\right)}{h_{a,i} + \dfrac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]} \quad (13)$$

$e_{4,1}$ = 2,641 m

$e_{4,2}$ = 3,657 m

RAKENTEEN OMAPAINO

Resultantti (kuva 4)

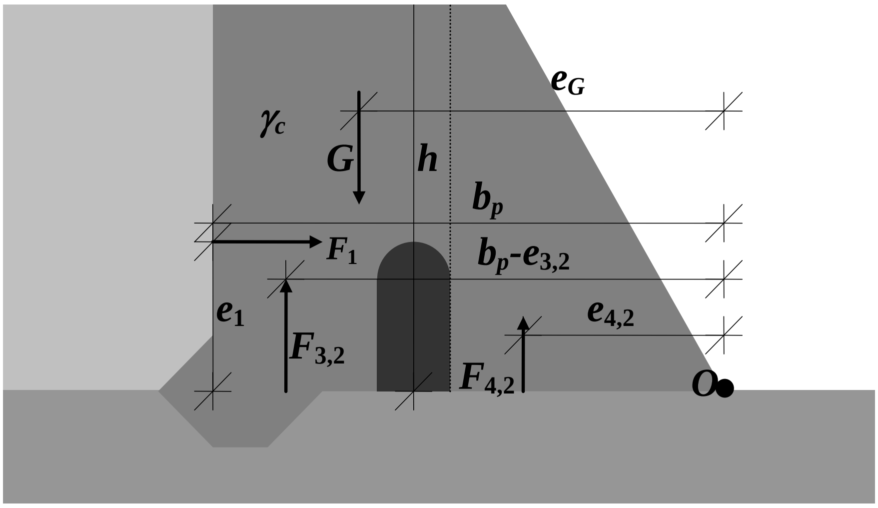
$$G = [hd + h(b_p - d)/2]\gamma_c \quad (14)$$

= 1 245 073 N/m

Vaikutuskohdan etäisyys origosta

$$e_G = \frac{\gamma_c}{G}\left[hd\left(b_p - \frac{d}{2}\right) + \frac{1}{2}h\left(b_p - d\right)\frac{2}{3}\left(b_p - d\right)\right] \quad (15)$$

= 5,125 m



Kuva 4.

<u>VASTAUS</u>: **Vedenpaineen resultantit ja vaikutuskohdat *x*-*y*- koordinaatistossa on esitetty taulukossa 1.**

Taulukko 1.

		F_x [N/m]	y [m]	F_y [N/m]	x [m]
Vedenpaine	**Ylävesi**	**397 305**	**-3,000**		
	Alavesi	**-38 231**	**1,000**	**22 073**	**-0,577**
Noste	**Yläveden puoli**			**-83 876**	**-7,880**
	Alaveden puoli			**-145 277**	**-2,641**

Kaatumisvarmuusehto pisteen *O* suhteen (*RIL 123-1979*, kohta 2.42, s. 63)

$$n = \frac{\sum M_p}{\sum M_k} = \frac{Ge_G}{F_1 e_1 + F_{3,2}(b_p - e_{3,2}) + F_{4,2}e_{4,2}} \qquad (16)$$

$$= \quad 2{,}807 \quad > \quad 1{,}5 \quad \text{OK}$$

Liukuvarmuusehto (kynnystä ei ole otettu huomioon)

$$n = \frac{\mu\left(\sum V - N\right)}{\sum H} \qquad (17)$$

$$\Rightarrow n = \frac{\mu\left[G - (F_{3,2} + F_{4,2})\right]}{F_1} \qquad (18)$$

$$= \quad 1{,}860 \quad < \quad 2 \quad \text{Liukuu!}$$

5. TEHTÄVÄ

Kaksi-aukkoisen sillan päätytuet on perustettu kalliolle ja keskituki hiekalle (kuva 1). Sillan keskituki painuu matkan

δ = 0,010 m.

Määritä sillan taivutusmomentti- ja leikkausvoimakuvio omasta painosta ennen ja jälkeen painuman!

Sillan pituus

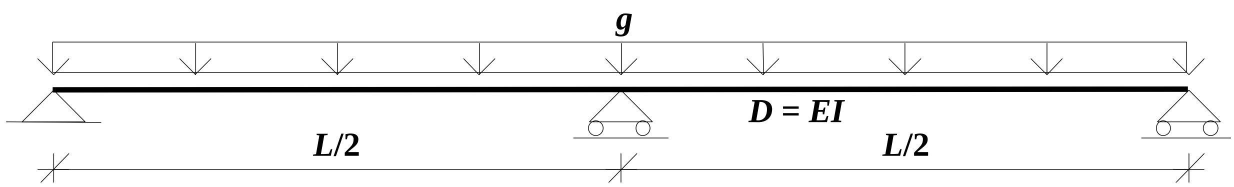
L = 16 m

Sillan taivutusjäykkyys

D = 120 MNm^2

Sillan omapaino

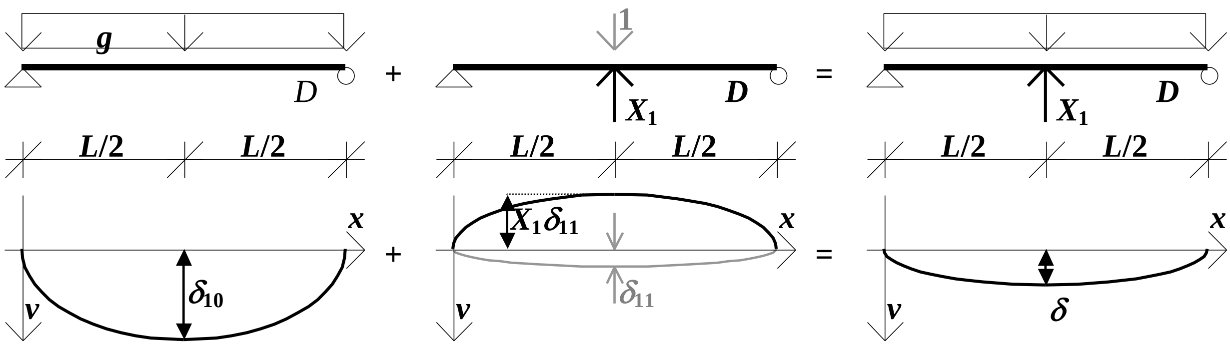
g = 0,010 MN/m



Kuva 1.

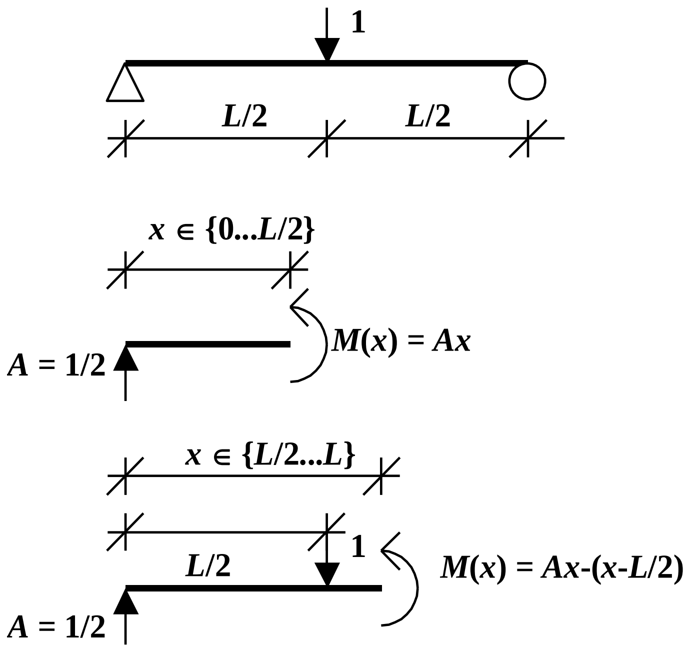
Staattisesti määrättyyn perusmuotoon ulkoisesta kuormasta (g) aiheutuvan taipuman (δ_{10}) ja keskituen tuntemattoman tukireaktion (X_1) aiheuttaman taipuman ($-X_1\delta_{11}$) summa on yhtä suuri kuin keskituen painuma (δ). Tässä δ_{11} on keskituen kohdalla alaspäin vaikuttavan yksikkövoiman aiheuttama taipuma (kuva 2).

$$\delta_{10}-X_1\delta_{11} = \delta \qquad (1)$$



Kuva 2.

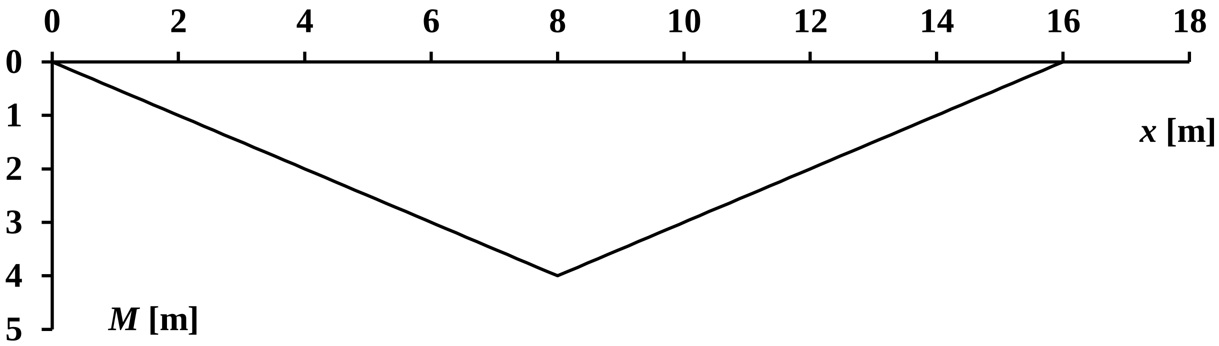
Taivutusmomentti yksikkövoimasta jänteen keskellä on ratkaistu kuvassa 3.



Kuva 3.

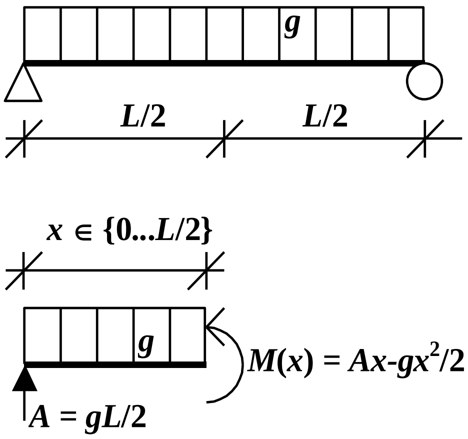
Taivutusmomentti on (kuva 4)

$$M_1 = \begin{cases} \dfrac{x}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{L-x}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \qquad \text{(2a, b)}$$



Kuva 4.

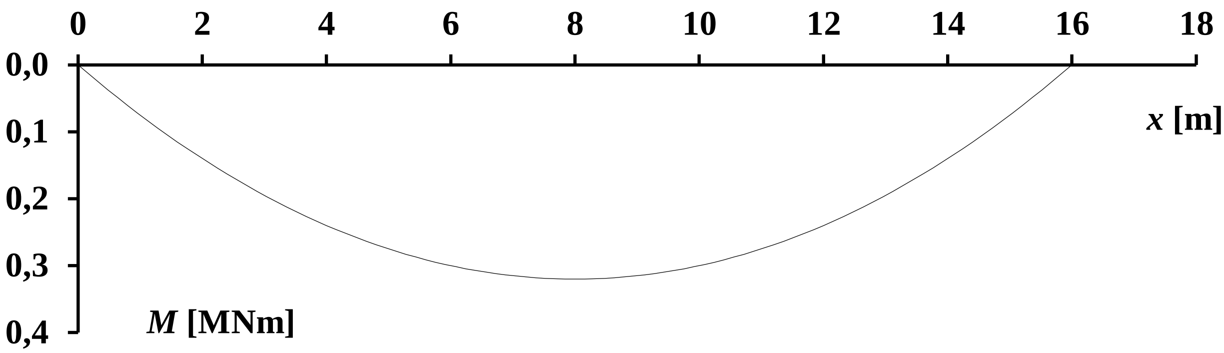
Taivutusmomentti omasta painosta (kuva 6) on ratkaistu kuvassa 5.



Kuva 5.

Taivutusmomentti on (kuva 6)

$$M_0 = \frac{g}{2}\left(Lx - x^2\right), \quad x = [0, L] \qquad \text{(3)}$$



Kuva 6.

Siirtymät

$$\begin{cases} \delta_{10} = \int_0^L \frac{M_1 M_0}{D} dx \\ \delta_{11} = \int_0^L \frac{M_1 M_1}{D} dx \end{cases} \tag{4a, b}$$

Sijoittamalla momenttien lausekkeet saadaan

$$\begin{cases} \delta_{10} = \frac{5gL^4}{384D} \\ \delta_{11} = \frac{L^3}{48D} \end{cases} \tag{5a, b}$$

$$\Rightarrow \begin{cases} \delta_{10} = & 0{,}071 \text{ m} \\ \delta_{11} = & 0{,}711 \text{ m/MN} \end{cases}$$

Tukireaktio

$$X_{1,i} = \frac{\delta_{10} - \delta_i}{\delta_{11}} \tag{6}$$

Ennen painumaa (alkutila, $i = a$)

$$\delta_a = 0{,}000 \text{ m}$$

$$X_{1,a} = 0{,}100 \text{ MN}$$

Painuman jälkeen (lopputila, $i = l$)

$$X_{1,l} = 0{,}086 \text{ MN}$$

Taivutusmomentti

$$M_i(x) = M_0 - X_{1,i}M_1 \tag{7}$$

$$= \begin{cases} \dfrac{g}{2}\left(Lx - x^2\right) - X_{1,i}\,\dfrac{x}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{g}{2}\left(Lx - x^2\right) - X_{1,i}\,\dfrac{L-x}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \tag{8a, b}$$

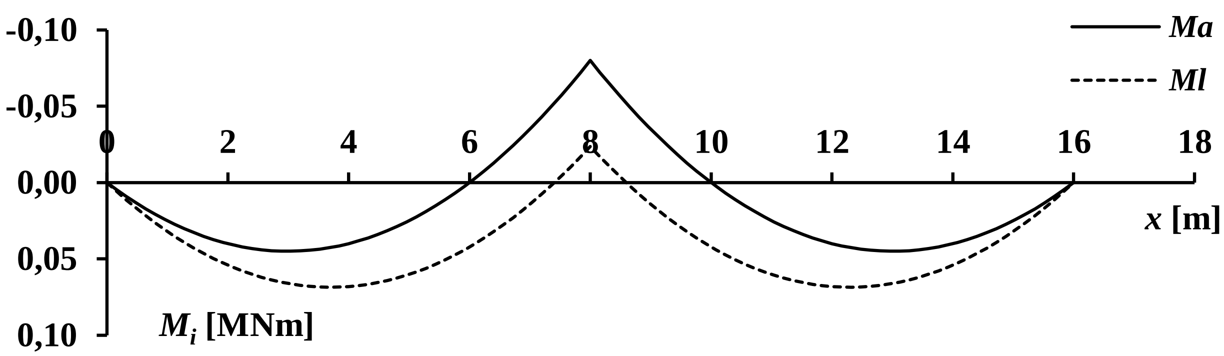
Leikkausvoima

$$Q_i(x) = \frac{dM}{dx} \tag{9}$$

$$= \begin{cases} \dfrac{g}{2}\left(L - 2x\right) - \dfrac{X_{1,i}}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{g}{2}\left(L - 2x\right) + \dfrac{X_{1,i}}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \tag{10a, b}$$

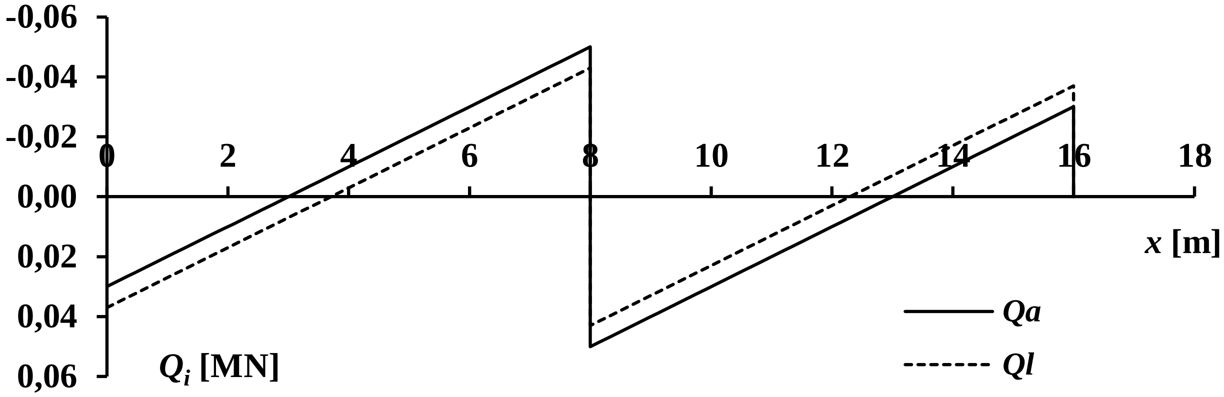
<u>VASTAUS</u>:

Taivutusmomenttikuvio alku- (a) ja lopputilassa (l) on esitetty kuvassa 7.



Kuva 7.

Leikkausvoimakuvio alku- (*a*) ja lopputilassa (*l*) on esitetty kuvassa 8.



Kuva 8.

6. TEHTÄVÄ

Määritä yksiaukkoisen sillan (yksinkertaisen palkin) taipumakuvio junasta, jonka aiheuttama kuormitus on kuvan 1 kaavion mukainen! Vertaa *EN 1991-2*, kohta 6.3.2, kuormamalli 71, kun kerroin

α = 1,46

Etäisyys

a_0 = 3 m

Sillan pituus

L = 16 m

Sillan taivutusjäykkyys

D = 12 000 MNm^2

Pistevoima veturin akselista

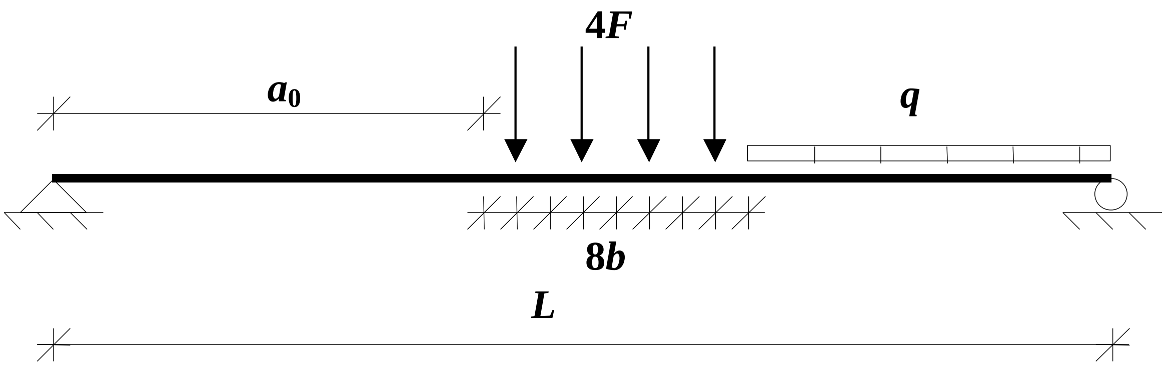
F = 0,370 MN

Kuormatut vaunut (mielivaltainen pituus)

q = 0,120 MN/m

Mitta

b = 0,800 m



Kuva 1.

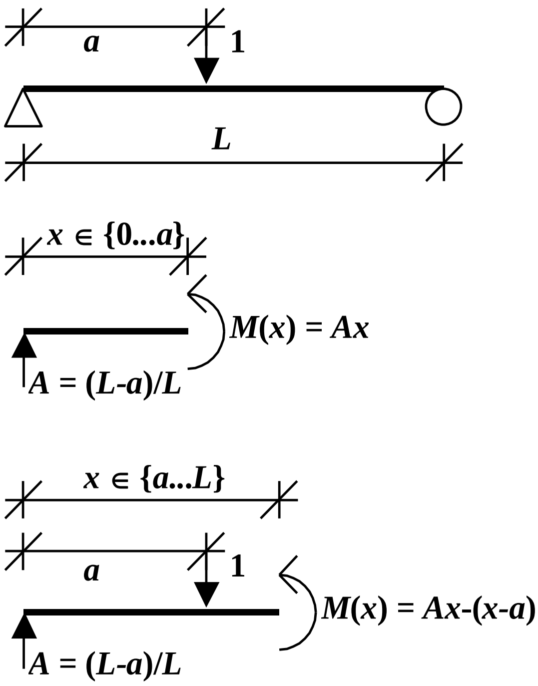
Taipumaviivan differentiaaliyhtälö integroidaan määrätysti kahteen kertaan

$$v''(x) = \frac{-M(x)}{EI} \tag{1}$$

$$\Rightarrow v'(x) = \frac{-1}{D}\int_0^x M(x)dx + A = \phi(x) \tag{2}$$

$$\Rightarrow \quad v(x) = \frac{-1}{D}\int_0^x\int_0^x M(x)(dx)^2 + Ax + B \tag{3}$$

PISTEVOIMA *a* -MITAN PÄÄSSÄ VASEMMALTA TUELTA



Kuva 2.

Taivutusmomentti (kuva 2)

$$M(x) = \begin{cases} \dfrac{x}{L}(L-a), & x \in \{0 \ldots a\} \\ \dfrac{a}{L}(L-x), & x \in \{a \ldots L\} \end{cases} \tag{4a, b}$$

Taipuma x:n funktiona

$$v'(x)=\begin{cases} v_1'(x), & x\in\{0..a\} \\ v_2'(x), & x\in\{a..L\}\end{cases} \qquad (5a, b)$$

$$=\begin{cases} \dfrac{x^2}{2DL}(a-L)+A_1, & x\in\{0..a\} \\ \dfrac{ax}{2DL}(x-2L)+A_2, & x\in\{a..L\}\end{cases} \qquad (6a, b)$$

$$\Rightarrow v(x)=\begin{cases} v_1(x), & x\in\{0..a\} \\ v_2(x), & x\in\{a..L\}\end{cases} \qquad (7a, b)$$

$$=\begin{cases} \dfrac{x^3}{6DL}(a-L)+A_1x+B_1, & x\in\{0..a\} \\ \dfrac{a}{6DL}\left(x^3-3Lx^2\right)+A_2x+B_2, & x\in\{a..L\}\end{cases} \qquad (8a, b)$$

Reunaehdot

$$v_1'(a) = v_2'(a) \qquad (9)$$

$$v_1(0) = 0 \qquad (10)$$

$$v_2(L) = 0 \qquad (11)$$

$$v_1(a) = v_2(a) \qquad (12)$$

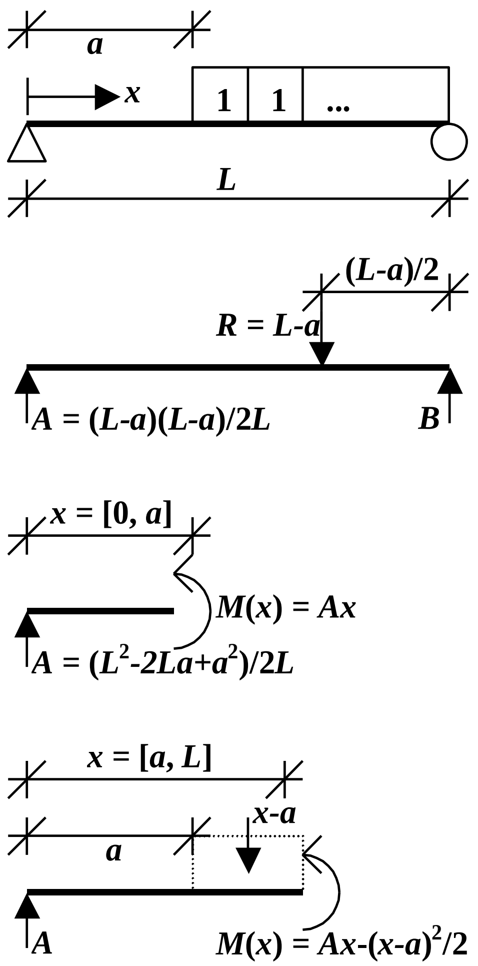
Reunaehdoista saadaan integroimisvakiot

$$\begin{cases} A_1=\dfrac{a}{6DL}\left(2L^2-3La+a^2\right) & (13) \\ B_1=0 & (14) \\ A_2=\dfrac{a}{6DL}\left(2L^2+a^2\right) & (15) \\ B_2=\dfrac{-a^3}{6D} & (16)\end{cases}$$

Taipumaviiva pistevoimasta F, joka vaikuttaa kohdassa $x = a$, on

$$v_F(x)=\begin{cases} \dfrac{F}{6DL}\left[(a-L)x^3+a(2L^2-3La+a^2)x\right], & x\in\{0...a\} \\ \dfrac{F}{6DL}\left[ax^3-3Lax^2+a(2L^2+a^2)x-La^3\right], & x\in\{a...L\}\end{cases} \qquad (17a, b)$$

VÄLILLÄ [*a*, *L*] VAIKUTTAVA, PITUUSYKSIKKÖÄ KOHTI TASAISESTI JAKAUTUNUT VOIMA



Kuva 3.

Taivutusmomentti (kuva 3)

$$M(x) = \begin{cases} \dfrac{x}{2L}\left(L^2 - 2La + a^2\right), & x \in \{0 \ldots a\} \\ \dfrac{1}{2L}\left[-Lx^2 + \left(L^2 + a^2\right)x - La^2\right], & x \in \{a \ldots L\} \end{cases} \tag{18a, b}$$

Taipuma x :n funktiona

$$v'(x)=\begin{cases} v_1'(x), & x\in\{0\ldots a\} \\ v_2'(x), & x\in\{a\ldots L\} \end{cases} \tag{5a, b}$$

$$=\begin{cases} \dfrac{x^2}{4DL}\left(-L^2+2La-a^2\right)+A_1, & x\in\{0\ldots a\} \\ \dfrac{1}{12DL}\left[2Lx^3+3\left(L^2+a^2\right)x^2-6La^2x\right]+A_2, & x\in\{a\ldots L\} \end{cases} \tag{19a, b}$$

$$\Rightarrow v(x)=\begin{cases} v_1(x), & x\in\{0\ldots a\} \\ v_2(x), & x\in\{a\ldots L\} \end{cases} \tag{7a, b}$$

$$=\begin{cases} \dfrac{x^3}{12DL}\left(-L^2+2La-a^2\right)+A_1x+B_1, & x\in\{0\ldots a\} \\ \\ \dfrac{1}{24DL}\left[Lx^4-2\left(L^2+a^2\right)x^3+6La^2x^2\right]+A_2x+B_2, \\ \quad x\in\{a\ldots L\} \end{cases} \tag{20a, b}$$

Reunaehdoista 9…12 saadaan integroimisvakiot

$$\begin{cases} A_1=\dfrac{1}{24DL}\left(L^4-4L^2a^2+4La^3-a^4\right) & (21) \\ B_1=0 & (22) \\ A_2=\dfrac{1}{24DL}\left(L^4-4L^2a^2-a^4\right) & (23) \\ B_2=\dfrac{a^4}{24D} & (24) \end{cases}$$

Taipumaviiva tasaisesti jakautuneesta kuormasta q on

$$v_q(x)=\begin{cases} \dfrac{q}{24DL}\left[2\left(-L^2+2La-a^2\right)x^3+\right. \\ \left.+\left(L^4-4L^2a^2+4La^3-a^4\right)x\right], & x\in\{0\ldots a\} \\ \\ \dfrac{q}{24DL}\left[Lx^4-2\left(L^2+a^2\right)x^3+6La^2x^2+\right. \\ \left.+\left(L^4-4L^2a^2-a^4\right)x+La^4\right], & x\in\{a\ldots L\} \end{cases} \tag{25a, b}$$

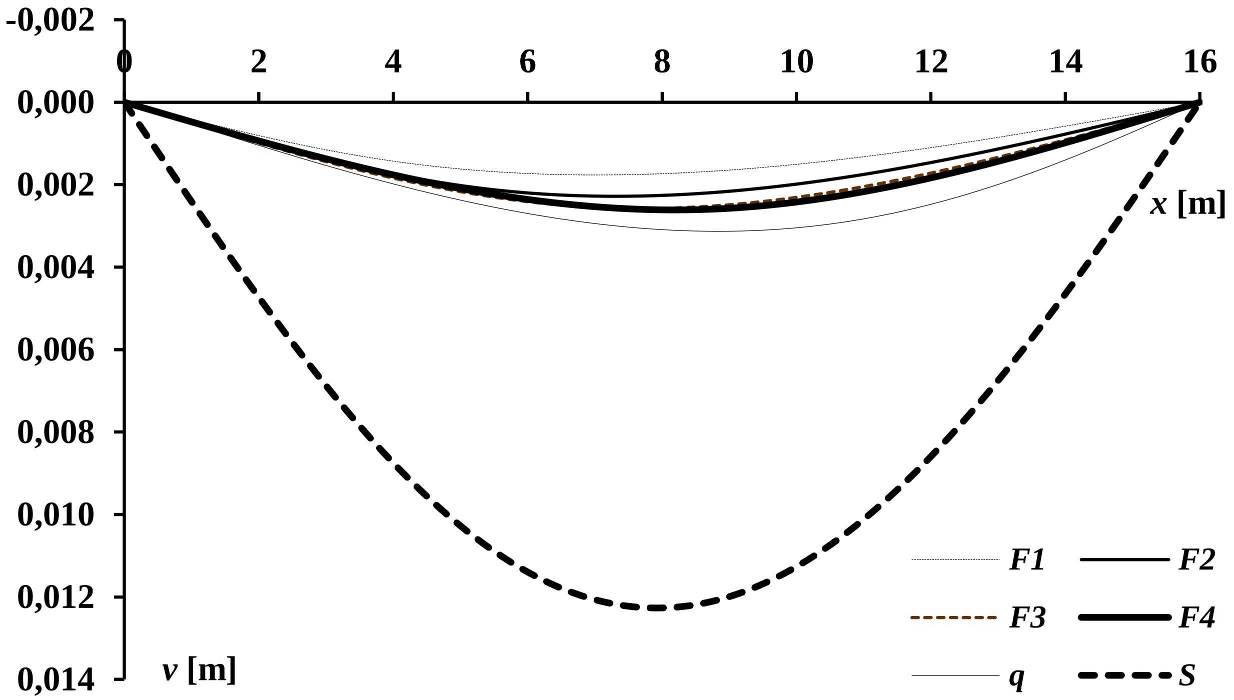
SUPERPOSITIIO

Taulukko 1. Pistekuormien ja viivakuorman vaikutuskohdat.

		a_i m
Pistekuorma F_1	$a_1 = a_0+b =$	3,800
Pistekuorma F_2	$a_2 = a_0+3b =$	5,400
Pistekuorma F_3	$a_3 = a_0+5b =$	7,000
Pistekuorma F_4	$a_4 = a_0+7b =$	8,600
Viivakuorma q	$a_5 = a_0+8b =$	9,400

Sijoittamalla vaikutuskohdat $a_1 \ldots a_4$ (taulukko 1) yhtälöihin 17 ja a_5 yhtälöön 25 sekä laskemalla taipumat yhteen saadaan sillan taipumakuvio (kuva 4).

<u>VASTAUS</u>: Taipumakuvio on esitetty kuvassa 4.



Kuva 4. Taipumakuvio.

7. TEHTÄVÄ

Määritä se ajoneuvon kuorma-asento (mitta *a*), jolla kuvan 1 kaksiaukkoisen sillan keskitukeen tulee suurin mahdollinen tukireaktio sekä tämän tukireaktion suuruus!

Kuormamalli vastaa kevyen liikenteen sillan tilapäistä huoltoajoneuvoa *Eurokoodin* mukaan (*EN 1991-2* § 5.6.3).

Sillan pituus

L = 16 m

Sillan taivutusjäykkyys

D = 120 MNm^2

Nauhakuorma koko sillan pituudella sillan omasta painosta

p = 0,009 MN/m

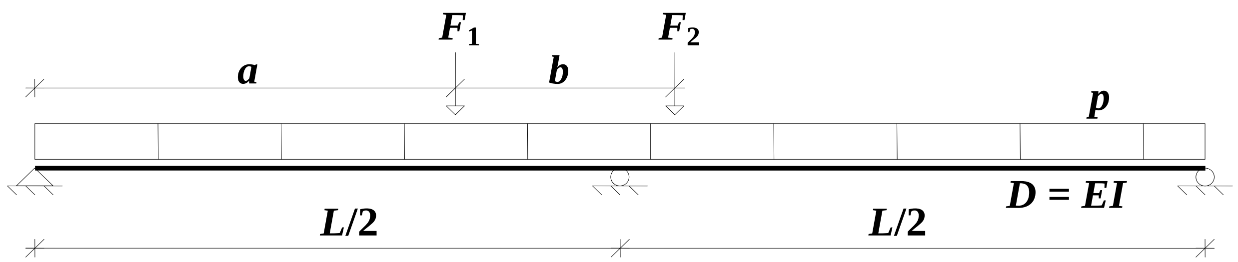
Akselikuorma

F_1 = 0,080 MN

F_2 = 0,040 MN

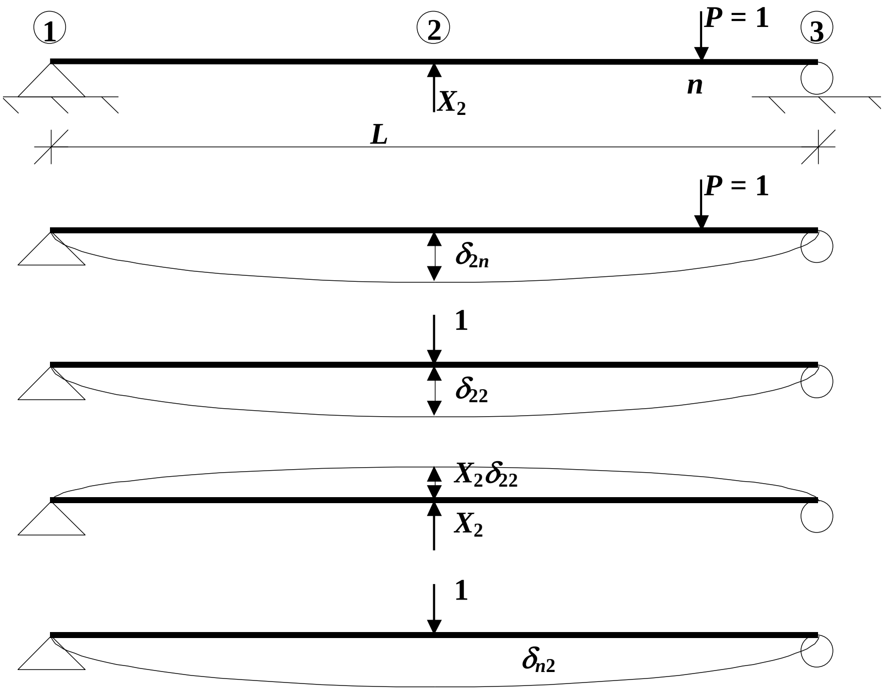
Akseliväli

b = 3,000 m



Kuva 1.

Poistamalla keskituki saadaan rakenteen staattisesti määrätty perusmuoto (kuva 2).



Kuva 2.

Pisteessä n (x-koordinaatin arvo) vaikuttavasta ykkösen suuruisesta voimasta syntyvä keskipisteen (2) taipuma δ_{2n} ja tukireaktion X_2 aiheuttama keskipisteen taipuma $X_2\delta_{22}$ asetetaan toistensa vastaluvuiksi. Tässä δ_{22} on keskipisteen taipuma samassa pisteessä vaikuttavasta ykkösen suuruisesta voimasta.

$$0 = \delta_{2n} - X_2\delta_{22} \tag{1}$$

$$\Rightarrow X_2 = \frac{\delta_{2n}}{\delta_{22}} \tag{2}$$

Miinus-merkki yhtälössä 1 osoittaa tukireaktion suunnan olevan ylöspäin. Yhtälön 2 mukaan tukireaktio X_2 saa maksimiarvon, kun taipuma δ_{2n} saa maksimiarvon.

Maxwellin säännöstä: pisteessä n vaikuttavasta ykkösen suuruisesta voimasta keskipisteeseen syntyvä taipuma δ_{2n} on yhtä suuri kuin keskipisteessä vaikuttavasta ykkösen suuruisesta voimasta pisteeseen n syntyvä taipuma δ_{n2}.

$$\delta_{2n} = \delta_{n2} \tag{3}$$

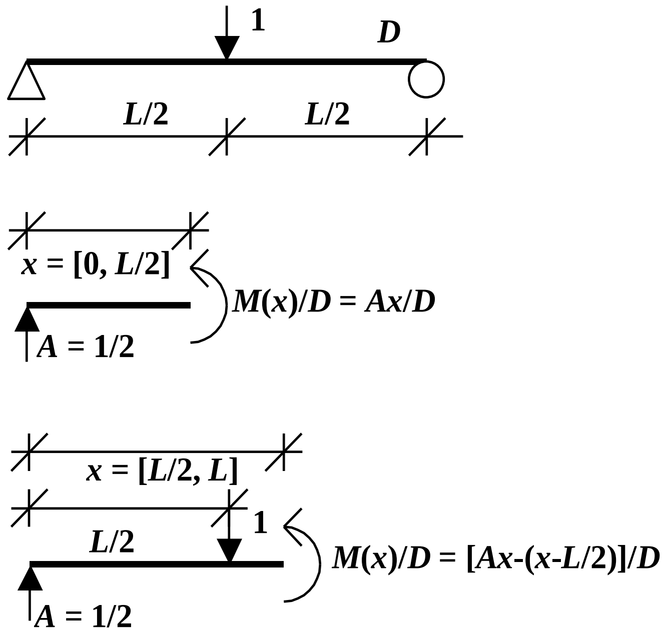
Yksikkövoima aiheuttaman tukireaktion vaikutusviiva saadaan keskipisteessä vaikuttavan ykkösen suuruisen voiman aiheuttaman taipumaviivan avulla.

$$X_2 = \frac{\delta_{n2}}{\delta_{22}} \tag{4}$$

Ratkaistaan taipuma $\delta_{n2} = v(x)$ Mohrin menetelmällä.

Yksikkövoiman aiheuttaman taivutusmomentin suhde taivutusjäykkyyteen (kuva 3)

$$\frac{M}{D}(x) = \begin{cases} \dfrac{x}{2D}, & x \in \{0 \ldots L/2\} \\ \dfrac{L-x}{2D}, & x \in \{L/2 \ldots L\} \end{cases} \tag{5a, b}$$



Kuva 3.

M/D-pinnalla kuormitetun rakenteen $M_{M/D}$-pinta on sen taipumaviiva (kuva 4)

$$v(x) = M_{M/D}(x) \tag{6}$$

$$T = \frac{L^2}{16D}$$

$$e_1 = \frac{x}{3}$$

$$R_1 = \frac{x^2}{4D}$$

$$e_2 = x - \frac{L}{3}$$

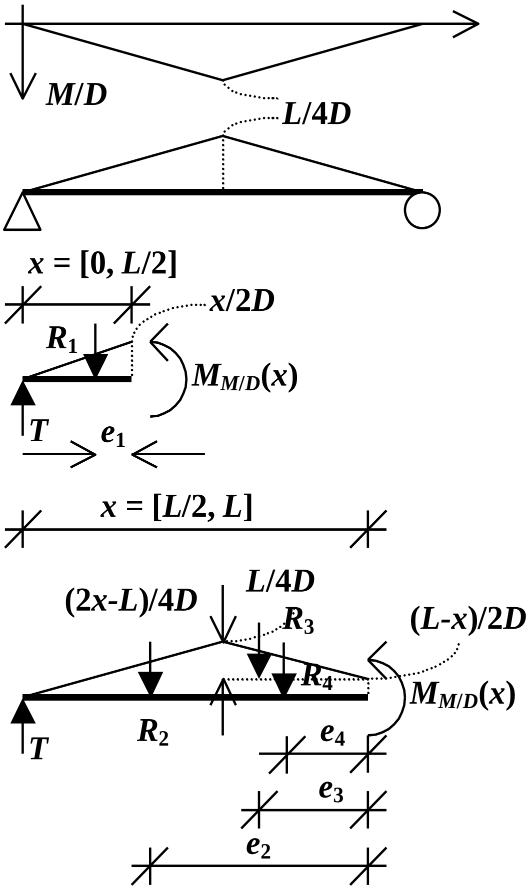
$$R_2 = \frac{L^2}{16D}$$

$$e_3 = \frac{2}{3}\left(x - \frac{L}{2}\right)$$

$$R_3 = \frac{2x - L}{8D}\left(x - \frac{L}{2}\right)$$

$$e_4 = \frac{1}{2}\left(x - \frac{L}{2}\right)$$

$$R_4 = \frac{L - x}{2D}\left(x - \frac{L}{2}\right)$$



Kuva 4.

Taipuma jänteen puolivälissä sijaitsevasta yksikkövoimasta kohdassa *x*

$$v(x) = \begin{cases} Tx - R_1 e_1, & x \in \{0 \ldots L/2\} \\ Tx - R_2 e_2 - R_3 e_3 - R_4 e_4, & x \in \{L/2 \ldots L\} \end{cases} \qquad \text{(7a, b)}$$

$$= \begin{cases} \dfrac{1}{48D}\left(-4x^3 + 3L^2 x\right), & x \in \{0 \ldots L/2\} \\ \dfrac{1}{48D}\left(4x^3 - 12Lx^2 + 9L^2 x - L^3\right), & x \in \{L/2 \ldots L\} \end{cases} \qquad \text{(8a, b)}$$

Taipuma kohdassa x jänteen puolivälissä sijaitsevasta pistevoimasta F

$$v(x,F)=\begin{cases}\dfrac{F}{48D}\left(-4x^3+3L^2x\right), & x\in\{0 \ldots L/2\}\\ \dfrac{F}{48D}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2 \ldots L\}\end{cases} \qquad \text{(9a, b)}$$

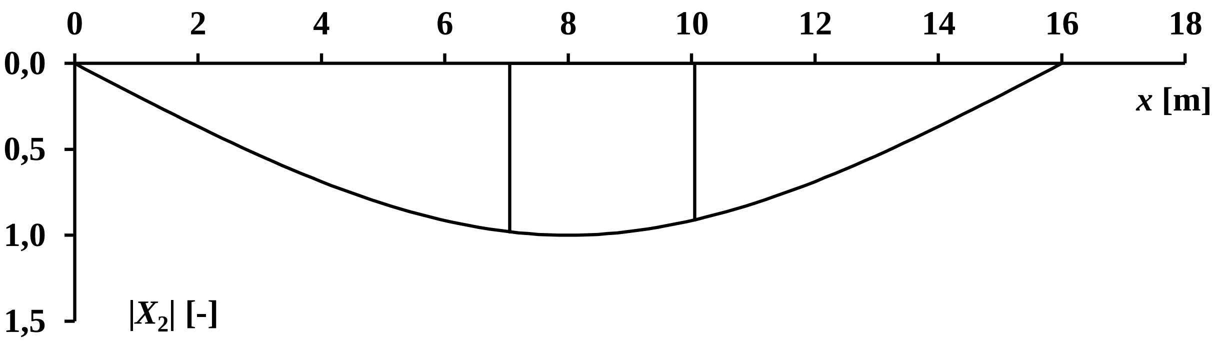
Keskipisteen taipuma samassa pisteessä vaikuttavasta ykkösen suuruisesta voimasta saadaan yhtälöstä 8

$$\delta_{22}=v\left(x=\frac{L}{2}\right) \qquad (10)$$

$$=\frac{L^3}{48D} \qquad (11)$$

Sijoittamalla yhtälöön 4 yhtälöt 8 ja 11 saadaan tukireaktion vaikutusviivaksi yksikkövoimasta. Ks. kuva 5.

$$X_2(x)=\begin{cases}\dfrac{1}{L^3}\left(-4x^3+3L^2x\right), & x\in\{0 \ldots L/2\}\\ \dfrac{1}{L^3}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2 \ldots L\}\end{cases} \qquad \text{(12a, b)}$$



Kuva 5.

Tukireaktion vaikutusviiva akselikuormasta F_i on

$$X_{2,F_i}(x)=\begin{cases}\dfrac{F_i}{L^3}\left(-4x^3+3L^2x\right), & x\in\{0 \ldots L/2\}\\ \dfrac{F_i}{L^3}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2 \ldots L\}\end{cases} \qquad \text{(13a, b)}$$

Tukireaktion suuruus tasaisesti jakautuneesta voimasta *p* saadaan kertomalla yhtälö 12 voimalla *p* ja integroimalla voiman vaikutusmatkan yli

$$X_{2,p} = \int_0^{L/2} \frac{p}{L^3}\left(-4x^3 + 3L^2x\right)dx + \tag{14}$$

$$+ \int_{L/2}^{L} \frac{p}{L^3}\left(4x^3 - 12Lx^2 + 9L^2x - L^3\right)dx$$

$$= \frac{5}{8}pL \tag{15}$$

Iteroimalla löydetään suurin tukireaktio ja sen sijainti (kaavat 13 ja 15, taulukko 1)

a = 7,050 m

Taulukko 1.

		x_i [m]	$X_{2,i}$ [MN]
Pistevoima F_1	x_1 = a =	7,050	0,078
Pistevoima F_2	$x_2 = a + b$ =	10,050	0,036
Viivavoima p			0,090
		Σ	0,205

VASTAUS: Kuorma-asento: mitta

a = 7,050 m

Tukireaktio

X_2 = 0,205 MN

8. TEHTÄVÄ

Määritä kellarin seinään (kuva 1) kohdistuvan maanpaineen jakauma ja resultantti, kun maanpinnan kaltevuuskulma vaakatasoon nähden on

A) $\beta_A =$ **0 °**

B) $\beta_B =$ **20 °**

Täyte on tiivistetty seuraavasti:

Tiivistyskone	**400**	**kg:n tärylevy**
Tiivistyskerrat	**4**	**kpl**
Kerrospaksuus	**0,350**	**m**

Seinän korkeus

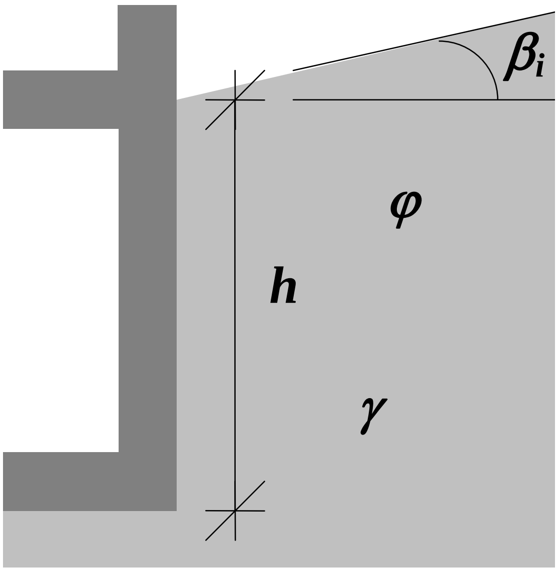
$h =$ **2,500 m**

Maan sisäinen kitkakulma

$\varphi =$ **34 °**

Maan tilavuuspaino

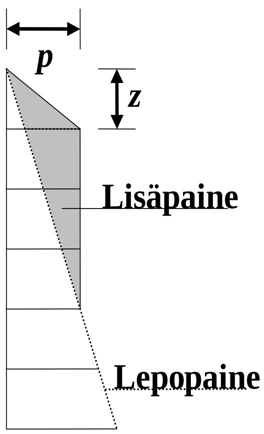
$\gamma =$ **0,019 MN/m^3**



Kuva 1.

Taulukko 1. Siirtymättömän tukirakenteen takana tehtävän täytön aiheuttama pysyvä maanpaine (kuva 2).[1]

Tiivistyskone	Tiivistys-koneen paino [kg]	Tiivistys-kertojen määrä kpl	Kerros-paksuus *h* [m]	Taite-syvyys *z* [m]	Maan-paine *p* [MN/m²]
Täryjyrä	3 000	6	0,400	0,500	0,019
Tärylevy	400	4	0,350	0,500	0,016
Tärylevy	100	4	0,200	0,500	0,012



Kuva 2. Painekuvio.

1) *Geotekninen suunnittelu, RIL 207-2009,* kohta 9.5.5.

Liikkumattomaan rakenteeseen kohdistuu lepopaine, joka vaikuttaa horisontaalisuunnassa.

A) Kaltevuuskulma on nolla

Lepopaineen maanpaineluku

$$K_o = 1\text{-}\sin\varphi \qquad (1)$$

$$= 0{,}441$$

Paine seinän alapinnan tasossa ilman tiivistystä (kuva 3)

$$p_o = K_o \gamma h \qquad (2)$$

$$= 0{,}021 \text{ MN/m}^2$$

Lepopaineen resultantti

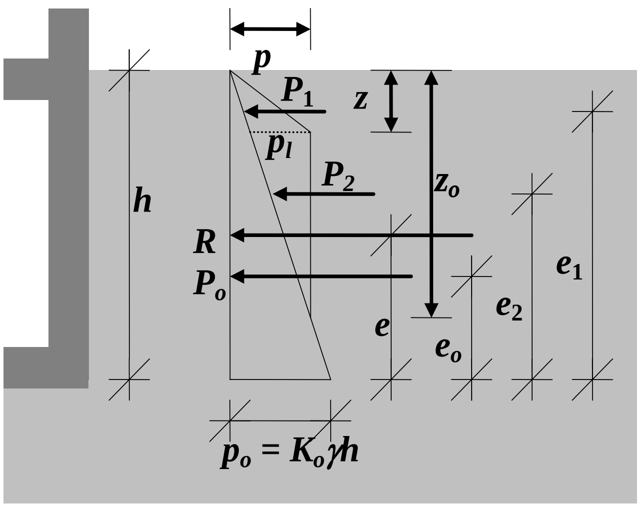
$$P_o = \frac{1}{2} p_o h \qquad (3)$$

$$= 0{,}026 \text{ MN/m}$$

Resultantin etäisyys seinän alapinnan tasosta

$$e_o = h/3 \qquad (4)$$

$$= 0{,}833 \text{ m}$$



Kuva 3.

Tiivistyksen jälkeinen paine syvyydellä *z* (kuvat 2 ja 3)

$$p = 0{,}016\ \mathrm{MN/m^2}$$

Tiivistyksen aiheuttaman painejakauman taitekohdan etäisyys maan pinnalta

$$z = 0{,}500\ \mathrm{m}$$

Tiivistyksen aiheuttaman painejakauman vaikutuksen päättymiskohdan etäisyys maan pinnan tasalta (vrt. kaava 2)

$$z_o = \frac{p}{K_o\gamma} \tag{5}$$

$$= 1{,}910\ \mathrm{m}$$

$$< h$$

Tiivistyksen aiheuttama lisä syvyydellä *z*

$$p_l = p - K_o\gamma z \tag{6}$$

$$= 0{,}012\ \mathrm{MN/m^2}$$

Painelisän resultantti kohdan *z* yläpuolella ($_1$)

$$P_1 = p_l z/2 \tag{7}$$

$$= 0{,}003\ \mathrm{MN/m}$$

Resultantin etäisyys perustamistasosta

$$e_1 = h - \frac{2}{3}z \tag{8}$$

$$= 2{,}167\ \mathrm{m}$$

Painelisän resultantti kohdan *z* alapuolella ($_2$)

$$P_2 = p_l(z_o - z)/2 \tag{9}$$

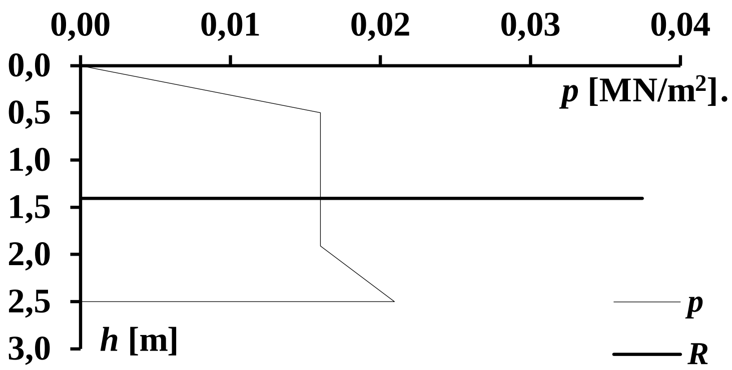
$$= 0{,}008\ \mathrm{MN/m}$$

Resultantin etäisyys perustamistasosta

$$e_2 = h - z - \frac{1}{3}(z_o - z) \quad (10)$$

$$= \quad 1{,}530 \text{ m}$$

<u>VASTAUS</u>: **Maanpainejakauma ja sen resultantti on esitetty kuvassa 4.**



Kuva 4.

Maanpaineen resultantti

$$R = P_o + P_1 + P_2 \quad (11)$$

$$= \quad 0{,}0375 \text{ MN/m}$$

Resultantin etäisyys perustamistasosta

$$e = \frac{P_o e_o + P_1 e_1 + P_2 e_2}{R} \quad (12)$$

$$= \quad 1{,}093 \text{ m}$$

B) Kaltevuuskulma poikkeaa nollasta

Lepopaineen maanpaineluku (vertaa kaava 1)

$$K_{oB} = (1-\sin\varphi)(1+\sin\beta) \quad (13)$$

$$= 0{,}592$$

Paine seinän alapinnan tasossa ilman tiivistystä (kuva 5)

$$p_{oB} = K_{oB}\,\gamma h \quad (14)$$

$$= 0{,}028 \text{ MN/m}^2$$

Lepopaineen resultantti

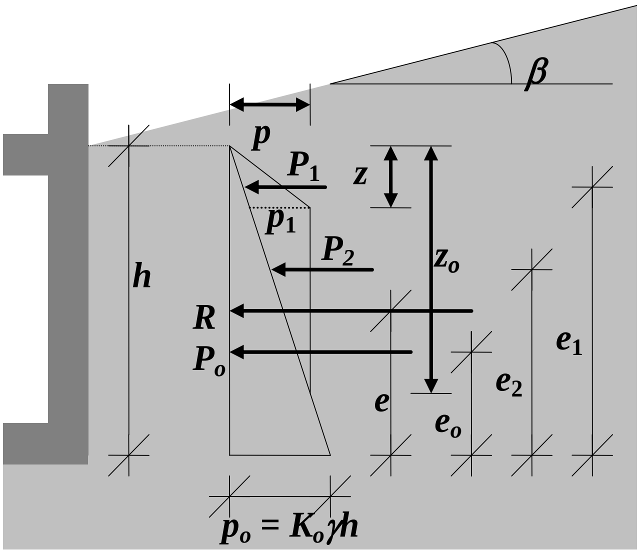
$$P_{oB} = \frac{1}{2} p_{oB} h \quad (15)$$

$$= 0{,}035 \text{ MN/m}$$

Resultantin etäisyys seinän alapinnan tasosta

$$e_{oB} = h/3 \quad (16)$$

$$= 0{,}833 \text{ m}$$



Kuva 5.

Tiivistyksen aiheuttaman painejakauman vaikutuksen päättymiskohdan etäisyys maan pinnan tasalta (vrt. kaava 2)

$$z_{oB} = \frac{p}{K_{oB}\gamma} \tag{17}$$

= 1,424 m

< h

Tiivistyksen aiheuttama lisä syvyydellä z

$$p_{IB} = p - K_{oB}\gamma z \tag{18}$$

= 0,010 MN/m^2

Painelisän resultantti kohdan z yläpuolella ($_1$)

$$P_{1B} = p_{IB} z/2 \tag{19}$$

= 0,003 MN/m

Resultantin etäisyys perustamistasosta

$$e_{1B} = h - \frac{2}{3}z \tag{20}$$

= 2,167 m

Painelisän resultantti kohdan z alapuolella ($_2$)

$$P_{2B} = p_{IB}(z_{oB} - z)/2 \tag{21}$$

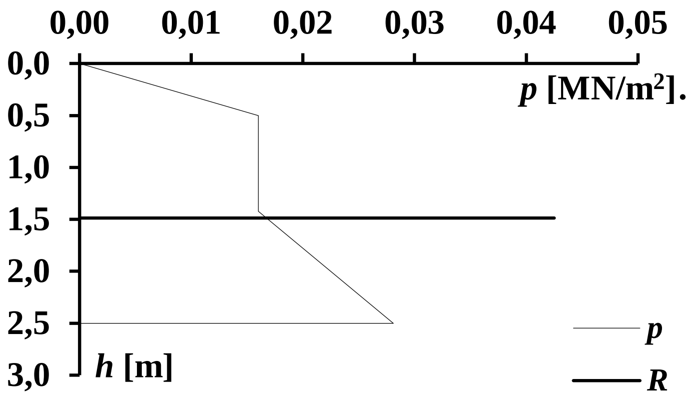
= 0,005 MN/m

Resultantin etäisyys perustamistasosta

$$e_{2B} = h - z - \frac{1}{3}(z_{oB} - z) \tag{22}$$

= 1,692 m

<u>VASTAUS</u>: Maanpainejakauma ja sen resultantti on esitetty kuvassa 6.



Kuva 6.

Maanpaineen resultantti

$$R = P_{oB} + P_{1B} + P_{2B} \tag{23}$$
$$= 0{,}0425 \text{ MN/m}$$

Resultantin etäisyys perustamistasosta

$$e = \frac{P_{oB}e_{oB} + P_{1B}e_{1B} + P_{2B}e_{2B}}{R} \tag{24}$$
$$= 1{,}012 \text{ m}$$

9. TEHTÄVÄ

Laske oheiseen betonista valmistettuun kasuuniin kohdistuva maanpaine (aktiivisen maanpaineen horisontaalikomponentti), kun taustatäyttö on tehty merihiekasta!

Mitat

a =	1	m
h_1 =	2	m
h_2 =	1	m
h_3 =	6	m

Maanpinnan kaltevuuskulman tangentin käänteisluku

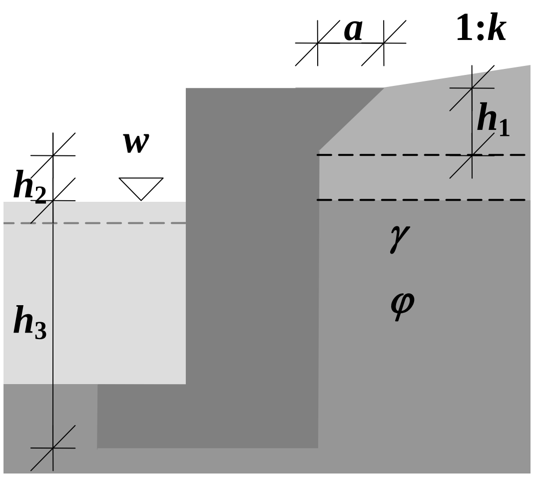
k =	10

Maan sisäinen kitkakulma

φ =	38 °

Maan tilavuuspaino

γ =	0,018	MN/m^3



Kuva 1.

Ohje: Maanpaine lasketaan kolmessa kerroksessa, jotka kuvassa 1 on erotettu toisistaan katkoviivoin.

Maanpinnan kaltevuus vaakatasoon nähden (kuva 2)

$$\beta = \arctan\left(\frac{1}{k}\right) \tag{1}$$

$$= \quad 5{,}711\ °$$

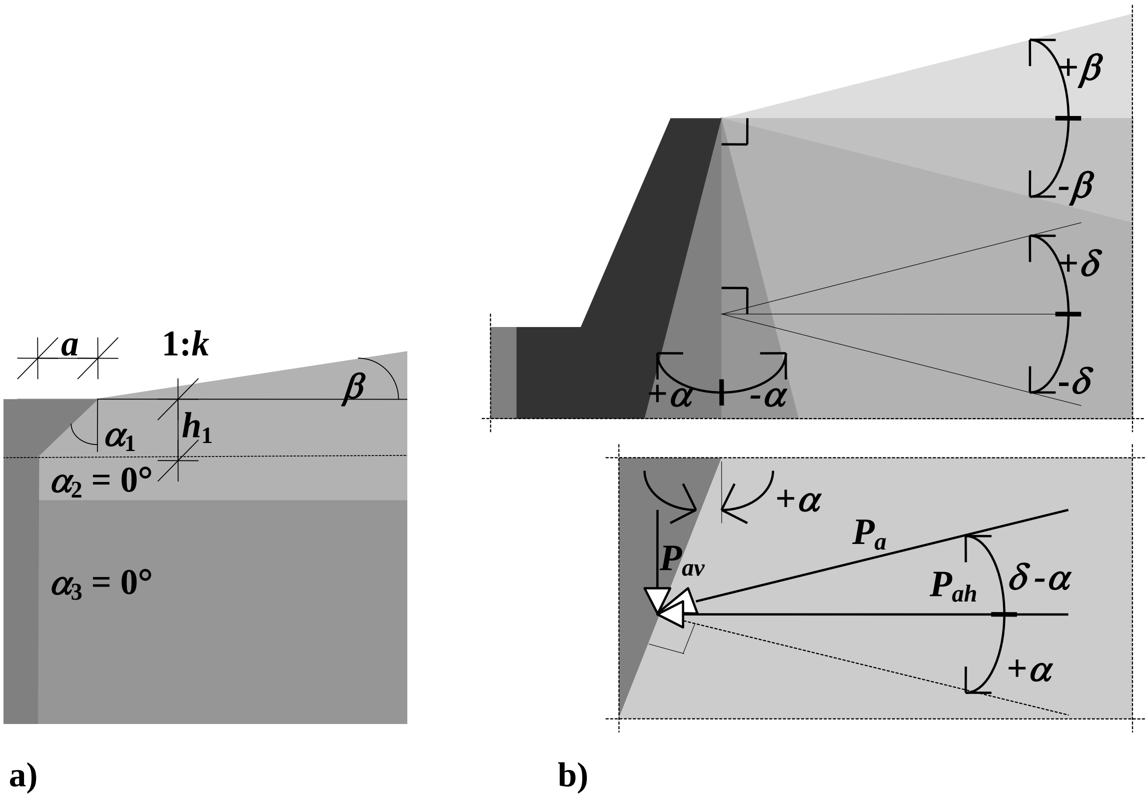
Tukimuurin kaltevuus pystytasoon verrattuna kussakin kerroksessa (kuva 2)

$$\alpha_1 = \arctan\left(\frac{a}{h_1}\right) \tag{2}$$

$$= \quad 26{,}565\ °$$

$$\alpha_2 = \quad 0{,}000\ °$$

$$\alpha_3 = \quad 0{,}000\ °$$



Kuva 2.

Seinäkitkakulma, kun liukupinta on betonirakenteen ja maan välissä

$$\delta = \frac{3}{4}\varphi \tag{3}$$

$$= \quad 28{,}500\ °$$

Aktiivisen maanpaineen horisontaalikomponentin maanpaineluku

$$K_{ahi} = \frac{\cos^2(\varphi+\alpha_i)}{\cos^2\alpha_i\left(1+\sqrt{\frac{\sin(\varphi+\delta)\sin(\varphi-\beta)}{\cos(\alpha_i-\delta)\cos(\alpha_i+\beta)}}\right)^2} \tag{4}$$

$K_{ah\,1}$ = 0,074

$K_{ah\,2}$ = 0,203

$K_{ah\,3}$ = 0,203

Maan tilavuuspaino pohjavedenpinnan yläpuolella

$\gamma_1 = \gamma_2 =$ 0,018 MN/m^3

Maan tilavuuspaino pohjavedenpinnan alapuolella (taulukko 1 tehtävän lopussa)

$\gamma_3 =$ 0,012 MN/m^3

Maakerroksen n aktiivipaine p_n maakerroksen ylä- (y) ja alaosassa (a)

$$\begin{cases} p_{ny} = \begin{cases} 0, & n=1 \\ K_n \sum_{i=1}^{n-1}(\gamma_i h_i), & n \geq 2 \end{cases}, \quad n \in N \\ p_{na} = K_n \sum_{i=1}^{n}(\gamma_i h_i), \quad n \geq 1 \end{cases} \tag{5a, b}$$

p_{1y} = 0 MN/m^2

$p_{1a} = K_{ah\,1}(\gamma_1 h_1)$ (6)

= 0,002675 MN/m^2

$p_{2y} = K_{ah\,2}(\gamma_1 h_1)$ (7)

= 0,007312 MN/m^2

$p_{2a} = K_{ah\,2}(\gamma_1 h_1+\gamma_2 h_2)$ (8)

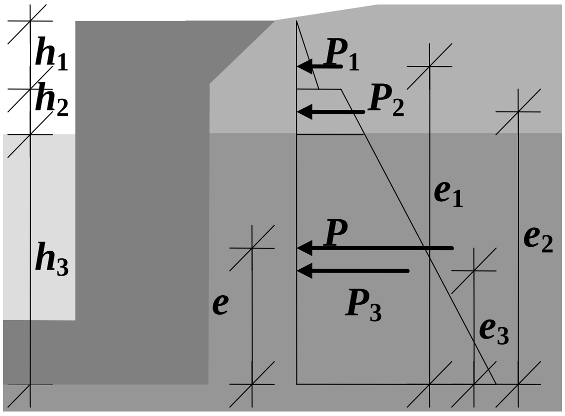
0,010968 MN/m^2

$p_{3y} = K_{ah\,3}(\gamma_1 h_1+\gamma_2 h_2)$ (9)

0,010968 MN/m^2

$p_{3a} = K_{ah\,3}(\gamma_1 h_1+\gamma_2 h_2+\gamma_3 h_3)$ (10)

= 0,025593 MN/m^2



Kuva 3.

Maakerroksen n aktiivipaineen resultantti P_n (kuva 3)

$$P_n = p_{ny}h_n + \frac{1}{2}(p_{na} - p_{ny})h_n \tag{11}$$

$P_1 =$ 0,003 MN/m

$P_2 =$ 0,009 MN/m

$P_3 =$ 0,110 MN/m

Aktiivipaineen resultanttien P_n etäisyydet e_n tukimuurin alaosasta

$$e_n = \frac{p_{ny}\frac{h_n}{2} + \frac{1}{2}\left(p_{na} - p_{ny}\right)\frac{h_n}{3}}{p_{ny} + \frac{1}{2}\left(p_{na} - p_{ny}\right)} + \begin{cases} \sum_{i=n+1}^{n_{max}} h_i, & n = [1, \quad n_{max}] \\ 0, & n = n_{max} \end{cases} \tag{12}$$

$e_1 =$ 7,667 m

$e_2 =$ 6,467 m

$e_3 =$ 2,600 m

<u>VASTAUS</u>:

Maanpaineen resultantti

$$P = P_1 + P_2 + P_3 \tag{13}$$

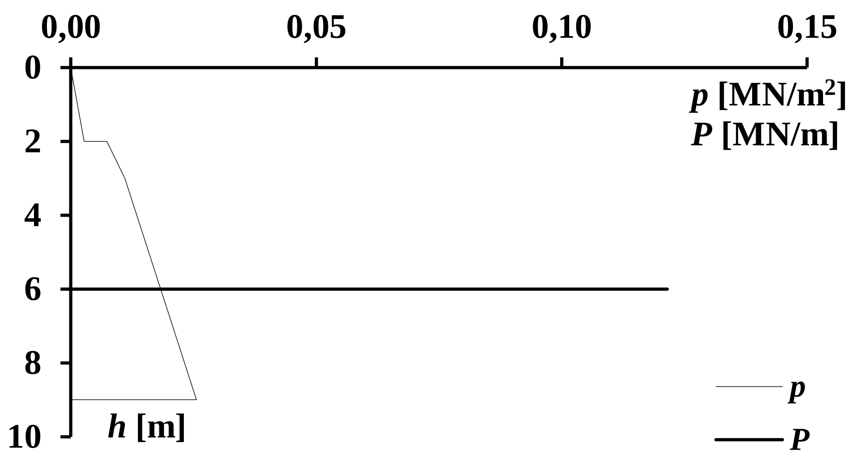
$= $ 0,121 MN/m

Etäisyys tukimuurin alaosasta

$$e = \frac{\sum_{n=1}^{n_{max}} P_n e_n}{\sum_{n=1}^{n_{max}} P_n} \tag{14}$$

$$= \quad 3{,}002 \text{ m}$$

Painejakauma ja paineresultantti on esitetty kuvassa 4.



Kuva 4.

Taulukko 1. Karkearakeisten maalajien arviointi rakeisuuden perusteella.[1)]

Maalaji		Tilavuuspaino pohjavedenpinnan yläpuol. $[kN/m^3]$	Tilavuuspaino pohjavedenpinnan alapuol. $[kN/m^3]$	Kitka-kulma ϕ [°]
Hieno hiekka	Löyhä	15…17	9…	30
(hHk)	Keskitiivis			33
$d_{10} \le 0{,}06$	Tiivis	16…18	11	36
Hiekka (Hk)	L	16…18	10…	32
$d_{10} > 0{,}06$	K			35
	T	17…19	12	38
Sora (Sr)	L	17…19	10…	34
	K			37
	T	18…20	12	40
Moreeni (Mr)	Hyvin löyhä	16…19	10…12	…34
	L	17…20	10…12	…36
	K	18…21	11…13	…38
	T	19…23	11…14	…40
Tiivistetty täyte	Louhe	15…18	9…11	45
perustusten alla[2)]	Murske	19…22	11…13	42
	Sora	18…21	11…13	40

1) Lähde: Tielaitos: *Pohjarakennusohjeet sillansuunnittelussa.* TIEL 2172068-99. Helsinki 1999. 71 s. ISBN 951-726-583-2. Taulukko 1. s. 9.

2) Näiden arvojen käyttö edellyttää, että työn suoritus ja materiaalit ovat *Sillanrakentamisen yleisten laatuvaatimusten - SYL 2* kohdan 2.7.1.2 /24/ mukaiset.

10. TEHTÄVÄ

Laske ja piirrä kuvan 1 mukaiseen betonista valmistettuun tukimuuriin kohdistuvan lepopaineen ja aktiivisen maanpaineen (horisontaalikomponentti) jakaumat sekä lisäksi maanpaineen resultantti ja sen sijainti! Täyttömateriaali on hiekka.

Mitat

a	=	2,200	m
b	=	5,200	m
c	=	1,000	m
d	=	1,000	m
h_1	=	6,200	m
h_2	=	2,100	m

Kuormitus

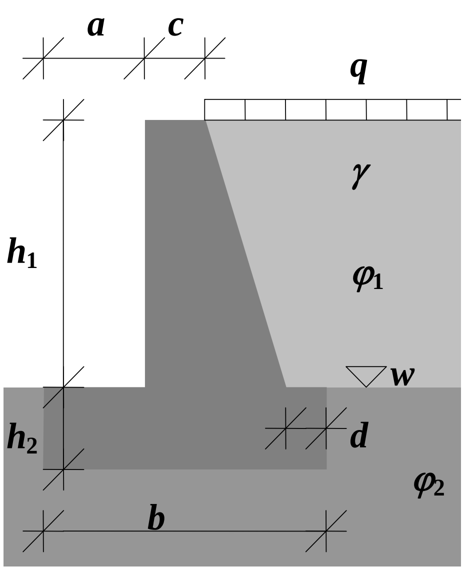
q	=	0,012	MN/m^2

Maakerroksen tilavuuspaino

γ	=	0,018	MN/m^3

Maakerroksen sisäinen kitkakulma

φ_1	=	32	°
φ_2	=	34	°



Kuva 1.

Maan tilavuuspaino pohjavedenpinnan yläpuolella

$$\gamma_1 = 0{,}018 \text{ MN/m}^3$$

Maan tilavuuspaino pohjavedenpinnan alapuolella (taulukko 1 tehtävässä 9; *Pohjarakennusohjeet sillansuunnittelussa,* taulukko 1)

$$\gamma_2 = 0{,}011 \text{ MN/m}^3$$

LEPOPAINE

Lepopainekerroin

$$K_{oi} = 1\text{-}\sin\varphi_i \tag{1}$$

$$K_{o1} = 0{,}470$$

$$K_{o2} = 0{,}441$$

Maakerroksen n lepopaine p_{on} maakerroksen ylä- (y) ja alaosassa (a) (kuva 2)

$$\begin{cases} p_{ony} = \begin{cases} K_{on}q\,, & n = 1 \\ K_{on}\left(\sum_{i=1}^{n-1}\gamma_i h_i + q\right), & n \geq 2 \end{cases} \\ p_{ona} = K_{on}\left(\sum_{i=1}^{n}\gamma_i h_i + q\right), \quad n \geq 1 \end{cases}, \quad n \in N \tag{2a, b}$$

$$p_{o1y} = K_{o1}q \tag{3}$$

$$= 0{,}005641 \text{ MN/m}^2$$

$$p_{o1a} = K_{o1}(\gamma_1 h_1 + q) \tag{4}$$

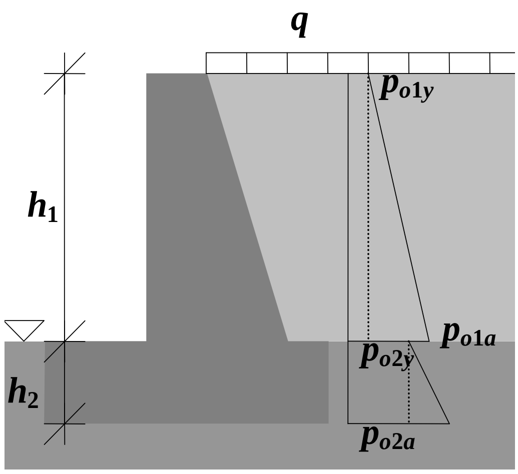
$$= 0{,}058102 \text{ MN/m}^2$$

$$p_{o2y} = K_{o2}(\gamma_1 h_1 + q) \tag{5}$$

$$= 0{,}054484 \text{ MN/m}^2$$

$$p_{o2a} = K_{o2}(\gamma_1 h_1 + \gamma_2 h_2 + q) \tag{6}$$

$$= 0{,}064666 \text{ MN/m}^2$$



Kuva 2.

Maakerroksen n lepopaineen resultantti P_{on} sekä näiden summa P_o (kuva 3)

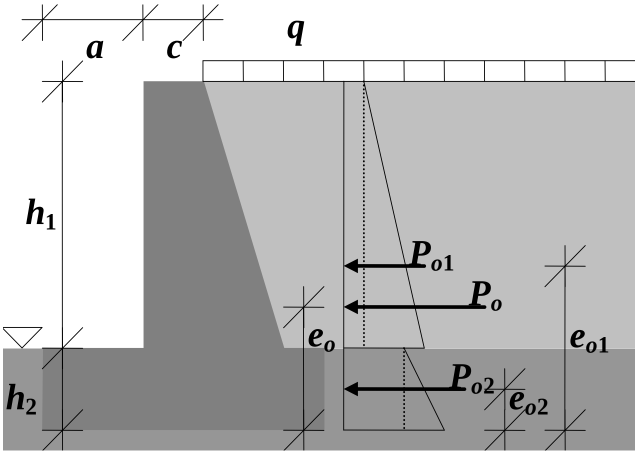
$$P_{on} = p_{ony}h_n + \frac{1}{2}(p_{ona} - p_{ony})h_n \tag{7}$$

$$P_{o1} = 0{,}198 \text{ MN/m}$$

$$P_{o2} = 0{,}125 \text{ MN/m}$$

$$P_o = P_{o1} + P_{o2} \tag{8}$$

$$= 0{,}323 \text{ MN/m}$$



Kuva 3.

Lepopaineen resultanttien P_{on} ja P_o etäisyydet e_{on} ja e_o tukimuurin alaosasta (kuva 3)

$$e_{on} = \frac{p_{ony}\frac{h_n}{2} + \frac{1}{2}\left(p_{ona} - p_{ony}\right)\frac{h_n}{3}}{p_{ony} + \frac{1}{2}\left(p_{ona} - p_{ony}\right)} + \begin{cases} \sum_{i=n+1}^{n_{\max}} h_i, & n = [1, \quad n_{\max} - 1] \\ 0, & n = n_{\max} \end{cases} \tag{9}$$

$e_{o\,1}$ = 4,350 m

$e_{o\,2}$ = 1,020 m

$$e_o = \frac{\sum_{n=1}^{n_{\max}} P_{on} e_{on}}{\sum_{n=1}^{n_{\max}} P_{on}} \tag{10}$$

= 3,059 m

AKTIIVIPAINE

Maanpinnan kaltevuus vaakatasoon nähden (kuva 4)

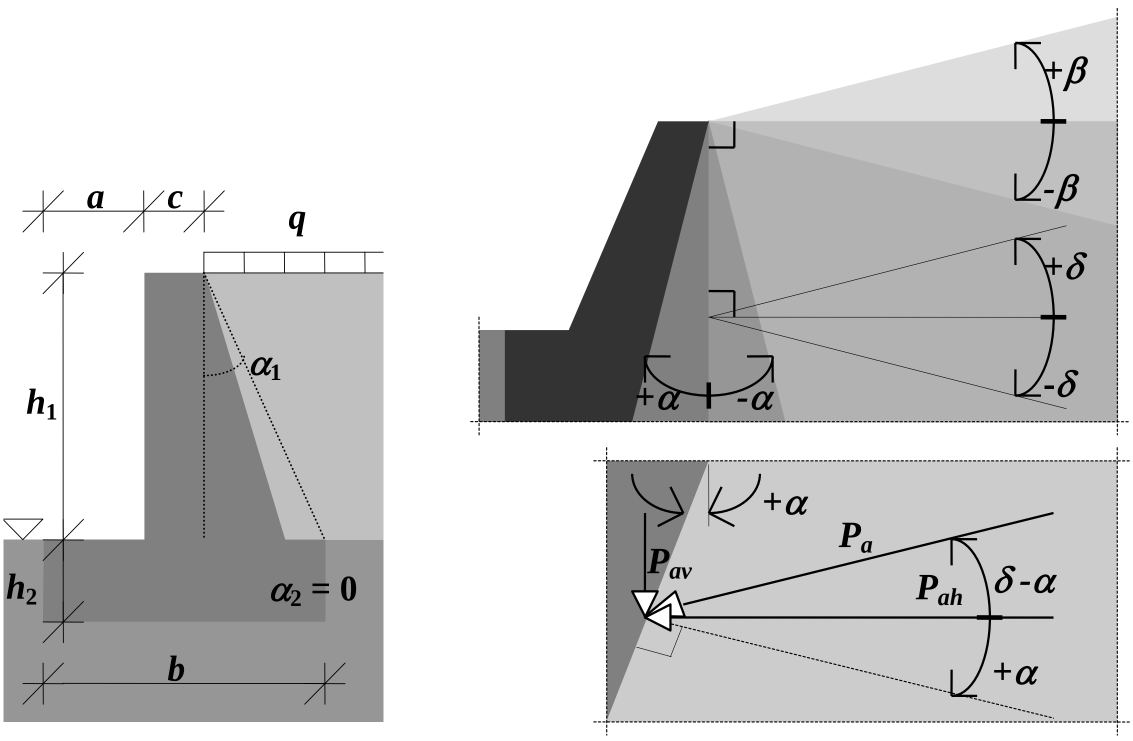
$$\beta = 0\,°$$

Liukupinnan kaltevuus pystytasoon verrattuna (kuva 4)

$$\alpha_1 = -\arctan\left(\frac{b-a-c}{h_1}\right) \tag{11}$$

$$= -17{,}879\,°$$

$$\alpha_2 = 0{,}000\,°$$



a) b)

Kuva 4.

Seinäkitkakulma, kun liukupinta ei ole rakenteen ja maan välissä

$$\delta_1 = \varphi_1 \tag{12}$$

$$= 32{,}000\,°$$

Seinäkitkakulma, kun liukupinta on betonirakenteen ja maan välissä

$$\delta_2 = \frac{3}{4}\varphi_2 \qquad (13)$$

$$= \quad 25{,}500\ °$$

Aktiivisen maanpaineen horisontaalikomponentin maanpaineluku

$$K_{ahi} = \frac{\cos^2(\varphi_i + \alpha_i)}{\cos^2\alpha_i\left(1 + \sqrt{\frac{\sin(\varphi_i+\delta_i)\sin(\varphi_i-\beta)}{\cos(\alpha_i-\delta_i)\cos(\alpha_i+\beta)}}\right)^2} \qquad (14)$$

$$K_{ah\,1} = \quad 0{,}293$$

$$K_{ah\,2} = \quad 0{,}229$$

Maakerroksen n aktiivipaine p_n maakerroksen ylä- (y) ja alaosassa (a)

$$\begin{cases} p_{ny} = \begin{cases} K_n q, & n = 1 \\ K_n\left(\sum_{i=1}^{n-1}\gamma_i h_i + q\right), & n \geq 2 \end{cases} \\ p_{na} = K_n\left(\sum_{i=1}^{n}\gamma_i h_i + q\right), \quad n \geq 1 \end{cases}, \quad n \in N \qquad (15a, b)$$

$$p_{1y} = K_{ah\,1}q \qquad (16)$$

$$= \quad 0{,}003521\ \text{MN/m}^2$$

$$p_{1a} = K_{ah\,1}(\gamma_1 h_1 + q) \qquad (17)$$

$$= \quad 0{,}036263\ \text{MN/m}^2$$

$$p_{2y} = K_{ah\,2}(\gamma_1 h_1 + q) \qquad (18)$$

$$= \quad 0{,}028363\ \text{MN/m}^2$$

$$p_{2a} = K_{ah\,2}(\gamma_1 h_1 + \gamma_2 h_2 + q) \qquad (19)$$

$$= \quad 0{,}033664\ \text{MN/m}^2$$

Maakerroksen n aktiivipaineen resultantti P_n sekä näiden summa P (kuva 5)

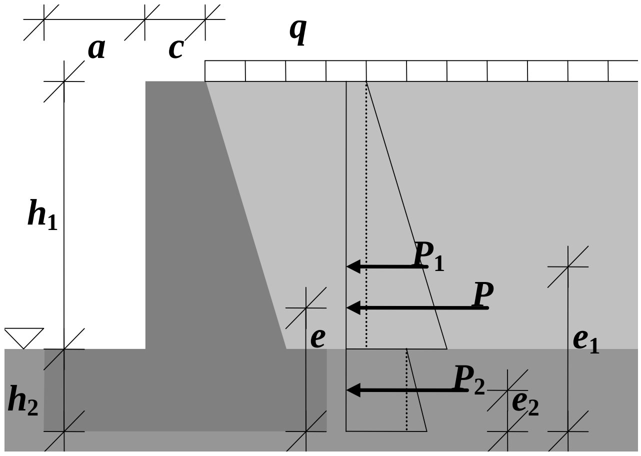
$$P_n = p_{ny}h_n + \frac{1}{2}(p_{na} - p_{ny})h_n \qquad (20)$$

$$P_1 = \quad 0{,}123\ \text{MN/m}$$

$$P_2 = 0{,}065 \ \mathrm{MN/m}$$

$$P = P_1 + P_2 \tag{21}$$

$$= 0{,}188 \ \mathrm{MN/m}$$



Kuva 5.

Aktiivipaineen resultanttien P_n ja P etäisyydet e_n ja e tukimuurin alaosasta (kuva 5)

$$e_n = \frac{p_{ny}\dfrac{h_n}{2} + \dfrac{1}{2}\left(p_{na} - p_{ny}\right)\dfrac{h_n}{3}}{p_{ny} + \dfrac{1}{2}\left(p_{na} - p_{ny}\right)} + \begin{cases} \sum\limits_{i=n+1}^{n_{\max}} h_i, & n = [1, \quad n_{\max}] \\ 0, & n = n_{\max} \end{cases} \tag{22}$$

$$e_1 = 4{,}350 \ \mathrm{m}$$

$$e_2 = 1{,}020 \ \mathrm{m}$$

$$e = \frac{\sum\limits_{n=1}^{n_{\max}} P_n e_n}{\sum\limits_{n=1}^{n_{\max}} P_n} \tag{23}$$

$$= 3{,}199 \ \mathrm{m}$$

VASTAUS:

Maanpaineen resultantti ja sen etäisyys tukimuurin alaosasta

Lepopaine

P_o = 0,323 MN/m

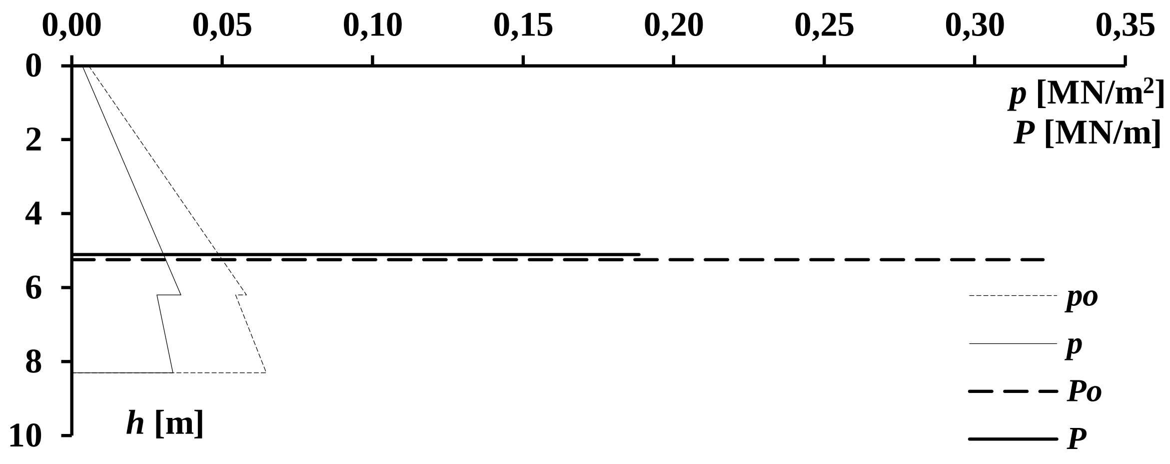
e_o = 3,059 m

Aktiivipaine

P = 0,188 MN/m

e = 3,199 m

Maanpainejakauma on esitetty kuvassa 6.



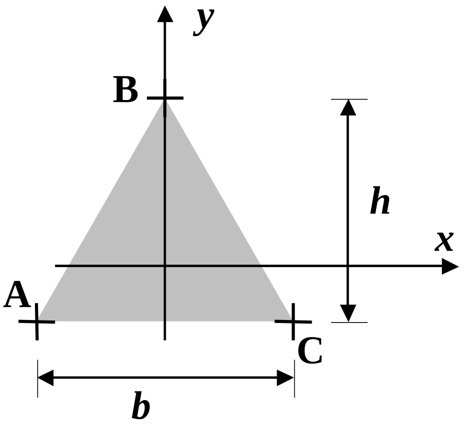
Kuva 6.

11. TEHTÄVÄ

Määritä kuvan 1 homogeenisen tasakylkisen kolmiopoikkileikkauksen sydänkuvio!

Mitat

b	=	2 m
h	=	3 m



Kuva 1.

Sijoitetaan koordinaatisto siten, että koordinaattiakselit yhtyvät poikkileikkauksen pääjäyhyysakseleihin.

Pinta-ala

$$A = bh/2 \tag{1}$$
$$= 3{,}000 \text{ m}^2$$

Jäyhyysmomentti x - ja y -akselin suhteen

$$I_x = \frac{bh^3}{36} \tag{2}$$
$$= 1{,}500 \text{ m}^4$$
$$I_y = \frac{hb^3}{48} \tag{3}$$
$$= 0{,}500 \text{ m}^4$$

Homogeeniselle poikkileikkaukselle jäyhyyssäde s -akselin suhteen on

$$i_s = \sqrt{\frac{I_s}{A}} \tag{4}$$

x - ja y -akselien suhteen olevien jäyhyyssäteiden neliöt

$$i_x^2 = \frac{I_x}{A} \tag{5}$$
$$= 0{,}500 \text{ m}^2$$
$$i_y^2 = \frac{I_y}{A} \tag{6}$$
$$= 0{,}167 \text{ m}^2$$

Ulkonurkkien A ja B koordinaatit

$$x_{\text{Ap}} = -b/2 \tag{7}$$
$$= -1{,}000 \text{ m}$$
$$y_{\text{Ap}} = -h/3 \tag{8}$$
$$= -1{,}000 \text{ m}$$
$$x_{\text{Bp}} = 0{,}000 \text{ m}$$
$$y_{\text{Bp}} = 2h/3 \tag{9}$$
$$= 2{,}000 \text{ m}$$

Sydänkuviota rajoittavien suorien yhtälöt ovat muotoa

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \qquad (10)$$

$$\Rightarrow y(x) = \alpha x + \beta \qquad (11)$$

jossa

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \qquad (12)$$

$$\beta = -\frac{i_x^2}{y_{ip}} \qquad (13)$$

Sijoittamalla saadaan

	α_ι		β_ι	
$y_A(x) =$	-3,000 x	+	0,500	[m]
$y_B(x) =$	0,000 x	+	-0,250	[m]

Suorien *i* ja *j* leikkauspisteen *x* -koordinaatti

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \qquad (14)$$

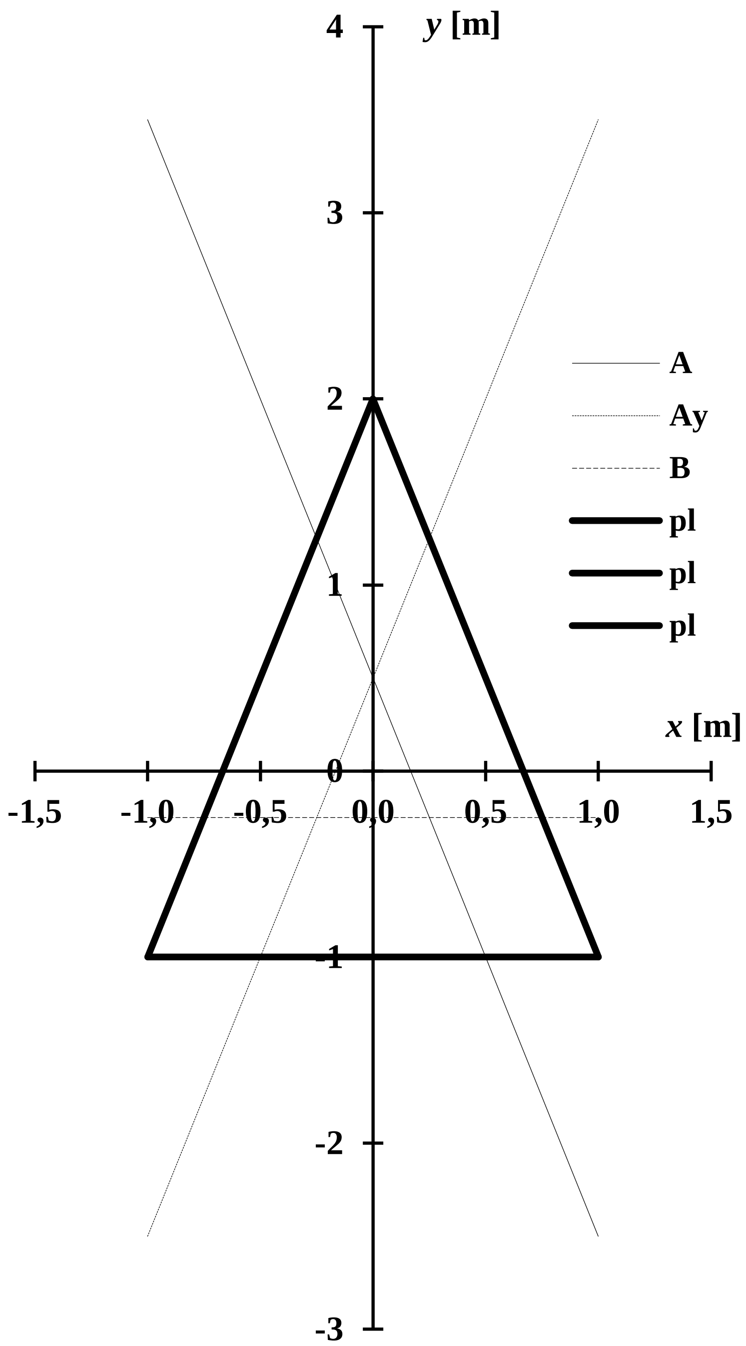
Suorien *i* ja *j* leikkauspisteet on laskettu taulukossa 1.

Taulukko 1.

ij	x_{ij} m	y_{ij} m
AB	0,250	-0,250
A*x*	0,167	0,000
A*y*	0,000	0,500

VASTAUS:

Sydänkuviota rajoittavat suorat: A, sen peilaus *y* :n suhteen Ay ja B on esitetty kuvassa 2.



Kuva 2.

12. TEHTÄVÄ

Kalliolla oleva peruslaatta tehdään sisä- ja ulkoreunaltaan ellipsin muotoisesta teräsrenkaasta, joka valetaan täyteen betonia niin, että valmis laatta toimii liittorakenteena (kuva 1). Määritä perustuksen sydänkuvio ennen betonointia ja betonin kovetuttua!

Mitat

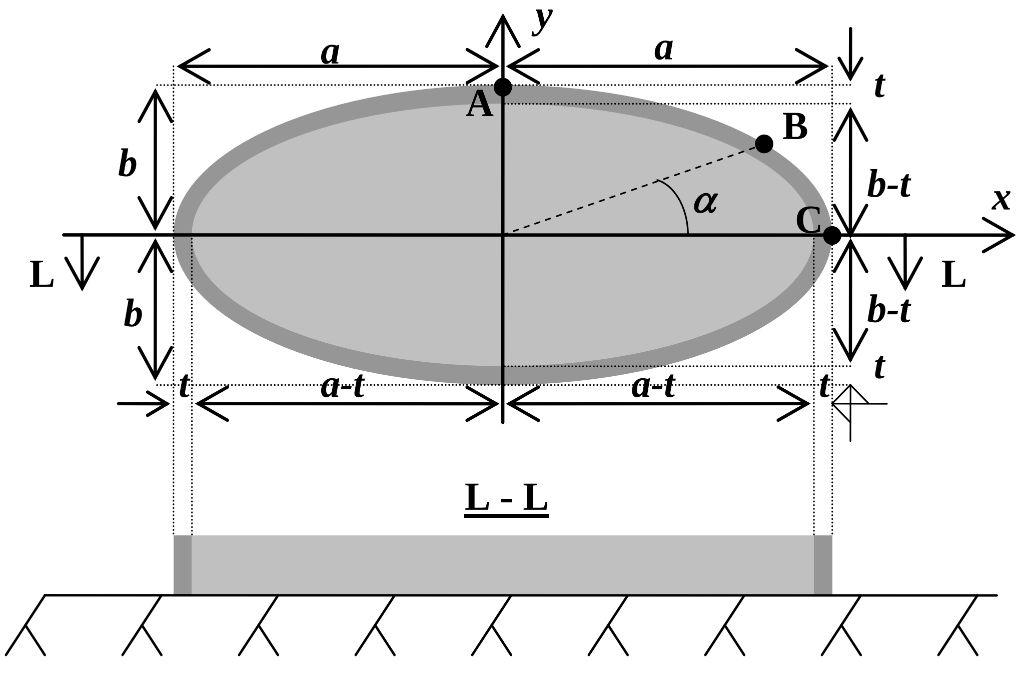
a = π m
≈ 3,142 m
b = $\pi/2$ m
≈ 1,571 m
t = $1/\pi^{\pi}$ m
≈ 0,027 m

Betonin (*c*) ja teräksen (*s*) kimmokertoimet

E_c = 30 000 MN/m^2
E_s = 210 000 MN/m^2

Ohje: Arvioi sydänkuvion muoto määrittämällä kuvaan merkittyjä pisteitä A, B ja C vastaavat sydänkuviota rajoittavat suorat. Kulma

α = $\pi/8$



Kuva 1.

Pinta-alat

Betoni

$$A_c = \pi(a-t)(b-t) \tag{1}$$
$$= 15{,}099\ \mathrm{m}^2$$

Teräs

$$A_s = \pi ab - A_c \tag{2}$$
$$= 0{,}404\ \mathrm{m}^2$$

Poikkileikkaus

$$A = A_c + A_s \tag{3}$$
$$= 15{,}503\ \mathrm{m}^2$$

Jäyhyysmomentit *x*- ja *y*- akselien suhteen

Betoni

$$I_{cx} = \pi(a-t)(b-t)^3/4 \tag{4}$$
$$= 8{,}992\ \mathrm{m}^4$$
$$I_{cy} = \pi(b-t)(a-t)^3/4 \tag{5}$$
$$= 36{,}609\ \mathrm{m}^4$$

Teräs

$$I_{sx} = \pi ab^3/4 - I_{cx} \tag{6}$$
$$= 0{,}571\ \mathrm{m}^4$$
$$I_{sy} = \pi ba^3/4 - I_{cy} \tag{7}$$
$$= 1{,}644\ \mathrm{m}^4$$

TERÄSRENGAS

Jäyhyyssäteen neliö *x* -akselin ja *y*- akselin suhteen

$$i_{sx}^2 = I_{sx}/A_s \tag{8}$$
$$= 1{,}416\ \mathrm{m}^2$$
$$i_{sy}^2 = I_{sy}/A_s \tag{9}$$
$$= 4{,}072\ \mathrm{m}^2$$

Suoran kulmakerroin

$$k = \tan(\alpha) \approx 0{,}414 \quad (10)$$

Ulkonurkkien koordinaatit

$$x_{\mathrm{Ap}} = 0{,}000 \text{ m}$$

$$y_{\mathrm{Ap}} = b = 1{,}571 \text{ m}$$

$$\begin{cases} \dfrac{x_{Bp}^2}{a^2} + \dfrac{y_{Bp}^2}{b^2} = 1 \\ y_{Bp} = kx_{Bp} \end{cases} \quad (11a, b)$$

$$\Rightarrow \quad x_{Bp} = \begin{matrix} + \\ (-) \end{matrix} \sqrt{\frac{a^2 b^2}{k^2 a^2 + b^2}} \quad (12)$$

$$= 2{,}419 \text{ m}$$

$$y_{\mathrm{Bp}} = 1{,}002 \text{ m}$$

$$x_{\mathrm{Cp}} = a = 3{,}142 \text{ m}$$

$$y_{\mathrm{Cp}} = 0{,}000 \text{ m}$$

Sydänkuviota rajoittavien suorien yhtälöt ovat muotoa

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \quad (13)$$

$$\Rightarrow y(x) = \alpha x + \beta \quad (14)$$

jossa

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \quad (15)$$

$$\beta = -\frac{i_x^2}{y_{ip}} \quad (16)$$

Sijoittamalla saadaan

	α_ι		β_ι	
$y_{As}(x) =$	$0{,}000\ x$	+	$-0{,}901$	[m]
$y_{Bs}(x) =$	$-0{,}839\ x$	+	$-1{,}413$	[m]

Kun $y_{ip} = 0$,

$$x = -i_y^{\ 2}/x_{ip} \tag{17}$$

$$x_{Cs} = -1{,}296 \text{ m}$$

Suorien i ja j leikkauspisteen x -koordinaatti

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \tag{18}$$

Suorien i ja j leikkauspisteet on laskettu taulukossa 1.

Taulukko 1.

ij	x_{ij} m	y_{ij} m
AB	-0,609	-0,901
BC	-1,296	-0,325

Oletetaan, että sydänkuvio on ellipsi, jonka puoliakselit ovat

$$n_s = -x_{Cs}$$
$$= 1{,}296 \text{ m}$$
$$m_s = -\beta_{As}$$
$$= 0{,}901 \text{ m}$$

Pistettä B vastaavan sydänkuviota rajoittavan suoran ja em. ellipsin leikkauspiste on

$$\begin{cases} \dfrac{x_o^2}{n_s^2} + \dfrac{y_o^2}{m_s^2} = 1 \\ y_o = \alpha_B x_o + \beta_B \end{cases} \tag{19a, b}$$

$$\Rightarrow x_o = \frac{-n_s^2 \alpha_B \beta_B \pm n_s m_s \sqrt{n_s^2 \alpha_B^2 + m_s^2 - \beta_B^2}}{m_s^2 + n_s^2 \alpha_B^2} \tag{20}$$

x_o = -0,998 m
y_o = -0,575 m

Ellipsin tangentti tässä pisteessä on

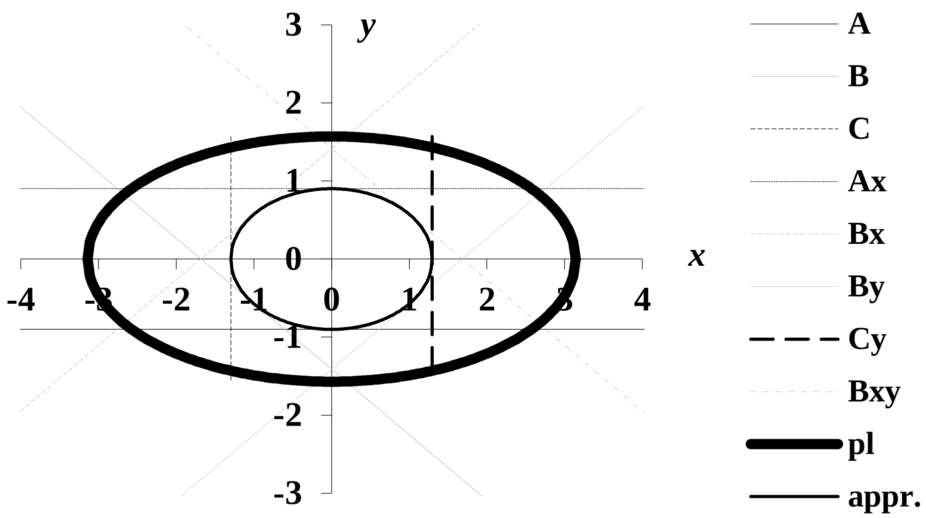
$$\frac{x_o}{n_s^2}x+\frac{y_o}{m_s^2}y=1 \tag{21}$$

$$\Rightarrow y=-\frac{m_s^2 x_o}{n_s^2 y_o}x+\frac{m_s^2}{y_o} \tag{22}$$

= -0,839 x + -1,413 [m]

joka on sama kuin edellä saatu sydänkuviota rajoittava suora $y_B(x)$.

<u>VASTAUS</u>: ♥-kuviota rajoittavat suorat A, B ja C sekä näiden peilaukset x- ja y-akselien suhteen on esitetty kuvassa 2.



Kuva 2.

Huom! Sydänkuvio ei ole yhdenmuotoinen teräsrenkaan ulko- tai sisäpinnan kanssa:

a/b = 2,000
$(a-t)/(b-t)$ = 2,018
n_s/m_s = 1,438

LIITTORAKENNE

Aksiaalijäykkyys

Betoni

$$C_c = E_c A_c \tag{23}$$
$$= 452\,984 \text{ MN}$$

Teräs

$$C_s = E_s A_s \tag{24}$$
$$= 84\,768 \text{ MN}$$

Liittorakenne

$$C = C_c + C_s \tag{25}$$
$$= 537\,753 \text{ MN}$$

Taivutusjäykkyys

Betoni

$$D_{cx} = E_c I_{cx} \tag{26}$$
$$= 269\,751 \text{ MNm}^2$$
$$D_{cy} = E_c I_{cy} \tag{27}$$
$$= 1\,098\,265 \text{ MNm}^2$$

Teräs

$$D_{sx} = E_s I_{sx} \tag{28}$$
$$= 119\,994 \text{ MNm}^2$$
$$D_{sy} = E_s I_{sy} \tag{29}$$
$$= 3{,}45\text{E}{+}05 \text{ MNm}^2$$

Liittorakenne

$$D_x = D_{cx} + D_{sx} \tag{30}$$
$$= 389\,746 \text{ MNm}^2$$
$$D_y = D_{cy} + D_{sy} \tag{31}$$
$$= 1{,}44\text{E}{+}06 \text{ MNm}^2$$

Jäyhyyssäteen neliö x -akselin ja y- akselin suhteen

$$i_x^{\,2} = D_x / C \tag{32}$$
$$= 0{,}725 \text{ m}^2$$

$$i_y^{\,2} = D_y / C \qquad (33)$$
$$= \quad 2{,}684 \text{ m}^2$$

Sydänkuviota rajoittavien suorien yhtälöt

	α_ι		β_ι
$y_A(x) =$	$0{,}000\ x$	+	$-0{,}461$ [m]
$y_B(x) =$	$-0{,}652\ x$	+	$-0{,}723$ [m]

Kun $y_{ip} = 0$,

$$x_C = \quad -0{,}854 \text{ m}$$

Suorien i ja j leikkauspisteet on laskettu taulukossa 2.

Taulukko 2.

ij	x_{ij} m	y_{ij} m
AB	-0,402	-0,461
BC	-0,854	-0,166

Oletetaan, että sydänkuvio on ellipsi, jonka puoliakselit ovat

$$n = -x_C$$
$$= \quad 0{,}854 \text{ m}$$
$$m = -\beta_A$$
$$= \quad 0{,}461 \text{ m}$$

Pistettä B vastaavan sydänkuviota rajoittavan suoran ja em. ellipsin leikkauspiste on

$$\begin{cases} x_o = \dfrac{-n^2\alpha_B\beta_B \pm nm\sqrt{n^2\alpha_B^2 + m^2 - \beta_B^2}}{m^2 + n^2\alpha_B^2} \\ y_o = \alpha_B x_o + \beta_B \end{cases} \qquad (34a, b)$$

$$\Rightarrow \begin{cases} x_o = & -0{,}658 \text{ m} \\ y_o = & -0{,}294 \text{ m} \end{cases}$$

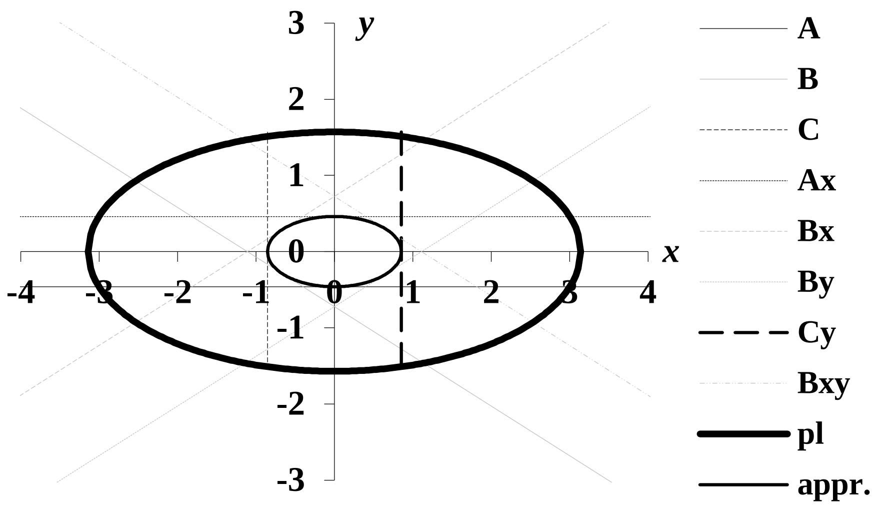
Ellipsin tangentti on

$$y = \frac{-m^2 x_o}{n^2 y_o} x + \frac{m^2}{y_o} \tag{35}$$

$$= \quad -0{,}652\ x \quad + \quad -0{,}723\ [\mathrm{m}]$$

joka on sama kuin edellä saatu sydänkuviota rajoittava suora $y_B(x)$.

<u>VASTAUS</u>: ♥-kuviota rajoittavat suorat A, B ja C sekä näiden peilaukset *x* - ja *y* -akselien suhteen on esitetty kuvassa 3.



Kuva 3.

Huom! Sydänkuvio ei ole yhdenmuotoinen teräsrenkaan ulko- tai sisäpinnan kanssa:

a/b	=	**2,000**
$(a\text{-}t)/(b\text{-}t)$	=	**2,018**
n/m	=	**1,852**

13. TEHTÄVÄ

Määritä y -z -tasossa se alue, jolla pystykuorma voi sijaita niin, että kaikki paalut ovat puristettuja (sydänkuvio)! Paalut ovat pystysuoria sekä poikkileikkaukseltaan, materiaaliltaan ja pituudeltaan samanlaisia. Ks. kuva 1 ja taulukko 1.

Käytä vastauksessa paaluryhmän pääjäyhyyssuunnan koordinaatistoa (ψ, ζ) tai painopistekoordinaatistoa (y , z), jonka akselit ovat annetun (y_0, z_0) suuntaiset!

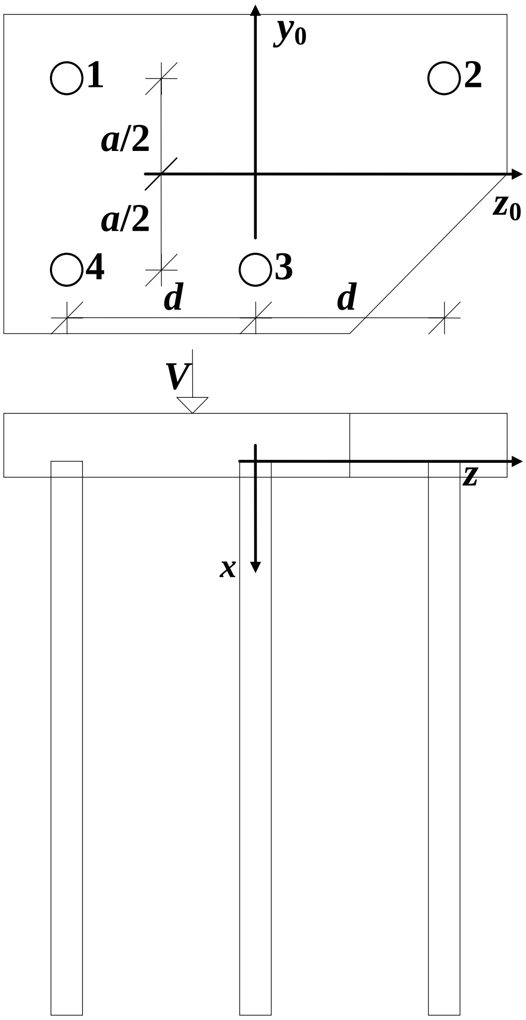
Mitat

a = 1,5 m

d = 1,5 m

Taulukko 1. Paalujen yläpään koordinaatit.

Symboli	i	z_{0i}	y_{0i}
Yksikkö	-	m	m
Lukuarvo	1	-1,500	0,750
	2	1,500	0,750
	3	0,000	-0,750
	4	-1,500	-0,750



Kuva 1.

Oletetaan, että paalun poikkileikkauksen omalla painopistelinjalla (y', z') sen tulo- ja jäyhyysmomentit häviävät

$$I_{z'i} = 0 \quad (1)$$

$$I_{y'i} = 0 \quad (2)$$

$$I_{y'z'i} = 0 \quad (3)$$

Merkitään paalun pinta-alaa dimensiottomalla luvulla

$$A_i = 1 \quad (4)$$

Koko paaluryhmän pinta-ala

$$A = 4A_i \quad (5)$$

$$= 4$$

Painopiste

$$z_{0p} = \Sigma z_i A_i / A \quad (6)$$

$$= -0{,}375 \text{ m}$$

$$y_{0p} = \Sigma y_i A_i / A \quad (7)$$

$$= 0{,}000 \text{ m}$$

Paalujen z-koordinaatit painopistekoordinaatistossa

$$z_i = z_{0i} - z_{0p} \quad (8)$$

$$z_1 = -1{,}125 \text{ m}$$

$$z_2 = 1{,}875 \text{ m}$$

$$z_3 = 0{,}375 \text{ m}$$

$$z_4 = -1{,}125 \text{ m}$$

Paalujen y-koordinaatit painopistekoordinaatistossa

$$y_i = y_{0i} - y_{0p} \quad (9)$$

$$y_1 = 0{,}750 \text{ m}$$

$$y_2 = 0{,}750 \text{ m}$$

$$y_3 = -0{,}750 \text{ m}$$

$$y_4 = -0{,}750 \text{ m}$$

Paalujen jäyhyysmomentit painopisteakselin z suhteen (Steinerin sääntö)

$$I_{zi} = I_{z'i} + A_i y_{pi}^2 \quad (10)$$

$$I_{z1} = 0{,}563 \text{ m}^2$$

$$I_{z2} = 0{,}563 \text{ m}^2$$

$$I_{z3} = 0{,}563 \text{ m}^2$$

$$I_{z4} = 0{,}563 \text{ m}^2$$

Paalujen jäyhyysmomentit painopisteakselin y suhteen

$$I_{yi} = I_{y'i} + A_i z_{pi}^2 \tag{11}$$

$I_{y1} =$ 1,266 m^2
$I_{y2} =$ 3,516 m^2
$I_{y3} =$ 0,141 m^2
$I_{y4} =$ 1,266 m^2

Paalujen tulomomentit painopistekoordinaatistossa (y, z)

$$I_{yzi} = I_{y'z'i} + A_i y_{pi} z_{pi} \tag{12}$$

$I_{yz1} =$ -0,844 m^2
$I_{yz2} =$ 1,406 m^2
$I_{yz3} =$ -0,281 m^2
$I_{yz4} =$ 0,844 m^2

Koko paaluryhmän tulo- ja jäyhyysmomentit painopisteakselien z ja y suhteen

$$I_z = \Sigma I_{zi} \tag{13}$$

= 2,250 m^2

$$I_y = \Sigma I_{yi} \tag{14}$$

= 6,188 m^2

$$I_{yz} = \Sigma I_{yzi} \tag{15}$$

= 1,125 m^2

Pääjäyhyyssuunnan kulma

$$\alpha = \frac{1}{2}\arctan\frac{2I_{yz}}{I_y - I_z} \tag{16}$$

= 0,260 rad
= 14,872 °

Pääjäyhyysmomentit

$$I_\zeta = I_z\cos^2\alpha + I_y\sin^2\alpha - I_{yz}\sin(2\alpha) \tag{17}$$

= 1,951 m^2

$$I_\psi = I_z\sin^2\alpha + I_y\cos^2\alpha + I_{yz}\sin(2\alpha) \tag{18}$$

= 6,486 m^2

Jäyhyyssäteen neliöt ζ- ja ψ-akselien suhteen

$$i_\zeta^2 = I_\zeta / A \tag{19}$$

= 0,488 m^2

$$i_\psi{}^2 = I_\psi/A \tag{20}$$
$$= 1{,}622\ \mathrm{m}^2$$

Paalujen ζ-koordinaatit

$$\zeta_i = y_i \sin\alpha + z_i \cos\alpha \tag{21}$$
$$\zeta_1 = -0{,}895\ \mathrm{m}$$
$$\zeta_2 = 2{,}005\ \mathrm{m}$$
$$\zeta_3 = 0{,}170\ \mathrm{m}$$
$$\zeta_4 = -1{,}280\ \mathrm{m}$$

Paalujen ψ-koordinaatit

$$\psi_i = y_i \cos\alpha - z_i \sin\alpha \tag{22}$$
$$\psi_1 = 1{,}014\ \mathrm{m}$$
$$\psi_2 = 0{,}244\ \mathrm{m}$$
$$\psi_3 = -0{,}821\ \mathrm{m}$$
$$\psi_4 = -0{,}436\ \mathrm{m}$$

Sydänkuviota rajoittavien suorien yhtälöt ψ-ζ-koordinaatistossa ovat muotoa

$$0 = 1 + \frac{\varsigma_{ip}}{i_\psi^2}\varsigma + \frac{\psi_{ip}}{i_\varsigma^2}\psi \tag{23}$$

$$\Rightarrow \psi(\varsigma) = A\varsigma + B \tag{24}$$

jossa

$$A = -\frac{\varsigma_{ip} i_\varsigma^2}{\psi_{ip} i_\psi^2} \tag{25}$$

$$B = -\frac{i_\varsigma^2}{\psi_{ip}} \tag{26}$$

Sijoittamalla saadaan

	A_ι		B_ι
$\psi_1(\zeta) =$	$0{,}266\ \zeta$	+	$-0{,}481$ [m]
$\psi_2(\zeta) =$	$-2{,}475\ \zeta$	+	$-2{,}002$ [m]
$\psi_3(\zeta) =$	$0{,}062\ \zeta$	+	$0{,}594$ [m]
$\psi_4(\zeta) =$	$-0{,}883\ \zeta$	+	$1{,}119$ [m]

Suorien *i* ja *j* leikkauspisteen ζ-koordinaatti

$$\varsigma_{ij} = \frac{B_i - B_j}{A_j - A_i} \tag{27}$$

Suorien *i* ja *j* leikkauspisteen koordinaatit on esitetty taulukossa 2.

Taulukko 2.

ij	ζ_{ij} m	ψ_{ij} m	z_{ij} m	y_{ij} m
12	-0,555	-0,629	-0,375	-0,750
23	-1,023	0,530	-1,125	0,250
34	0,555	0,629	0,375	0,750
41	1,393	-0,111	1,375	0,250

Sydänkuviota rajoittavien suorien yhtälöt y-z-koordinaatistossa ovat muotoa

$$y(z) = Cz + D \tag{28}$$

jossa

$$C = \tan(\alpha + \arctan A) \tag{29}$$

$$D = B(\cos\alpha + C\sin\alpha) \tag{30}$$

Sijoittamalla saadaan

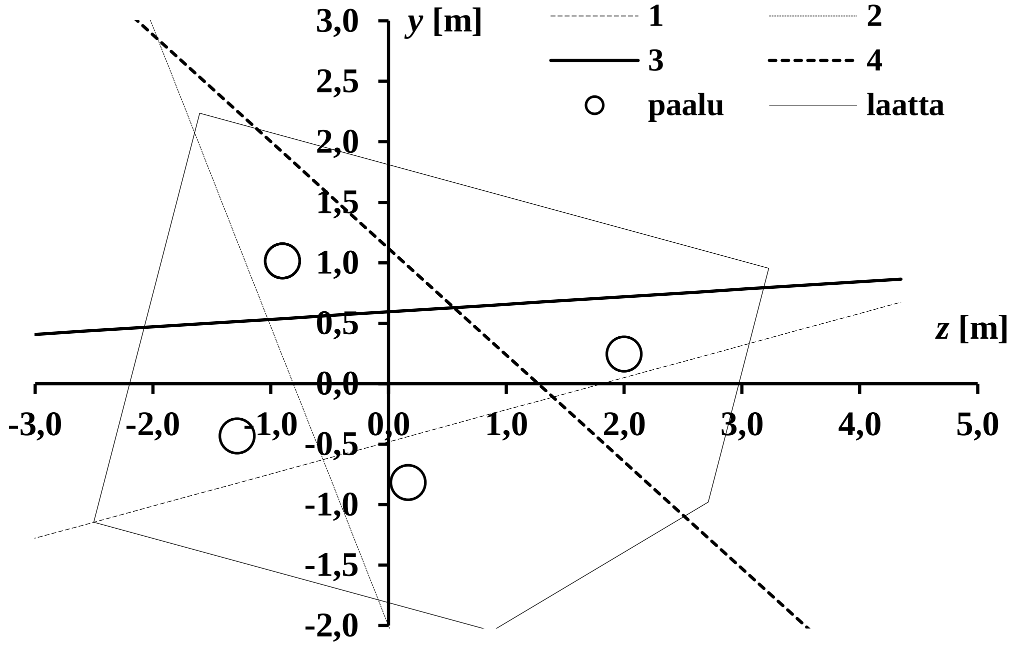
	C_ι			D_ι	
$y_1(z) =$	0,571	z	+	-0,536	[m]
$y_2(z) =$	-1,333	z	+	-1,250	[m]
$y_3(z) =$	0,333	z	+	0,625	[m]
$y_4(z) =$	-0,500	z	+	0,938	[m]

Suorien *i* ja *j* leikkauspisteen z-koordinaatti

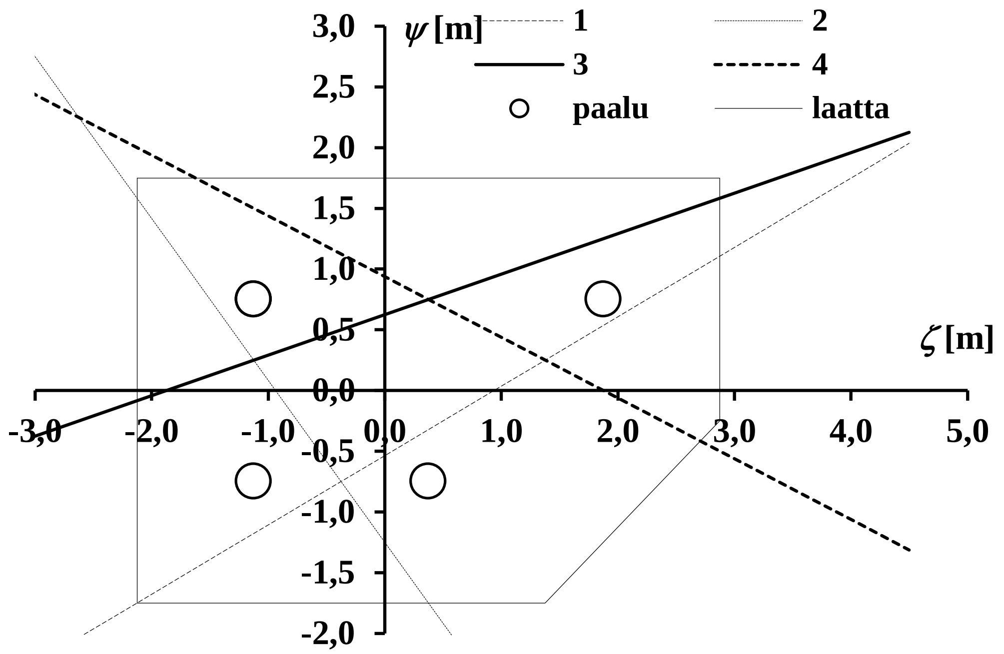
$$z_{ij} = \frac{D_i - D_j}{C_j - C_i} \tag{31}$$

Suorien *i* ja *j* leikkauspisteen koordinaatit on esitetty taulukossa 2.

<u>VASTAUS</u>: ♥-kuviota rajoittavat suorat 1, 2, 3 ja 4 on esitetty kuvissa 2 ja 3.



Kuva 2. Pääjäyhyyssuunnan koordinaatisto.



Kuva 3. Painopistekoordinaatisto.

TOINEN TAPA (vertaa esimerkkitehtävä 29)

Painopiste lasketaan kuten edellä

$$z_{0p} = -0{,}375 \text{ m}$$
$$y_{0p} = 0{,}000 \text{ m}$$

Taulukko 3. Jäykkyysmatriisin alkioiden laskenta.

Muuttuja	Plu	Sijaintikoordinaatit			Kulma	Jäykkyys
Symboli	*i*	x_i	y_i	z_i	α_i	k_i
Yksikkö		m	m	m	°	-
Lukuarvo	1	0	0,750	-1,125	0	1
	2	0	0,750	1,875	0	1
	3	0	-0,750	0,375	0	1
	4	0	-0,750	-1,125	0	1

⋮

Muuttuja	Plu	Suuntakulman kosinit			Norm.voiman momenttivarsi		
Symboli	*i*	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Yksikkö		-	-	-	m	m	m
Lukuarvo	1	1,000	0,000	0,000	0,000	-1,125	-0,750
	2	1,000	0,000	0,000	0,000	1,875	-0,750
	3	1,000	0,000	0,000	0,000	0,375	0,750
	4	1,000	0,000	0,000	0,000	-1,125	0,750

⋮

Muuttuja	Plu	Paalun jäykkyysmatriisin alkiot					
Symboli	*i*	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Lukuarvo	1	1,000	-1,125	-0,750	1,266	0,844	0,563
	2	1,000	1,875	-0,750	3,516	-1,406	0,563
	3	1,000	0,375	0,750	0,141	0,281	0,563
	4	1,000	-1,125	0,750	1,266	-0,844	0,563
Yksikkö		MN/m	MN	MN	MNm	MNm	MNm
Lukuarvo		4,000	0,000	0,000	6,188	-1,125	2,250
Symboli		k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Muuttuja		Paaluryhmän jäykkyysmatriisin alkiot					

Jäykkyysmatriisi, jonka alkioiden laskenta on esitetty taulukossa 3.

$$[K]=\begin{bmatrix} 4{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 6{,}188 & -1{,}125 \\ 0{,}000 & -1{,}125 & 2{,}250 \end{bmatrix} \quad \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

Jäykkyysmatriisin käänteismatriisi

$$[K]^{-1}=\begin{bmatrix} 0{,}250 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}178 & 0{,}089 \\ 0{,}000 & 0{,}089 & 0{,}489 \end{bmatrix} \quad \begin{bmatrix} \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{1/N} & \mathrm{1/N} & \mathrm{1/Nm} \end{bmatrix}$$

Tasapainoehdosta

$$\{F\} = [K]\{\delta\} \tag{32}$$

saadaan paaluryhmän siirtymävektori

$$\{\delta\} = [K]^{-1}\{F\} \tag{33}$$

$$\Rightarrow \begin{Bmatrix} u \\ \varphi \\ \theta \end{Bmatrix} = [K]^{-1} \begin{Bmatrix} 1 \\ z \\ -y \end{Bmatrix} \quad \begin{matrix} \text{(Pystyvoima } x \text{-akselin suuntaan [MN])} \\ \text{(Momentti } y \text{-akselin ympäri [MNm])} \\ \text{(Momentti } z \text{-akselin ympäri [MNm])} \end{matrix} \tag{34}$$

$$\Rightarrow \begin{cases} u = 0{,}000\,y + 0{,}000\,z + 0{,}250 \\ \varphi = -0{,}089\,y + 0{,}178\,z + 0{,}000 \\ \theta = -0{,}489\,y + 0{,}089\,z + 0{,}000 \end{cases}$$

Paaluvoima

$$N_i = k_i \Delta_i \tag{35}$$

$$= k_i (p_{x,i}\,u + r_{y,i}\,\varphi + r_{z,i}\,\theta) \tag{36}$$

$$N_1 = 0{,}467\,y + -0{,}267\,z + 0{,}250 = 0$$

$$N_2 = 0{,}200\,y + 0{,}267\,z + 0{,}250 = 0$$

$$N_3 = -0{,}400\,y + 0{,}133\,z + 0{,}250 = 0$$

$$N_4 = -0{,}267\,y + -0{,}133\,z + 0{,}250 = 0$$

Sydänkuviota rajoittavien suorien yhtälöt painopistekoordinaatistossa

$$y_1(z) = 0{,}571\,z + -0{,}536 \text{ [m]}$$

$$y_2(z) = -1{,}333\,z + -1{,}250 \text{ [m]}$$

$$y_3(z) = 0{,}333\,z + 0{,}625 \text{ [m]}$$

$$y_4(z) = -0{,}500\,z + 0{,}938 \text{ [m]}$$

VASTAUS: **Kuten edellä painopistekoordinaatistossa (kuva 3).**

14. TEHTÄVÄ

Määritä kuvan 1 liittopoikkileikkauksen (peruslaatan) sydänkuvio!

Betonin lujuusluokka

$K_1 =$ 60 MN/m^2

$K_2 =$ 20 MN/m^2

Mitat

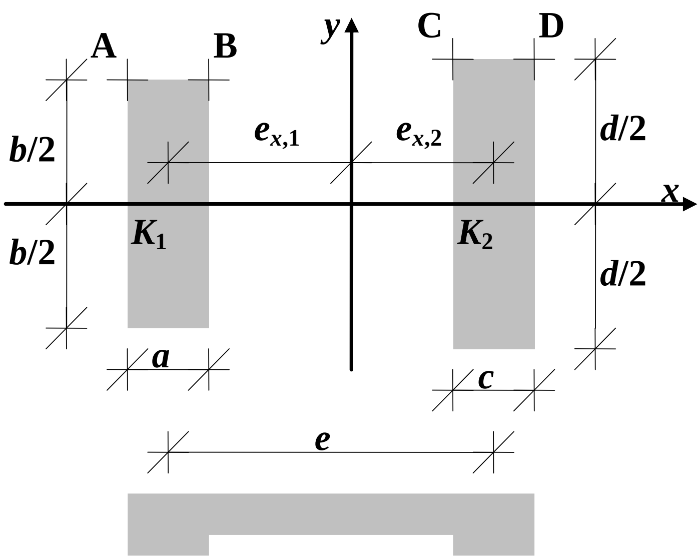
$a =$ 1 m

$b =$ 3 m

$c =$ 1 m

$d =$ 3 m

$e =$ 4 m



Kuva 1.

Betonin kimmokerroin

$$E_i = k\sqrt{K_i K_0} \tag{1}$$

jossa k = 1 ja $K_0 = 25 \cdot 10^6$ MN/m². Osan i kimmokerroin

$$E_1 = 38730 \text{ MN/m}^2$$

$$E_2 = 22361 \text{ MN/m}^2$$

Osan i pinta-ala

$$A_1 = ab \tag{2}$$

$$= 3{,}000 \text{ m}^2$$

$$A_2 = cd \tag{3}$$

$$= 3{,}000 \text{ m}^2$$

Osan i puristusjäykkyys

$$C_i = E_i A_i \tag{4}$$

$$C_1 = 116\,190 \text{ MN}$$

$$C_2 = 67\,082 \text{ MN}$$

Poikkileikkauksen puristusjäykkyys

$$C = \sum_{i=1}^{n} C_i \tag{5}$$

$$= 183\,272 \text{ MN}$$

Koordinaatistona käytetään pääjäykkyyskoordinaatistoa, jonka origo sijaitsee kimmokertoimella painotetussa painopisteessä.

Osan jäyhyysmomentti sen oman x -akselin suuntaisen painopisteakselin x' suhteen (symmetriasyistä pääjäyhyysakseli yhtyy x -akseliin)

$$I_{x',1} = \frac{ab^3}{12} \tag{6}$$

$$= 2{,}250 \text{ m}^4$$

$$I_{x',2} = \frac{cd^3}{12} \tag{7}$$

$$= 2{,}250 \text{ m}^4$$

Osan jäyhyysmomentti sen oman *y* -akselin suuntaisen pääjäyhyysakselin *y* ' suhteen

$$I_{y',1} = \frac{ba^3}{12} \qquad (8)$$

$$= \quad 0{,}250 \text{ m}^4$$

$$I_{y',2} = \frac{dc^3}{12} \qquad (9)$$

$$= \quad 0{,}250 \text{ m}^4$$

Yleisesti homogeeniselle poikkileikkauksella osan *i* jäyhyysmomentti *r* -akselin suhteen on Steinerin säännön mukaan

$$I_{r,i} = I_{r',i} + A_i e_{s,i}^2 \qquad (10)$$

jossa $I_{r',i}$ on osan i jäyhyysmomentti *r* -akselin suuntaisen osan oman *r'* -painopisteakselin suhteen ja $e_{s,i}$ on *r* - ja *r* '-akselien välinen etäisyys (*s* -akseli on kohtisuorassa *r* -akselia vastaan).

Liittorakenteelle osan *i* taivutusjäykkyys on vastaavasti

$$D_{r,i} = D_{r',i} + C_i e_{s,i}^2 \qquad (11)$$

Osan *i* taivutusjäykkyys *x'* - ja *y'* -akselin (yleisesti *r* '-akselin) suhteen

$$D_{r',i} = E_i I_{r',i} \qquad (12)$$

$$D_{x',1} = \quad 87\ 142 \text{ MNm}^2$$

$$D_{x',2} = \quad 50\ 312 \text{ MNm}^2$$

$$D_{y',1} = \quad 9\ 682 \text{ MNm}^2$$

$$D_{y',2} = \quad 5\ 590 \text{ MNm}^2$$

Puristusjäykkyyden aiheuttama momenttitasapainoyhtälö *y* -akselin eli koko poikkileikkauksen kimmokertoimella painotetun pääjäyhyysakselin suhteen on

$$C_1 e_{x,1} = C_2 e_{x,2} \qquad (13)$$

$$\Rightarrow C_1 e_{x,1} = C_2 (e - e_{x,1}) \qquad (14)$$

Osien 1 ja 2 *y* '-akselien ja *y* -akselin väliset etäisyydet

$$e_{x,1} = \frac{eC_2}{C_1 + C_2} \qquad (15)$$

$$= \quad 1{,}464 \text{ m}$$

$$e_{x,2} = e - e_{x,1} \qquad (16)$$

$$= \quad 2{,}536 \text{ m}$$

Osan *i* taivutusjäykkyys *x* - ja *y* -akselin suhteen yhtälöstä 11

$D_{x,\,1}$ = 87 142 $\mathrm{MNm^2}$

$D_{x,\,2}$ = 50 312 $\mathrm{MNm^2}$

$D_{y,\,1}$ = 258 746 $\mathrm{MNm^2}$

$D_{y,\,2}$ = 436 980 $\mathrm{MNm^2}$

Poikkileikkauksen taivutusjäykkyys *x* - ja *y* -akselin (yleisesti *r* -akselin) suhteen

$$D_r = \sum_{i=1}^{n} D_{r,i} \tag{17}$$

D_x = 137 454 $\mathrm{MNm^2}$

D_y = 695 726 $\mathrm{MNm^2}$

Homogeeniselle poikkileikkaukselle jäyhyyssäde *s* -akselin suhteen on

$$i_s = \sqrt{\frac{I_s}{A}} \tag{18}$$

Liittorakenteelle vastaavasti

$$i_s = \sqrt{\frac{D_s}{C}} \tag{19}$$

***x* - ja *y* -akselien suhteen olevien jäyhyyssäteiden neliöt**

$$i_x^2 = \frac{D_x}{C} \tag{20}$$

= 0,750 $\mathrm{m^2}$

$$i_y^2 = \frac{D_y}{C} \tag{21}$$

= 3,796 $\mathrm{m^2}$

***y* -akselin positiivisella puolella olevien ulkonurkkien koordinaatit**

$$x_{\mathrm{Ap}} = -e_{x,\,1} - a/2 \tag{22}$$

= -1,964 m

$$y_{\mathrm{Ap}} = b/2 \tag{23}$$

= 1,500 m

$$x_{\mathrm{Bp}} = -e_{x,\,1} + a/2 \tag{24}$$

= -0,964 m

$$y_{\mathrm{Bp}} = b/2 \tag{25}$$

= 1,500 m

$$x_{\mathrm{Cp}} = e_{x,2} - c/2 \tag{26}$$
$$= 2{,}036 \text{ m}$$
$$y_{\mathrm{Cp}} = d/2 \tag{27}$$
$$= 1{,}500 \text{ m}$$
$$x_{\mathrm{Dp}} = e_{x,2} + c/2 \tag{28}$$
$$= 3{,}036 \text{ m}$$
$$y_{\mathrm{Dp}} = d/2 \tag{29}$$
$$= 1{,}500 \text{ m}$$

Sydänkuviota rajoittavien suorien yhtälöt ovat muotoa

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \tag{30}$$

$$\Rightarrow y(x) = \alpha x + \beta \tag{31}$$

jossa

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \tag{32}$$

$$\beta = -\frac{i_x^2}{y_{ip}} \tag{33}$$

Sijoittamalla saadaan

	α_ι			β_ι	
$y_{\mathrm{A}}(x) =$	0,259	x	+	-0,500	[m]
$y_{\mathrm{B}}(x) =$	0,127	x	+	-0,500	[m]
$y_{\mathrm{C}}(x) =$	-0,268	x	+	-0,500	[m]
$y_{\mathrm{D}}(x) =$	-0,400	x	+	-0,500	[m]

Suorien i ja j leikkauspisteen x-koordinaatti

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \tag{34}$$

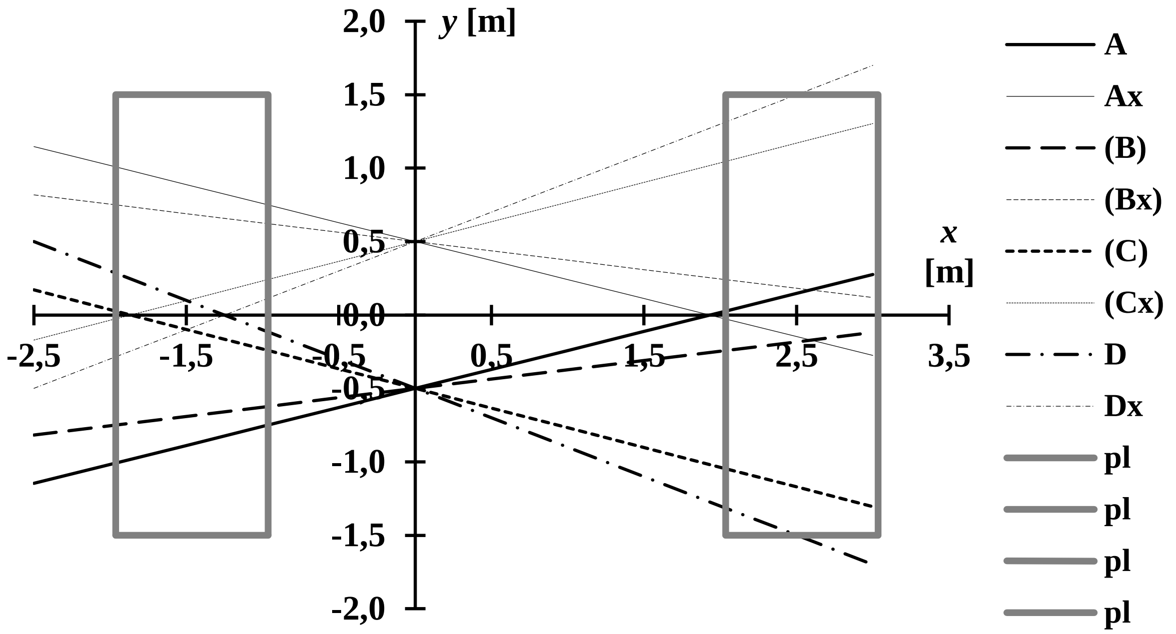
Suorien *i* ja *j* leikkauspisteen koordinaatit on esitetty taulukossa 1.

Taulukko 1.

ij	x_{ij} m	y_{ij} m
A*x*	1,933	0,000
B*x*	3,937	0,000
C*x*	-1,865	0,000
D*x*	-1,250	0,000
AB	0,000	-0,500
BC	0,000	-0,500
CD	0,000	-0,500

<u>VASTAUS</u>:

Sydänkuviota rajoittavat suorat: A ja D sekä näiden peilaukset *x* -akselin suhteen (Ax, Dx). (Suorat C ja D rajoittavat vain yhdessä pisteessä.)



Kuva 2.

15. TEHTÄVÄ

Monikomponenttisauvan koostuu n :stä sisäkkäisestä yhdensuuntaisesta putkesta. Poikkileikkauksessa (x-y -taso) kunkin renkaan sisä- ja ulkoreunat ovat ellipsin muotoisia siten, että kaikkien renkaiden pääjäyhyyskoordinaatistot yhtyvät (kuva 1). Sisin rengas on reiätön ellipsialue.

Putken numero i ulkoreunaellipsin puolisäteet ovat

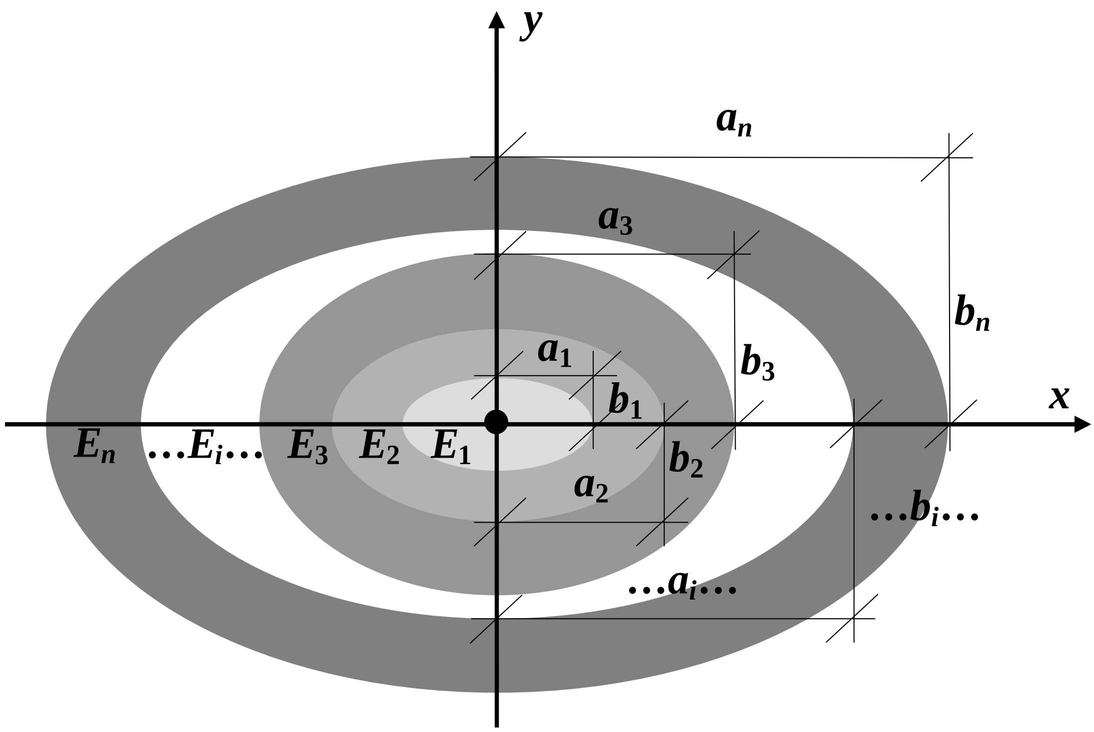
$$a_i > 0 \wedge r_i \in \mathrm{R}$$
$$b_i > 0 \wedge r_i \in \mathrm{R}$$

ja kimmokerroin on

$$E_i \geq 0 \wedge E_i \in \mathrm{R}$$

Näissä $i \in \mathrm{N} \wedge i \in \{1 \ldots n\}$. Raja-arvolla $E_i = 0$ materiaalia ei ole (esim. tyhjiö).

Määritä poikkileikkauksen sydänkuvio ja sovella tulosta (umpinaiseen) homogeeniseen sauvaan sekä (sisältä tyhjään) äärettömän ohueen putkeen, jonka poikkileikkaus on ympyrän muotoinen!



Kuva 1.

Ellipsin yhtälö on (kuva 2)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$

Ellipsin pistettä (x_p, y_p) vastaava sydänkuviota rajoittava suora on

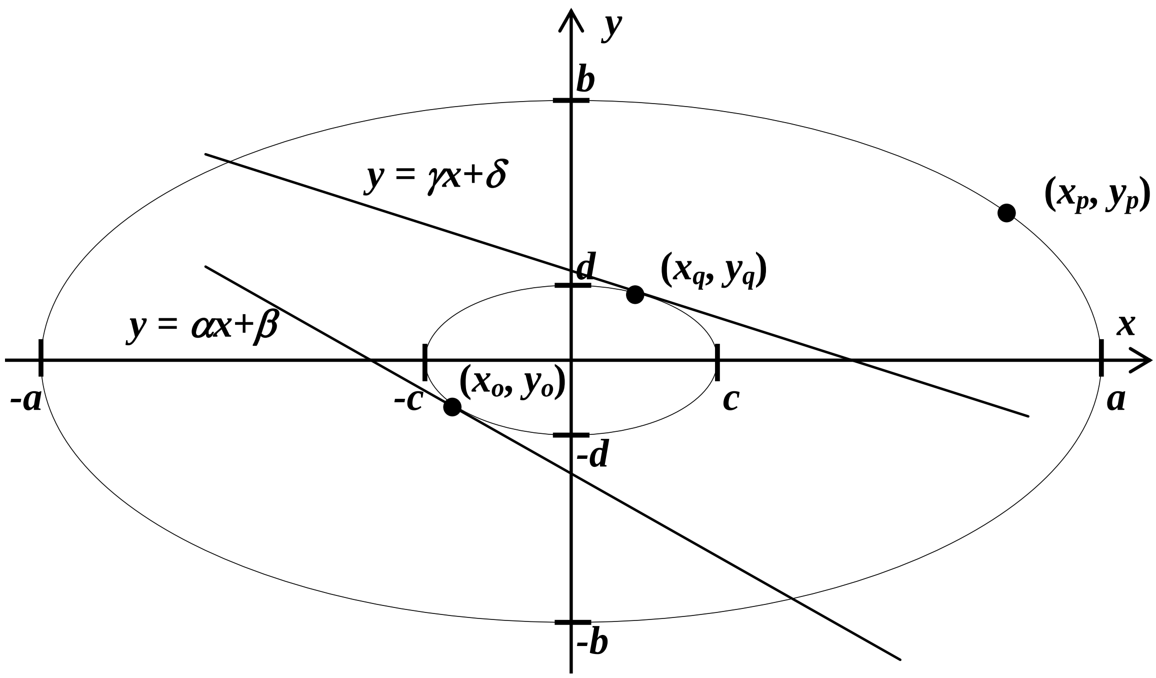
$$1+\frac{x_p}{i_y^2}x+\frac{y_p}{i_x^2}y=0 \tag{2}$$

$$\Rightarrow \qquad y=\alpha x+\beta \tag{3}$$

jossa

$$\alpha=-\frac{x_p}{y_p}\frac{i_x^2}{i_y^2} \tag{4}$$

$$\beta=-\frac{i_x^2}{y_p} \tag{5}$$



Kuva 2.

Oletetaan, että ellipsialueen (reuna: yhtälö 1) sydänkuvion reunan muoto on ellipsi

$$\frac{x^2}{c^2}+\frac{y^2}{d^2}=1 \tag{6}$$

Tämän oletetun sydänkuvioellipsin tangentti pisteessä (x_q, y_q) on

$$\frac{x_q}{c^2}x + \frac{y_q}{d^2}y = 1 \tag{7}$$

$$\Rightarrow \qquad y = \gamma x + \delta \tag{8}$$

jossa

$$\gamma = -\frac{x_q}{y_q}\frac{d^2}{c^2} \tag{9}$$

$$\delta = \frac{d^2}{y_q} \tag{10}$$

Ellipsin (yhtälö 1) pistettä (a, 0) vastaava sydänkuviota rajoittava suora on

$$1 + \frac{a}{i_y^2}x = 0 \tag{11}$$

josta oletetun sydänkuvioellipsin x- akselin suuntaiseksi puolisäteeksi pisteessä (-c, 0) saadaan

$$c = \frac{i_y^2}{a} \tag{12}$$

Vastaavasti pistettä (0, b) vastaava sydänkuviota rajoittava suora on

$$1 + \frac{b}{i_x^2}y = 0 \tag{13}$$

josta oletetun sydänkuvioellipsin y- akselin suuntaiseksi puolisäteeksi pisteessä (0, -d) saadaan

$$d = \frac{i_x^2}{b} \tag{14}$$

Puolisäteiden c ja d avulla voidaan yhtälöt 4 ja 5 kirjoittaa muotoon

$$\alpha = -\frac{x_p}{y_p}\frac{bd}{ac} \tag{15}$$

$$\beta = -\frac{bd}{y_p} \tag{16}$$

Ellipsin (yhtälö 1) pistettä (x_p, y_p) vastaavan sydänkuviota rajoittavan suoran (yhtälö 3) ja oletetun sydänkuvioellipsin (yhtälö 6) leikkauspiste (x_o, y_o) on

$$\begin{cases} \dfrac{x_o^2}{c^2} + \dfrac{y_o^2}{d^2} = 1 \\ y_o = \alpha x_o + \beta \end{cases} \tag{17a, b}$$

$$\Rightarrow \begin{cases} x_o = \dfrac{-c^2\alpha\beta \pm cd\sqrt{q}}{d^2 + c^2\alpha^2} \\ y_o = \dfrac{d^2\beta \pm cd\sqrt{q}}{d^2 + c^2\alpha^2} \end{cases} \tag{18a, b}$$

jossa

$$q = d^2 + c^2\alpha^2 - \beta^2 \tag{19}$$

$$\Rightarrow q = d^2 + c^2\left(\frac{x_p}{y_p}\frac{bd}{ac}\right)^2 - \left(\frac{bd}{y_p}\right)^2 \quad \Bigg| \cdot \frac{y_p^2}{b^2 d^2} \tag{20}$$

$$\Rightarrow \frac{y_p^2}{b^2 d^2} q = \frac{y_p^2}{b^2} + \frac{x_p^2}{a^2} - 1 \tag{21}$$

$$\Rightarrow q = 0 \tag{22}$$

Eli leikkauspiste on (yhtälöistä 18 ja 22)

$$\begin{cases} x_o = \dfrac{-c^2\alpha\beta}{d^2 + c^2\alpha^2} \\ y_o = \dfrac{d^2\beta}{d^2 + c^2\alpha^2} \end{cases} \tag{23a, b}$$

Koska piste $(x_q,\ y_q)$ on oletetun sydänkuvioellipsin (yhtälö 6) mielivaltainen piste, voimme sijoittaa

$$\begin{cases} x_q = x_o \\ y_q = y_o \end{cases} \tag{24a, b}$$

Gamma (yhtälöistä 9, 24 ja 23) on

$$\gamma = -\frac{-c^2\alpha\beta}{d^2 + c^2\alpha^2}\frac{d^2 + c^2\alpha^2}{d^2\beta}\frac{d^2}{c^2} \tag{25}$$

$$\Rightarrow \gamma = \alpha \tag{26}$$

Delta (yhtälöistä 10, 24b, 23b ja 15) on

$$\delta = d^2\frac{d^2 + c^2\alpha^2}{d^2\beta} \tag{27}$$

$$\Rightarrow \delta = \frac{d^2}{\beta}\left[1 + \frac{c^2}{d^2}\left(\frac{x_p}{y_p}\frac{bd}{ac}\right)^2\right] \tag{28}$$

$$\Rightarrow \delta = \frac{d^2}{\beta}\left[1 + \frac{x_p^2}{y_p^2}\frac{b^2}{a^2}\right] \tag{29}$$

Koska yhtälöstä 1 saadaan

$$\frac{x_p^2}{y_p^2}\frac{b^2}{a^2} + 1 = \frac{b^2}{y_p^2} \tag{30}$$

niin (yhtälöstä 16)

$$\delta = \frac{d^2}{\beta}\frac{b^2}{y_p^2} \tag{31}$$

$$\Rightarrow \delta = \beta \tag{32}$$

Joten ellipsin (yhtälö 1) pistettä (x_p, y_p) vastaava sydänkuviota rajoittava suora (yhtälö 3) ja oletetun sydänkuvioellipsin tangentti (yhtälö 8) yhtyvät. Ellipsialueen sydänkuvio on siis ellipsialue.

Monikomponenttisauvan sydänkuvioellipsin puolisäde x -akselin suunnassa on (yhtälö 12)

$$c = \frac{D_y}{a_n C} \tag{33}$$

$$\Rightarrow c = \frac{\sum_{i=1}^{n} E_i I_{yi}}{a_n \sum_{i=1}^{n} E_i A_i} \tag{34}$$

$$\Rightarrow c = \frac{\sum_{i=1}^{n} E_i \left(b_i a_i^3 - b_{i-1} a_{i-1}^3 \right)}{4 a_n \sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1} \right)} \tag{35}$$

Näissä D_y on taivutusjäykkyys y- akselien suhteen ja C on aksiaalijäykkyys koko poikkileikkaukselle. I_{yi} on vastaava taivutusjäyhyys ja A_i pinta-ala renkaalle i.

Vastaavasti saadaan puolisäde y -akselin suunnassa (yhtälö 14)

$$d = \frac{D_x}{b_n C} \tag{36}$$

$$\Rightarrow d = \frac{\sum_{i=1}^{n} E_i I_{xi}}{b_n \sum_{i=1}^{n} E_i A_i} \tag{37}$$

$$\Rightarrow d = \frac{\sum_{i=1}^{n} E_i \left(a_i b_i^3 - a_{i-1} b_{i-1}^3 \right)}{4 b_n \sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1} \right)} \tag{38}$$

Homogeenisen, umpinaisen poikkileikkauksen tapauksessa $n = 1$ ja

$$c = \frac{a_1}{4} \tag{39}$$

$$d = \frac{b_1}{4} \tag{40}$$

Sydänkuvioalue on yhdenmuotoinen poikkileikkausalueen kanssa

$$\frac{a_1}{c} = \frac{b_1}{d} = 4 \tag{41}$$

Homogeenisen, reiällisen renkaan tapauksessa $n = 2$ ja $E_1 = 0$ ja

$$c = \frac{b_2 a_2^3 - b_1 a_1^3}{4a_2\left(a_2 b_2 - a_1 b_1\right)} \tag{42}$$

$$d = \frac{a_2 b_2^3 - a_1 b_1^3}{4b_2\left(a_2 b_2 - a_1 b_1\right)} \tag{43}$$

Ohuen ympyrärenkaan tapauksessa $b_i = a_i$ ja $a_1 \to a_2$ ja

$$\left.\begin{matrix} c \\ d \end{matrix}\right\} = \frac{a_2}{2} = r \tag{44}$$

jossa r on sydänkuvioympyrän säde.

<u>VASTAUS</u>:

Ellipsisymmetrisen n -osaisen moniomponenttisauvan poikkileikkauksen sydänkuvio on ellipsiala, jonka yhtälö poikkileikkauksen pääjäykkyyskoordinaatistossa on

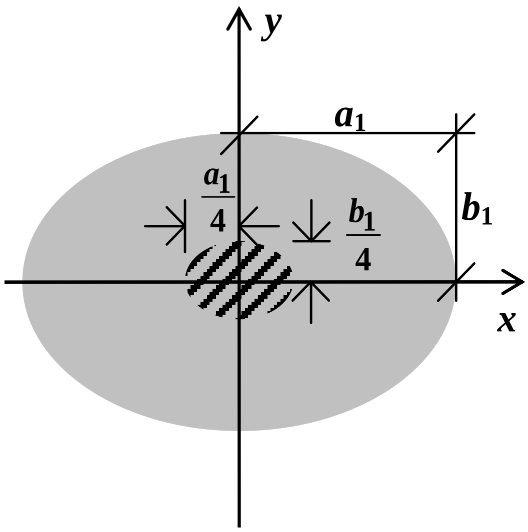
$$16a_n^2 \frac{\left[\sum_{i=1}^{n} E_i\left(a_i b_i - a_{i-1} b_{i-1}\right)\right]^2}{\left[\sum_{i=1}^{n} E_i\left(b_i a_i^3 - b_{i-1} a_{i-1}^3\right)\right]^2} x^2 + 16b_n^2 \frac{\left[\sum_{i=1}^{n} E_i\left(a_i b_i - a_{i-1} b_{i-1}\right)\right]^2}{\left[\sum_{i=1}^{n} E_i\left(a_i b_i^3 - a_{i-1} b_{i-1}^3\right)\right]^2} y^2 \le 1 \tag{45}$$

Homogeenisen poikkileikkauksen tapauksessa sydänkuvion yhtälö on (kuva 3)

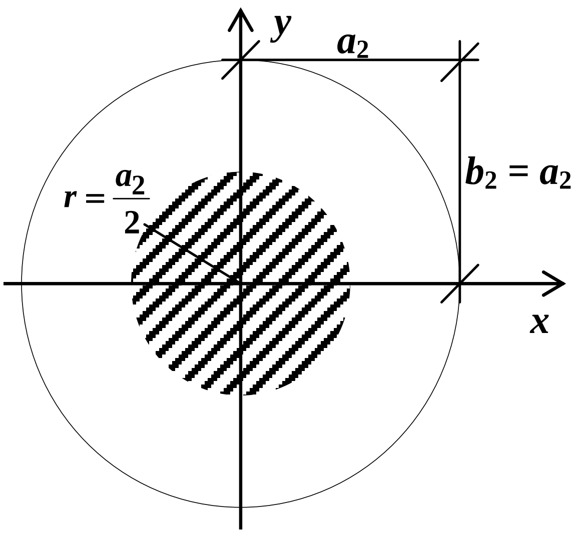
$$\frac{16}{a_1^2} x^2 + \frac{16}{b_1^2} y^2 \le 1 \tag{46}$$

Äärettömän ohuen ympyrärenkaan tapauksessa sydänkuvion yhtälö on (kuva 4)

$$x^2 + y^2 \leq \frac{a_2^2}{4} \tag{47}$$



Kuva 3.



Kuva 4.

16. TEHTÄVÄ

Kallionvaraiseen peruslaattaan (kuva 1) vaikuttaa keskeinen pystykuorma ja momentti

F_{zo} = 2 MN
M_{xo} = 0,350 MNm

A) Etsi peruslaatan pienin sivumitta a, jolla koko pohjan ala on puristettu. Mikä on tällöin suurin jännitys?

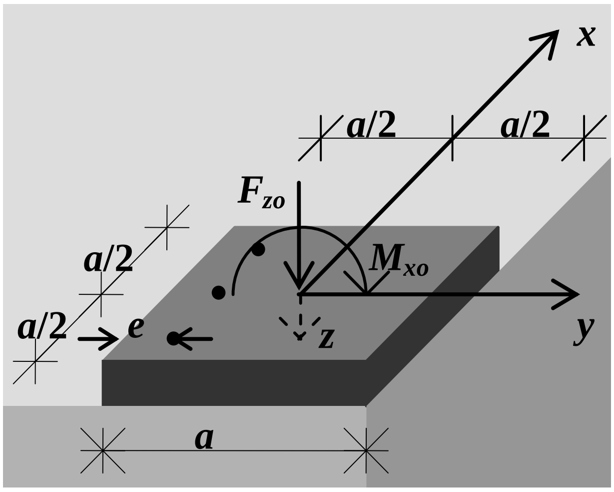
B) Etsi pienin sivumitta a siinä tapauksessa, että jännitys kallion pinnassa saa olla enintään (*RIL 121-2004*, kohta 5.5.3.1, s. 82)

σ_{max} = 10 MN/m^2

C) Etsi pienin sivumitta a B-kohdan tapauksessa, kun käytetään lisäksi hyväksi ankkurointia, jonka etäisyys reunasta on

e = 0,10 m

Kuinka suuri ankkurointivoima tällöin tarvitaan?

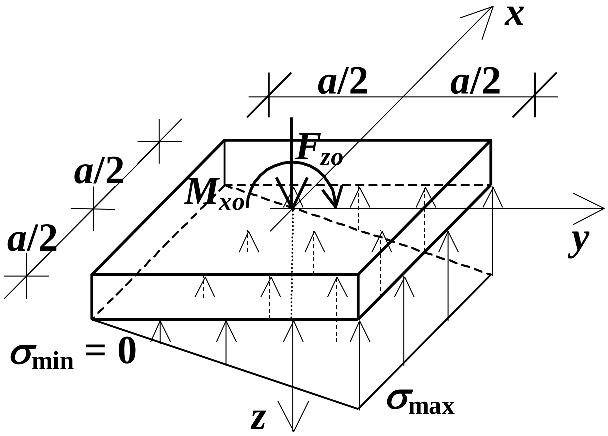


Kuva 1.

A)

Normaalivoiman ja taivutusmomentin aiheuttama normaalijännitys (kuva 2)

$$\sigma(y) = \frac{F_z}{A} + \frac{M_x}{I_x} y \tag{1}$$



Kuva 2. Jännityskuvaaja on kolmisivuinen suora särmiö.

Neliöpoikkileikkauksen ala

$$A = a^2 \tag{2}$$

Jäyhyysmomentti x -akselin suhteen

$$I_x = a^4/12 \tag{3}$$

Yhtälöistä 1, 2 ja 3

$$\sigma(y) = \frac{F_{zo}}{a^2} + 12\frac{M_{xo}}{a^4} y \tag{4}$$

Normaalijännitys reunalla

$$\sigma_{min} = \sigma\left(y = \frac{-a}{2}\right) = 0 \tag{5}$$

$$\Rightarrow \quad 0 = \frac{F_{zo}}{a^2} - 6\frac{M_{xo}}{a^3} \tag{6}$$

Sivumitta

$$a = 6\frac{M_{xo}}{F_{zo}} \tag{7}$$

$$= \quad 1{,}050 \text{ m}$$

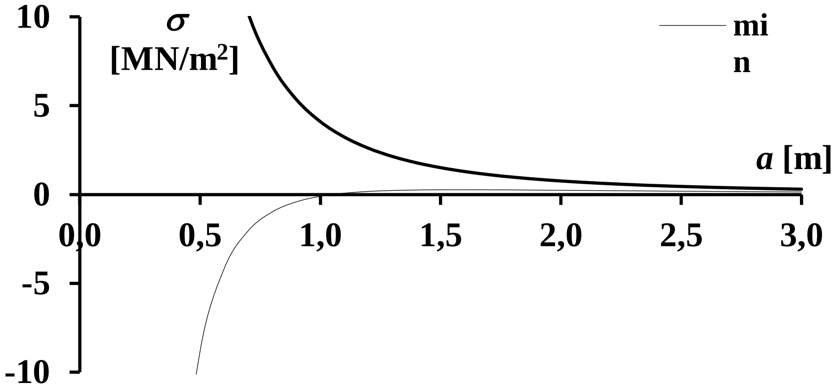
Jännitys vastakkaisella reunalla

$$\sigma_{max} = \sigma\left(\frac{a}{2}\right) \tag{8}$$

$$= \frac{F_{zo}}{a^2} + 6\frac{M_{xo}}{a^3} \tag{9}$$

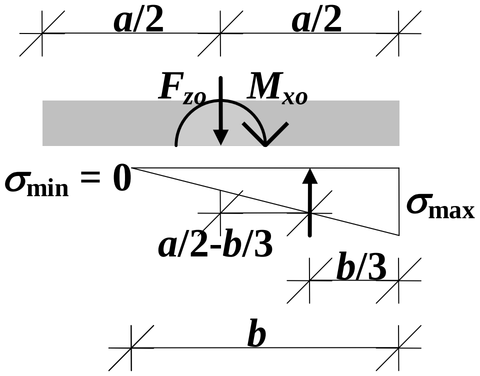
$$= \quad 3{,}628 \ \text{MN/m}^2$$

Reunajännitykset laatan leveyden funktiona on esitetty kuvassa 3.



Kuva 3.

B)



Kuva 4.

Tasapainoehdot (kuva 4)

$$\begin{cases} \sum F_z = 0 \\ \sum M_x = 0 \end{cases} \tag{10a, b}$$

$$\Rightarrow \quad \begin{cases} 0 = F_{zo} - \dfrac{ab}{2}\sigma_{\max} \\ 0 = M_{xo} - \left(\dfrac{a}{2} - \dfrac{b}{3}\right)\dfrac{ab}{2}\sigma_{\max} \end{cases} \tag{11a, b}$$

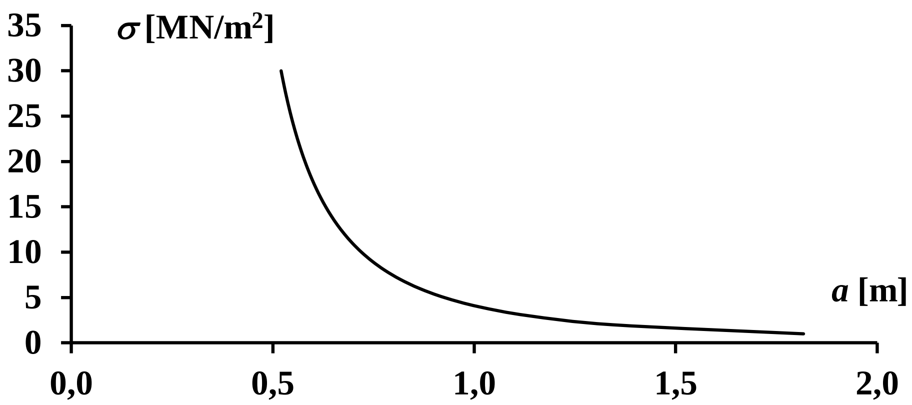
$$\Rightarrow \quad \begin{cases} b = \dfrac{2F_{zo}}{a\sigma_{\max}} \\ a = \dfrac{-M_{xo} \overset{(+)}{-} \sqrt{M_{xo}^2 + \dfrac{4}{3}\dfrac{F_{zo}^3}{\sigma_{\max}}}}{-F_{zo}} \end{cases} \tag{12a, b}$$

Sivumitta *a* ja puristetun alueen pituus *b*

a = 0,720 m

b = 0,555 m

Maksimireunajännitys laatan leveyden funktiona on esitetty kuvassa 5.

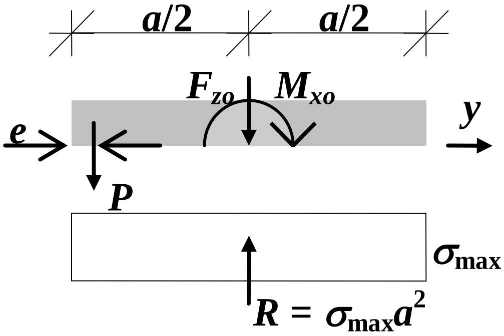


Kuva 5.

C)

Pystyvoimien ja momenttien tasapainoehdot (kuva 6)

$$\begin{cases} R = F_{zo} + P \\ M_{xo} = P\left(\dfrac{a}{2} - e\right) \end{cases} \qquad (13a, b)$$



Kuva 6.

Ratkaistaan jälkimmäisestä ankkurointivoima

$$P = \frac{M_{xo}}{\dfrac{a}{2} - e} \qquad (14)$$

Sijoitetaan pystyvoimien tasapainoehtoon

$$\sigma_{max} a^2 = F_{zo} + \frac{M_{xo}}{\dfrac{a}{2} - e} \quad \Big| \cdot (a - 2e) \qquad (15)$$

$$\Rightarrow \quad \sigma_{max} a^2 (a - 2e) = F_{zo}(a - 2e) + 2M_{xo} \qquad (16)$$

$$\Rightarrow \sigma_{max} a^3 - 2e\sigma_{max} a^2 - F_{zo} a + 2eF_{zo} - 2M_{xo} = 0 \qquad (17)$$

Yhtälö saa arvon

0,000 MNm

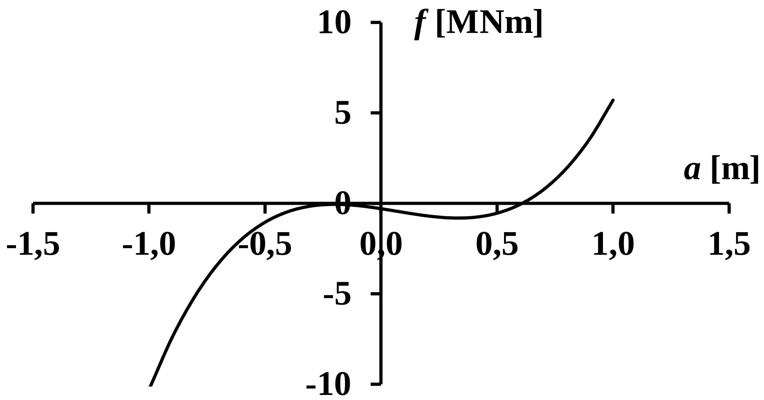
kun

a = 0,609 m

Yhtälöstä 14 saadaan tällöin ankkurointivoimaksi

$$P = 1{,}711 \text{ MN}$$

Yhtälön 17 vasemman puolen lausekkeen arvo laatan leveyden funktiona on esitetty kuvassa 7.



Kuva 7.

<u>VASTAUS</u>:

A) **Peruslaatan pienin sivumitta, jolla koko ala on puristettu, on**

$$a = 1{,}050 \text{ m.}$$

Tällöin suurin puristusjännitys on

$$\sigma_{max} = 3{,}628 \text{ MN/m}^2.$$

B) **Pienin sivumitta, kun jännitys kallion pinnassa saa olla enintään**

$$\sigma_{max} = 10 \text{ MN/m}^2,$$

on

$$a = 0{,}720 \text{ m.}$$

C) **Pienin sivumitta, kun lisäksi käytetään ankkurointia, on**

$$a = 0{,}609 \text{ m.}$$

Tällöin ankkurointivoima on

$$P = 1{,}711 \text{ MN}$$

17. TEHTÄVÄ

Kallionvaraiseen perusmuuriin (kuva 1) vaikuttaa pystykuorma

N = 0,750 MN

Rakenne ankkuroidaan esijännitysteräksillä kallioon.

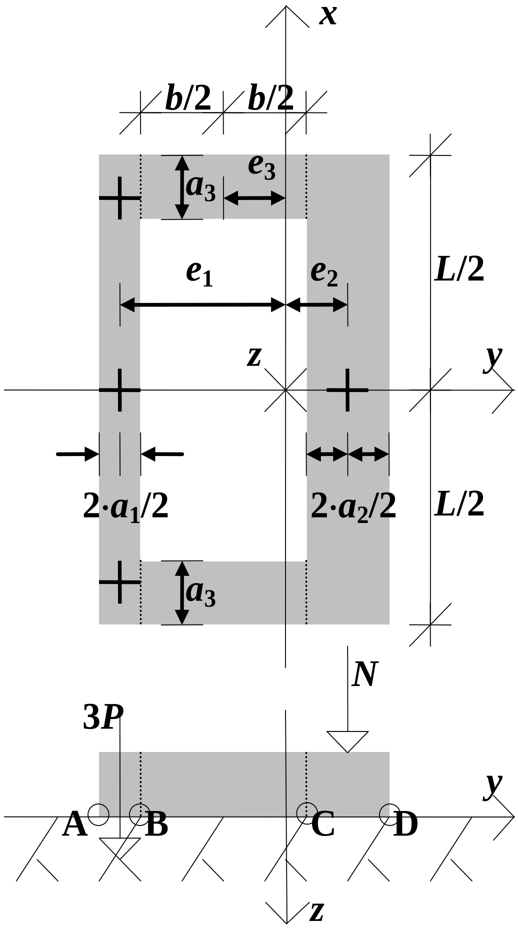
A) Minkälainen on pohjapainejakauma, kun esijännitysvoima

P = 0 MN?

B) Kuinka suuri esijännitysvoima tarvitaan, että pohjapaine olisi tasaisesti jakautunut?

Perusmuurin mitat

a_1 =	0,400	m
a_2 =	0,800	m
a_3 =	0,600	m
b =	2,000	m
L =	6,000	m



Kuva 1.

Pinta-ala

$$A_1 = a_1 L \tag{1}$$
$$= 2{,}400 \text{ m}^2$$
$$A_2 = a_2 L \tag{2}$$
$$= 4{,}800 \text{ m}^2$$
$$A_3 = a_3 b \tag{3}$$
$$= 1{,}200 \text{ m}^2$$
$$A = A_1 + A_2 + 2A_3 \tag{4}$$
$$= 9{,}600 \text{ m}^2$$

Sijoitetaan koordinaatiston origo painopisteeseen.

Koko poikkileikkauksen painopisteen etäisyys (*x*- akseli) osan 1 painopisteestä

$$e_1 = \frac{A_1 \cdot 0 + A_2(e_1 + e_2) + 2A_3(e_1 - e_3)}{A} \tag{5}$$
$$= \frac{A_2\left(\frac{a_1}{2} + b + \frac{a_2}{2}\right) + 2A_3\left(\frac{a_1}{2} + \frac{b}{2}\right)}{A} \tag{6}$$
$$= 1{,}600 \text{ m}$$

Vastaavasti

$$e_2 = \frac{a_1}{2} + b + \frac{a_2}{2} - e_1 \tag{7}$$
$$= 1{,}000 \text{ m}$$
$$e_3 = e_1 - \frac{a_1}{2} - \frac{b}{2} \tag{8}$$
$$= 0{,}400 \text{ m}$$

A)

Jäyhyysmomentti x -akselin suhteen (Steinerin sääntö)

$$I_x = \frac{La_1^3}{12} + A_1e_1^2 + \frac{La_2^3}{12} + A_2e_2^2 + 2\left(\frac{a_3b^3}{12} + A_3e_3^2\right) \tag{9}$$

$$= \quad 12{,}416 \text{ m}^4$$

Taivutusmomentti x -akselin suhteen

$$M_x = Ne_2 \tag{10}$$

$$= \quad 0{,}750 \text{ MNm}$$

Jännitys pisteessä A

$$\sigma(y) = \frac{M_x}{I_x}y + \frac{N}{A} \tag{11}$$

$$\Rightarrow \quad \sigma_A = \frac{M_x}{I_x}\left(\frac{-a_1}{2} - e_1\right) + \frac{N}{A} \tag{12}$$

$$= \quad -0{,}031 \text{ MN/m}^2 \qquad < 0, \text{ vetoa}$$

Vetoa ei voi esiintyä. Lasketaan jännitys pisteessä B siten, että osa 1 jää pois.

$$A_B = A_2 + 2A_3 \tag{13}$$

$$= \quad 7{,}200 \text{ m}^2$$

$$e_{2B} = \frac{2A_3\left(\frac{a_2}{2} + \frac{b}{2}\right)}{A_B} \tag{14}$$

$$= \quad 0{,}467 \text{ m}$$

$$e_{3B} = \frac{a_2}{2} + \frac{b}{2} - e_{2B} \tag{15}$$

$$= \quad 0{,}933 \text{ m}$$

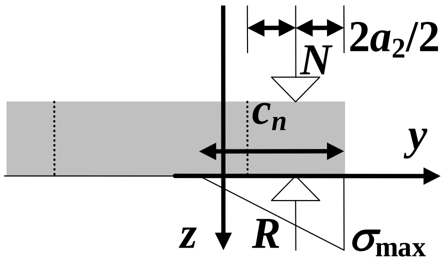
$$I_{xB} = \frac{La_2^3}{12} + A_2e_{2B}^2 + 2\left(\frac{a_3b^3}{12} + A_3e_{3B}^2\right) \tag{16}$$

$$= \quad 4{,}192 \text{ m}^4$$

$$M_{xB} = Ne_{2B} \quad (17)$$
$$= \quad 0{,}350 \ \text{MNm}$$
$$\sigma_B = \frac{M_{xB}}{I_{xB}}\left(\frac{-b}{2} - e_{3B}\right) + \frac{N}{A_B} \quad (18)$$
$$= \quad -0{,}057 \ \text{MN/m}^2 \qquad < 0, \text{ vetoa} \quad \text{(a)}$$

Neutraaliakselin sijainti, jos etumuuri on tarpeeksi leveä, on

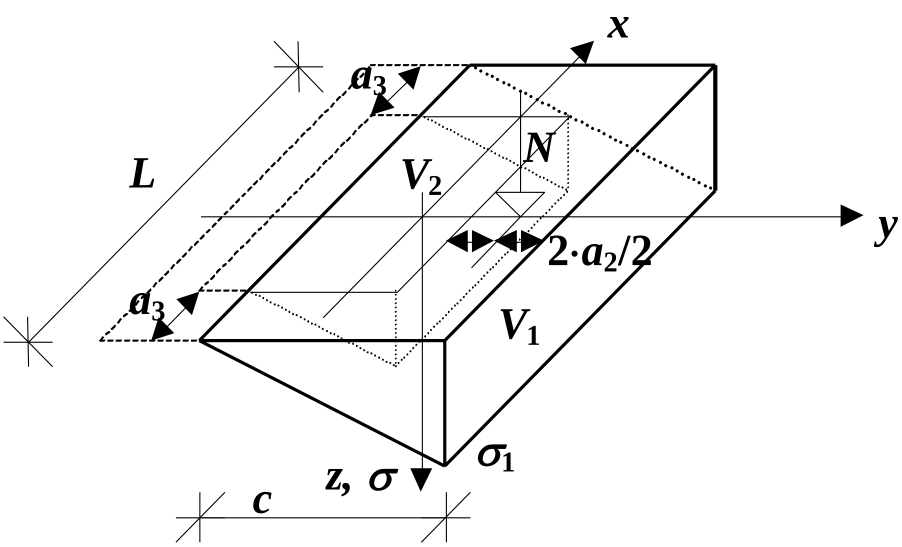
$$c_n = 3a_2/2 \quad (19)$$
$$= \quad 1{,}200 \ \text{m} \qquad > \qquad a_2 \quad \text{(b)}$$



Kuva 2.

Kohtien a ja b perusteella neutraaliakseli kulkee pisteiden B ja C välissä (kuva 2).

Jännityskuvaaja on esitetty kuvassa 3.



Kuva 3.

Vertikaalivoimien tasapainoehto

$$\sum V = 0 \tag{20}$$

$$\Rightarrow N = V_1 - V_2 \tag{21}$$

$$\Rightarrow N = \frac{1}{2} Lc\sigma_1 - \frac{1}{2}(L - 2a_3)(c - a_2)\left(\frac{c - a_2}{c}\right)\sigma_1 \tag{22}$$

$$\Rightarrow \sigma_1 = \frac{2Nc}{Lc^2 - (L - 2a_3)(c - a_2)^2} \tag{23}$$

Momenttitasapainoehto pisteen D suhteen

$$\sum M = 0 \tag{24}$$

$$\Rightarrow N\frac{a_2}{2} = V_1\frac{c}{3} - V_2\left(\frac{c - a_2}{3} + a_2\right) \tag{25}$$

$$\Rightarrow N\frac{a_2}{2} = \frac{1}{6} Lc^2\sigma_1 - \frac{1}{6c}(L - 2a_3)(c - a_2)^2(c + 2a_2)\sigma_1 \tag{26}$$

$$\Rightarrow N\frac{a_2}{2} = \frac{\sigma_1}{6c}\left[Lc^3 - (L - 2a_3)(c - a_2)^2(c + 2a_2)\right] \tag{27}$$

Yhtälöistä 23 ja 27 seuraa

$$\frac{a_2}{2} = \frac{Lc^3 - (L - 2a_3)(c - a_2)^2(c + 2a_2)}{3Lc^2 - 3(L - 2a_3)(c - a_2)^2} \tag{28}$$

Ratkaisun iterointi yhtälöstä 28 (vertaa kolmannen asteen yhtälön ratkaisu tehtävän lopussa): vakio

$a_2/2 =$ **0,400 m**

kun

$c =$ **1,600 m**

Tällöin jännitys pisteessä D yhtälöstä 23 on

$\sigma_1 =$ **0,195 MN/m²**

VASTAUS:

Pohjapainejakauma on kahden kolmisivuisen suoran särmiön erotus. Kuvassa 3

$$c = 1{,}600 \text{ m}$$

$$\sigma_1 = 0{,}195 \text{ MN/m}^2$$

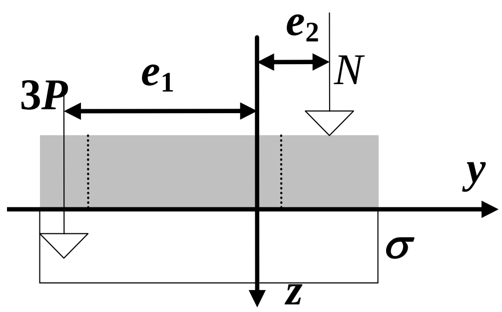
B)

Poikkileikkaus on tasaisesti puristettu, kun pohjapintaan vaikuttavien voimien resultantti vaikuttaa painopisteessä (kuva 4).

$$Ne_2 = 3Pe_1 \tag{29}$$

$$\Rightarrow \quad P = \frac{Ne_2}{3e_1} \tag{30}$$

$$= 0{,}156 \text{ MN}$$



Kuva 4.

VASTAUS:

Tarvittava esijännitysvoima on

$$P = 0{,}156 \text{ MN}$$

KOLMANNEN ASTEEN YHTÄLÖN RATKAISU

Kolmannen asteen yhtälö on muotoa

$$\alpha c^3 + \beta c^2 + \gamma c + \delta = 0 \qquad \textbf{(I)}$$

Merkitään

$$\mu = \frac{\lambda^2}{4} + \frac{\kappa^3}{27} \qquad \textbf{(II)}$$

jossa

$$\kappa = \frac{3\alpha\gamma - \beta^2}{3\alpha^2} \qquad \textbf{(III)}$$

$$\lambda = \frac{2\beta^3 - 9\alpha\beta\gamma + 27\alpha^2\delta}{27\alpha^3} \qquad \textbf{(IV)}$$

Jos μ < 0, yhtälöllä on kolme reaalijuurta

$$c_n = 2\sqrt{\frac{-\kappa}{3}}\cos\frac{\phi + 2n\pi}{3} - \frac{\beta}{3\alpha}, \quad n \in \{1, 2, 3\} \qquad \textbf{(Va, b, c)}$$

joissa

$$\phi = \arccos\left(\frac{-\lambda}{2}\sqrt{\frac{-27}{\kappa^3}}\right) \qquad \textbf{(VI)}$$

Jos μ = 0, yhtälöllä on kaksi erisuurta reaalijuurta

$$c_1 = 2 \cdot \sqrt[3]{\frac{-\lambda}{2}} - \frac{\beta}{3\alpha}$$

$$\left.\begin{matrix} c_2 \\ c_3 \end{matrix}\right\} = \sqrt[3]{\frac{\lambda}{2}} - \frac{\beta}{3\alpha} \qquad \textbf{(VIIa, b)}$$

joista jälkimmäinen on kaksoisjuuri.
Jos lisäksi λ = 0, yhtälöllä on yksi reaalinen kolmoisjuuri.

Jos μ > 0, yhtälöllä on yksi reaalijuuri

$$c = \sqrt[3]{\frac{-\lambda}{2} + \sqrt{\mu}} + \sqrt[3]{\frac{-\lambda}{2} - \sqrt{\mu}} - \frac{\beta}{3\alpha} \qquad \textbf{(VIII)}$$

ja kaksi kompleksijuurta.

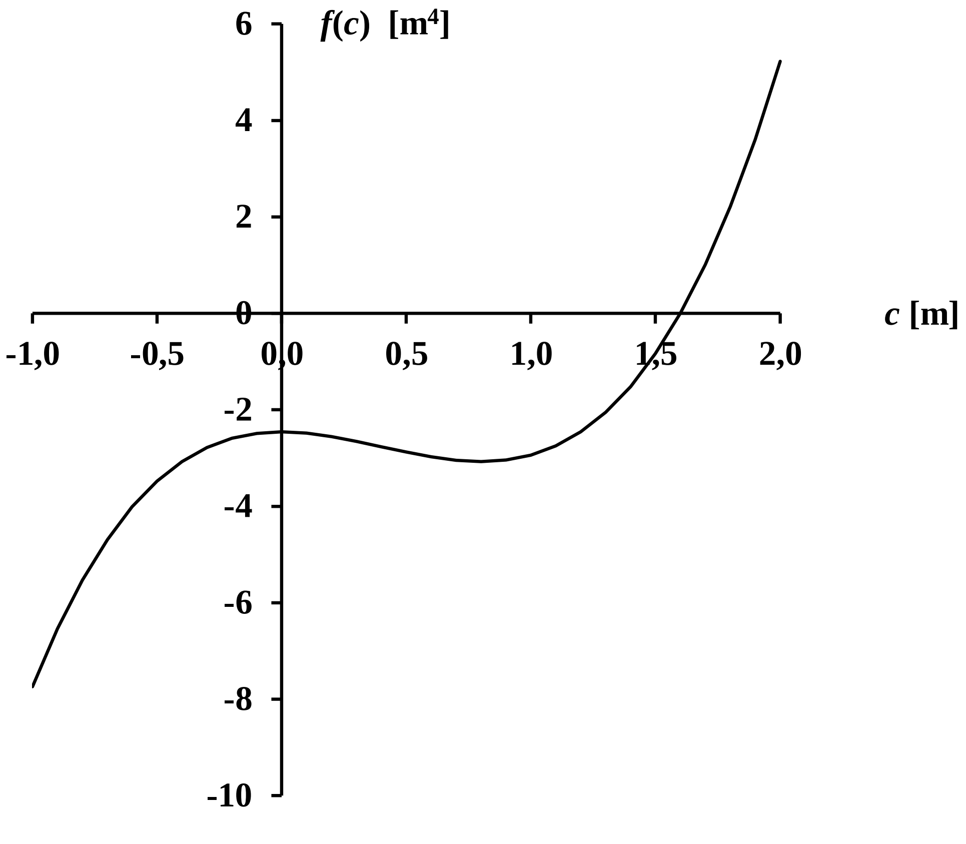
Tehtävän tapauksessa yhtälö 20 voidaan kirjoittaa yhtälön I muotoon, missä

$$\alpha = 4a_3 \tag{IX}$$
$$= 2{,}400 \text{ m}$$
$$\beta = -6a_2a_3 \tag{X}$$
$$= -2{,}880 \text{ m}^2$$
$$\gamma = 0{,}000 \text{ m}^3$$
$$\delta = a_2^{\,3}(2a_3 - L) \tag{XI}$$
$$= -2{,}458 \text{ m}^4$$

Sijoittamalla lukuarvot saadaan

$$\kappa = -0{,}480 \text{ m}^2$$
$$\lambda = -1{,}152 \text{ m}^3$$
$$\mu = 0{,}328 \text{ m}^6$$
$$c = 1{,}600 \text{ m}$$

Vertaa kuva I.



Kuva I.

18. TEHTÄVÄ

Määritä kallionvaraisen teräsbetonisen tukimuurin kaatumis- ja liukuvarmuus kokonaisvarmuusmenetelmällä (kuva 1)!

Mitat

a = 1,000 m

b = 2,000 m

h = 5,000 m

t = 0,500 m

Pintavoima (hyötykuorma)

q = 0,010 MN/m^2

Maan sisäinen kitkakulma

φ = 36,000 °

Maan tilavuuspaino

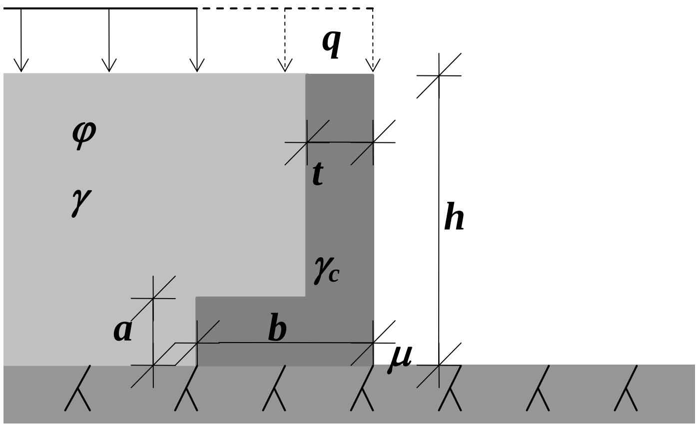
γ = 0,018 MN/m^3

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Kitkakerroin kallion ja betonin välissä

μ = 0,750



Kuva 1.

Kallionvaraisen rakenteen kokonaisvarmuusluvun minimiarvo kaatumista ja liukumista vastaan on (*RIL 121-2004*, kohta 5.1.2.2, s. 58)

$$n = 1{,}500$$

Lepopaineen maanpaineluku

$$K_o = 1\text{-}\sin\varphi \tag{1}$$

$$= 0{,}412$$

Maamassan aiheuttaman paineen resultantti ja sen etäisyys tukimuurin alapinnasta (kuva 2)

$$P_\gamma = \frac{1}{2} K_0 \gamma h^2 \tag{2}$$

$$= 0{,}093 \text{ MN/m}$$

$$e_\gamma = h/3 \tag{3}$$

$$= 1{,}667 \text{ m}$$

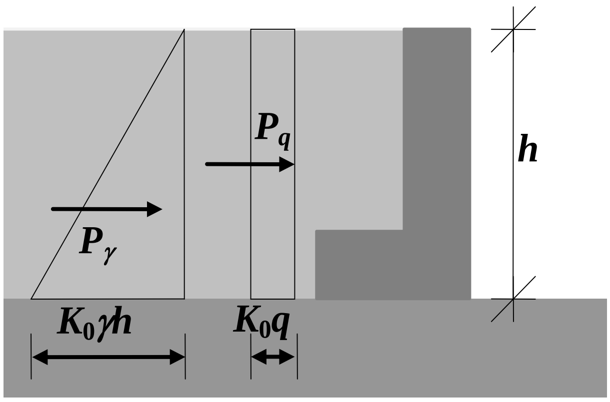
Pintakuorman aiheuttaman paineen resultantti ja sen etäisyys tukimuurin alapinnasta (kuva 2)

$$P_q = K_0 q h \tag{4}$$

$$= 0{,}021 \text{ MN/m}$$

$$e_q = h/2 \tag{5}$$

$$= 2{,}500 \text{ m}$$



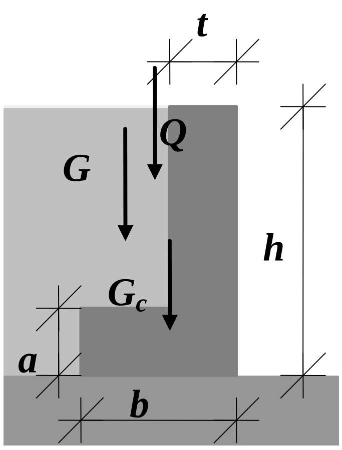
Kuva 2.

Kaatava momentti (kuva 4)

$$M_k = P_\gamma e_\gamma + P_q e_q \tag{6}$$
$$= 0{,}206 \text{ MNm/m}$$

Tukimuurin omanpainon resultantti ja sen etäisyys etunurkasta (kuva 3)

$$G_c = \gamma_c [t(h-a)+ab] \tag{7}$$
$$= 0{,}100 \text{ MN/m}$$
$$e_c = \frac{\gamma_c\left[\frac{t}{2}t(h-a)+\frac{b}{2}ab\right]}{G_c} \tag{8}$$
$$= 0{,}625 \text{ m}$$



Kuva 3.

Maamassan painon resultantti ja sen etäisyys etunurkasta (kuva 3)

$$G = \gamma(b-t)(h-a) \tag{9}$$
$$= 0{,}108 \text{ MN/m}$$
$$e = t + \frac{b-t}{2} \tag{10}$$
$$= 1{,}250 \text{ m}$$

Pintakuorman (hyötykuorman) ei oleteta vaikuttavan tukimuurin yläpuolella (katkoviiva kuvassa 1); näin saadaan vaarallisin kuormitustapaus.

$$Q = 0 \cdot qb \tag{11}$$

$$= 0{,}000 \text{ MN/m}$$

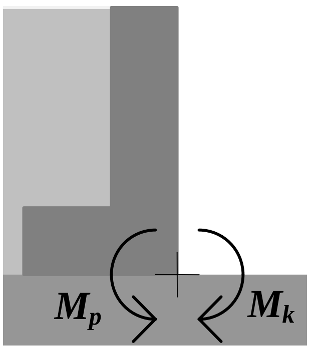
$$e_Q = b/2 \tag{12}$$

$$= 1{,}000 \text{ m}$$

Pystyssä pitävä momentti (kuva 4)

$$M_p = G_c e_c + Ge + Qe_Q \tag{13}$$

$$= 0{,}198 \text{ MNm/m}$$



Kuva 4.

Kaatumisvarmuus (alaindeksi *t* englanninkielen sanasta 'tilting')

$$n_t = \frac{M_p}{M_k} \tag{14}$$

$$= 0{,}958 \quad < n = \quad 1{,}500 \qquad \text{Kaatuu!}$$

Ankkuroidaan rakenne teräksellä (kuva 5) reunaetäisyydeltä

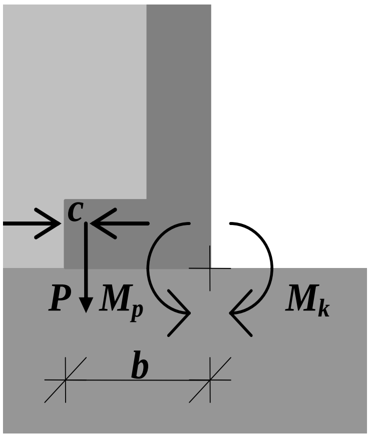
$$c = 0{,}100 \text{ m}$$

Teräksen vetokapasiteetin tulee olla (yhtälöstä 14)

$$\frac{P(b-c) + M_p}{M_k} \geq n \tag{15}$$

$$\Rightarrow \qquad P \geq \frac{nM_k - M_p}{b-c} \tag{16}$$

$$\Rightarrow P \geq \quad 0{,}059 \text{ MN/m}$$



Kuva 5.

Liukuvarmuus (alaindeksi *s* englanninkielen sanasta 'sliding')

$$n_s = \frac{\mu V}{H} \tag{17}$$

$$= \frac{\mu\left(G_c + G + Q\right)}{P_\gamma + P_q} \tag{18}$$

$$= \quad 1{,}376 \quad < n = \quad 1{,}500 \qquad \text{Liukuu!}$$

Ankkuroidaan tukimuuri pystysuoralla esijännitysvoimalla P. Kun otetaan lisäksi huomioon esijännitysteräksen leikkauskapasiteetti Q_p, saadaan

$$\frac{\mu(V + P)}{H - Q_p} \geq n \tag{19}$$

Jos oletetaan, että leikkauskapasiteetti

$$Q_p = 0$$

niin esijännitysvoima on

$$P \geq \frac{nH - \mu V}{\mu} \tag{20}$$

$$\Rightarrow P \geq \frac{n(P_\gamma + P_q) - \mu(G_c + G + Q)}{\mu} \tag{21}$$

$$\Rightarrow P \geq \quad 0{,}019 \text{ MN/m}$$

<u>VASTAUS</u>:

Kaatumis- ja liukuvarmuusehdot eivät täyty: rakenne tulee ankkuroida esimerkiksi pystysuoralla esijännitysvoimalla

$$P = 0{,}060 \text{ MN/m}$$

reunaetäisyyden ollessa

$$c = 0{,}100 \text{ m}$$

19. TEHTÄVÄ

Määritä kuvan 1 perustuksen pienin mitta *a* maan kantokyvyn mukaan Eurokoodin (*SFS-EN 1997-1*) mukaan! Käytä murtorajatilan (GEO) mitoitustapaa 2 (kohta 2.4.7.3.4.3) sekä liitteitä A (kohta A.3) ja D!

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400

k_H = 0,000

k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

γ_m = 0,020 MN/m^3

Maan sisäinen kitkakulma

φ = 30,000 °

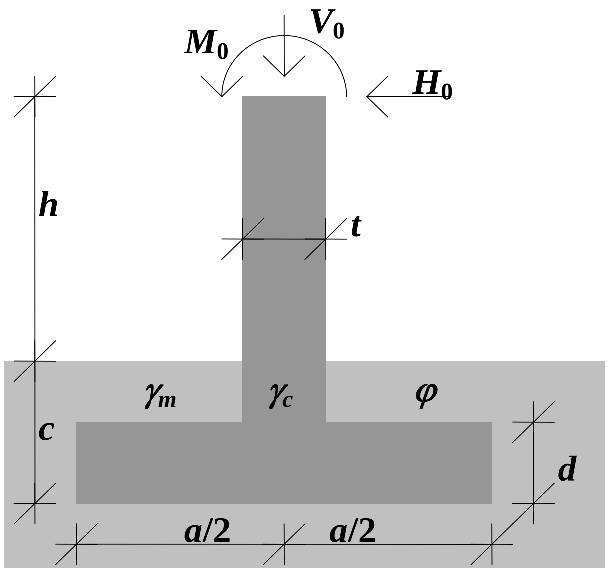
Mitat

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Ratkaisussa on käytetty Eurokoodin *SFS-EN 1997-1* esittämää tapaa.
Kansallisessa liitteessä kuormien yhdistely ja osavarmuuskertoimet saattavat poiketa siitä; näin on tilanne Suomen kansallisen liitteen kohdalla.

Mitoitustavan 2 osavarmuuslukujen yhdistelmä on

A1 "+" M1 "+" R2

jossa

- **A viittaa kuormiin tai kuormien vaikutuksiin (liitteen A taulukko A.3),**
- **M viittaa maaparametreihin (taulukko A.4),**
- **R viittaa kestävyyteen (taulukko A.5),**
- **numerot 1 ja 2 viittaavat sarjoihin 1 ja 2 ja**
- **"+" tarkoittaa yhdistettynä.**

Kuormien yhdistely murtorajatilassa

$$q_d = \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} \tag{1}$$

voidaan esittää muodossa

$$q_d = \left.\begin{matrix} 1{,}35 \\ 1{,}35 \\ 1{,}35 \\ 1{,}35 \\ 1{,}0 \\ 1{,}0 \\ 1{,}0 \\ 1{,}0 \end{matrix}\right\}(g_c + g_m) + \left.\begin{matrix} 1{,}5 \\ 1{,}5 \\ 0 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \\ 0 \end{matrix}\right\} q_{k1} + \left.\begin{matrix} 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \end{matrix}\right\} q_{k2} \quad \begin{matrix} (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f) \\ (g) \\ (h) \end{matrix} \tag{2a…h}$$

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 2{,}100 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa.
Alaindeksit viittaavat kaavaan 1.

Perustuksen omapaino (*a*- mitan funktio)

$$V_{gc} = \gamma_c[(h+c-d)t+ad]L \qquad (3)$$

$$= 0{,}250 \text{ MN}$$

Maan omapaino (*a*- mitan funktio)

$$V_{gm} = \gamma_m(a-t)(c-d)L \qquad (4)$$

$$= 0{,}122 \text{ MN}$$

Pysyvä pystykuorma

$$V_g = k_V V_0 \qquad (5)$$

$$= 0{,}600 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1-k_V)V_0 \qquad (6)$$

$$= 0{,}900 \text{ MN}$$

Pysyvä vaakakuorma

$$H_g = k_H H_0 \qquad (7)$$

$$= 0{,}000 \text{ MN}$$

Muuttuva vaakasuora tuulikuorma

$$H_{q2} = (1-k_H)H_0 \qquad (8)$$

$$= 0{,}130 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0 + k_H H_0 (c + h) \tag{9}$$
$$= 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1 - k_M) M_0 \tag{10}$$
$$= 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

$$M_{q2} = (1 - k_H) H_0 (c + h) \tag{11}$$
$$= 0{,}481 \text{ MNm}$$

Kuormien laskenta-arvot

Pystykuorma

$$V_d = 1{,}35(V_{gm} + V_g + V_{gc}) + 1{,}5 V_{q1} \tag{12}$$
$$= 2{,}663 \text{ MN}$$

Vaakakuorma

$$H_d = 1{,}35 H_g + 1{,}5 H_{q2} \tag{13}$$
$$= 0{,}195 \text{ MN}$$

Momentti

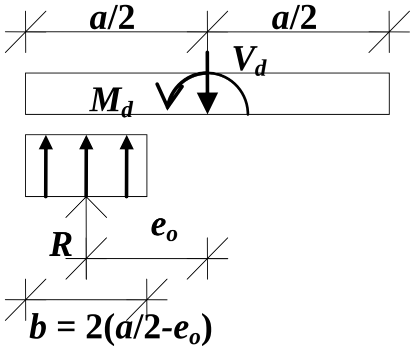
$$M_d = 1{,}35 M_g + 1{,}5 M_{q1} + 1{,}5 M_{q2} \tag{14}$$
$$= 1{,}067 \text{ MNm}$$

Momenttitasapainosta kiertymispisteen suhteen (kuva 2)

$$M_d - V_d e_o = 0 \tag{15}$$

saadaan kuormaepäkeskisyys

$$e_o = \frac{M_d}{V_d} \tag{16}$$
$$= 0{,}401 \text{ m}$$



Kuva 2.

Tasaisen pohjapaineen vaikutusalueen pituus *a* -mitan suunnassa

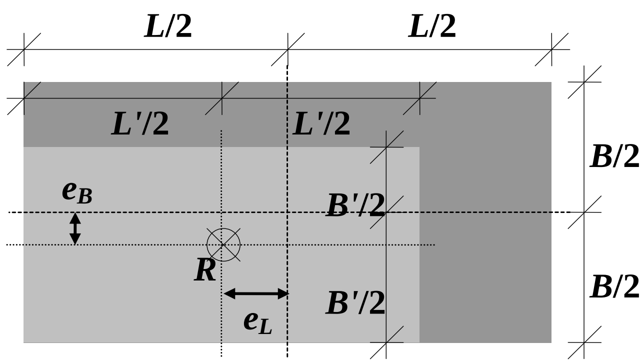
$$b = 2\left(\frac{a}{2} - e_o\right) \qquad (17)$$

$$= \quad 1{,}298 \text{ m}$$

Peruslaatan tehokkaan alueen pienempi sivumitta (kuva 3)

$$B' = \min\begin{cases} b \\ L \end{cases} \qquad (18)$$

$$= \quad 1{,}298 \text{ m}$$



Kuva 3.

Peruslaatan tehokkaan alueen suurempi sivumitta

$$L' = \max\begin{cases} b \\ L \end{cases} \qquad (19)$$

$$= \quad 4{,}000 \text{ m}$$

Tehokkaan alueen ala (kuva 3)

$$A' = B'L' \qquad (20)$$

$$= \quad 5{,}194 \text{ m}^2$$

Pohjapaineen suuruus

$$q_d = \frac{V_d}{A'} \qquad (21)$$

$$= \quad 0{,}513 \text{ MN/m}^2$$

Koheesion laskenta-arvo

$$c' = \quad 0 \text{ MN/m}^2$$

Perustamistason yläpuolinen tilavuuspaino

$$\gamma = \gamma_m \qquad (22)$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Perustamistason alapuolinen tilavuuspaino

$$\gamma' = \gamma_m \qquad (23)$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Tehokas mitoituspaine perustamistasolla

$$q' = c\,\gamma \qquad (24)$$

$$= \quad 0{,}030 \text{ MN/m}^2$$

Pohjan kaltevuus

$$\alpha = \quad 0\ ^\circ$$

Maakerroksen lujuusparametrin osavarmuusluku murtorajatilassa (*EN 1997-1* , Liite A. Taulukko A.4).

$$\gamma_\varphi = 1{,}000$$

Kitkakulman laskenta-arvo

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{25}$$

$$= 30{,}000\ ^\circ$$

Kantavuuskertoimet

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} \tag{26}$$

$$= 18{,}401$$

$$N_\gamma = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} - 1\right]\tan\varphi' \tag{27}$$

$$= 20{,}093$$

Perustuksen pohjan kaltevuuskertoimet

$$\left.\begin{matrix} b_q \\ b_\gamma \end{matrix}\right\} = (1 - \alpha \tan\varphi')^2 \tag{28a, b}$$

$$= 1{,}000$$

Peruslaatan muodon vaikutuskertoimet

$$s_q = 1 + \frac{B'}{L'}\sin\varphi' \tag{29}$$

$$= 1{,}162$$

$$s_\gamma = 1 - 0{,}3\frac{B'}{L'} \tag{30}$$

$$= 0{,}903$$

Parametri *m*

$$m = m_B \quad (31)$$

$$= \frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}} \quad (32)$$

$$= \quad 1{,}755$$

Kuormitusresultantin kaltevuuden vaikutuskertoimet

$$i_q = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^m \quad (33)$$

$$= \quad 0{,}875$$

$$i_\gamma = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^{m+1} \quad (34)$$

$$= \quad 0{,}811$$

Kestävyyden osavarmuuskerroin (*EN 1997-1* , liite A, taulukko A.5)

$$\gamma_R = \quad 1{,}4$$

Kantokyky

$$q_m = \frac{R}{A'} \quad (35)$$

$$= c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \quad (36)$$

$$= \quad 0{,}752 \ \mathrm{MN/m^2}$$

Kantokyvyn laskenta-arvo

$$q_{md} = \frac{q_m}{\gamma_R} \quad (37)$$

$$= \quad 0{,}537 \ \mathrm{MN/m^2}$$

Kantavuuden suhde pohjapaineeseen

$$n = q_{md} / q_d \qquad (38)$$

$$= 1{,}048 > 1\text{, OK}$$

Peruslaatan leveys saadaan iteroimalla.

$$a = 2{,}100 \text{ m}$$

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

KAIKKI KUORMITUSTAPAUKSET

Perustuksen *a* -mitat eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain *a* :sta ja/tai osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

VASTAUS:

Peruslaatan leveys

$$a = 2{,}400 \text{ m}$$

Taulukko 1.

Suure	Yht.	Kuormitustapaukset								Yks.
i		a	b	c	d	e	f	g	h	
a		2,055	1,347	2,124	0,784	2,042	1,250	2,358	0,660	m
ext(a)								Max	Min	
a_{round}		2,100	1,400	2,200	0,800	2,100	1,300	2,400	0,700	m
γ_g	2	1,35	1,35	1,35	1,35	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,50	1,50	0,00	0,00	1,50	1,50	0,00	0,00	
$\gamma_{qk\,2}$	2	1,50	0,00	1,50	0,00	1,50	0,00	1,50	0,00	
V_{gc}	3	0,247	0,205	0,251	0,171	0,247	0,199	0,265	0,164	MN
V_{gm}	4	0,119	0,068	0,124	0,028	0,118	0,061	0,141	0,019	MN
V_d	12	2,655	2,529	1,317	1,078	2,315	2,210	1,006	0,782	MN
H_d	13	0,195	0,000	0,195	0,000	0,195	0,000	0,195	0,000	MN
M_d	14	1,067	0,346	0,851	0,130	1,034	0,312	0,818	0,096	MNm
e_0	16	0,402	0,137	0,646	0,120	0,446	0,141	0,812	0,123	m
b	17	1,251	1,074	0,832	0,544	1,149	0,968	0,733	0,414	m
B'	18	1,251	1,074	0,832	0,544	1,149	0,968	0,733	0,414	m
L'	19	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	20	5,005	4,294	3,327	2,174	4,597	3,871	2,934	1,657	m^2
q_d	21	0,530	0,589	0,396	0,496	0,504	0,571	0,343	0,472	MN/m^2
s_q	29	1,156	1,134	1,104	1,068	1,144	1,121	1,092	1,052	
s_γ	30	0,906	0,919	0,938	0,959	0,914	0,927	0,945	0,969	
m	31	1,762	1,788	1,828	1,880	1,777	1,805	1,845	1,906	
i_q	33	0,874	1,000	0,746	1,000	0,855	1,000	0,672	1,000	
i_γ	34	0,810	1,000	0,636	1,000	0,783	1,000	0,542	1,000	
q_m	36	0,743	0,824	0,554	0,694	0,705	0,799	0,480	0,661	MN/m^2
q_{md}	37	0,530	0,589	0,396	0,496	0,504	0,571	0,343	0,472	MN/m^2
n	38	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

20. TEHTÄVÄ

Määritä kuvan 1 perustuksen pienin mitta *a* maan kantokyvyn mukaan Eurokoodin (*NA SFS-EN 1997-1*) mukaan! Käytä murtorajatilan (GEO) mitoitustapaa 2 (kohta 2.4.7.3.4.3) sekä liitteitä A (kohta A.3) ja D Suomen kansallisesta liitteestä!

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400

k_H = 0,000

k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

γ_m = 0,020 MN/m^3

Maan sisäinen kitkakulma

φ = 30,000 °

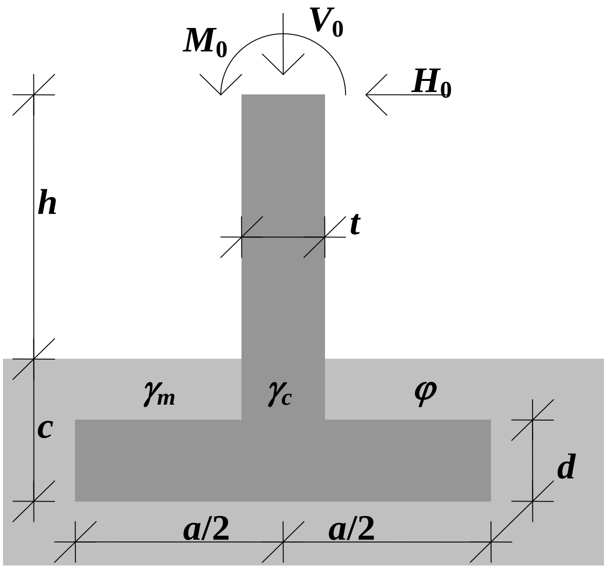
Mitat

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Ratkaisussa on käytetty *Suomen kansallista liitettä standardiin SFS-EN 1997-1* .
Siinä esitetty kuormien yhdistely ja osavarmuuskertoimet poikkeavat standardissa *SFS-EN 1997-1* esitetystä.

Mitoitustavan 2 osavarmuuslukujen yhdistelmä on

A1 "+" M1 "+" R2

jossa (*Kansallista liite standardiin SFS-EN 1997-1*)

- **A viittaa kuormiin tai kuormien vaikutuksiin (liitteen A taulukko A.3),**
- **M viittaa maaparametreihin (taulukko A.4),**
- **R viittaa kestävyyteen (taulukko A.5),**
- **numerot 1 ja 2 viittaavat sarjoihin 1 ja 2 ja**
- **"+" tarkoittaa yhdistettynä.**

Yhdistelyarvo hyötykuormalle (*Kansallinen liite standardiin SFS-EN 1990* , taulukko A1.1, luokka A)

$$\psi_{0,imposed} = 0{,}7$$

ja tuulikuormalle

$$\psi_{0,wind} = 0{,}6$$

jolloin

$$1{,}5\psi_{0,imposed} = 1{,}05$$

$$1{,}5\psi_{0,wind} = 0{,}90$$

Kun kuormakerroin

$$K_{FI} = 1$$

niin kuormien yhdistely murtorajatilassa on

$$F_d = \left.\begin{matrix}1{,}15\\1{,}35\end{matrix}\right\} G_{kj,\mathrm{sup}} + 0{,}9 G_{kj,\mathrm{inf}} + \left.\begin{matrix}1{,}5\\0\end{matrix}\right\} Q_{k1} + \left.\begin{matrix}1{,}5\\0\end{matrix}\right\} \sum \psi_{0i} Q_{ki} \tag{1}$$

joka voidaan esittää muodossa

$$q_d = \left.\begin{matrix}1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}35\\0{,}9\\0{,}9\\0{,}9\\0{,}9\\0{,}9\end{matrix}\right\}(g_c + g_m) + \left.\begin{matrix}1{,}5\\1{,}5\\1{,}05\\0\\0\\1{,}5\\1{,}5\\1{,}05\\0\\0\end{matrix}\right\}q_{k1} + \left.\begin{matrix}0{,}9\\0\\1{,}5\\1{,}5\\0\\0{,}9\\0\\1{,}5\\1{,}5\\0\end{matrix}\right\}q_{k2} \qquad \begin{matrix}(a)\\(b)\\(c)\\(d)\\(e)\\(f)\\(g)\\(h)\\(i)\\(j)\end{matrix} \qquad (2a\ldots j)$$

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 1{,}900 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa. Alaindeksit viittaavat kaavaan 1.

Perustuksen ja maan omapaino (*a*- mitan funktio)

$$V_{gc,m} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \qquad (3)$$

$$= 0{,}346 \text{ MN}$$

Pysyvä pystykuorma

$$V_g = k_V V_0 \qquad (4)$$

$$= 0{,}600 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1\text{-}k_V)V_0 \tag{5}$$
$$= 0{,}900 \text{ MN}$$

Pysyvä vaakakuorma

$$H_g = k_H H_0 \tag{6}$$
$$= 0{,}000 \text{ MN}$$

Muuttuva vaakasuora tuulikuorma

$$H_{q2} = (1\text{-}k_H)H_0 \tag{7}$$
$$= 0{,}130 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0 + k_H H_0(c+h) \tag{8}$$
$$= 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1\text{-}k_M)M_0 \tag{9}$$
$$= 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

$$M_{q2} = (1\text{-}k_H)H_0(c+h) \tag{10}$$
$$= 0{,}481 \text{ MNm}$$

Kuormien laskenta-arvot: pystykuorma, vaakakuorma ja momentti

$$V_d = 1{,}15(V_{gc,m} + V_g) + 1{,}5V_{q1} \tag{11}$$
$$= 2{,}438 \text{ MN}$$
$$H_d = 1{,}15H_g + 0{,}9H_{q2} \tag{12}$$
$$= 0{,}117 \text{ MN}$$
$$M_d = 1{,}15M_g + 1{,}5M_{q1} + 0{,}9M_{q2} \tag{13}$$
$$= 0{,}759 \text{ MNm}$$

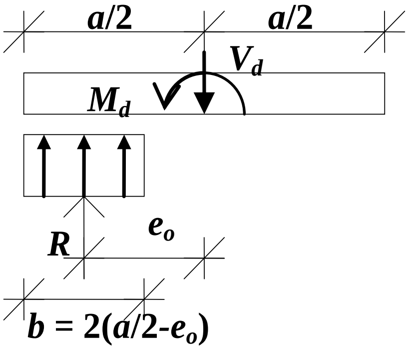
Momenttitasapainosta kiertymispisteen suhteen (kuva 2)

$$M_d - V_d e_o = 0 \tag{14}$$

saadaan kuormaepäkeskisyys

$$e_o = \frac{M_d}{V_d} \tag{15}$$

$$= \quad 0{,}311 \text{ m}$$



Kuva 2.

Tasaisen pohjapaineen vaikutusalueen pituus *a* -mitan suunnassa

$$b = 2\left(\frac{a}{2} - e_o\right) \tag{16}$$

$$= \quad 1{,}277 \text{ m}$$

Peruslaatan tehokkaan alueen pienempi sivumitta (kuva 3)

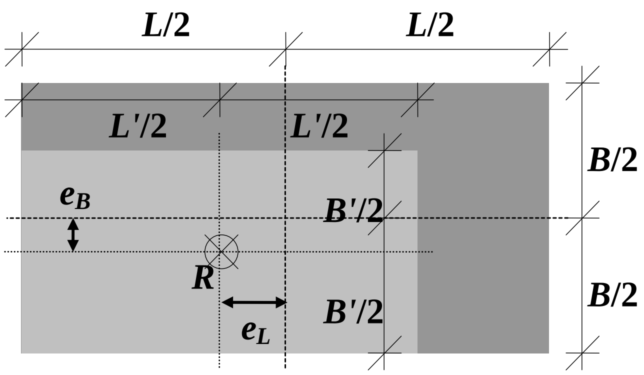
$$B' = \min\begin{cases} b \\ L \end{cases} \tag{17}$$

$$= \quad 1{,}277 \text{ m}$$

Peruslaatan tehokkaan alueen suurempi sivumitta

$$L' = \max\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= \quad 4{,}000 \text{ m}$$



Kuva 3.

Tehokkaan alueen ala (kuva 3)

$$A' = B'L' \tag{19}$$

$$= 5{,}108 \ \mathrm{m}^2$$

Pohjapaineen suuruus

$$q_d = \frac{V_d}{A'} \tag{20}$$

$$= 0{,}477 \ \mathrm{MN/m}^2$$

Koheesion laskenta-arvo

$$c' = 0 \ \mathrm{MN/m}^2$$

Perustamistason yläpuolinen tilavuuspaino

$$\gamma = \gamma_m \tag{21}$$

$$= 0{,}020 \ \mathrm{MN/m}^3$$

Perustamistason alapuolinen tilavuuspaino

$$\gamma' = \gamma_m \tag{22}$$

$$= 0{,}020 \ \mathrm{MN/m}^3$$

Tehokas mitoituspaine perustamistasolla

$$q' = c\,\gamma \tag{23}$$

$$= 0{,}030 \ \mathrm{MN/m}^2$$

Pohjan kaltevuus

α = 0 °

Maakerroksen lujuusparametrin osavarmuusluku murtorajatilassa (*Kansallinen liite standardiin SFS-EN 1997-1* , Liite A. Taulukko A.4)

γ_φ = 1,000

Kitkakulman laskenta-arvo

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{24}$$

= 30,000 °

Kantavuuskertoimet

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} \tag{25}$$

= 18,401

$$N_\gamma = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} - 1\right]\tan\varphi' \tag{26}$$

= 20,093

Perustuksen pohjan kaltevuuskertoimet

$$\left.\begin{matrix} b_q \\ b_\gamma \end{matrix}\right\} = (1 - \alpha \tan\varphi')^2 \tag{27a, b}$$

= 1,000

Peruslaatan muodon vaikutuskertoimet

$$s_q = 1 + \frac{B'}{L'}\sin\varphi' \tag{28}$$

= 1,160

$$s_\gamma = 1 - 0{,}3\frac{B'}{L'} \tag{29}$$

= 0,904

Parametri *m*

$$m = m_B \tag{30}$$

$$= \frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}} \tag{31}$$

$$= \quad 1{,}758$$

Kuormitusresultantin kaltevuuden vaikutuskertoimet

$$i_q = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^m \tag{32}$$

$$= \quad 0{,}917$$

$$i_\gamma = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^{m+1} \tag{33}$$

$$= \quad 0{,}873$$

Kestävyyden osavarmuuskerroin (*Kansallinen liite standardiin SFS-EN 1997-1* , Liite A. Taulukko A.5)

$$\gamma_R = \quad 1{,}55$$

Kantokyky

$$q_m = \frac{R}{A'} \tag{34}$$

$$= c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{35}$$

$$= \quad 0{,}790 \ \text{MN/m}^2$$

Kantokyvyn laskenta-arvo

$$q_{md} = \frac{q_m}{\gamma_R} \tag{36}$$

$$= \quad 0{,}510 \ \text{MN/m}^2$$

Kantavuuden suhde pohjapaineeseen

$$n = q_{md} / q_{d} \qquad (37)$$

$$= \quad 1{,}068 > 1\text{, OK}$$

Peruslaatan leveys saadaan iteroimalla.

$$a = \quad 1{,}900 \text{ m}$$

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

KAIKKI KUORMITUSTAPAUKSET

Perustuksen *a* -mitat eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain *a* :sta ja/tai osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

<u>VASTAUS</u>: Peruslaatan leveys

$$a = \quad 2{,}600 \text{ m}$$

Taulukko 1.

Suure	Yht.	Kuormitustapaukset										Yks.
i		a	b	c	d	e	f	g	h	i	j	
a		1,839	1,377	2,110	2,292	0,834	1,803	1,302	2,128	2,517	0,658	m
ext(a)										Max	Min	
a_{round}		1,900	1,400	2,200	2,300	0,900	1,900	1,400	2,200	2,600	0,700	m
γ_g	2	1,15	1,15	1,15	1,15	1,35	0,90	0,90	0,90	0,90	0,90	
$\gamma_{qk\,1}$	2	1,50	1,50	1,05	0,00	0,00	1,50	1,50	1,05	0,00	0,00	
$\gamma_{qk\,2}$	2	0,90	0,00	1,50	1,50	0,00	0,90	0,00	1,50	1,50	0,00	
$V_{gc,m}$	3	0,338	0,277	0,374	0,398	0,205	0,333	0,267	0,376	0,427	0,182	MN
V_d	11	2,429	2,358	2,065	1,147	1,087	2,190	2,130	1,823	0,925	0,704	MN
H_d	12	0,117	0,000	0,195	0,195	0,000	0,117	0,000	0,195	0,195	0,000	MN
M_d	13	0,759	0,326	0,983	0,832	0,130	0,735	0,302	0,959	0,808	0,086	MNm
e_0	15	0,313	0,138	0,476	0,725	0,119	0,336	0,142	0,526	0,874	0,123	m
b	16	1,213	1,100	1,158	0,842	0,595	1,132	1,018	1,076	0,769	0,413	m
B'	17	1,213	1,100	1,158	0,842	0,595	1,132	1,018	1,076	0,769	0,413	m
L'	18	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	19	4,853	4,400	4,631	3,369	2,382	4,527	4,071	4,304	3,077	1,651	m^2
q_d	20	0,500	0,536	0,446	0,341	0,456	0,484	0,523	0,424	0,301	0,426	MN/m^2
s_q	28	1,152	1,138	1,145	1,105	1,074	1,141	1,127	1,135	1,096	1,052	
s_γ	29	0,909	0,917	0,913	0,937	0,955	0,915	0,924	0,919	0,942	0,969	
m	31	1,767	1,784	1,776	1,826	1,870	1,779	1,797	1,788	1,839	1,906	
i_q	32	0,916	1,000	0,838	0,712	1,000	0,907	1,000	0,817	0,647	1,000	
i_γ	33	0,872	1,000	0,759	0,591	1,000	0,858	1,000	0,730	0,511	1,000	
q_m	34	0,776	0,831	0,691	0,528	0,707	0,750	0,811	0,657	0,466	0,661	MN/m^2
q_{md}	36	0,501	0,536	0,446	0,341	0,456	0,484	0,523	0,424	0,301	0,426	MN/m^2
n	37	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

21. TEHTÄVÄ

Määritä kuvan 1 perustuksen pienin mitta *a* maan kantokyvyn mukaan.

Käytä Eurokoodin murtorajatilan (GEO) mitoitustapaa DA2*:

- ***Suomen kansallinen liite standardiin SFS-EN 1997-1*, kohta 4 ja taulukko A.3.**
- ***SFS-EN 1997-1+AC*, Liite D; tai kurssin *Kaavakokoelma*, kohta 2.1.**

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN
H_o = 0,130 MN
M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400
k_H = 0,000
k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

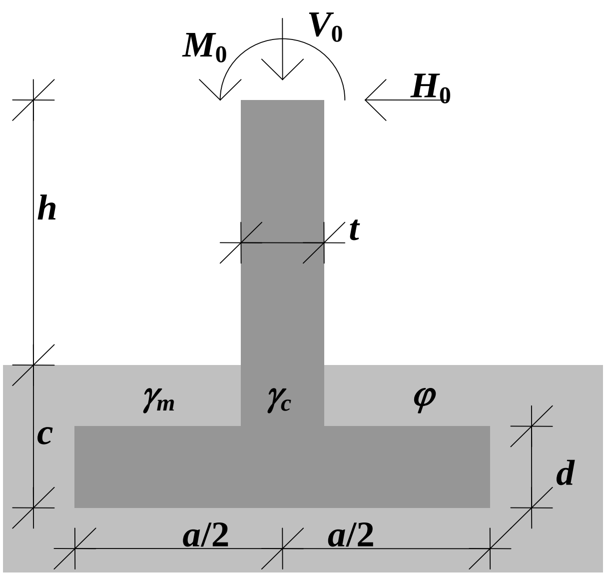
γ_m = 0,018 MN/m^3

Maan sisäinen kitkakulma

φ = 34,000 °

Mitat

c = 1,500 m
h = 2,200 m
d = 0,600 m
t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Ratkaisussa on käytetty *Suomen kansallista liitettä standardiin SFS-EN 1997-1* .

Siinä esitetty kuormien yhdistely ja osavarmuuskertoimet poikkeavat standardissa *SFS-EN 1997-1* esitetystä.

Mitoitustavan 2 osavarmuuslukujen yhdistelmä on

A1 "+" M1 "+" R2

jossa (*Kansallista liite standardiin SFS-EN 1997-1*)

- **A viittaa kuormiin tai kuormien vaikutuksiin (liitteen A taulukko A.3),**
- **M viittaa maaparametreihin (taulukko A.4),**
- **R viittaa kestävyyteen (taulukko A.5),**
- **numerot 1 ja 2 viittaavat sarjoihin 1 ja 2 ja**
- **"+" tarkoittaa yhdistettynä.**

Yhdistelyarvo hyötykuormalle (*Kansallinen liite standardiin SFS-EN 1990* , taulukko A1.1, luokka A)

$$\psi_{0,imposed} = 0{,}7$$

ja tuulikuormalle

$$\psi_{0,wind} = 0{,}6$$

jolloin

$$1{,}5\psi_{0,imposed} = 1{,}05$$

$$1{,}5\psi_{0,wind} = 0{,}90$$

Kun kuormakerroin

$$K_{FI} = 1$$

niin kuormien yhdistely murtorajatilassa on

$$F_d = \left.\begin{matrix}1{,}15\\1{,}35\end{matrix}\right\}G_{kj,\sup} + 0{,}9G_{kj,\inf} + \left.\begin{matrix}1{,}5\\0\end{matrix}\right\}Q_{k1} + \left.\begin{matrix}1{,}5\\0\end{matrix}\right\}\sum\psi_{0i}Q_{ki} \qquad (1)$$

joka voidaan esittää muodossa

$$q_d = \left.\begin{matrix}1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}35\\0{,}9\\0{,}9\\0{,}9\\0{,}9\\0{,}9\end{matrix}\right\}(g_c+g_m)+\left.\begin{matrix}1{,}5\\1{,}5\\1{,}05\\0\\0\\1{,}5\\1{,}5\\1{,}05\\0\\0\end{matrix}\right\}q_{k1}+\left.\begin{matrix}0{,}9\\0\\1{,}5\\1{,}5\\0\\0{,}9\\0\\1{,}5\\1{,}5\\0\end{matrix}\right\}q_{k2}\quad\begin{matrix}(a)\\(b)\\(c)\\(d)\\(e)\\(f)\\(g)\\(h)\\(i)\\(j)\end{matrix} \qquad (2a\ldots j)$$

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 1{,}800 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa. Alaindeksit viittaavat kaavaan 1.

Perustuksen ja maan omapaino (*a*- mitan funktio)

$$V_{gc,m} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \qquad (3)$$

$$= 0{,}323 \text{ MN}$$

Pysyvä pystykuorma

$$V_g = k_V V_0 \qquad (4)$$

$$= 0{,}600 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1-k_V)V_0 \tag{5}$$
$$= 0{,}900 \text{ MN}$$

Pysyvä vaakakuorma

$$H_g = k_H H_0 \tag{6}$$
$$= 0{,}000 \text{ MN}$$

Muuttuva vaakasuora tuulikuorma

$$H_{q2} = (1-k_H)H_0 \tag{7}$$
$$= 0{,}130 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0 + k_H H_0(c+h) \tag{8}$$
$$= 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1-k_M)M_0 \tag{9}$$
$$= 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

$$M_{q2} = (1-k_H)H_0(c+h) \tag{10}$$
$$= 0{,}481 \text{ MNm}$$

Kuormien ominaisarvot: pystykuorma, vaakakuorma ja momentti

$$V_k = V_{gc,m} + V_g + V_{q1} \tag{11}$$
$$= 1{,}823 \text{ MN}$$
$$H_k = H_g + H_{q2} \tag{12}$$
$$= 0{,}130 \text{ MN}$$
$$M_k = M_g + M_{q1} + M_{q2} \tag{13}$$
$$= 0{,}721 \text{ MNm}$$

Pystykuorman laskenta-arvo

$$V_d = 1{,}15(V_{gc,m} + V_g) + 1{,}5V_{q1} \tag{14}$$

$$= 2{,}411 \text{ MN}$$

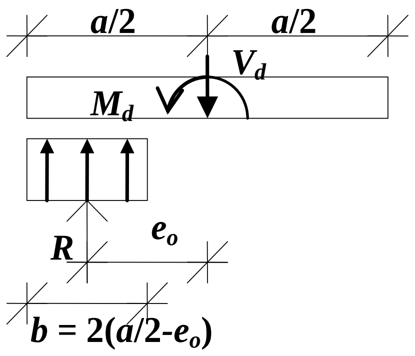
Momenttitasapainosta kiertymispisteen suhteen (kuva 2)

$$M_k - V_k e_o = 0 \tag{15}$$

saadaan kuormaepäkeskisyys

$$e_o = \frac{M_k}{V_k} \tag{16}$$

$$= 0{,}396 \text{ m}$$



Kuva 2.

Tasaisen pohjapaineen vaikutusalueen pituus *a* -mitan suunnassa

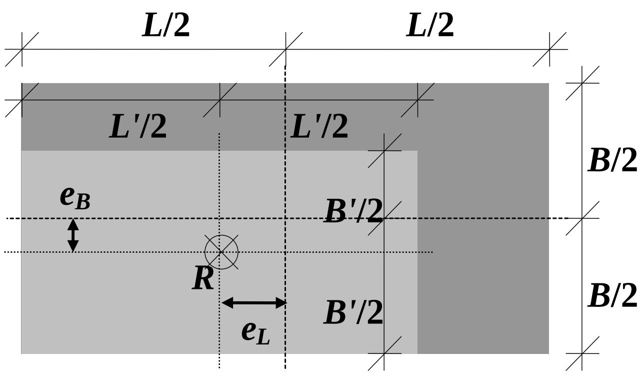
$$b = 2\left(\frac{a}{2} - e_o\right) \tag{17}$$

$$= 1{,}009 \text{ m}$$

Peruslaatan tehokkaan alueen pienempi sivumitta (kuva 3)

$$B' = \min\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= 1{,}009 \text{ m}$$



Kuva 3.

Peruslaatan tehokkaan alueen suurempi sivumitta

$$L' = \max\begin{cases} b \\ L \end{cases} \qquad (19)$$

$$= \quad 4{,}000 \text{ m}$$

Tehokkaan alueen ala (kuva 3)

$$A' = B'L' \qquad (20)$$

$$= \quad 4{,}035 \text{ m}^2$$

Pohjapaineen suuruus

$$q_d = \frac{V_d}{A'} \qquad (21)$$

$$= \quad 0{,}597 \text{ MN/m}^2$$

Koheesion laskenta-arvo

$$c' = \quad 0 \text{ MN/m}^2$$

Perustamistason yläpuolinen tilavuuspaino

$$\gamma = \gamma_m \qquad (22)$$

$$= \quad 0{,}018 \text{ MN/m}^3$$

Perustamistason alapuolinen tilavuuspaino

$$\gamma' = \gamma_m \tag{23}$$

$$= \quad 0{,}018 \ \mathrm{MN/m^3}$$

Tehokas mitoituspaine perustamistasolla

$$q' = c\,\gamma \tag{24}$$

$$= \quad 0{,}027 \ \mathrm{MN/m^2}$$

Pohjan kaltevuus

$$\alpha = \quad 0\ °$$

Maakerroksen lujuusparametrin osavarmuusluku murtorajatilassa (*Kansallinen liite standardiin SFS-EN 1997-1* , Liite A. Taulukko A.4)

$$\gamma_\varphi = \quad 1{,}000$$

Kitkakulman laskenta-arvo

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{25}$$

$$= \quad 34{,}000\ °$$

Kantavuuskertoimet

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} \tag{26}$$

$$= \quad 29{,}440$$

$$N_\gamma = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} - 1\right]\tan\varphi' \tag{27}$$

$$= \quad 38{,}366$$

Perustuksen pohjan kaltevuuskertoimet

$$\left.\begin{array}{l} b_q \\ b_\gamma \end{array}\right\} = (1 - \alpha\tan\varphi')^2 \tag{28a, b}$$

$$= \quad 1{,}000$$

Peruslaatan muodon vaikutuskertoimet

$$s_q = 1 + \frac{B'}{L'}\sin\varphi' \tag{29}$$

$$= \quad 1{,}141$$

$$s_\gamma = 1 - 0{,}3\frac{B'}{L'} \tag{30}$$

$$= \quad 0{,}924$$

Parametri *m*

$$m = m_B \tag{31}$$

$$= \frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}} \tag{32}$$

$$= \quad 1{,}799$$

Kuormitusresultantin kaltevuuden vaikutuskertoimet

$$i_q = \left(1 - \frac{H_k}{V_k + A'c'\cot\varphi'}\right)^m \tag{33}$$

$$= \quad 0{,}875$$

$$i_\gamma = \left(1 - \frac{H_k}{V_k + A'c'\cot\varphi'}\right)^{m+1} \tag{34}$$

$$= \quad 0{,}813$$

Kestävyyden osavarmuuskerroin (*Kansallinen liite standardiin SFS-EN 1997-1* , Liite A. Taulukko A.5)

$$\gamma_R \; = \quad 1{,}55$$

Kantokyky

$$q_m = \frac{R}{A'} \tag{35}$$

$$= c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{36}$$

$= \quad 1{,}056 \ \mathrm{MN/m^2}$

Kantokyvyn laskenta-arvo

$$q_{md} = \frac{q_m}{\gamma_R} \tag{37}$$

$= \quad 0{,}681 \ \mathrm{MN/m^2}$

Kantavuuden suhde pohjapaineeseen

$$n = \frac{q_{md}}{q_d} \tag{38}$$

$= \quad 1{,}140 > 1$, OK

Peruslaatan leveys saadaan iteroimalla.

$a = 1{,}800$ m

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

KAIKKI KUORMITUSTAPAUKSET

Kantavuuden suhde pohjapaineeseen eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

VASTAUS: **Peruslaatan leveys**

$a = 1{,}800$ m

Taulukko 1.

	Yht.	Kuormitustapaukset										Yks.
i		a	b	c	d	e	f	g	h	i	j	
a		1,707	1,063	1,588	1,794	0,663	1,642	1,007	1,519	1,706	0,540	m
ext(a)					Max						Min	
a_{round}		1,800	1,100	1,600	1,800	0,700	1,700	1,100	1,600	1,800	0,600	m
γ_{cm}	2	1,15	1,15	1,15	1,15	1,35	0,90	0,90	0,90	0,90	0,90	
$\gamma_{qk\,1}$	2	1,50	1,50	1,05	0,00	0,00	1,50	1,50	1,05	0,00	0,00	
$\gamma_{qk\,2}$	2	0,90	0,00	1,50	1,50	0,00	0,90	0,00	1,50	1,50	0,00	
γ_{cmk}	2	1	1	1	1	1	1	1	1	1	1	
$\gamma_{qk\,1k}$	2	1	1	1	0	0	1	1	1	0	0	
$\gamma_{qk\,2k}$	2	1	0	1	1	0	1	0	1	1	0	
$V_{gc,m}$	3	0,311	0,231	0,296	0,322	0,181	0,303	0,224	0,288	0,311	0,165	MN
V_k	11	1,811	1,731	1,796	0,922	0,781	1,803	1,724	1,788	0,911	0,765	MN
H_k	12	0,130	0,000	0,130	0,130	0,000	0,130	0,000	0,130	0,130	0,000	MN
M_k	13	0,721	0,240	0,721	0,577	0,096	0,721	0,240	0,721	0,577	0,096	MNm
V_d	14	2,398	2,305	1,976	1,060	1,054	2,163	2,091	1,744	0,820	0,689	MN
e_0	16	0,398	0,139	0,401	0,626	0,123	0,400	0,139	0,403	0,633	0,125	m
b	17	0,911	0,786	0,786	0,542	0,417	0,842	0,728	0,713	0,439	0,289	m
B'	18	0,911	0,786	0,786	0,542	0,417	0,842	0,728	0,713	0,439	0,289	m
L'	19	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	20	3,643	3,143	3,142	2,167	1,666	3,368	2,913	2,851	1,757	1,155	m^2
q_d	21	0,658	0,734	0,629	0,489	0,633	0,642	0,718	0,612	0,467	0,596	MN/m^2
s_q	29	1,127	1,110	1,110	1,076	1,058	1,118	1,102	1,100	1,061	1,040	
s_γ	30	0,932	0,941	0,941	0,959	0,969	0,937	0,945	0,947	0,967	0,978	
m	32	1,815	1,836	1,836	1,881	1,906	1,826	1,846	1,849	1,901	1,933	
i_q	33	0,874	1,000	0,871	0,751	1,000	0,872	1,000	0,870	0,746	1,000	
i_γ	34	0,811	1,000	0,808	0,645	1,000	0,809	1,000	0,806	0,640	1,000	
q_m	36	1,020	1,137	0,975	0,758	0,981	0,995	1,113	0,948	0,723	0,925	MN/m^2
q_{md}	37	0,658	0,734	0,629	0,489	0,633	0,642	0,718	0,612	0,467	0,596	MN/m^2
n	38	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

22. TEHTÄVÄ

Määritä kuvan 1 perustuksen pienin mitta *a* maan kantokyvyn mukaan murtorajatilamenetelmällä teosten *RIL-144-2002 Rakenteiden kuormitusohjeet* ja *RIL121-2004 Pohjarakennusohjeet* mukaan!

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400

k_H = 0,000

k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

γ_m = 0,020 MN/m^3

Maan sisäinen kitkakulma

φ = 30,000 °

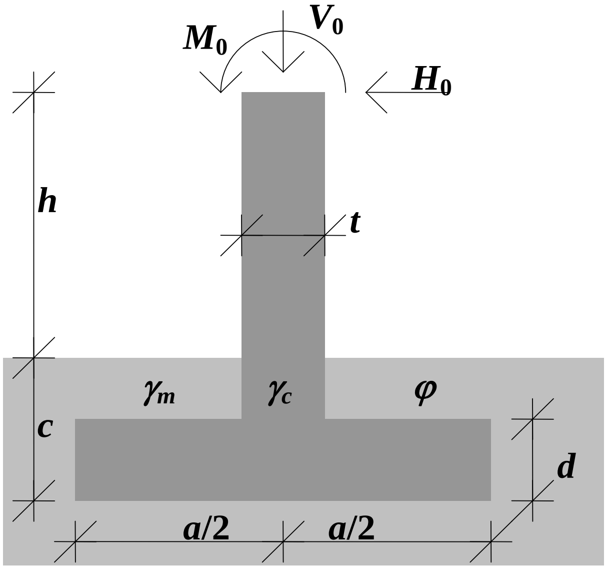
Mitat

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Kuormien yhdistely murtorajatilassa (*RIL 144-2002*, kohta 8.21, s. 149)

$$q_d = \sum_{i=1}^{m} \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} + \sum_{i=3}^{n} \gamma_{qi} q_{ki} \tag{1}$$

voidaan esittää muodossa

$$q_d = 1{,}0 g_c + \left.\begin{matrix} 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{matrix}\right\} g + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \end{matrix}\right\} q_{k1} + \left.\begin{matrix} 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \end{matrix}\right\} q_{k2} \quad \begin{matrix} (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f) \\ (g) \\ (h) \end{matrix} \tag{2a...h}$$

Osavarmuuskertoimet ovat ohjeista

- ***RIL 144-2002*, taulukko 8.21a, s. 150, ja**
- ***RIL 121-2004*, kohta 5.1.2.3, s. 60.**

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 2{,}700 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa. Alaindeksit viittaavat kaavaan 1.

Perustuksen ja maan omapaino (*a*- mitan funktio)

$$V_{gc} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \qquad (3)$$

$$= \quad 0{,}452 \text{ MN}$$

Pysyvä pystykuorma

$$V_g = k_V V_0 \qquad (4)$$

$$= \quad 0{,}600 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1-k_V)V_0 \qquad (5)$$

$$= \quad 0{,}900 \text{ MN}$$

Pysyvä vaakakuorma

$$H_g = k_H H_0 \qquad (6)$$

$$= \quad 0{,}000 \text{ MN}$$

Muuttuva vaakasuora tuulikuorma

$$H_{q2} = (1-k_H)H_0 \qquad (7)$$

$$= \quad 0{,}130 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0+H_g(c+h) \qquad (8)$$

$$= \quad 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1-k_M)M_0 \qquad (9)$$

$$= \quad 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

$$M_{q2} = (1-k_H)H_0(c+h) \qquad (10)$$

$$= \quad 0{,}481 \text{ MNm}$$

Kuormien laskenta-arvot: pystykuorma, vaakakuorma ja momentti

$$V_d = 1{,}0V_{gc} + 1{,}2V_g + 1{,}6V_{q1} \quad (11)$$

$$= 2{,}612 \text{ MN}$$

$$H_d = 1{,}2H_g + 1{,}6H_{q2} \quad (12)$$

$$= 0{,}208 \text{ MN}$$

$$M_d = 1{,}2M_g + 1{,}6M_{q1} + 1{,}6M_{q2} \quad (13)$$

$$= 1{,}115 \text{ MNm}$$

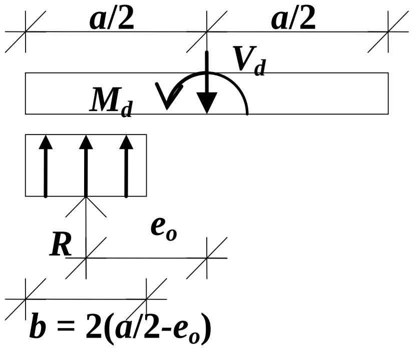
Momenttitasapainosta kiertymispisteen suhteen (kuva 2)

$$M_d - V_d e_o = 0 \quad (14)$$

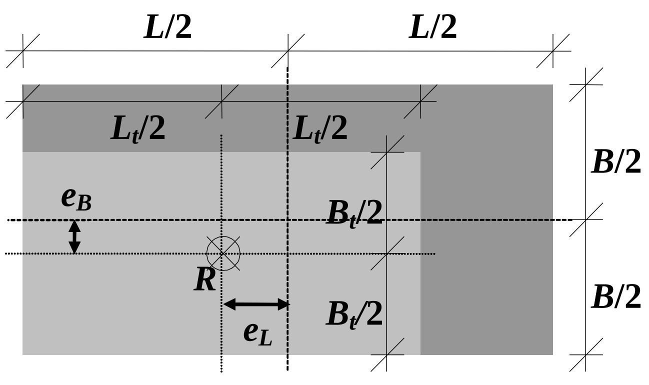
saadaan kuormaepäkeskisyys

$$e_o = \frac{M_d}{V_d} \quad (15)$$

$$= 0{,}427 \text{ m}$$



Kuva 2.



Kuva 3.

Tasaisen pohjapaineen vaikutusalueen pituus *a* -mitan suunnassa

$$b = 2\left(\frac{a}{2} - e_o\right) \tag{16}$$

$$= \quad 1{,}846 \text{ m}$$

Peruslaatan tehokkaan alueen pienempi sivumitta (kuva 3)

$$B_t = \min\begin{cases} b \\ L \end{cases} \tag{17}$$

$$= \quad 1{,}846 \text{ m}$$

Peruslaatan tehokkaan alueen suurempi sivumitta

$$L_t = \max\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= \quad 4{,}000 \text{ m}$$

Tehokkaan alueen ala

$$A_t = B_t L_t \tag{19}$$

$$= \quad 7{,}384 \text{ m}^2$$

Pohjapaineen suuruus

$$q_d = \frac{V_d}{A_t} \tag{20}$$

$$= \quad 0{,}354 \text{ MN/m}^2$$

Peruslaatan pienin perustamissyvyys

$$D = c \tag{21}$$

$$= \quad 1{,}500 \text{ m}$$

Koheesion laskenta-arvo

$$c_d = \quad 0 \text{ MN/m}^2$$

Perustamistason yläpuolinen tilavuuspaino

$$\gamma'_1 = \gamma_m \tag{22}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Perustamistason alapuolinen tilavuuspaino

$$\gamma'_2 = \gamma_m \tag{23}$$

$$= \quad 0{,}020 \ \mathrm{MN/m^3}$$

Maakerroksen lujuusparametrin osavarmuusluku murtorajatilassa (*RIL 121-2004*, taulukko 7, s. 60)

$$\gamma_\varphi = \quad 1{,}250$$

Kitkakulman laskenta-arvo

$$\varphi_d = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{24}$$

$$= \quad 24{,}791\ °$$

Kantavuuskertoimet

$$N_D = \tan^2(45° + \frac{\varphi_d}{2})e^{\pi \tan\varphi_d} \tag{25}$$

$$= \quad 10{,}431$$

$$N_B = 1{,}5\left[\tan^2\left(45° + \frac{\varphi_d}{2}\right)e^{\pi \tan\varphi_d} - 1\right]\tan\varphi_d \tag{26}$$

$$= \quad 6{,}534$$

Peruslaatan muodon vaikutuskertoimet

$$s_B = 1 - 0{,}4\left(\frac{B_t}{L_t}\right) \tag{27}$$

$$= \quad 0{,}815$$

$$s_D = 1 + 0{,}2\left(\frac{B_t}{L_t}\right) \tag{28}$$

$$= \quad 1{,}092$$

Kuormitusresultantin kaltevuuden vaikutuskertoimet

$$i_D = \left(1 - \frac{H_d}{V_d + A_t c_d \cot \varphi_d}\right)^2 \tag{29}$$

$= \quad 0,847$

$$i_B = \left(1 - \frac{H_d}{V_d + A_t c_d \cot \varphi_d}\right)^4 \tag{30}$$

$= \quad 0,718$

Kantokyvyn laskenta-arvo

$$q_{md} = c_d N_c s_c i_c + \gamma_1' D N_D s_D i_D + \frac{1}{2}\gamma_2' B_t N_B s_B i_B \tag{31}$$

$= \quad 0,360 \ \mathrm{MN/m^2}$

Kantavuuden suhde pohjapaineeseen

$$n = q_{md} / q_d \tag{32}$$

$= \quad 1,018 > 1$, OK

Peruslaatan leveys saadaan iteroimalla.

$a = \quad 2,700$ m

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

KAIKKI KUORMITUSTAPAUKSET

Perustuksen *a* -mitat eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain *a* :sta ja/tai osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

<u>VASTAUS</u>: **Peruslaatan leveys**

$a = \quad 2,800$ m

Taulukko 1.

Suure	Yht.	Kuormitustapaukset								Yks.
i		a	b	c	d	e	f	g	h	
a		2,673	1,763	2,666	0,890	2,630	1,666	2,787	0,755	
ext(a)								Max	Min	
a_{round}		2,700	1,800	2,700	0,900	2,700	1,700	2,800	0,800	
γ_m	2	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	
γ_g	2	1,20	1,20	1,20	1,20	0,90	0,90	0,90	0,90	
$\gamma_{qk\,1}$	2	1,60	1,60	0,00	0,00	1,60	1,60	0,00	0,00	
$\gamma_{qk\,2}$	2	1,60	0,00	1,60	0,00	1,60	0,00	1,60	0,00	
V_{gc}	3	0,448	0,328	0,447	0,213	0,442	0,315	0,463	0,195	MN
V_d	11	2,608	2,488	1,167	0,933	2,422	2,295	1,003	0,735	MN
H_d	12	0,208	0,000	0,208	0,000	0,208	0,000	0,208	0,000	MN
M_d	13	1,115	0,346	0,885	0,115	1,086	0,317	0,856	0,086	MNm
e_0	15	0,428	0,139	0,758	0,124	0,448	0,138	0,853	0,118	m
b	16	1,818	1,485	1,150	0,643	1,733	1,390	1,081	0,520	m
B_t	17	1,818	1,485	1,150	0,643	1,733	1,390	1,081	0,520	m
L_t	18	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A_t	19	7,271	5,941	4,599	2,573	6,931	5,559	4,322	2,080	m^2
q_d	20	0,359	0,419	0,254	0,362	0,349	0,413	0,232	0,353	MN/m^2
s_B	27	0,818	0,851	0,885	0,936	0,827	0,861	0,892	0,948	
s_D	28	1,091	1,074	1,057	1,032	1,087	1,069	1,054	1,026	
i_D	29	0,847	1,000	0,675	1,000	0,836	1,000	0,628	1,000	
i_B	30	0,717	1,000	0,456	1,000	0,698	1,000	0,395	1,000	
q_{md}	31	0,359	0,419	0,254	0,362	0,350	0,413	0,232	0,353	MN/m^2
n	32	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

23. TEHTÄVÄ

Määritä perustuksen (kuva 1) pienin mitta *a* maan kantokyvyn mukaan kokonaisvarmuus(luku)menetelmällä käyttäen teoksen *RIL 121-2004* kantavuuskaavaa siten, että kokonaisvarmuus maapohjan murtumista vastaan on

n = 2

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400

k_H = 0,000

k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

γ_m = 0,020 MN/m^3

Maan sisäinen kitkakulma

φ = 30,000 °

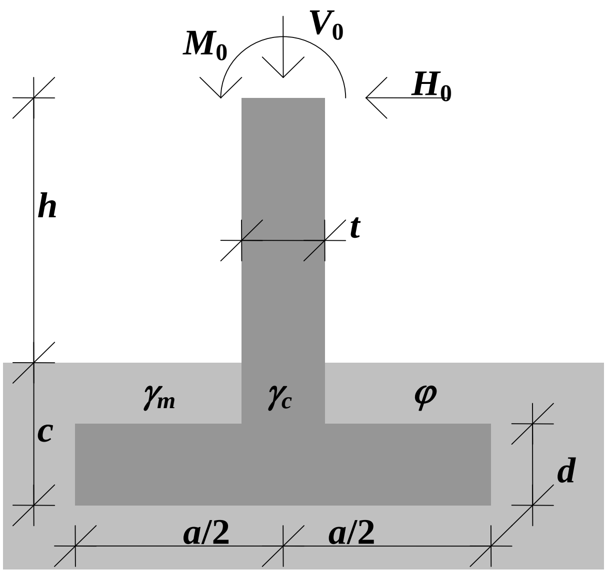
Mitat

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Kuormien yhdistely

$$q_d = \sum_{i=1}^{m} g_i + q_{k1} + q_{k2} + \sum_{i=3}^{n} q_{ki} \tag{1}$$

voidaan esittää muodossa

$$q_d = g + \left.\begin{matrix}1\\1\\0\\0\end{matrix}\right\} q_{k1} + \left.\begin{matrix}1\\0\\1\\0\end{matrix}\right\} q_{k2} \qquad \begin{matrix}\text{(a)}\\\text{(b)}\\\text{(c)}\\\text{(d)}\end{matrix} \tag{2a...d}$$

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 2{,}300 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa. Alaindeksit viittaavat kaavaan 1.

Yläpuolisen rakenteen, perustuksen ja maan omapaino (a- mitan funktio)

$$V_g = k_V V_0 + \gamma_c[(h+c-d)t + ad]L + \gamma_m(a-t)(c-d)L \tag{3}$$

$$= 0{,}999 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1-k_V)V_0 \tag{4}$$

$$= 0{,}900 \text{ MN}$$

Pysyvä vaakakuorma

$$H_g = k_H H_0 \qquad (5)$$
$$= \quad 0{,}000 \text{ MN}$$

Muuttuva vaakasuora tuulikuorma

$$H_{q2} = (1\text{-}k_H)H_0 \qquad (6)$$
$$= \quad 0{,}130 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0 + H_g(b+h) \qquad (7)$$
$$= \quad 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1\text{-}k_M)M_0 \qquad (8)$$
$$= \quad 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

$$M_{q2} = (1\text{-}k_H)H_0(c+h) \qquad (9)$$
$$= \quad 0{,}481 \text{ MNm}$$

Kuormien laskenta-arvot: pystykuorma, vaakakuorma ja momentti

$$V_d = V_g + V_{q1} \qquad (10)$$
$$= \quad 1{,}899 \text{ MN}$$
$$H_d = H_g + H_{q2} \qquad (11)$$
$$= \quad 0{,}130 \text{ MN}$$
$$M_d = M_g + M_{q1} + M_{q2} \qquad (12)$$
$$= \quad 0{,}721 \text{ MNm}$$

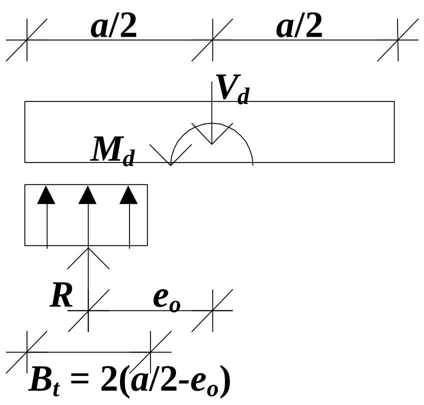
Momenttitasapainosta kiertymispisteen suhteen (kuva 2)

$$M_d - V_d e_o = 0 \qquad (13)$$

saadaan kuormaepäkeskisyys

$$e_o = \frac{M_d}{V_d} \quad (14)$$

$= \quad 0,380 \text{ m}$



Kuva 2.

Tasaisen pohjapaineen vaikutusalueen pituus *a* -mitan suunnassa

$$b = 2(\frac{a}{2} - e_o) \quad (15)$$

$= \quad 1,541 \text{ m}$

Peruslaatan tehokkaan alueen pienempi sivumitta (kuva 3)

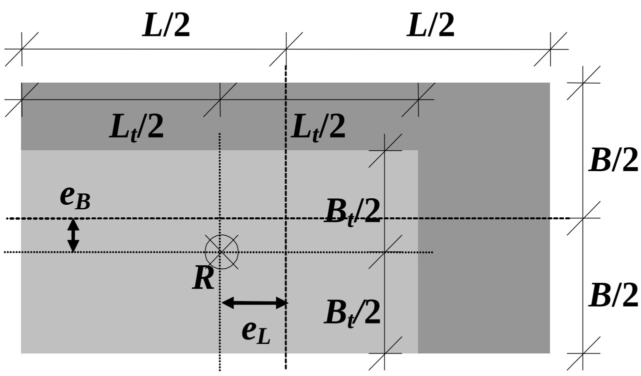
$$B_t = \min\begin{cases} b \\ L \end{cases} \quad (16)$$

$= \quad 1,541 \text{ m}$

Peruslaatan tehokkaan alueen suurempi sivumitta

$$L_t = \max\begin{cases} b \\ L \end{cases} \quad (17)$$

$= \quad 4,000 \text{ m}$



Kuva 3.

Tehokkaan alueen ala

$$A_t = B_t L_t \tag{18}$$

$$= \quad 6{,}162 \text{ m}^2$$

Pohjapaineen suuruus

$$q_d = \frac{V_d}{A_t} \tag{19}$$

$$= \quad 0{,}308 \text{ MN/m}^2$$

Peruslaatan pienin perustamissyvyys

$$D = c \tag{20}$$

$$= \quad 1{,}500 \text{ m}$$

Koheesion laskenta-arvo

$$c_d = \quad 0 \text{ MN/m}^2$$

Perustamistason yläpuolinen tilavuuspaino

$$\gamma'_1 = \gamma_m \tag{21}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Perustamistason alapuolinen tilavuuspaino

$$\gamma'_2 = \gamma_m \tag{22}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Kitkakulman laskenta-arvo

$$\varphi_d = \varphi \tag{23}$$

$$= 30{,}000\ °$$

Kantavuuskertoimet

$$N_D = \tan^2(45° + \frac{\varphi_d}{2})e^{\pi \tan\varphi_d} \tag{24}$$

$$= 18{,}401$$

$$N_B = 1{,}5\left[\tan^2\left(45° + \frac{\varphi_d}{2}\right)e^{\pi \tan\varphi_d} - 1\right]\tan\varphi_d \tag{25}$$

$$= 15{,}070$$

Peruslaatan muodon vaikutuskertoimet

$$s_B = 1 - 0{,}4\left(\frac{B_t}{L_t}\right) \tag{26}$$

$$= 0{,}846$$

$$s_D = 1 + 0{,}2\left(\frac{B_t}{L_t}\right) \tag{27}$$

$$= 1{,}077$$

Kuormitusresultantin kaltevuuden vaikutuskertoimet

$$i_D = \left(1 - \frac{H_d}{V_d + A_t c_d \cot\varphi_d}\right)^2 \tag{28}$$

$$= 0{,}868$$

$$i_B = \left(1 - \frac{H_d}{V_d + A_t c_d \cot\varphi_d}\right)^4 \tag{29}$$

$$= 0{,}753$$

Kantokyvyn laskenta-arvo

$$q_{md} = c_d N_c s_c i_c + \gamma_1' D N_D s_D i_D + \frac{1}{2}\gamma_2' B_t N_B s_B i_B \tag{30}$$

$$= 0{,}664\ \mathrm{MN/m^2}$$

Kantavuuden suhde pohjapaineeseen

$$n = q_{md} / q_d \qquad (31)$$

$$= 2{,}154 \quad > \quad 2{,}000 \quad \text{OK}$$

Peruslaatan leveys saadaan iteroimalla.

$$a = a_1 = 2{,}300 \text{ m}$$

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

KAIKKI KUORMITUSTAPAUKSET

Perustuksen *a* -mitat eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain *a* :sta ja/tai osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

VASTAUS: **Peruslaatan leveys**

$$a = 2{,}300 \text{ m}$$

Taulukko 1.

Suure	Yht.	Kuormitustapaukset				Yksikkö
i		a	b	c	d	
a		2,209	1,470	2,141	0,853	m
ext(a)		Max			Min	
a_{round}		2,300	1,500	2,200	0,900	m
γ_g	2	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,00	1,00	0,00	0,00	
$\gamma_{qk\,2}$	2	1,00	0,00	1,00	0,00	
V_g	3	0,987	0,889	0,978	0,808	MN
V_d	10	1,887	1,789	0,978	0,808	MN
H_d	11	0,130	0,000	0,130	0,000	MN
M_d	12	0,721	0,240	0,577	0,096	MNm
e_0	14	0,382	0,134	0,590	0,119	m
b	15	1,445	1,202	0,961	0,616	m
B_t	16	1,445	1,202	0,961	0,616	m
L_t	17	4,000	4,000	4,000	4,000	m
A_t	18	5,779	4,807	3,843	2,463	m^2
q_d	19	0,327	0,372	0,254	0,328	MN/m^2
s_B	26	0,856	0,880	0,904	0,938	
s_D	27	1,072	1,060	1,048	1,031	
i_D	28	0,867	1,000	0,752	1,000	
i_B	29	0,752	1,000	0,565	1,000	
q_{md}	30	0,653	0,745	0,509	0,656	MN/m^2
n	31	2,000	2,000	2,000	2,000	

24. TEHTÄVÄ

Määritä perustuksen (kuva 1) pienin mitta *a* maan kantokyvyn mukaan sallittujen jännitysten menetelmällä (vrt. *Eurokoodin* ohjaileviin sääntöihin perustuva menetelmä, *EN 1997-1:2004*, 6.4(5)P), kun

σ_{sall} = 0,400 MN/m^2

Yläpuolisesta rakenteesta aiheutuvan kuormituksen ominaisarvot

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Pysyvän kuorman osuus kokonaiskuormasta on

k_V = 0,400

k_H = 0,000

k_M = 0,400

ja loput on muuttuvaa kuormaa. Pystykuorman V_o ja momentin M_o muuttuva osuus aiheutuu hyötykuormasta ja vaakakuorman H_o tuulikuormasta.

Teräsbetonin tilavuuspaino

γ_c = 0,025 MN/m^3

Maan tilavuuspaino

γ_m = 0,020 MN/m^3

Maan sisäinen kitkakulma

φ = 30,000 °

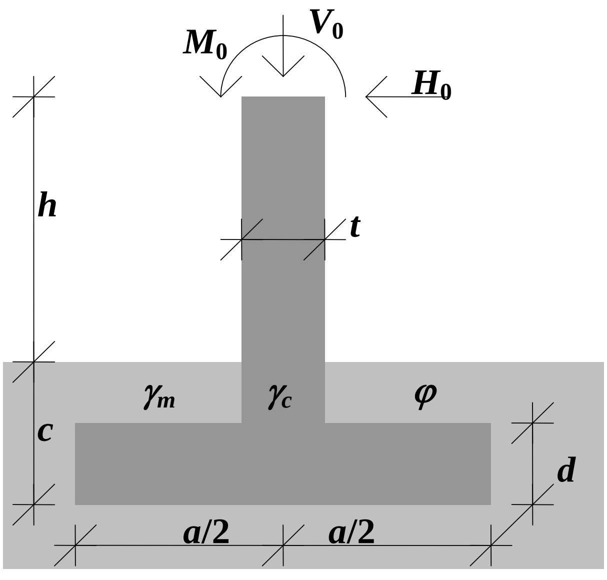
Mitat

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m



Kuva 1.

Perustuksen toinen sivumitta

L = 4,000 m

Tässä sivusuunnassa kuormitus on keskeinen.

KUORMIEN YHDISTELY

Kuormien yhdistely

$$q_d = \sum_{i=1}^{m} g_i + q_{k1} + q_{k2} + \sum_{i=3}^{n} q_{ki} \qquad (1)$$

voidaan esittää muodossa

$$q_d = g + \left.\begin{matrix}1\\1\\0\\0\end{matrix}\right\} q_{k1} + \left.\begin{matrix}1\\0\\1\\0\end{matrix}\right\} q_{k2} \qquad \begin{matrix}(a)\\(b)\\(c)\\(d)\end{matrix} \qquad (2a\ldots d)$$

Kertoimet vaakariveittäin vastaavat samaa tapausta.

Tutkitaan alkuun kuormitustapaus a ja sen jälkeen kaikki muut.

KUORMITUSTAPAUS a

Valitaan

$$a = 2{,}400 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Lasketaan ominaiskuormat peruslaatan alapinnan tasossa. Alaindeksit viittaavat kaavaan 1.

Yläpuolisen rakenteen, perustuksen ja maan omapaino (*a*- mitan funktio)

$$V_g = k_V V_0 + \gamma_c[(h + c - d)t + ad]L + \gamma_m (a - t)(c - d)L \qquad (3)$$
$$= 1{,}012 \text{ MN}$$

Muuttuva pystysuora hyötykuorma

$$V_{q1} = (1 - k_V)V_0 \qquad (4)$$
$$= 0{,}900 \text{ MN}$$

Pysyvien kuormien aiheuttama momentti

$$M_g = k_M M_0 + k_H H_0 (c+h) \qquad (5)$$
$$= 0{,}096 \text{ MNm}$$

Hyötykuorman aiheuttama momentti

$$M_{q1} = (1-k_M) M_0 \qquad (6)$$
$$= 0{,}144 \text{ MNm}$$

Tuulikuorman aiheuttava momentti

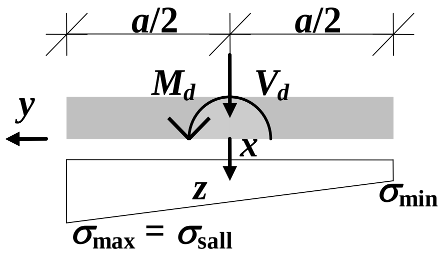
$$M_{q2} = (1-k_H) H_0 (c+h) \qquad (7)$$
$$= 0{,}481 \text{ MNm}$$

Kuormien laskenta-arvot: pystykuorma ja momentti

$$V_d = V_g + V_{q1} \qquad (8)$$
$$= 1{,}912 \text{ MN}$$
$$M_d = M_g + M_{q1} + M_{q2} \qquad (9)$$
$$= 0{,}721 \text{ MNm}$$

Normaalivoiman ja taivutusmomentin aiheuttama normaalijännitys (kuva 2)

$$\sigma(y) = \frac{V_d}{A} + \frac{M_d}{I_x} y \qquad (10)$$



Kuva 2.

Suorakaidepoikkileikkauksen ala

$$A = aL \qquad (11)$$
$$= 9{,}600 \text{ m}^2$$

Suorakaidepoikkileikkauksen jäyhyysmomentti x -akselin suhteen

$$I_x = \frac{La^3}{12} \tag{12}$$

$$= \quad 4{,}608 \ \mathrm{m}^4$$

Maksimijännitys (kuva 2)

$$\sigma_{max} = \sigma(y = a/2) \tag{13}$$

$$= \quad 0{,}387 \ \mathrm{MN/m^2} \quad < \quad \sigma_{sall}$$

Peruslaatan leveys saadaan iteroimalla.

$$a = \quad 2{,}400 \ \mathrm{m}$$

Arvo on pyöristetty ylöspäin 0,1 m:n tarkkuudella.

Minimijännitys (kuva 2)

$$\sigma_{min} = \sigma(y = -a/2) \tag{14}$$

$$= \quad 0{,}011 \ \mathrm{MN/m^2} \quad \text{puristusta}$$

KAIKKI KUORMITUSTAPAUKSET

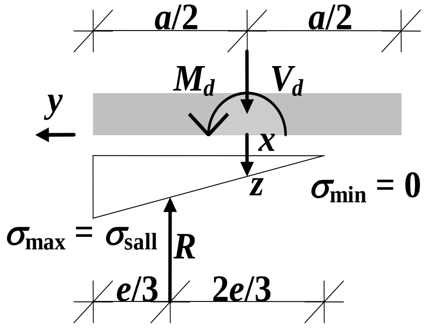
Perustuksen a -mitat eri kuormitustapauksissa on laskettu taulukossa 1. Mukana ovat vain a :sta ja/tai osavarmuuskertoimista riippuvat suureet; muut suureet on laskettu edellä.

KUORMITUSTAPAUS c

Kuormitustapauksessa c saadaan perustuksen toiseen reunaan vetoa. Vetoa ei voi esiintyä. Tarkastellaan tilanne, kun poikkileikkaus on osittain puristettu (kuva 3).

Taulukko 1.

Suure	Yht.	Kuormitustapaukset				Yksikkö
i		a	b	c	d	
a		2,343	1,673	1,791	0,907	m
ext(a)		Max			Min	
a_{round}		2,400	1,700	1,800	1,000	m
γ_g	2	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,00	1,00	0,00	0,00	
$\gamma_{qk\,2}$	2	1,00	0,00	1,00	0,00	
V_g	3	1,004	0,916	0,932	0,815	MN
V_d	8	1,904	1,816	0,932	0,815	MN
M_d	9	0,721	0,240	0,577	0,096	MNm
A	11	9,370	6,692	7,163	3,626	m^2
I_x	12	4,285	1,561	1,914	0,248	m^4
σ_{max}	13	0,400	0,400	0,400	0,400	MN/m^2
σ_{min}	14	0,006	0,143	-0,140	0,049	MN/m^2
ext(σ_{min})			Max	Min		



Kuva 3.

Valitaan

$$a_c = \quad 2{,}100 \text{ m}$$

Tässä oleva arvo on iteroinnin tuloksena saatu lopullinen ratkaisu.

Yläpuolisen rakenteen, perustuksen ja maan omapaino (kaava 3, a_c - mitan funktio)

$$V_{g\,c} = \quad 0{,}972 \text{ MN}$$

Kuormien laskenta-arvot: pystykuorma ja momentti

$$V_{d\,c} = V_{g\,c} \tag{15}$$

$$= \quad 0{,}972 \text{ MN}$$

$$M_{d\,c} = M_g + M_{q\,2} \tag{16}$$

$$= \quad 0{,}577 \text{ MNm}$$

Pystyvoimien tasapainoehdosta (kuva 3)

$$V_{dc} = \frac{1}{2} e L \sigma_{sall} \tag{17}$$

saadaan puristetun alueen leveys

$$e = \frac{2V_{dc}}{L\sigma_{sall}} \tag{18}$$

$$= \quad 1{,}216 \text{ m}$$

Momenttitasapainoehto origon suhteen

$$M_d = V_{dc}\left(\frac{a}{2} - \frac{e}{3}\right) \tag{19}$$

$$= \quad 0{,}627 \text{ MNm}$$

Pohjapaineen aiheuttaman momentin suhde ulkoiseen momenttiin

$$n = M_d / M_{d\,c} \tag{20}$$

$$= \quad 1{,}087 \quad \geq \quad 1$$

Peruslaatan leveys saadaan iteroimalla.

$$a = a_c = \quad 2{,}100 \text{ m}$$

VASTAUS: **Peruslaatan leveys**

$$a = \quad 2{,}400 \text{ m}$$

25. TEHTÄVÄ

Laske kuvan 1 lyöntipaaluryhmän paaluvoimat!

Kaikilla paaluilla on sama puristusjäykkyys

$$C_i = E_i A_i$$

Paalujen pituudet

$L_1 =$ 14,000 m

$L_2 =$ 10,500 m

$L_3 =$ 7,000 m

Paalujen lukumäärät riveittäin

$n_1 =$ 4 kpl

$n_2 =$ 3 kpl

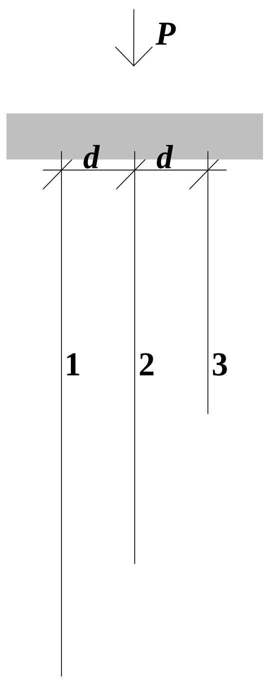
$n_3 =$ 3 kpl

Pystyvoima

$P =$ 6 MN

Mitta

$d =$ 1,000 m



Kuva 1.

Paalujen suhteelliset jäykkyydet

$$k_i = \frac{C_i / L_i}{C_1 / L_1} = \frac{L_1}{L_i} \qquad (1)$$

$k_1 =$ 1,000

$k_2 =$ 1,333

$k_3 =$ 2,000

Jäykkyydet otetaan huomioon painopisteakselia laskettaessa.

Painopisteakselin etäisyys paalurivistä (kuva 2)

$$r_1 = \frac{\sum_{i=2}^{3} n_i k_i z_i}{\sum_{i=1}^{3} n_i k_i} \qquad (2)$$

$$= \frac{n_2 k_2 d + n_3 k_3 \cdot 2d}{n_1 k_1 + n_2 k_2 + n_3 k_3} \qquad (3)$$

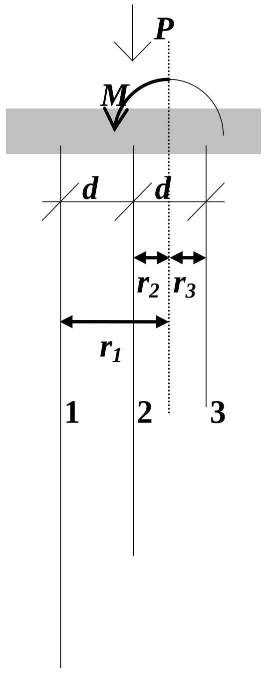
= 1,143 m

$r_2 = r_1 - d$ (4)

= 0,143 m

$r_3 = r_1 - 2d$ (5)

= -0,857 m



Kuva 2.

Momentti painopisteakselilla

$M = Pr_2$ (6)

= 0,857 MNm

Paaluvoima rivissä *i*

$$N_i = \frac{k_i}{\sum n_i k_i} P + \frac{k_i r_i}{\sum n_i k_i r_i^2} M \qquad (7)$$

$$= \frac{k_i P}{n_1 k_1 + n_2 k_2 + n_3 k_3} + \frac{k_i r_i M}{n_1 k_1 r_1^2 + n_2 k_2 r_2^2 + n_3 k_3 r_3^2} \qquad (8)$$

<u>**VASTAUS**</u>**: Paaluvoimat riveittäin**

$N_1 =$ 0,529 MN

$N_2 =$ 0,588 MN

$N_3 =$ 0,706 MN

26. TEHTÄVÄ

Määritä kuvan 1 lyöntipaaluryhmän pienin ja suurin paaluvoima (taulukot 1 ja 2) sekä suhteelliset siirtymät ja kiertymä!

Kuormitus

F_x = 8 MN
F_z = 2 MN
M_y = 1 MNm

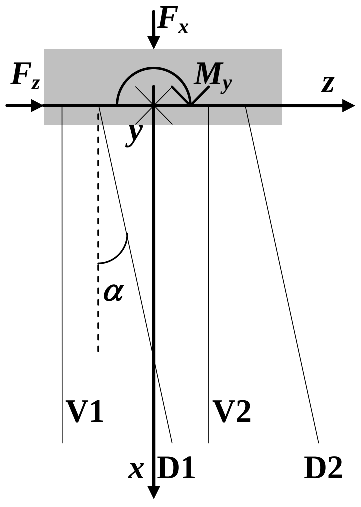
Taulukko 1. Vertikaalipaalut (z_{Vi} paalun yläpäässä).

Muuttuja	Rivi	Paalujen lkm	Vaaka-sijainti	Suht. jäykkyys
Symboli	i_V	n_{Vi}	z_{Vi}	k_{Vi}
Yksikkö	-	kpl	m	-
Lukuarvo	1	10	-3,000	0,600
	2	9	3,000	0,700

Taulukko 2. Diagonaalipaalut (z_{Di} paalun yläpäässä), joiden kaltevuus on

κ = 1/tanα
= 3,500 .

Muuttuja	Rivi	Paalujen lkm	Vaaka-sijainti	Suht. jäykkyys
Symboli	i_D	n_{Di}	z_{Di}	k_{Di}
Yksikkö	-	kpl	m	-
Lukuarvo	1	8	-3,000	0,962
	2	7	2,500	0,962



Kuva 1.

Pysty- ja vinopaalujen suhteelliset jäykkyydet

$$k_i = \frac{\dfrac{E_i A_i}{L_i}}{\dfrac{E_0 A_0}{L_0}}$$

ovat keskenään verrannollisia.

Diagonaalien kaltevuuskulma

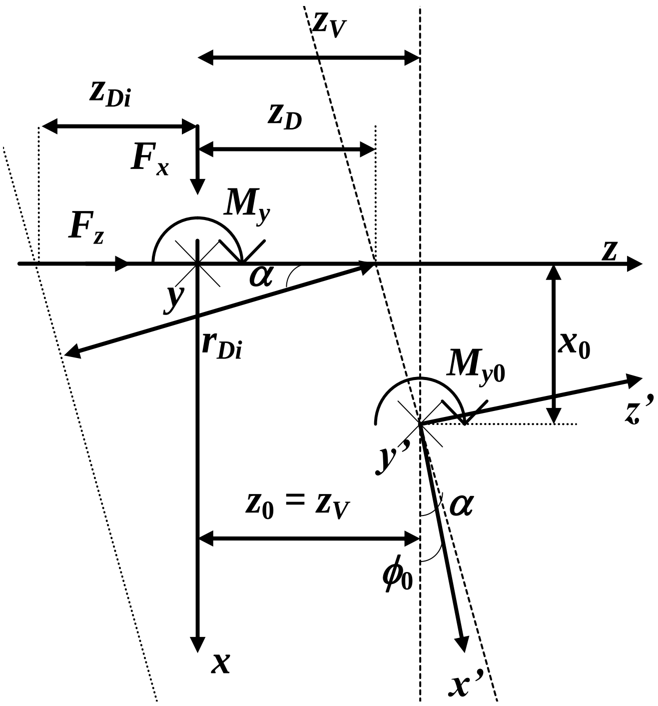
$$\alpha = \arctan(1/\kappa) \tag{1}$$
$$= 15{,}945\ °$$

Vertikaali- ja diagonaalipaalujen aksiaaliset jäykkyydet

$$A_V = \Sigma n_{Vi} k_{Vi} \tag{2}$$
$$= 12{,}300$$
$$A_D = \Sigma n_{Di} k_{Di} \tag{3}$$
$$= 14{,}423$$

Vertikaali ja diagonaalipaalujen painopisteakselien *z*- koordinaatit paalun katkaisutasolla (kuva 2)

$$z_V = \frac{\sum n_{Vi} k_{Vi} z_{Vi}}{A_V} \tag{4}$$
$$= 0{,}073\ \text{m}$$
$$z_D = \frac{\sum n_{Di} k_{Di} z_{Di}}{A_D} \tag{5}$$
$$= -0{,}433\ \text{m}$$



Kuva 2.

Kiertokeskiön sijainti (kuva 2)

$$\begin{cases} x_0 = \dfrac{z_V - z_D}{\tan\alpha} \\ z_0 = z_V \end{cases} \qquad \textbf{(6a, b)}$$

$$\Rightarrow \begin{cases} x_0 = & 1{,}773 \text{ m} \\ z_0 = & 0{,}073 \text{ m} \end{cases}$$

Pääjäykkyyssuunnan kulma myötäpäivään (ei tarvita tehtävässä)

$$\phi_0 = \frac{1}{2}\arctan\left[\frac{-2A_D\cos\alpha\sin\alpha}{A_V + A_D\cos 2\alpha}\right] \qquad \textbf{(7)}$$

$$= \quad -8{,}623\ °$$

$$= \quad -0{,}150 \text{ rad}$$

Vertikaalipaalujen kohtisuorat etäisyydet (posit./negat.) kiertokeskiöstä

$$r_{Vi} = z_{Vi} - z_V \qquad \textbf{(8)}$$

$$r_{V1} = \quad -3{,}073 \text{ m}$$

$$r_{V2} = \quad 2{,}927 \text{ m}$$

Diagonaalipaalujen kohtisuorat etäisyydet (posit./negat.) kiertokeskiöstä

$$r_{Di} = (z_{Di} - z_D)\cos\alpha \qquad \textbf{(9)}$$

$$r_{D1} = \quad -2{,}468 \text{ m}$$

$$r_{D2} = \quad 2{,}820 \text{ m}$$

Jäyhyysmomentti kiertokeskiön suhteen

$$\begin{cases} I_V = \sum n_{Vi} k_{Vi} r_{Vi}^2 \\ I_D = \sum n_{Di} k_{Di} r_{Di}^2 \end{cases} \qquad \textbf{(10a, b)}$$

$$\Rightarrow \begin{cases} I_V = & 110{,}634 \text{ m}^2 \\ I_D = & 100{,}393 \text{ m}^2 \end{cases}$$

$$I = I_V + I_D \qquad \textbf{(11)}$$

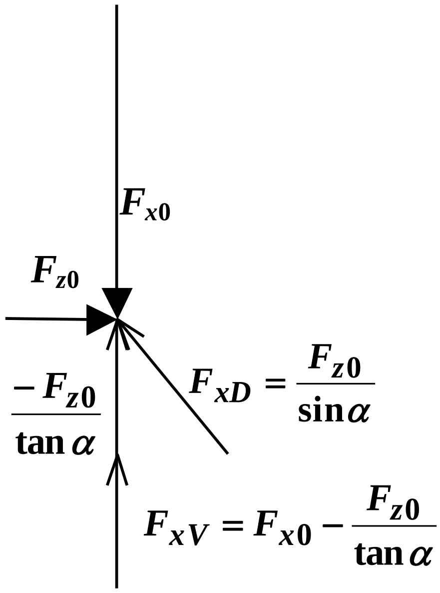
$$= \quad 211{,}027 \text{ m}^2$$

Kuormitus kiertokeskiössä

$$F_{x0} = F_x \tag{12}$$
$$= 8{,}000 \text{ MN}$$
$$F_{z0} = F_z \tag{13}$$
$$= 2{,}000 \text{ MN}$$
$$M_{y0} = M_y - F_x z_0 + F_z x_0 \tag{14}$$
$$= 3{,}960 \text{ MNm}$$

Kuormaosuudet pysty- ja vaakavoimista (kuva 3)

$$F_{xD} = \frac{F_{z0}}{\sin\alpha} \tag{15}$$
$$= 7{,}280 \text{ MN} \qquad \textbf{Puristusta}$$
$$F_{xV} = F_{x0} - \frac{F_{z0}}{\tan\alpha} \tag{16}$$
$$= 1{,}000 \text{ MN} \qquad \textbf{Puristusta}$$



Kuva 3.

Vertikaalipaalujen aksiaalivoimat[1]

$$N_{Vi} = k_{Vi}\left(\frac{F_{xV}}{A_V} + \frac{M_{y0}}{I} r_{Vi}\right) \tag{17}$$

$N_{V1} =$ 0,014 MN Puristusta

$N_{V2} =$ 0,095 MN Puristusta

Diagonaalipaalujen aksiaalivoimat

$$N_{Di} = k_{Di}\left(\frac{F_{xD}}{A_D} + \frac{M_{y0}}{I} r_{Di}\right) \tag{18}$$

$N_{D1} =$ 0,441 MN Puristusta

$N_{D2} =$ 0,536 MN Puristusta

Suuntakulmien kosinit

$$p_{xVi} = \cos(0) \tag{19}$$
$$= 1{,}000$$
$$p_{zVi} = \sin(0) \tag{20}$$
$$= 0{,}000$$
$$p_{xDi} = \cos\alpha \tag{21}$$
$$= 0{,}962$$
$$p_{zDi} = \sin\alpha \tag{22}$$
$$= 0{,}275$$

Jäykkyysmatriisin alkiot

$$k_{11} = \Sigma n_{Vi} k_{Vi} p_{xVi}^{\ 2} + \Sigma n_{Di} k_{Di} p_{xDi}^{\ 2} \tag{23}$$
$$= 25{,}634$$
$$k_{12} = \Sigma n_{Vi} k_{Vi} p_{xVi} p_{zVi} + \Sigma n_{Di} k_{Di} p_{xDi} p_{zDi} \tag{24}$$
$$= 3{,}810$$
$$k_{22} = \Sigma n_{Vi} k_{Vi} p_{zVi}^{\ 2} + \Sigma n_{Di} k_{Di} p_{zDi}^{\ 2} \tag{25}$$
$$= 1{,}089$$

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1) Vertaa yhtälöön

$$\sigma = \frac{F}{A} + \frac{M}{I} y$$

$$k_{33} = I \tag{26}$$
$$= \quad 211{,}027 \text{ m}^2$$

Jäykkyysmatriisi

$$\mathrm{K} = \begin{bmatrix} 25{,}634 & 3{,}810 & 0{,}000 \\ 3{,}810 & 1{,}089 & 0{,}000 \\ 0{,}000 & 0{,}000 & 211{,}027 \end{bmatrix}$$

Jäykkyysmatriisi on dimensioton lukuun ottamatta alkiota k_{33}.

Käänteismatriisi

$$\mathrm{K}^{-1} = \begin{bmatrix} 0{,}081 & -0{,}285 & 0{,}000 \\ -0{,}285 & 1{,}915 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}005 \end{bmatrix}$$

Voimavektorin $\{f\}$ alkiot

$$F_{x0} = 8{,}000 \text{ MN}$$
$$F_{z0} = 2{,}000 \text{ MN}$$
$$M_{y0} = 3{,}960 \text{ MNm}$$

Siirtymävektori koordinaatiston ollessa kiertokeskiössä

$$\{\delta\} = [K]^{-1}\{f\} \tag{27}$$
$$\Rightarrow \begin{cases} u_0 = 0{,}081 \text{ MN} \\ w_0 = 1{,}553 \text{ MN} \\ \varphi = 0{,}019 \text{ MNm} \end{cases}$$

Suhteelliset siirtymät alkuperäisessä koordinaatistossa

$$u = u_0 - z_0 \sin\varphi \tag{28}$$
$$= 0{,}080 \text{ MN}$$
$$w = w_0 + x_0 \sin\varphi \tag{29}$$
$$= 1{,}586 \text{ MN}$$

Todelliset siirtymät saadaan, kun käytetään todellisia jäykkyyksiä.

Paaluvoimat saadaan myös siirtymäsuureiden avulla pystypaaluille

$$N_{Vi} = k_{Vi}(p_{xVi}u_0 + p_{zVi}w_0 + r_{yVi}\varphi) \qquad (30)$$

$N_{V1} =$ **0,014 MN**

$N_{V2} =$ **0,095 MN**

ja vinopaaluille

$$N_{Di} = k_{Di}(p_{xDi}u_0 + p_{zDi}w_0 + r_{yDi}\varphi) \qquad (31)$$

$N_{D1} =$ **0,441 MN**

$N_{D2} =$ **0,536 MN**

VASTAUS:

Pienin ja suurin paaluvoima

$N_{\min} =$	0,014 MN	Puristusta	V1
$N_{\max} =$	0,536 MN	Puristusta	D2

Suhteelliset siirtymät ja kiertymä

$u =$ **0,080 MN**

$w =$ **1,586 MN**

$\varphi =$ **0,019 MNm**

V a i h t o e h t o i n e n t a p a p a a l u v o i m i e n l a s k e n t a a n

Momenttien kuormaosuudet

$$M_V = \frac{I_V}{I_V + I_D} M_{y0} \tag{32}$$

$= \quad$ **2,076 MNm**

$$M_D = M_{y0} - M_V \tag{33}$$

$= \quad$ **1,884 MNm**

Vertikaalipaalujen aksiaalivoimat

$$N_{Vi} = k_{Vi}\left(\frac{F_{xV}}{A_V} + \frac{M_V r_{Vi}}{I_V}\right) \tag{34}$$

$N_{V1} =$	0,014	MN	Puristusta
$N_{V2} =$	0,095	MN	Puristusta

Diagonaalipaalujen aksiaalivoimat

$$N_{Di} = k_{Di}\left(\frac{F_{xD}}{A_D} + \frac{M_D r_{Di}}{I_D}\right) \tag{35}$$

$N_{D1} =$	0,441	MN	Puristusta
$N_{D2} =$	0,536	MN	Puristusta

27. TEHTÄVÄ

Vanhaa puupaaluperustusta on myöhemmin täydennetty teräsbetonipaaluilla ja vielä tämän jälkeen yhdellä teräspaalulla. Ks. kuva 1 ja taulukko 1.

Tutki ovatko lyöntipaaluryhmän paaluvoimat hyväksyttäviä! Määritä myös lyöntipaaluryhmän pääjäykkyyssuunnan koordinaatisto ja pääjäykkyydet!

Taulukko 1. Paalurivien lukumäärä sekä näissä riveissä olevien paalujen lukumäärä, kaltevuus poikkileikkausala, kimmokerroin ja sijainti.

Muuttuja	Rivi	Paalujen lkm	Kaltevuus	Poikkil. ala	Kimmo-kerroin	Pysty-sijainti	Vaaka-sijainti
Symboli	i	j_i	$\tan\alpha_i$	A_i	E_i	x_i	z_i
Yksikkö	-	kpl	-	m^2	MN/m^2	m	m
Lukuarvo	1	5	0,000	0,031	5 600	0	-2,000
	2	5	0,000	0,031	5 600	0	0,000
	3	3	0,200	0,090	31 600	0	0,000
	4	1	0,250	0,018	210 000	0	2,000

Paalujen sallitut puristusjännitykset paalutusluokassa III ovat seuraavat:

Puu[1]

$\sigma_{w,\,sall} =$ 5 MN/m^2

Teräsbetoni[2]

$\sigma_{c,\,sall} =$ 5 MN/m^2

Teräs[3]

$\sigma_{s,\,sall} =$ 40 MN/m^2

---

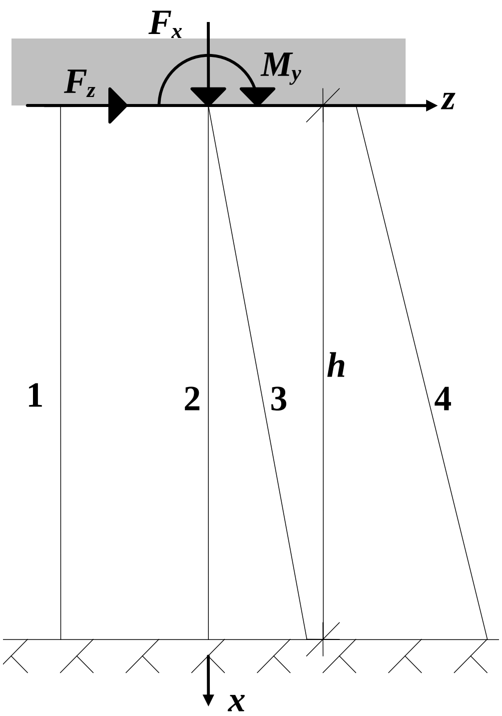
1) r = 100 mm, T40, AL A, KL 1.
2) a = 300 mm, K40.
3) r = 75 mm, Fe52C.

Kuormitus

$$F_x = 2{,}000 \text{ MN}$$
$$F_z = 0{,}400 \text{ MN}$$
$$M_y = 1{,}000 \text{ MNm}$$

Kovan pinnan syvyys

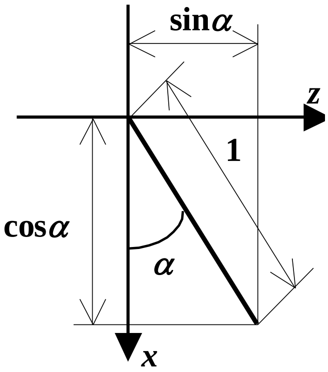
$$h = 8 \text{ m}$$



Kuva 1.

Pituus (kuva 2)

$$L_i = h_i / \cos\alpha_i \tag{1}$$



Kuva 2.

Jäykkyys

$$k_i = \frac{C_i}{L_i} \tag{2}$$

$$\Rightarrow k_i = \frac{E_i A_i}{L_i} \tag{3}$$

Suuntakulman kosini x - ja z -akselien suhteen (kuva 2)

$$p_{x,i} = \cos\alpha_i \tag{4}$$

$$p_{z,i} = \cos(90° - \alpha_i) \tag{5}$$

$$= \sin\alpha_i \tag{6}$$

Normaalivoiman momenttivarsi origon suhteen

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{7}$$

Paalurivien jäykkyysmatriisien alkiot

$$k_{11,i} = n_i k_i p_{x,i}^{\ 2} \tag{8}$$

$$k_{12,i} = k_{21,i} \tag{9}$$

$$= n_i k_i p_{x,i} p_{z,i} \tag{10}$$

$$k_{13,i} = k_{31,i} \tag{11}$$

$$= n_i k_i p_{x,i} r_{y,i} \tag{12}$$

$$k_{22,i} = n_i k_i p_{z,i}^{\;2} \qquad (13)$$

$$k_{23,i} = k_{32,i} \qquad (14)$$

$$= n_i k_i p_{z,i} r_{y,i} \qquad (15)$$

$$k_{33,i} = n_i k_i r_{y,i}^{\;2} \qquad (16)$$

Paaluryhmän jäykkyysmatriisin alkiot

$$k_{mn} = \sum_{i=0}^{\max} k_{mn,i} \qquad (17)$$

Taulukko 2. Jäykkyysmatriisin alkioiden laskenta.

Muuttuja	Rivi	Paalujen lkm	Pysty-sijainti	Vaaka-sijainti	Kalte-vuus	Kaltevuus-kulma	Pinta-ala
Symboli	i	j_i	x_i	z_i	$\tan\alpha_i$	α_i	A_i
Yksikkö	-	kpl	m	m	-	°	m^2
Lukuarvo	1	5	0,000	-2,000	0,000	0,000	0,031
	2	5	0,000	0,000	0,000	0,000	0,031
	3	3	0,000	0,000	0,200	11,310	0,090
	4	1	0,000	2,000	0,250	14,036	0,018
	⋮						

Muuttuja	Rivi	Kimmo-kerroin	Pituus	Jäykkyys	Suuntakulmien kosinit		Mom. varsi
Symboli	i	E_i	L_i	k_i	$p_{x,i}$	$p_{z,i}$	$r_{y,i}$
Kaava			(1)	(3)	(4)	(6)	(7)
Yksikkö	-	MN/m^2	m	MN/m	-	-	m
Lukuarvo	1	5 600	8,000	21,700	1,000	0,000	-2,000
	2	5 600	8,000	21,700	1,000	0,000	0,000
	3	31 600	8,158	348,596	0,981	0,196	0,000
	4	210 000	8,246	458,392	0,970	0,243	1,940
	⋮						

... *Muuttuja*	Rivi	Paalurivien jäykkyysmatriisien alkiot					
Symboli	i	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Kaava		(8)	(10)	(12)	(13)	(15)	(16)
Yksikkö	-	MN/m	MN/m	MN	MN/m	MN	MNm
Lukuarvo	1	108,500	0,000	-217,000	0,000	0,000	434,000
	2	108,500	0,000	0,000	0,000	0,000	0,000
	3	1005,567	201,113	0,000	40,223	0,000	0,000
	4	431,428	107,857	862,856	26,964	215,714	1725,712

⋮

Muuttuja	Paaluryhmän jäykkyysmatriisin alkiot					
Symboli	k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Kaava	(17)	(17)	(17)	(17)	(17)	(17)
Yksikkö	MN/m	MN/m	MN	MN/m	MN	MNm
Lukuarvo	1653,995	308,970	645,856	67,187	215,714	2159,712

Jäykkyysmatriisin alkioiden laskenta on esitetty taulukossa 2.

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{18}$$

$$=\begin{bmatrix} 1653,995 & 308,970 & 645,856 \\ 308,970 & 67,187 & 215,714 \\ 645,856 & 215,714 & 2159,712 \end{bmatrix} + \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

Tasapainoehdosta saadaan paaluryhmän siirtymävektori

$$\{f\} = [K]\{\delta\} \tag{19}$$

$$\Rightarrow \{\delta\} = [K]^{-1}\{f\} \tag{20}$$

Siirtymävektorin alkiot (paalulaatan siirtymät ja kiertymät)

$$u = 0{,}001 \text{ m}$$

$$w = 0{,}003 \text{ m}$$

$$\varphi = 0{,}000 \text{ rad}$$

Paaluvoimat on laskettu taulukossa 3.

$$N_i = k_i \Delta_i \tag{21}$$

$$= k_i (p_{x,i} u + p_{z,i} w + r_{y,i} \varphi) \tag{22}$$

$$N_{\max,i} = \sigma_{p,\,\mathrm{sall}} A_i \tag{23}$$

Taulukko 3. Paaluvoimat sekä sallitut ääriarvot.

Symboli	i	$N_{\min,i}$	N_i	$N_{\max,i}$	Sallituissa
Kaava			(21)	(23)	rajoissa
Yksikkö	-	MN	MN	MN	
Lukuarvo	1	0	0,015	0,155	OK
	2	0	0,013	0,155	OK
	3	0	0,435	0,450	OK
	4	0	0,595	0,720	OK

Kiertokeskiö

$$\begin{cases} x_0 = -\dfrac{\begin{vmatrix} k_{11} & k_{13} \\ k_{12} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{11}k_{23} - k_{12}k_{13}}{k_{11}k_{22} - k_{12}^2} \\ z_0 = -\dfrac{\begin{vmatrix} k_{12} & k_{13} \\ k_{22} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{12}k_{23} - k_{22}k_{13}}{k_{11}k_{22} - k_{12}^2} \end{cases} \tag{24a, b}$$

$x_0 =$ -10,0382 m

$z_0 =$ -1,4847 m

Pääjäykkyyssuunnan kulma (vastapäivään) radiaaneina

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{11} - k_{22}}\right) \tag{25}$$

= 0,186 rad

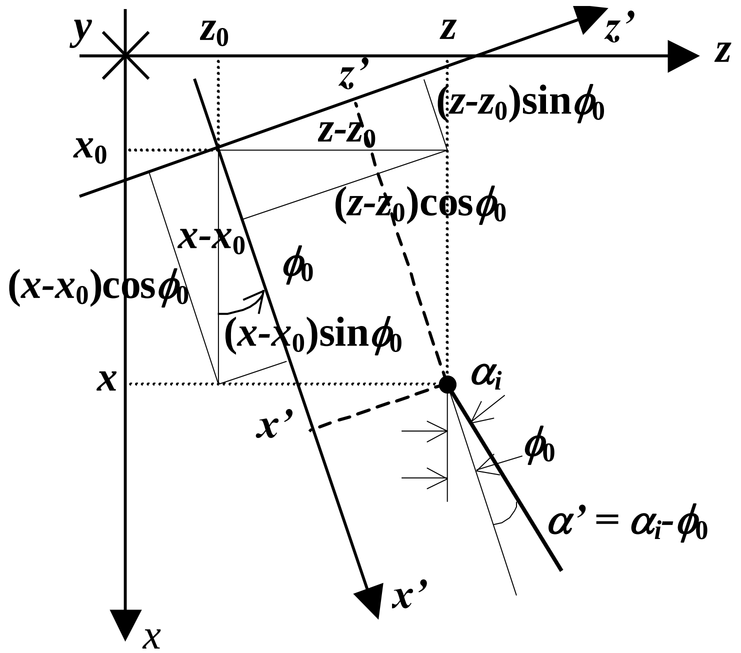
= 10,639 °

Paalun pysty- ja vaakasijainti sekä kaltevuuskulma pääjäykkyys-koordinaatistossa (kuva 3)

$$x_{ki} = (z_i - z_0)\sin\phi_0 + (x_i - x_0)\cos\phi_0 \tag{26}$$

$$z_{ki} = (z_i - z_0)\cos\phi_0 - (x_i - x_0)\sin\phi_0 \tag{27}$$

$$\alpha_{ki} = \alpha_i - \phi_0 \tag{28}$$



Kuva 3.

Taulukko 4. Jäykkyysmatriisin alkioiden laskenta.

Muuttuja	Rivi	Lkm	Sijaintikoordinaatit		Kulma
Symboli	$i_k = i$	$j_{ki} = j_i$	x_{ki}	z_{ki}	α_{ki}
Kaava			(26)	(27)	(28)
Yksikkö	-	kpl	m	m	°
Lukuarvo	1	5	9,771	-2,360	-10,639
	2	5	10,140	-0,394	-10,639
	3	3	10,140	-0,394	0,671
	4	1	10,509	1,572	3,398

⋮

Muuttuja	Rivi	Jäykkyys	Suuntak. kosinit		M.varsi
Symboli	$i_k = i$	$k_{ki} = k_i$	$p_{xk,i}$	$p_{zk,i}$	$r_{yk,i}$
Kaava			(4)	(6)	(7)
Yksikkö	-	MN/m	-	-	m
Lukuarvo	1	21,700	0,983	-0,185	-0,515
	2	21,700	0,983	-0,185	1,485
	3	348,596	1,000	0,012	-0,513
	4	458,392	0,998	0,059	0,946 …

… Muuttuja	Rivi	Paalurivien jäykkyysmatriisien alkiot					
Symboli	$i_k = i$	k_{11ki}	k_{12ki}	k_{13ki}	k_{22ki}	k_{23ki}	k_{33ki}
Yksikkö	-	MN/m	MN/m	MN	MN/m	MN	MNm
Lukuarvo	1	104,802	-19,686	-54,952	3,698	10,322	28,813
	2	104,802	-19,686	158,318	3,698	-29,739	239,162
	3	1045,646	12,253	-536,250	0,144	-6,284	275,011
	4	456,782	27,119	432,883	1,610	25,700	410,235

Muuttuja	Paaluryhmän jäykkyysmatriisin alkiot					
Symboli	k_{11k}	$k_{12k} = k_{21k}$	$k_{13k} = k_{31k}$	k_{22k}	$k_{23k} = k_{32k}$	k_{33k}
Yksikkö	MN/m	MN/m	MN	MN/m	MN	MNm
Lukuarvo	1712,032	0,000	0,000	9,149	0,000	953,222

Jäykkyysmatriisin alkiot on laskettu taulukossa 4.

$$[K_k] = \begin{bmatrix} k_{11k} & k_{12k} & k_{13k} \\ k_{21k} & k_{22k} & k_{23k} \\ k_{31k} & k_{32k} & k_{33k} \end{bmatrix} \tag{29}$$

$$= \begin{bmatrix} 1712{,}032 & 0{,}000 & 0{,}000 \\ 0{,}000 & 9{,}149 & 0{,}000 \\ 0{,}000 & 0{,}000 & 953{,}222 \end{bmatrix} \quad + \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

Pääjäykkyydet sijaitsevat lävistäjällä.

VASTAUS:

Paaluvoimat ovat sallituissa rajoissa.

Kiertokeskiö on

$x_0 =$ -10,0382 m

$z_0 =$ -1,4847 m

Pääjäykkyyssuunnan kulma on

$\phi_0 =$ 0,186 rad

Pääjäykkyydet ovat

$k_{11k} =$ 1 712 MN/m

$k_{22k} =$ 9 MN/m

$k_{33k} =$ 953 MNm

28. TEHTÄVÄ

Paaluille perustettua pilaria (kuva 1 ja taulukko 1) kuormittaa yläpäästä momentti

M_y = 1 MNm

Laske pilarin alapäähän aiheutuva, peruslaattaa kuormittava vaakavoima $F_{z\ 1\text{-}1}$ ja momentti $M_{y\ 1\text{-}1}$!

Taulukko 1. Paalurivien lukumäärä sekä näissä riveissä olevien paalujen lukumäärä, kaltevuus, jäykkyys sekä sijainti.

Muuttuja	Rivi	Paalujen lkm	Kaltevuus	Jäykkyys	Pysty-sijainti	Vaaka-sijainti
Symboli	i	j_i	$\tan\alpha_i$	k_i	x_i	z_i
Yksikkö	-	kpl	-	MN/m	m	m
Lukuarvo	1	1	-0,250	20,000	0,000	-0,800
	2	1	0,250	20,000	0,000	-0,800
	3	1	-0,250	20,000	0,000	0,800
	4	1	0,250	20,000	0,000	0,800

Pilarin poikkileikkaus on neliö, jonka sivun pituus on

a = 0,600 m

ja sen kimmokerroin on

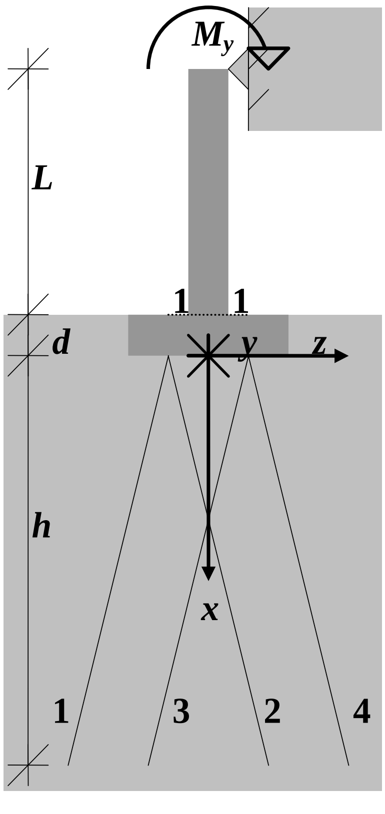
E = 30 000 MN/m²

Mitat

L = 6 m

d = 1 m

h = 10 m



Kuva 1.

Jäykkyysmatriisin alkiot on laskettu taulukossa 2.

Taulukko 2.

Muuttuja	Rivi	Paalujen lkm	Pysty-sijainti	Vaaka-sijainti	Kalte-vuus	Kaltevuus-kulma	Jäykkyys
Symboli	i	j_i	x_i	z_i	$\tan\alpha_i$	α_i	k_i
Yksikkö	-	kpl	m	m	-	°	MN/m
Lukuarvo	1	1	0,000	-0,800	-0,250	-14,036	20,000
	2	1	0,000	-0,800	0,250	14,036	20,000
	3	1	0,000	0,800	-0,250	-14,036	20,000
	4	1	0,000	0,800	0,250	14,036	20,000

⋮

Muuttuja	Rivi	Suuntak. kosinit		M.varsi
Symboli	i	$p_{x,i}$	$p_{z,i}$	$r_{y,i}$
Yksikkö	-	-	-	m
Lukuarvo	1	0,970	-0,243	-0,776
	2	0,970	0,243	-0,776
	3	0,970	-0,243	0,776
	4	0,970	0,243	0,776

⋮

Muuttuja	Rivi	Paalurivin pääjäykkyydet			Paalurivin muut matr. alkiot		
Symboli	i	$k_{11,i}$	$k_{22,i}$	$k_{33,i}$	$k_{12,i}=k_{21,i}$	$k_{13,i}=k_{31,i}$	$k_{23,i}=k_{32,i}$
Yksikkö	-	MN/m	MN/m	MNm	MN/m	MN	MN
Lukuarvo	1	18,824	1,176	12,047	-4,706	-15,059	3,765
	2	18,824	1,176	12,047	4,706	-15,059	-3,765
	3	18,824	1,176	12,047	-4,706	15,059	-3,765
	4	18,824	1,176	12,047	4,706	15,059	3,765

Muuttuja	Paaluryhmän pääjäykkyydet			Muut matriisin alkiot		
Symboli	k_{11}	k_{22}	k_{33}	$k_{12,i}=k_{21}$	$k_{13,i}=k_{31}$	$k_{23,i}=k_{32}$
Yksikkö	MN/m	MN/m	MNm	MN/m	MN	MN
Lukuarvo	75,294	4,706	48,188	0,000	0,000	0,000

Pääjäykkyyssuunnan kulma

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{22}-k_{11}}\right) \tag{1}$$

$$= \boxed{0{,}000} \text{ rad}$$

Kiertokeskiö

$$\begin{cases} x_0 = -\dfrac{\begin{vmatrix} k_{11} & k_{13} \\ k_{12} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{11}k_{23}-k_{12}k_{13}}{k_{11}k_{22}-k_{12}^2} \\ z_0 = -\dfrac{\begin{vmatrix} k_{12} & k_{13} \\ k_{22} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{12}k_{23}-k_{22}k_{13}}{k_{11}k_{22}-k_{12}^2} \end{cases} \tag{2a, b}$$

$$\Rightarrow \begin{cases} x_0 = 0{,}000 \text{ m} \\ z_0 = 0{,}000 \text{ m} \end{cases}$$

Jäykkyysmatriisi (taulukko 2)

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} 75{,}294 & 0{,}000 & 0{,}000 \\ 0{,}000 & 4{,}706 & 0{,}000 \\ 0{,}000 & 0{,}000 & 48{,}188 \end{bmatrix} \begin{bmatrix} \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN} & \text{MN} & \text{MNm} \end{bmatrix}$$

Jäykkyysmatriisin käänteismatriisi

$$[K]^{-1} = \begin{bmatrix} 0{,}013 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}213 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}021 \end{bmatrix}$$

Tasapainoehto

$$\{f\} = [K]\{\delta\} \tag{4}$$

Paaluryhmän siirtymävektori

$$\{\delta\} = [K]^{-1}\{f\} \tag{5}$$

Korvataan pilarin yläpään sivutuki vaakavoimalla

$$F_z = \quad -1 \text{ MN}$$

ja ratkaistaan todellisen tukireaktion suuruus.

$$H = cF_z \tag{6}$$

Momentti origossa vaakavoimasta

$$M_{y\,0,Fz} = F_z(L+d) \tag{7}$$

Voimavektorin alkiot origossa momentista (M_y = 1) ja vaakavoimasta (F_z = -1) on esitetty taulukossa 3.

Taulukko 3.

	$i = M_y$	$i = F_z$	
$F_{x\,0,i}$	0,000	0,000	MN
$F_{z\,0,i}$	0,000	-1,000	MN
$M_{y\,0,i}$	1,000	-7,000	MNm

Paalulaatan siirtymävektorin alkiot momentista (M_y = 1) ja vaakavoimasta (F_z = -1) on esitetty taulukossa 4.

Taulukko 4.

	$i = M_y$	$i = F_z$		
$u_{0,i}$	0,000	0,000	m	pystysiirtymä
$w_{0,i}$	0,000	-0,213	m	vaakasiirtymä
$\varphi_{0,i}$	0,021	-0,145	rad	kiertymä

Pilarin taivutusjäykkyys

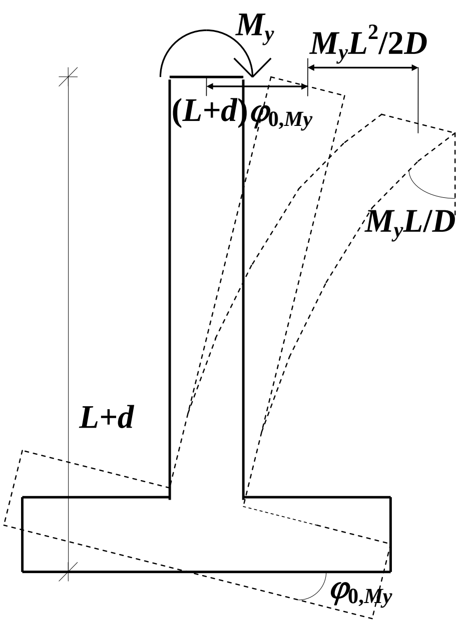
$$D = E\frac{a^4}{12} \tag{8}$$

$$= \quad 324 \ \mathrm{MN/m^2}$$

Momentin M_y aiheuttama vaakasiirtymä pilarin yläpäässä (kuva 2)

$$w_{My} = w_{0,My} + (L+d)\varphi_{0,My} + \frac{M_y L^2}{2D} \tag{9}$$

$$= \quad 0{,}201 \ \mathrm{m}$$



Kuva 2.

Vaakavoiman F_z aiheuttama vaakasiirtymä pilarin yläpäässä (kuva 3)

$$w_{Fz} = w_{0,Fz} + (L+d)\varphi_{0,Fz} + \frac{F_z L^3}{3D} \tag{10}$$

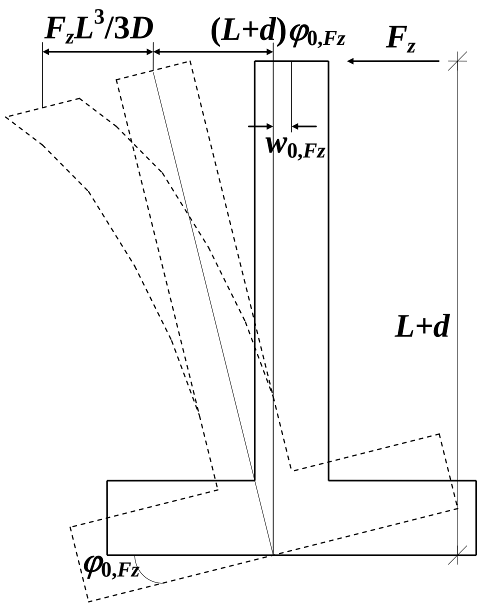
$$= \quad -1{,}452 \ \mathrm{m}$$

Ehto pilarin yläpäässä

$$w = w_{My} + cw_{Fz} = 0 \tag{11}$$

$$\Rightarrow c = \frac{-w_{My}}{w_{Fz}} \tag{12}$$

$$= \quad 0{,}138$$



Kuva 3.

Vaakavoima ja momentti pilarin alapäässä

$$F_{z\,1\text{-}1} = H = cF_z \tag{13}$$

$$= \quad -0{,}138 \text{ MN}$$

$$M_{y\,1\text{-}1} = M_y + HL \tag{14}$$

$$= \quad 0{,}170 \text{ MNm}$$

<u>VASTAUS</u>:

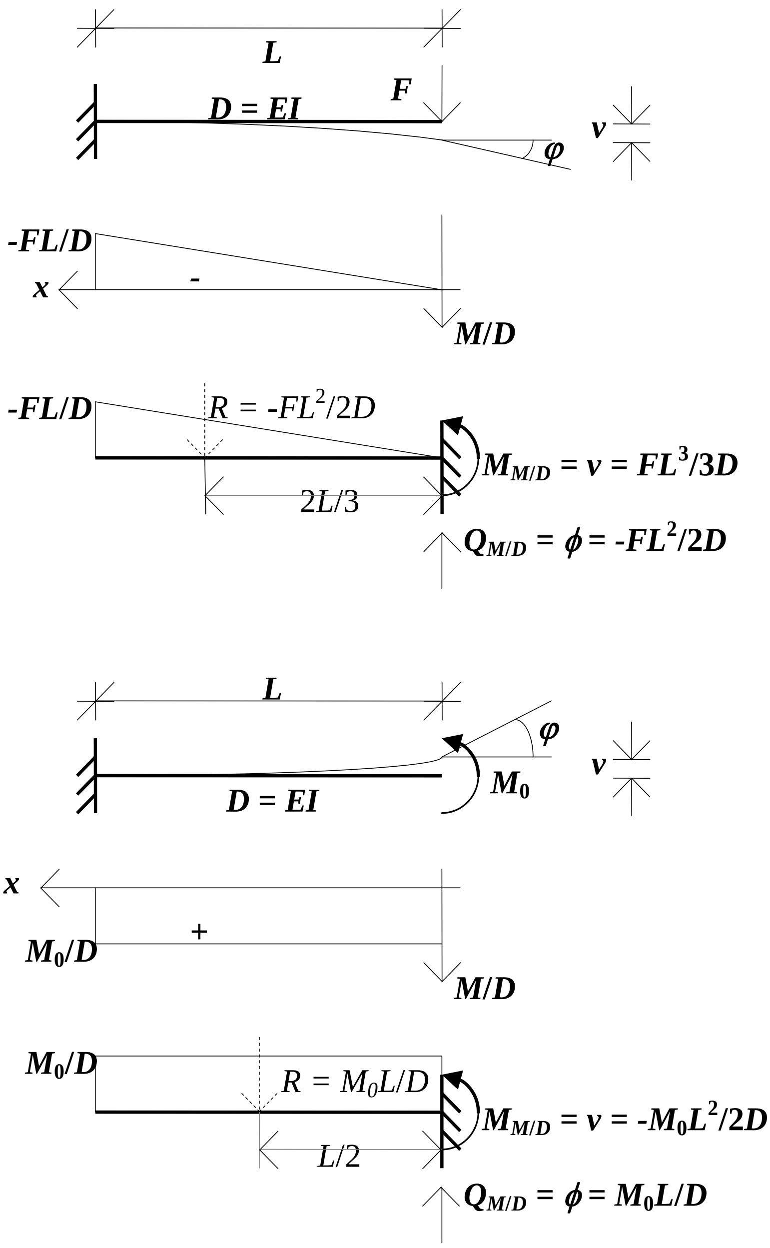
Vaakavoima ja momentti ovat

$$F_{z\,1\text{-}1} = \quad -0{,}138 \text{ MN}$$

$$M_{y\,1\text{-}1} = \quad 0{,}170 \text{ MNm}$$

ULOKKEEN TAIPUMIEN JA KIERTYMIEN JOHTAMINEN MOHRIN MENETELMÄLLÄ

Kuvassa A on esitetty ulokkeen pään taipuman ja kiertymän määrittäminen ulokkeen päässä vaikuttavasta pistevoimasta F ja pistemomentista M_0.



Kuva A.

Mohrin menetelmä

1) **Ratkaistaan ulkoisen kuorman rakenteeseen aiheuttaman taivutus-momentin $M(x)$ suhde rakenteen taivutusjäykkyyteen $D(x)$ (M/D -kuvio).**

2) **Rakenteen vapaakappalekuvion reunaehtoja modifioidaan siten, että alkuperäisen kuvion taipumat vastaavat uuden taivutusmomentteja ja vastaavasti kiertymät leikkausvoimia. Rakenteen modifioitua vapaakappalekuviota kuormitetaan M/D -jakaumalla.**

3) **M/D -jakaumalla kuormitetusta rakenteesta laskettava momenttikuvio $M_{M/D}(x)$ vastaa alkuperäisen rakenteen taipumakuviota $v(x)$ ja vastaavasti leikkausvoimakuvio $Q_{M/D}(x)$ kiertymäkuviota $\varphi(x)$.**

Ks. lisäesimerkkejä: Arvo Ylinen: *Kimmo ja lujuusoppi I*. WSOY. 2 p. Porvoo 1965. S. 268 - 273.

29. TEHTÄVÄ

Määritä y-z -tasossa se alue (sydänkuvio), jolla pystykuorma

V = 1 MN

voi sijaita niin, että kaikki paalut ovat puristettuja! Ks. kuva 1 ja taulukko 1.

Käytä avaruuspaalutuksen tasapainoyhtälöryhmää!

Paalut ovat pystysuoria.

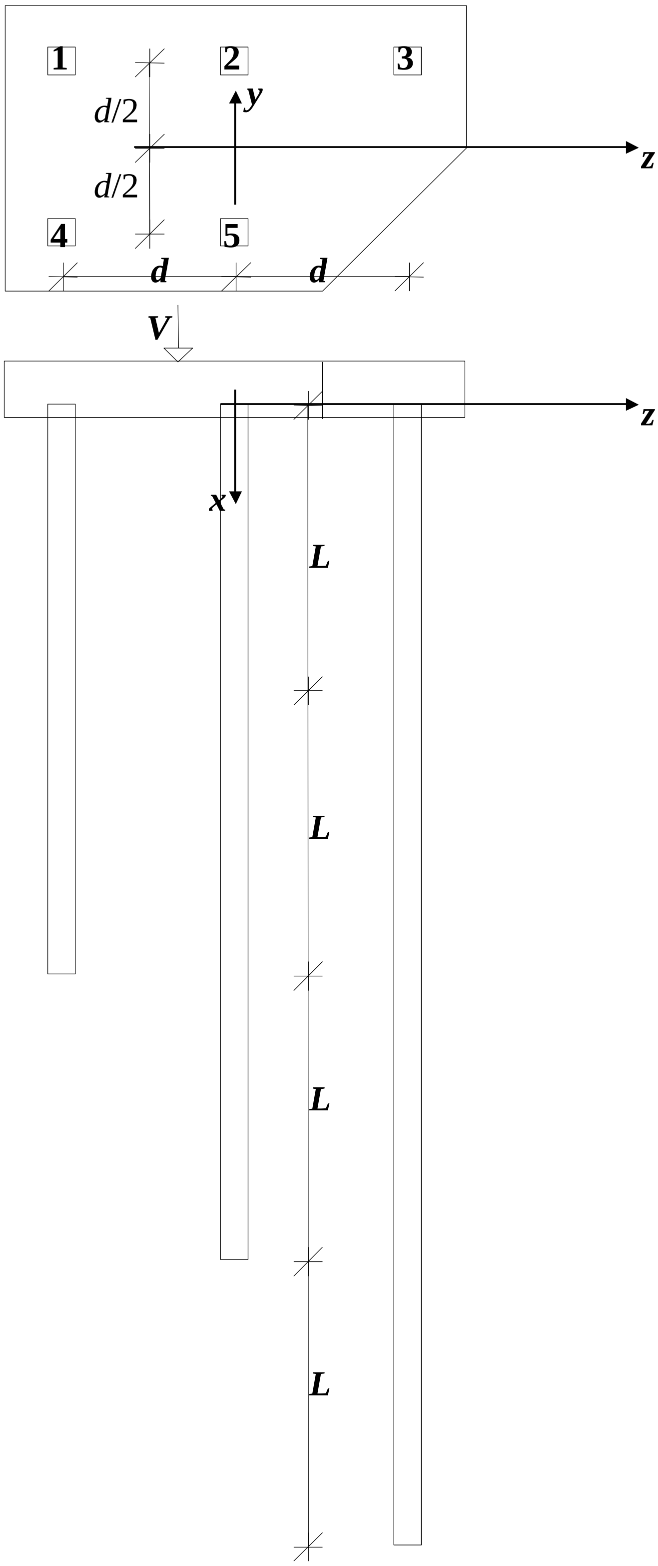
Mitat

d = 1,500 m

L = 5,000 m

Taulukko 1. Paalujen yläpään koordinaatit ja pituudet.

Symboli	i	x_i	y_i	z_i	L_i
Yksikkö	-	m	m	m	m
Lukuarvo	1	0,000	0,750	-1,500	10,000
	2	0,000	0,750	0,000	15,000
	3	0,000	0,750	1,500	20,000
	4	0,000	-0,750	-1,500	10,000
	5	0,000	-0,750	0,000	15,000



Kuva 1.

Suuntakulman kosini y - ja z -akselin sekä momenttivarsi x -akselin suhteen häviävät

$$p_{y,i} = \cos\alpha_{i,y} = 0 \tag{1}$$

$$p_{z,i} = \cos\alpha_{i,z} = 0 \tag{2}$$

$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} = 0 \tag{3}$$

Avaruuspaalutuksen jäykkkyysmatriisi sievenee 3 · 3 matriisiksi

$$\begin{array}{cccccc} 1 & & & & 2 & 3 \\ p_x & p_y & p_z & r_x & r_y & r_z \end{array}$$

$$[K]=\left[\begin{array}{cccccc} \sum kp_x^2 & 0 & 0 & 0 & \sum kp_x r_y & \sum kp_x r_z \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sum kr_y p_x & 0 & 0 & 0 & \sum kr_y^2 & \sum kr_y r_z \\ \sum kr_z p_x & 0 & 0 & 0 & \sum kr_z r_y & \sum kr_z^2 \end{array}\right] \begin{array}{cc} p_x & 1 \\ p_y & \\ p_z & \\ r_x & \\ r_y & 2 \\ r_z & 3 \end{array} \tag{4}$$

Paalujen suhteelliset aksiaalijäykkyydet

$$k_i = \frac{\dfrac{E_i A_i}{L_i}}{\dfrac{E_3 A_3}{L_3}} \tag{5}$$

$$\Rightarrow k_i = \frac{L_3}{L_i}$$

$$k_1 = 2{,}000$$

$$k_2 = 1{,}333$$

$$k_3 = 1{,}000$$

Paaluryhmän painopisteakselin paikka

$$y_0 = \frac{\sum k_i y_i}{\sum k_i} \tag{6}$$

$$= 0{,}098 \text{ m}$$

$$z_0 = \frac{\sum k_i z_i}{\sum k_i} \tag{7}$$

$$= -0{,}587 \text{ m}$$

Suuntakulman kosini *x* -akselin sekä momenttivarsi *y* - ja *z* -akselin suhteen

$$p_{x,i} = \cos\alpha_{i,x} \tag{8}$$

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{9}$$

$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{10}$$

Jäykkyysmatriisin alkion arvojen laskenta on esitetty taulukossa 2.

Taulukko 2. Jäykkyysmatriisin alkioiden laskenta.

Muuttuja	Plu	Sijaintikoordinaatit			Kulma	Jäykkyys
Symboli	i	x_i	y_i	z_i	α_i	k_i
Yksikkö		m	m	m	°	-
Lukuarvo	1	0,000	0,652	-0,913	0,000	2,000
	2	0,000	0,652	0,587	0,000	1,333
	3	0,000	0,652	2,087	0,000	1,000
	4	0,000	-0,848	-0,913	0,000	2,000
	5	0,000	-0,848	0,587	0,000	1,333
	⋮					

Muuttuja	Plu	Suuntakulman kosinit			Norm.voiman momenttivarsi		
Symboli	i	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Yksikkö		-	-	-	m	m	m
Lukuarvo	1	1,000	0,000	0,000	0,000	-0,913	-0,652
	2	1,000	0,000	0,000	0,000	0,587	-0,652
	3	1,000	0,000	0,000	0,000	2,087	-0,652
	4	1,000	0,000	0,000	0,000	-0,913	0,848
	5	1,000	0,000	0,000	0,000	0,587	0,848
	⋮						

⋮

Muuttuja	Plu	Paalun jäykkyysmatriisin alkiot					
Symboli	*i*	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Lukuarvo	1	2,000	-1,826	-1,304	1,667	1,191	0,851
	2	1,333	0,783	-0,870	0,459	-0,510	0,567
	3	1,000	2,087	-0,652	4,355	-1,361	0,425
	4	2,000	-1,826	1,696	1,667	-1,548	1,438
	5	1,333	0,783	1,130	0,459	0,664	0,958
Yksikkö		MN/m	MN	MN	MNm	MNm	MNm
Lukuarvo		7,667	0,000	0,000	8,609	-1,565	4,239
Symboli		k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Muuttuja		Paaluryhmän jäykkyysmatriisin alkiot					

Jäykkyysmatriisi

$$[K] = \begin{bmatrix} 7{,}667 & 0{,}000 & 0{,}000 \\ 0{,}000 & 8{,}609 & -1{,}565 \\ 0{,}000 & -1{,}565 & 4{,}239 \end{bmatrix} \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

Jäykkyysmatriisin käänteismatriisi

$$[K]^{-1} = \begin{bmatrix} 0{,}130 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}125 & 0{,}046 \\ 0{,}000 & 0{,}046 & 0{,}253 \end{bmatrix} \begin{bmatrix} \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{1/N} & \mathrm{1/N} & \mathrm{1/Nm} \end{bmatrix}$$

Tasapainoehdosta

$$\{F\} = [K]\{\delta\} \tag{11}$$

saadaan paaluryhmän siirtymävektori

$$\{\delta\} = [K]^{-1}\{F\} \tag{12}$$

$$\Rightarrow \begin{Bmatrix} u \\ \varphi \\ \theta \end{Bmatrix} = [K]^{-1} \begin{Bmatrix} 1 \\ z \\ -y \end{Bmatrix} \begin{matrix} \text{(Pystyvoima } x \text{ -akselin suuntaan [MN])} \\ \text{(Momentti } y \text{ -akselin ympäri [MNm])} \\ \text{(Momentti } z \text{ -akselin ympäri [MNm])} \end{matrix} \tag{13}$$

$$\Rightarrow \begin{cases} u = 0{,}000\, y + 0{,}000\, z + 0{,}130 \\ v = -0{,}046\, y + 0{,}125\, z + 0{,}000 \\ \theta = -0{,}253\, y + 0{,}046\, z + 0{,}000 \end{cases}$$

Paaluvoimat

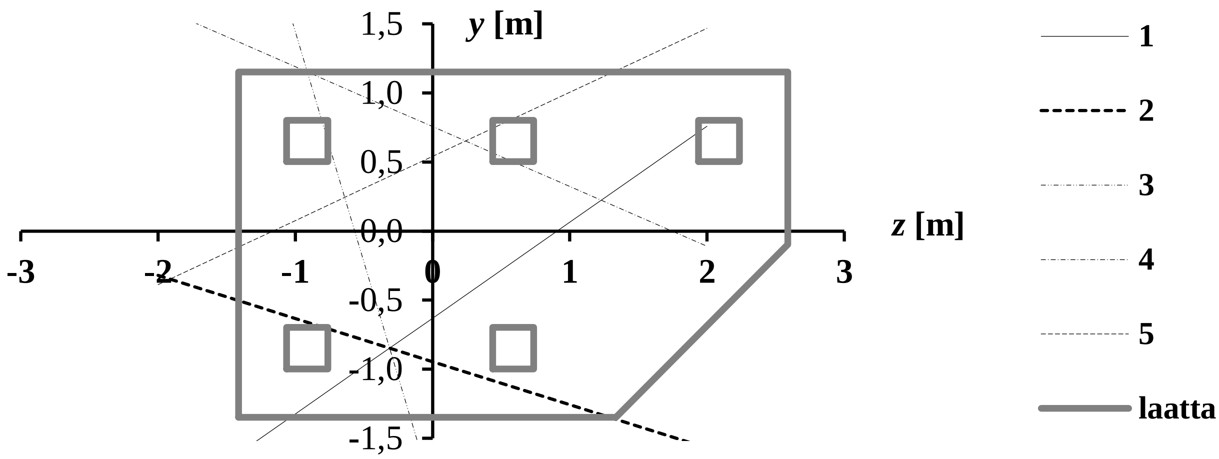
$$N_i = k_i \Delta_i \tag{14}$$

$$= k_i (p_{x,i}\, u + r_{y,i}\, \varphi + r_{z,i}\, \theta) \tag{15}$$

N_1 =	0,414	y	+	-0,287	z	+	0,261	= 0
N_2 =	0,184	y	+	0,057	z	+	0,174	= 0
N_3 =	0,069	y	+	0,230	z	+	0,130	= 0
N_4 =	-0,345	y	+	-0,149	z	+	0,261	= 0
N_5 =	-0,322	y	+	0,149	z	+	0,174	= 0

VASTAUS:

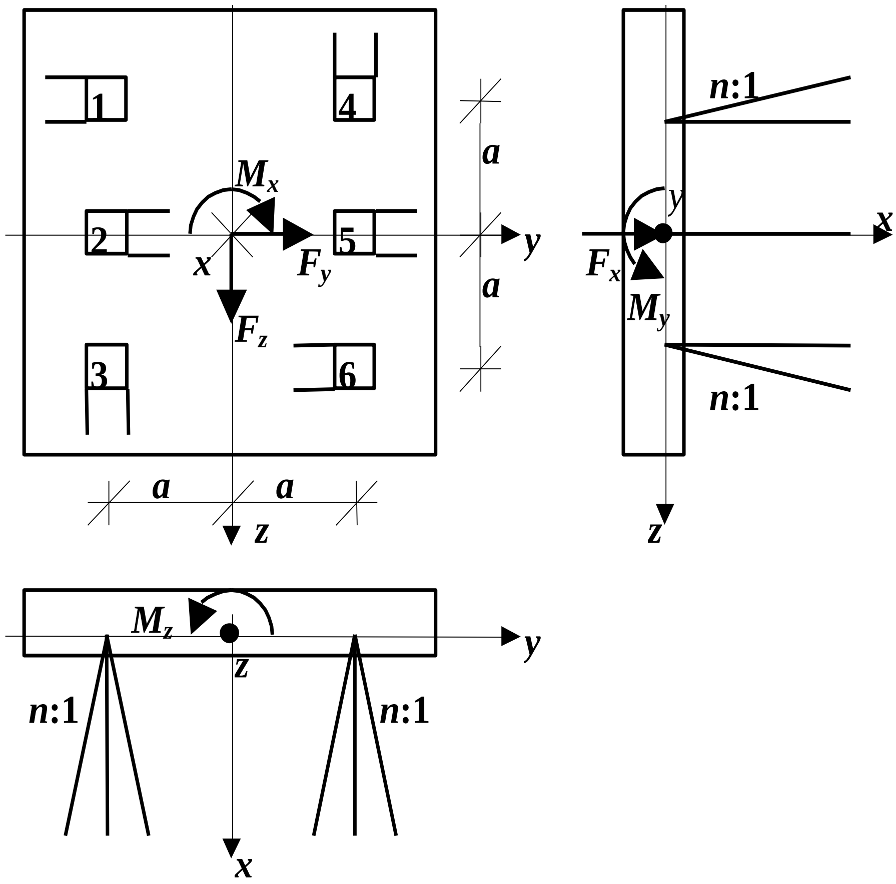
Sallittua aluetta (sydänkuvio) rajoittavat suorat on esitetty kuvassa 2.



Kuva 2.

30. TEHTÄVÄ

Määritä kuvan 1 lyöntipaaluryhmän paaluvoimat!



Kuva 1.

Kuormavektorin alkiot

F_x = 0,000 MN
F_y = 0,000 MN
F_z = 0,000 MN
M_x = 1,000 MNm
M_y = 0,000 MNm
M_z = 0,000 MNm

Vinopaalujen kaltevuus

n = 4

Paaluilla on sama suhteellinen jäykkyys EA/L.

Tasapainoyhtälöryhmä

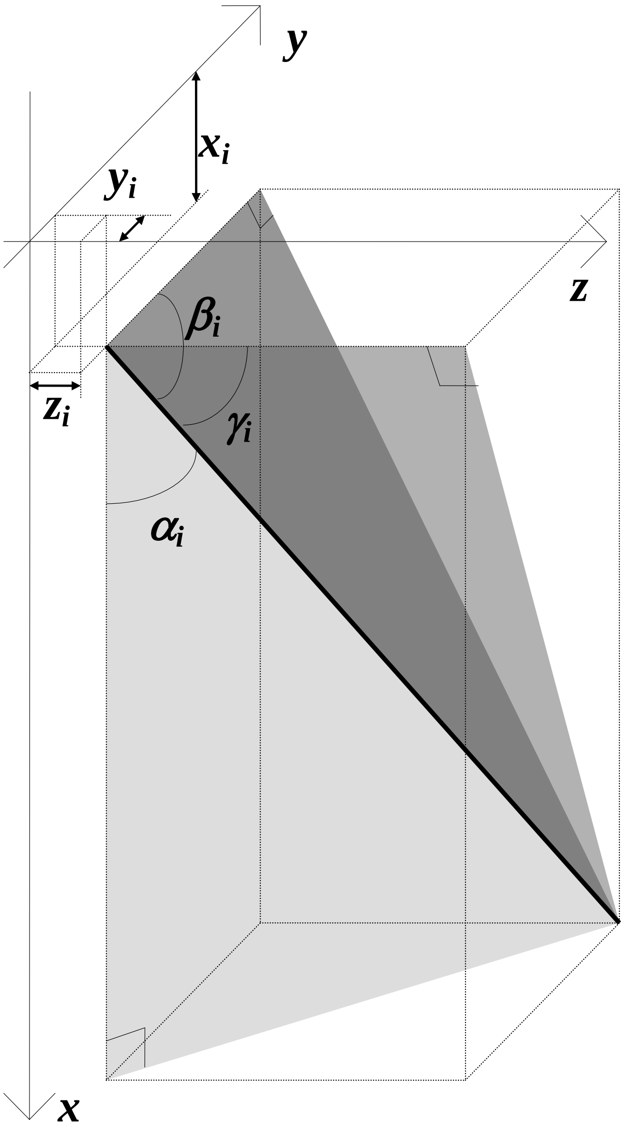
$$\{F\} = [K]\{\delta\} \tag{1}$$

jossa voimavektori

$$\{F\} = \{F_x \;\; F_y \;\; F_z \;\; M_x \;\; M_y \;\; M_z\}^{\mathrm{T}} \tag{2}$$

ja siirtymävektori

$$\{\delta\} = \{u \;\; v \;\; w \;\; \omega \;\; \varphi \;\; \theta\}^{\mathrm{T}} \tag{3}$$



Kuva 2.

ja paaluryhmän jäykkyysmatriisi on paalujen jäykkyysmatriisien summa

$$[K]=\sum_{i=1}^{n} k_i \begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} & p_{x,i}r_{x,i} & p_{x,i}r_{y,i} & p_{x,i}r_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} & p_{y,i}r_{x,i} & p_{y,i}r_{y,i} & p_{y,i}r_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 & p_{z,i}r_{x,i} & p_{z,i}r_{y,i} & p_{z,i}r_{z,i} \\ r_{x,i}p_{x,i} & r_{x,i}p_{y,i} & r_{x,i}p_{z,i} & r_{x,i}^2 & r_{x,i}r_{y,i} & r_{x,i}r_{z,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{y,i} & r_{y,i}p_{z,i} & r_{y,i}r_{x,i} & r_{y,i}^2 & r_{y,i}r_{z,i} \\ r_{z,i}p_{x,i} & r_{z,i}p_{y,i} & r_{z,i}p_{z,i} & r_{z,i}r_{x,i} & r_{z,i}r_{y,i} & r_{z,i}^2 \end{bmatrix} \tag{4}$$

jossa

$$p_{x,i} = \cos\alpha_i \tag{5}$$
$$p_{y,i} = \cos\beta_i \tag{6}$$
$$p_{z,i} = \cos\gamma_i \tag{7}$$
$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} \tag{8}$$
$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{9}$$
$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{10}$$

Kulmat α_i, β_i ja γ_i ilmenevät kuvasta 2.

Jäykkyysmatriisin alkioiden laskenta on esitetty taulukossa 1.

Taulukko 1. Jäykkyysmatriisin alkioiden laskenta.

Muuttuja	Paa-lu	Paalujen lkm	Pysty-sijainti	Vaakasijainti		Jäyk-kyys
Symboli	i	j_i	x_i	y_i	z_i	k_i
Yksikkö	-	kpl	m	m	m	MN/m
Lukuarvo	1	1	0,000	-2,000	-2,000	1,000
	2	1	0,000	-2,000	0,000	1,000
	3	1	0,000	-2,000	2,000	1,000
	4	1	0,000	2,000	-2,000	1,000
	5	1	0,000	2,000	0,000	1,000
	6	1	0,000	2,000	2,000	1,000
	⋮					

⋮

Muuttuja	Plu	Kaltevuus ja suuntakulmat					
Symboli	i	$\tan\alpha_i$	α_i	$\tan\beta_i$	β_i	$\tan\gamma_i$	γ_i
Yksikkö	-	-	°	-	°	-	°
Lukuarvo	1	-0,250	-14,036	-4,000	104,036	∞	90,000
	2	0,250	14,036	4,000	75,964	∞	90,000
	3	0,250	14,036	∞	90,000	4,000	75,964
	4	-0,250	-14,036	∞	90,000	-4,000	104,036
	5	0,250	14,036	4,000	75,964	∞	90,000
	6	-0,250	-14,036	-4,000	104,036	∞	90,000

⋮

Muuttuja	Plu	Suuntakulman kosinit			Norm.voiman momenttivarsi		
Symboli	i	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Yksikkö	-	-	-	-	m	m	m
Lukuarvo	1	0,970	-0,243	0,000	-0,485	-1,940	1,940
	2	0,970	0,243	0,000	0,000	0,000	1,940
	3	0,970	0,000	0,243	-0,485	1,940	1,940
	4	0,970	0,000	-0,243	-0,485	-1,940	-1,940
	5	0,970	0,243	0,000	0,000	0,000	-1,940
	6	0,970	-0,243	0,000	0,485	1,940	-1,940

⋮

Symboli	i	$K_{11,i}$	$K_{12,i}$	$K_{13,i}$	$K_{14,i}$	$K_{15,i}$	$K_{16,i}$
Lukuarvo	1	0,941	-0,235	0,000	-0,471	-1,882	1,882
	2	0,941	0,235	0,000	0,000	0,000	1,882
	3	0,941	0,000	0,235	-0,471	1,882	1,882
	4	0,941	0,000	-0,235	-0,471	-1,882	-1,882
	5	0,941	0,235	0,000	0,000	0,000	-1,882
	6	0,941	-0,235	0,000	0,471	1,882	-1,882
Yksikkö		MN/m	MN/m	MN/m	MN	MN	MN
Lukuarvo		5,647	0,000	0,000	-0,941	0,000	0,000
Symboli		K_{11}	$K_{12}=K_{21}$	$K_{13}=K_{31}$	$K_{14}=K_{41}$	$K_{15}=K_{51}$	$K_{16}=K_{61}$

⋮

⋮

Symboli	i	$K_{22,i}$	$K_{23,i}$	$K_{24,i}$	$K_{25,i}$	$K_{26,i}$
Lukuarvo	1	0,059	0,000	0,118	0,471	-0,471
	2	0,059	0,000	0,000	0,000	0,471
	3	0,000	0,000	0,000	0,000	0,000
	4	0,000	0,000	0,000	0,000	0,000
	5	0,059	0,000	0,000	0,000	-0,471
	6	0,059	0,000	-0,118	-0,471	0,471
Yksikkö		MN/m	MN/m	MN	MN	MN
Lukuarvo		0,235	0,000	0,000	0,000	0,000
Symboli		K_{22}	$K_{23}=K_{32}$	$K_{24}=K_{42}$	$K_{25}=K_{52}$	$K_{26}=K_{62}$

⋮

Symboli	i	$K_{33,i}$	$K_{34,i}$	$K_{35,i}$	$K_{36,i}$
Lukuarvo	1	0,000	0,000	0,000	0,000
	2	0,000	0,000	0,000	0,000
	3	0,059	-0,118	0,471	0,471
	4	0,059	0,118	0,471	0,471
	5	0,000	0,000	0,000	0,000
	6	0,000	0,000	0,000	0,000
Yksikkö		MN/m	MN	MN	MN
Lukuarvo		0,118	0,000	0,941	0,941
Symboli		K_{33}	$K_{34}=K_{43}$	$K_{35}=K_{53}$	$K_{36}=K_{63}$

⋮

Symboli	i	$K_{44,i}$	$K_{45,i}$	$K_{46,i}$	$K_{55,i}$	$K_{56,i}$	$K_{66,i}$
Lukuarvo	1	0,235	0,941	-0,941	3,765	-3,765	3,765
	2	0,000	0,000	0,000	0,000	0,000	3,765
	3	0,235	-0,941	-0,941	3,765	3,765	3,765
	4	0,235	0,941	0,941	3,765	3,765	3,765
	5	0,000	0,000	0,000	0,000	0,000	3,765
	6	0,235	0,941	-0,941	3,765	-3,765	3,765
Yksikkö		MNm	MNm	MNm	MNm	MNm	MNm
Lukuarvo		0,941	1,882	-1,882	15,059	0,000	22,588
Symboli		K_{44}	$K_{45}=K_{54}$	$K_{46}=K_{64}$	K_{55}	$K_{56}=K_{65}$	K_{66}

Jäykkyysmatriisi

$$K = \begin{bmatrix} 5{,}647 & 0{,}000 & 0{,}000 & -0{,}941 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}235 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}118 & 0{,}000 & 0{,}941 & 0{,}941 \\ -0{,}941 & 0{,}000 & 0{,}000 & 0{,}941 & 1{,}882 & -1{,}882 \\ 0{,}000 & 0{,}000 & 0{,}941 & 1{,}882 & 15{,}059 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}941 & -1{,}882 & 0{,}000 & 22{,}588 \end{bmatrix}$$

Siirtymävektorin alkiot

$u =$ 0,531 m

$v =$ 0,000 m

$w =$ 6,375 m

$\omega =$ 3,188 rad

$\varphi =$ -0,797 rad

$\theta =$ 0,000 rad

Paaluvoimat

$$N_i = k_i \Delta_i \qquad (11)$$

$$= k_i (p_{x,i} u + p_{y,i} v + p_{z,i} w + r_{x,i} \omega + r_{y,i} \varphi + r_{z,i} \theta) \qquad (12)$$

VASTAUS:

Paaluvoimat ovat

$N_1 =$ 0,515 MN

$N_2 =$ 0,515 MN

$N_3 =$ -1,031 MN

$N_4 =$ -1,031 MN

$N_5 =$ 0,515 MN

$N_6 =$ 0,515 MN

31. TEHTÄVÄ

Laske ja piirrä kuvan 1 suurpaalun leikkausvoima-, taivutusmomentti- ja taipumajakauma kolmea palkkielementtiä käyttäen!

Ympyränmuotoisen paalun poikkileikkauksen halkaisija

d = 0,700 m

Paalun betonin nimellislujuus

K = 45 MN/m^2

Mitta

a_1 = 3,500 m

a_2 = 3,500 m

a_3 = 3,500 m

Kuormitus

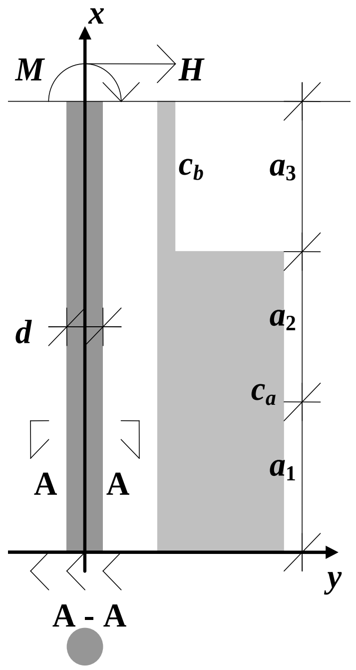
H = 1,500 MN

M = 0,200 MNm

Alustaluku

c_a = 40 MN/m^3

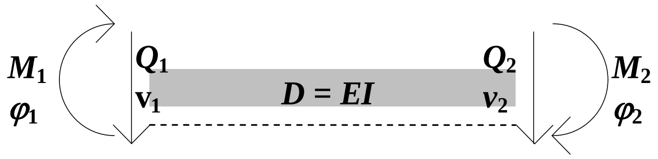
c_b = 5 MN/m^3



Kuva 1.

Palkkielementin tasapainoehto (positiiviset suunnat kuvassa 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix}$$



Kuva 2.

Jousivakio paalun osan *i* yläpäässä (kuva 3)

$$k_i = d \cdot \frac{a_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + a_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \tag{1}$$

k_{01} = ∞

= 1,0E+100 MN/m

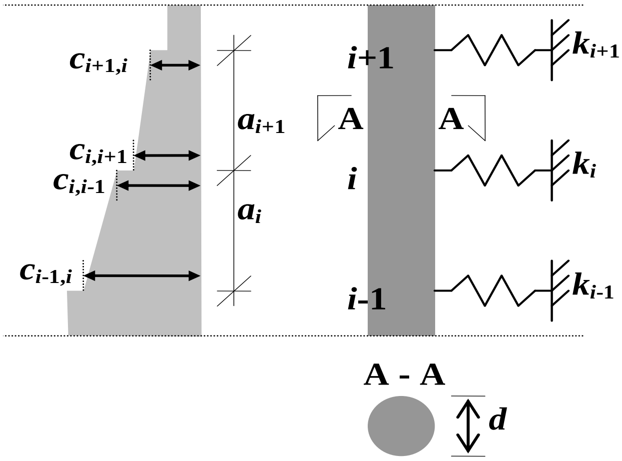
k_{02} = ∞

= 1,0E+100 MN/m

k_1 = 98,000 MN/m

k_2 = 55,125 MN/m

k_3 = 6,125 MN/m



Kuva 3.

Betonin kimmokerroin

$$E_i = k\sqrt{K_i K_0} \tag{2}$$

jossa k = 1 ja $K_0 = 25 \cdot 10^6$ MN/m^2.

E_i = 33 541 MN/m^2

Ympyrän jäyhyysmomentti

$$I = \frac{\pi(d/2)^4}{4} \tag{3}$$

= 0,011786 m^4

Pilarin jäykkyys

$$D = EI \tag{4}$$

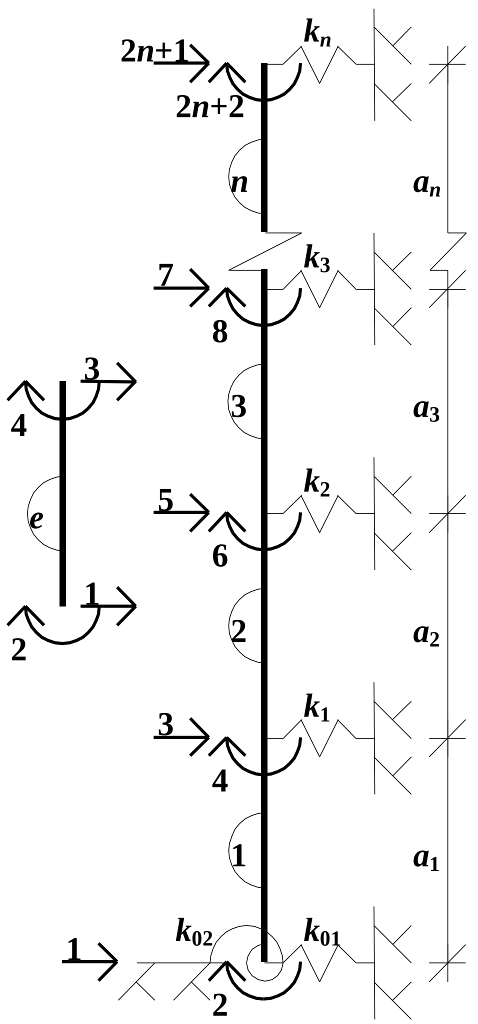
= 395 MNm^2

Elementin *i* jäykkyysmatriisi

$$K_i^e = \begin{bmatrix} 110{,}641 & 193{,}621 & -110{,}641 & 193{,}621 \\ 193{,}621 & 451{,}783 & -193{,}621 & 225{,}892 \\ -110{,}641 & -193{,}621 & 110{,}641 & -193{,}621 \\ 193{,}621 & 225{,}892 & -193{,}621 & 451{,}783 \end{bmatrix}$$

Yksiköt

$$\left[K_i^e\right] = \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MNm} & \mathrm{MN} & \mathrm{MNm} \\ \mathrm{MN/m} & \mathrm{MN} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MNm} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$



Kuva 4.

Systeemin tasapainoehdosta

$$\{F\} = [K]\{\delta\} \tag{5}$$

saadaan siirtymävektori

$$\{\delta\} = [K]^{-1}\{F\} \tag{6}$$

Systeemin jäykkyysmatriisi (kuva 4)

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} k_{11}^1 + k_{01} & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 & 0 & 0 \\ k_{21} = k_{12} & k_{22}^1 + k_{02} & k_{23}^1 & k_{24}^1 & 0 & 0 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 & 0 & 0 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 & 0 & 0 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^2 + k_{11}^3 + k_2 & k_{34}^2 + k_{12}^3 & k_{13}^3 & k_{14}^3 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^2 + k_{22}^3 & k_{23}^3 & k_{24}^3 \\ k_{71} = k_{18} & k_{72} = k_{27} & k_{73} = k_{37} & k_{74} = k_{47} & k_{75} = k_{57} & k_{76} = k_{67} & k_{33}^3 + k_3 & k_{34}^3 \\ k_{81} = k_{18} & k_{82} = k_{28} & k_{83} = k_{38} & k_{84} = k_{48} & k_{85} = k_{58} & k_{86} = k_{68} & k_{87} = k_{78} & k_{44}^3 \end{bmatrix} \tag{8}$$

Matriisi [*K*] lukuarvoina

1,0E+100	193,621	-110,641	193,621	0,000	0,000	0,000	0,000
193,621	1,0E+100	-193,621	225,892	0,000	0,000	0,000	0,000
-110,641	-193,621	319,282	0,000	-110,641	193,621	0,000	0,000
193,621	225,892	0,000	903,567	-193,621	225,892	0,000	0,000
0,000	0,000	-110,641	-193,621	276,407	0,000	-110,641	193,621
0,000	0,000	193,621	225,892	0,000	903,567	-193,621	225,892
0,000	0,000	0,000	0,000	-110,641	-193,621	116,766	-193,621
0,000	0,000	0,000	0,000	193,621	225,892	-193,621	451,783

Matriisin [*K*] yksiköt

MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm

Käänteismatriisi $[K]^{-1}$

0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,005	0,001	0,003	-0,001	-0,001	-0,001
0,000	0,000	0,001	0,002	0,002	0,000	0,001	0,000
0,000	0,000	0,003	0,002	0,012	0,002	0,017	0,001
0,000	0,000	-0,001	0,000	0,002	0,003	0,011	0,002
0,000	0,000	-0,001	0,001	0,017	0,011	0,076	0,020
0,000	0,000	-0,001	0,000	0,001	0,002	0,020	0,009

Käänteismatriisin $[K]^{-1}$ yksiköt

m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm

Systeemin kuormavektorin alkiot elementin i yläpäässä

$F_0 =$ 0,000 MN
$M_0 =$ 0,000 MNm
$F_1 =$ 0,000 MN
$M_1 =$ 0,000 MNm
$F_2 =$ 0,000 MN
$M_2 =$ 0,000 MNm
$F_3 =$ 1,500 MN
$M_3 =$ 0,200 MNm

Systeemin siirtymävektorin alkiot elementin i yläpäässä

$w_0 =$ 0,000 m
$\varphi_0 =$ 0,000 rad
$w_1 =$ -0,002 m
$\varphi_1 =$ 0,001 rad
$w_2 =$ 0,025 m
$\varphi_2 =$ 0,018 rad
$w_3 =$ 0,118 m
$\varphi_3 =$ 0,031 rad

Elementin i voimasuureet

$$F_i^e = K_i^e \delta_i^e \qquad (9)$$

1. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0{,}000 \text{ m} \\ 0{,}000 \\ -0{,}002 \text{ m} \\ 0{,}001 \end{bmatrix} = \begin{bmatrix} 0{,}407 \text{ MN} \\ 0{,}603 \text{ MNm} \\ -0{,}407 \text{ MN} \\ 0{,}822 \text{ MNm} \end{bmatrix}$$

2. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} -0{,}002 \text{ m} \\ 0{,}001 \\ 0{,}025 \text{ m} \\ 0{,}018 \end{bmatrix} = \begin{bmatrix} 0{,}601 \text{ MN} \\ -0{,}822 \text{ MNm} \\ -0{,}601 \text{ MN} \\ 2{,}926 \text{ MNm} \end{bmatrix}$$

3. elementti

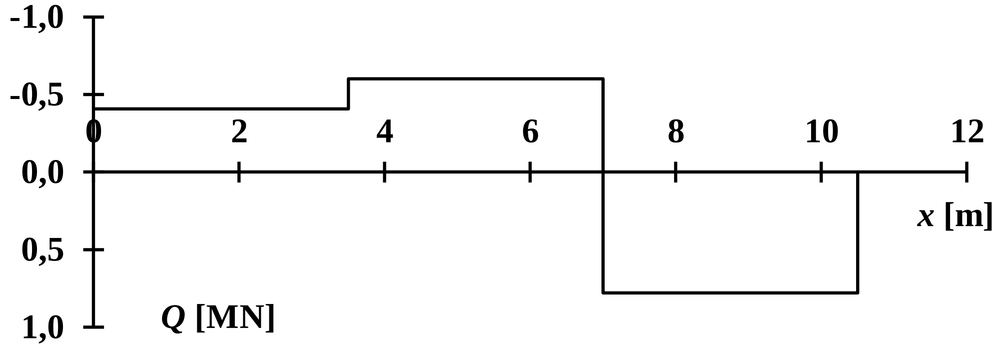
$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_3^e \cdot \begin{bmatrix} 0{,}025 \text{ m} \\ 0{,}018 \\ 0{,}118 \text{ m} \\ 0{,}031 \end{bmatrix} = \begin{bmatrix} -0{,}779 \text{ MN} \\ -2{,}926 \text{ MNm} \\ 0{,}779 \text{ MN} \\ 0{,}200 \text{ MNm} \end{bmatrix}$$

<u>VASTAUS</u>:

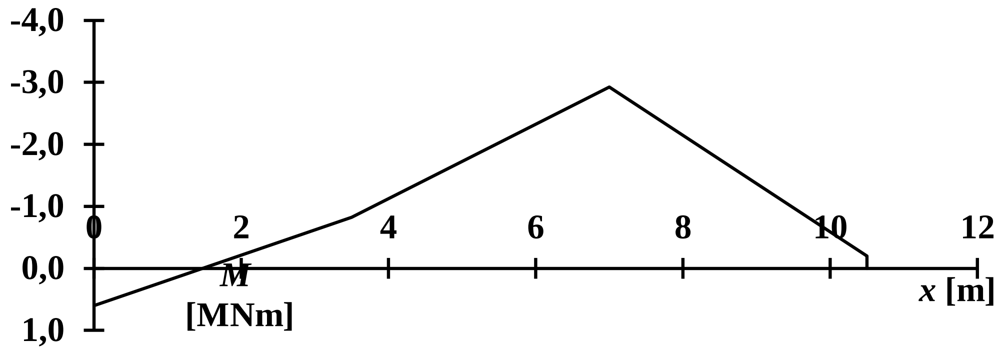
Systeemin leikkausvoima-, taivutusmomentti- ja taipumajakauma on esitetty taulukossa 1 sekä kuvissa 5, 6 ja 7. Positiiviset suunnat ilmenevät kuvasta 8.

Taulukko 1.

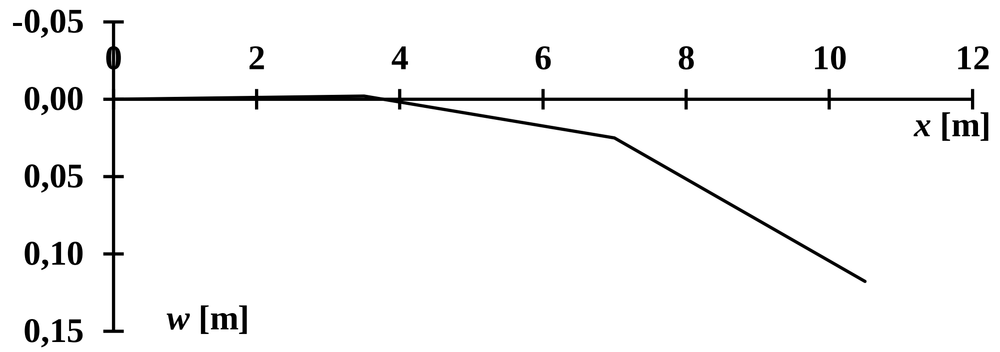
x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,407	0,603	0,000
3,500	-0,407	-0,822	-0,002
3,500	-0,601	-0,822	-0,002
7,000	-0,601	-2,926	0,025
7,000	0,779	-2,926	0,025
10,500	0,779	-0,200	0,118
10,500	0	0	



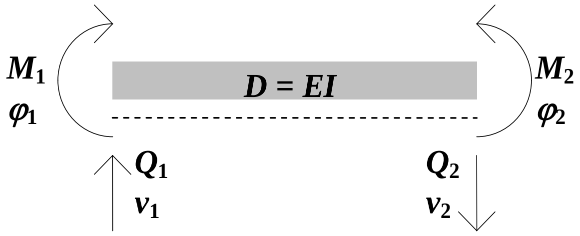
Kuva 5. Leikkausvoimakuvaaja.



Kuva 6. Taivutusmomenttikuvaaja.



Kuva 7. Taipumakuvaaja.



Kuva 8. Positiiviset suunnat (vrt. kuva 2).

32. TEHTÄVÄ

Laske ja piirrä kuvan 1 suurpaalun ja pilarin leikkausvoima-, taivutusmomentti- ja taipumajakauma käyttäen systeemin jäykkyysmatriisissa kolmea ylintä neljästä palkkielementistä!

Ympyränmuotoisen paalun poikkileikkauksen halkaisija

d = 0,700 m

Paalun betonin nimellislujuus

K = 45 MN/m^2

Mitta

a_1 = 3,500 m

a_2 = 3,500 m

a_3 = 3,500 m

a_4 = 3,500 m

Kuormitus

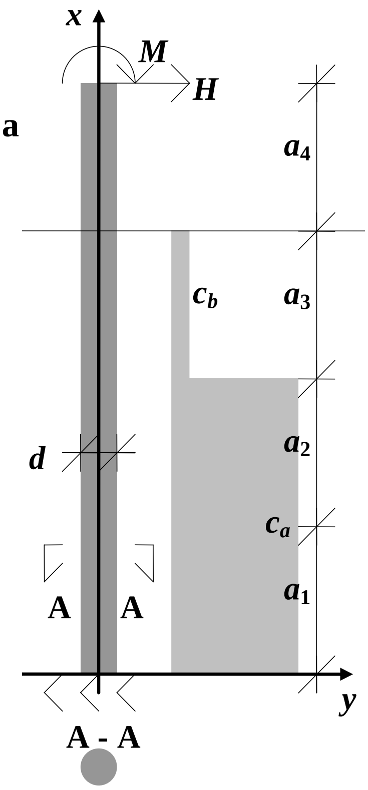
H = 1,500 MN

M = -5,050 MNm

Alustaluku

c_a = 40 MN/m^3

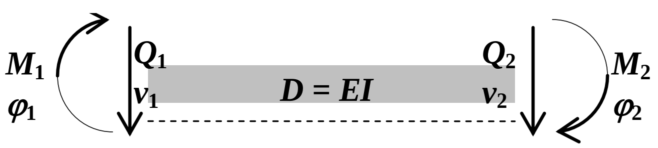
c_b = 5 MN/m^3



Kuva 1.

Palkkielementin tasapainoehto (positiiviset suunnat kuvassa 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix}$$



Kuva 2.

Jousivakio paalun osan *i* yläpäässä (kuva 3)

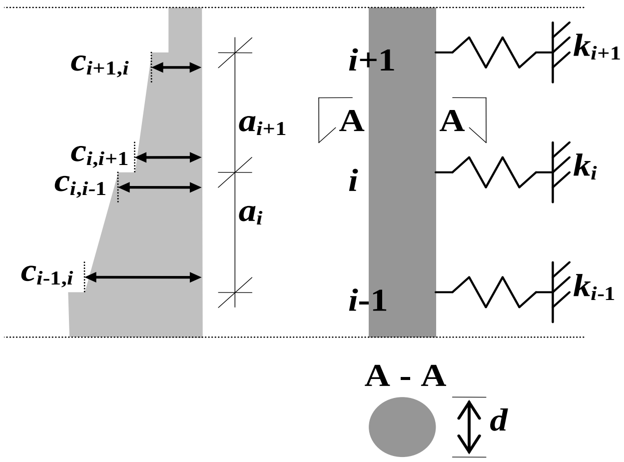
$$k_i = d \cdot \frac{a_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + a_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \tag{1}$$

$k_1 =$ 98,000 MN/m

$k_2 =$ 55,125 MN/m

$k_3 =$ 6,125 MN/m

$k_4 =$ 0 MN/m



Kuva 3.

Betonin kimmokerroin

$$E_i = k\sqrt{K_i K_0} \tag{2}$$

jossa $k = 1$ ja $K_0 = 25 \cdot 10^6$ MN/m²

$E_i =$ 33 541 MN/m²

Ympyrän jäyhyysmomentti

$$I = \frac{\pi (d/2)^4}{4} \tag{3}$$

$= 0{,}011786$ m⁴

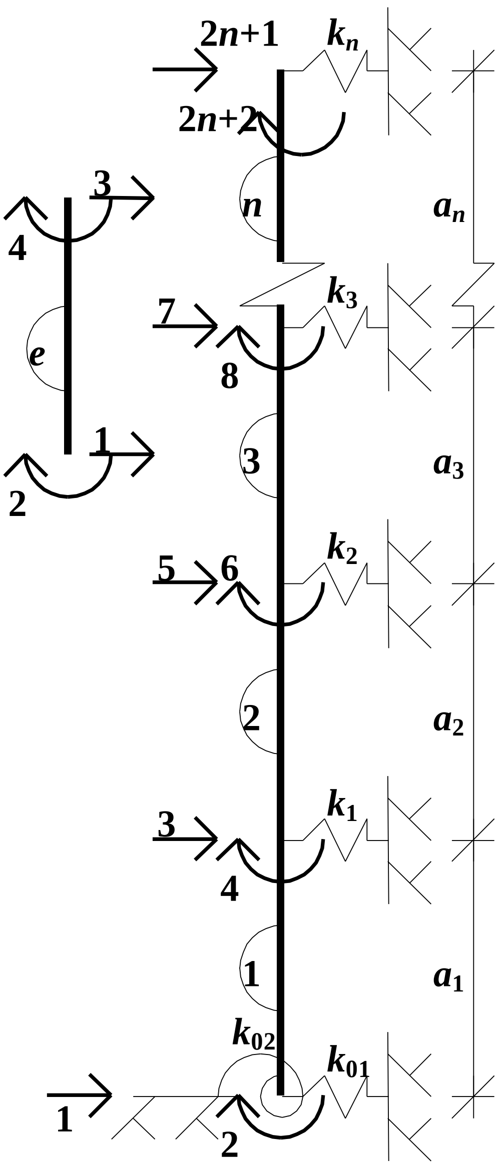
Pilarin jäykkyys

$$D = EI \tag{4}$$

$= 395{,}310$ MNm²

Elementin *i* jäykkyysmatriisi

$$K_i^e = \begin{bmatrix} 110{,}641 & 193{,}621 & -110{,}641 & 193{,}621 \\ 193{,}621 & 451{,}783 & -193{,}621 & 225{,}892 \\ -110{,}641 & -193{,}621 & 110{,}641 & -193{,}621 \\ 193{,}621 & 225{,}892 & -193{,}621 & 451{,}783 \end{bmatrix}$$



Kuva 4.

Systeemin tasapainoehdosta

$$\{F\} = [K]\{\delta\} \tag{5}$$

saadaan siirtymävektori

$$\{\delta\} = [K]^{-1}\{F\} \tag{6}$$

Systeemin jäykkyysmatriisi (kuva 4)

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 & 0 & 0 & 0 & 0 \\ k_{21} = k_{12} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 & 0 & 0 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^2 + k_{11}^3 + k_2 & k_{34}^2 + k_{12}^3 & k_{13}^3 & k_{14}^3 & 0 & 0 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^2 + k_{22}^3 & k_{23}^3 & k_{24}^3 & 0 & 0 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^3 + k_{11}^4 + k_3 & k_{34}^3 + k_{12}^4 & k_{13}^4 & k_{14}^4 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^3 + k_{22}^4 & k_{23}^4 & k_{24}^4 \\ k_{71} = k_{18} & k_{72} = k_{27} & k_{73} = k_{37} & k_{74} = k_{47} & k_{75} = k_{57} & k_{76} = k_{67} & k_{33}^4 + k_4 & k_{34}^4 \\ k_{81} = k_{18} & k_{82} = k_{28} & k_{83} = k_{38} & k_{84} = k_{48} & k_{85} = k_{58} & k_{86} = k_{68} & k_{87} = k_{78} & k_{44}^4 \end{bmatrix} \tag{8}$$

Matriisi $[K]$ lukuarvoina

319,282	0,000	-110,641	193,621	0,000	0,000	0,000	0,000
0,000	903,567	-193,621	225,892	0,000	0,000	0,000	0,000
-110,641	-193,621	276,407	0,000	-110,641	193,621	0,000	0,000
193,621	225,892	0,000	903,567	-193,621	225,892	0,000	0,000
0,000	0,000	-110,641	-193,621	227,407	0,000	-110,641	193,621
0,000	0,000	193,621	225,892	0,000	903,567	-193,621	225,892
0,000	0,000	0,000	0,000	-110,641	-193,621	110,641	-193,621
0,000	0,000	0,000	0,000	193,621	225,892	-193,621	451,783

Matriisin $[K]$ yksiköt

MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm

Käänteismatriisi $[K]^{-1}$

0,005	0,001	0,003	-0,001	-0,001	-0,001	-0,005	-0,001
0,001	0,002	0,002	0,000	0,001	0,000	-0,001	0,000
0,003	0,002	0,012	0,002	0,017	0,001	0,019	0,001
-0,001	0,000	0,002	0,003	0,011	0,002	0,019	0,002
-0,001	0,001	0,017	0,011	0,076	0,020	0,145	0,020
-0,001	0,000	0,001	0,002	0,020	0,009	0,052	0,009
-0,005	-0,001	0,019	0,019	0,145	0,052	0,363	0,068
-0,001	0,000	0,001	0,002	0,020	0,009	0,068	0,018

Käänteismatriisin $[K]^{-1}$ yksiköt

m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm

Systeemin kuormavektorin alkiot elementin *i* yläpäässä

$F_1 =$ 0,000 MN

$M_1 =$ 0,000 MNm

$F_2 =$ 0,000 MN

$M_2 =$ 0,000 MNm

$F_3 =$ 0,000 MN

$M_3 =$ 0,000 MNm

$F_4 =$ 1,500 MN

$M_4 =$ -5,050 MNm

Systeemin siirtymävektorin alkiot elementin *i* yläpäässä

$w_1 =$ -0,002 m

$\varphi_1 =$ 0,001 rad

$w_2 =$ 0,025 m

$\varphi_2 =$ 0,018 rad

$w_3 =$ 0,118 m

$\varphi_3 =$ 0,031 rad

$w_4 =$ 0,204 m

$\varphi_4 =$ 0,010 rad

Elementin *i* voimasuureet

$$F_i^{\,e} = K_i^{\,e}\,\delta_i^{\,e} \tag{9}$$

1. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0 \text{ m} \\ 0 \\ -0{,}002 \text{ m} \\ 0{,}001 \end{bmatrix} = \begin{bmatrix} 0{,}407 \text{ MN} \\ 0{,}603 \text{ MNm} \\ -0{,}407 \text{ MN} \\ 0{,}822 \text{ MNm} \end{bmatrix}$$

2. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} -0{,}002 \text{ m} \\ 0{,}001 \\ 0{,}025 \text{ m} \\ 0{,}018 \end{bmatrix} = \begin{bmatrix} 0{,}601 \text{ MN} \\ -0{,}822 \text{ MNm} \\ -0{,}601 \text{ MN} \\ 2{,}926 \text{ MNm} \end{bmatrix}$$

3. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_3^e \cdot \begin{bmatrix} 0{,}025 \text{ m} \\ 0{,}018 \\ 0{,}118 \text{ m} \\ 0{,}031 \end{bmatrix} = \begin{bmatrix} -0{,}779 \text{ MN} \\ -2{,}926 \text{ MNm} \\ 0{,}779 \text{ MN} \\ 0{,}200 \text{ MNm} \end{bmatrix}$$

4. elementti

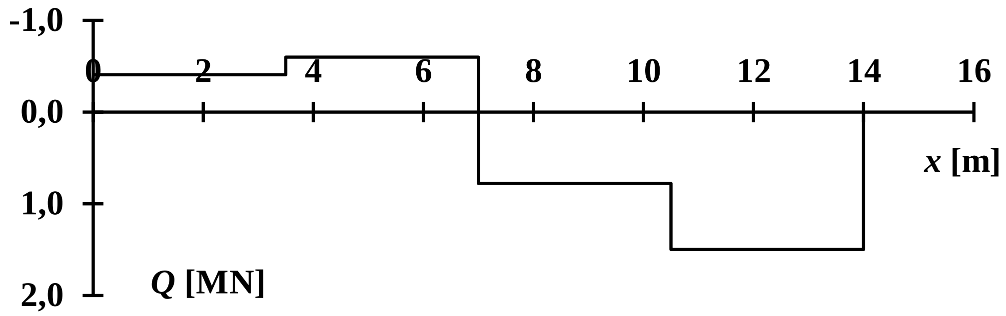
$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_4^e \cdot \begin{bmatrix} 0{,}118 \text{ m} \\ 0{,}031 \\ 0{,}204 \text{ m} \\ 0{,}010 \end{bmatrix} = \begin{bmatrix} -1{,}500 \text{ MN} \\ -0{,}200 \text{ MNm} \\ 1{,}500 \text{ MN} \\ -5{,}050 \text{ MNm} \end{bmatrix}$$

VASTAUS:

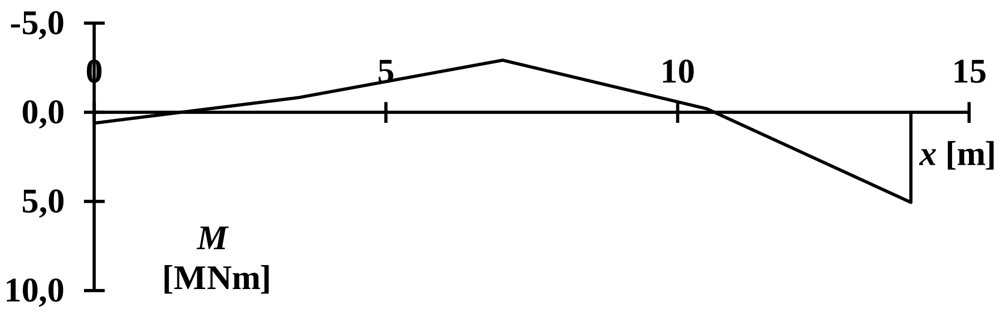
Systeemin leikkausvoima-, taivutusmomentti- ja taipumajakauma on esitetty taulukossa 1 sekä kuvissa 5, 6 ja 7. Positiiviset suunnat ilmenevät kuvasta 8.

Taulukko 1.

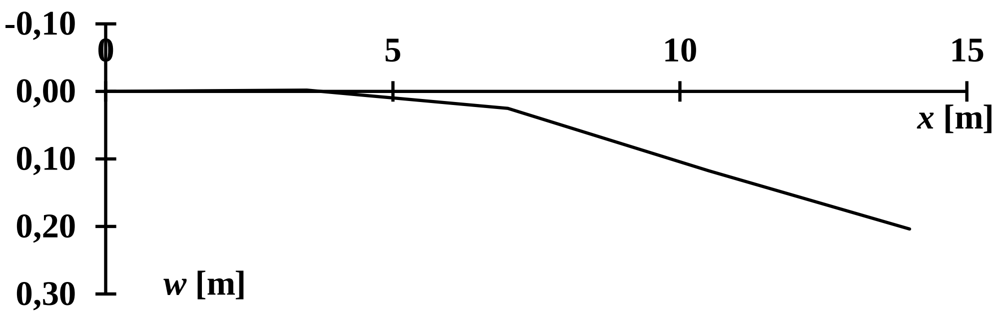
x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,407	0,603	0,000
3,500	-0,407	-0,822	-0,002
3,500	-0,601	-0,822	-0,002
7,000	-0,601	-2,926	0,025
7,000	0,779	-2,926	0,025
10,500	0,779	-0,200	0,118
10,500	1,500	-0,200	0,118
14,000	1,500	5,050	0,204
14,000	0	0	



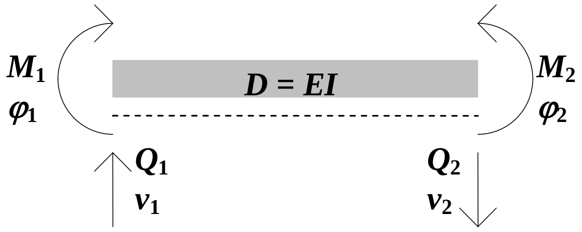
Kuva 5. Leikkausvoimakuvaaja.



Kuva 6. Taivutusmomenttikuvaaja.



Kuva 7. Taipumakuvaaja.



Kuva 8. Positiiviset suunnat (vrt. kuva 2).

33. TEHTÄVÄ

Sillan välitukeen kohdistuu tieliikennekuormien mukainen suurin jarrukuorma

J = 0,500 MN

Välituki koostuu kuvan 1 mukaisesti kaivinpaalusta (1) ja pilarista (2).
Kuinka paljon pilarin yläpää siirtyy jarrukuorman johdosta (*a* -mitta)?

Ratkaise tehtävä yksikkövoimamenetelmällä!

Maan kimmoisuus approksimoidaan (kuvan 1 mukaisesti) yhdellä jousella, jonka jousivakio on

k = 120 MN/m

Kaivinpaalun ja pilarin poikkileikkaus on ympyrä, jonka halkaisija on

d_1 = 1,200 m

d_2 = 0,800 m

Korkeus

h_1 = 10 m

h_2 = 8 m

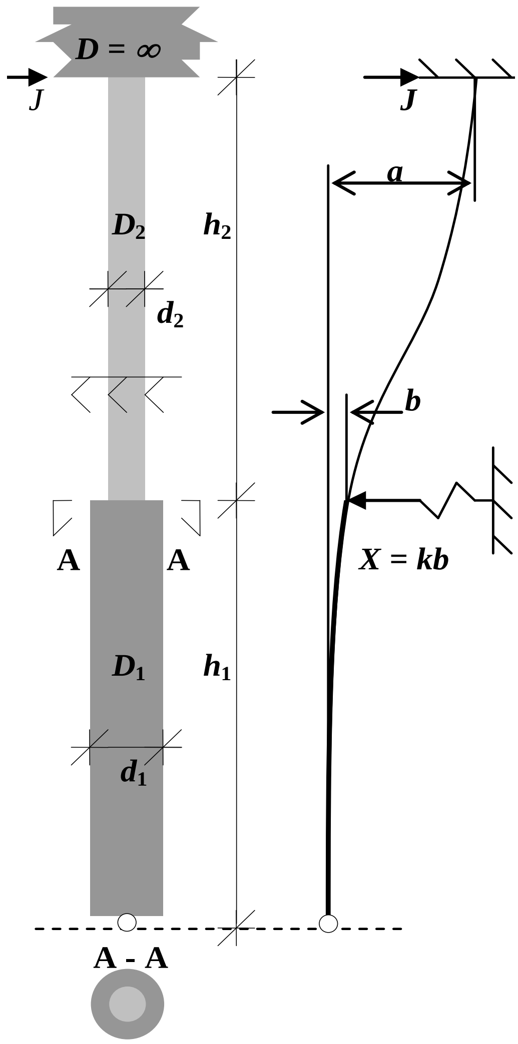
Teräsbetonin kimmokerroin

E = 30 000 MN/m^2

Sillan päällysrakennetta voidaan pitää taivutukseen nähden äärettömän jäykkänä.

$D = \infty$

Paalun alapään kiinnitystä voidaan pitää nivelellisenä.



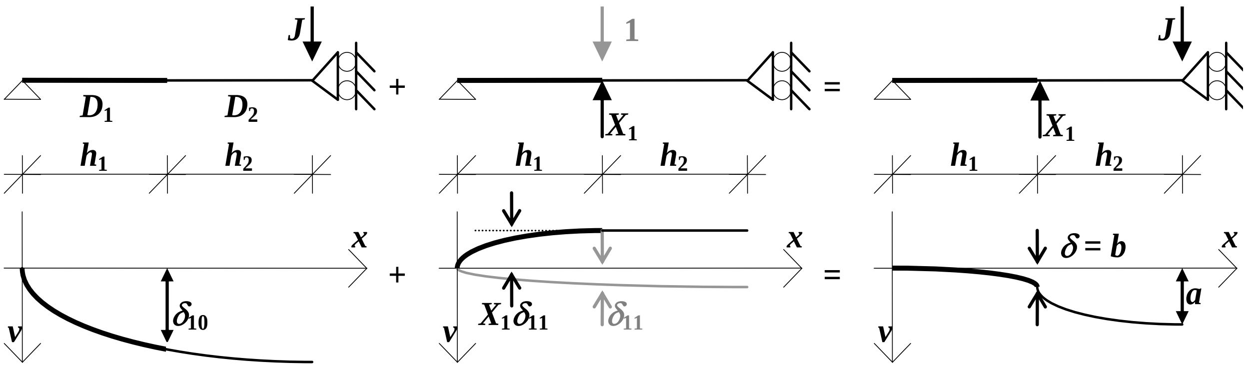
Kuva 1.

Kaivinpaalun ja pilarin jäykkyys

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$$D_1 = 3\,054 \text{ MNm}^2$$

$$D_2 = 603 \text{ MNm}^2$$



Kuva 2.

Yhteensopivuusehto (kuva 2)

$$\delta_{10} - X_1\delta_{11} = \delta \tag{2}$$

jossa

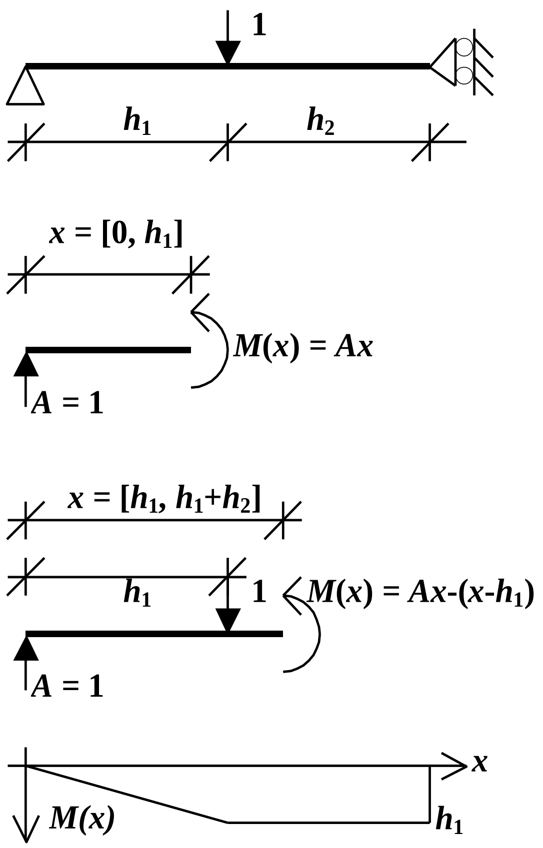
$$\delta = \frac{X_1}{k} \tag{3}$$

Taivutusmomentti yksikkövoimasta (kuva 3)

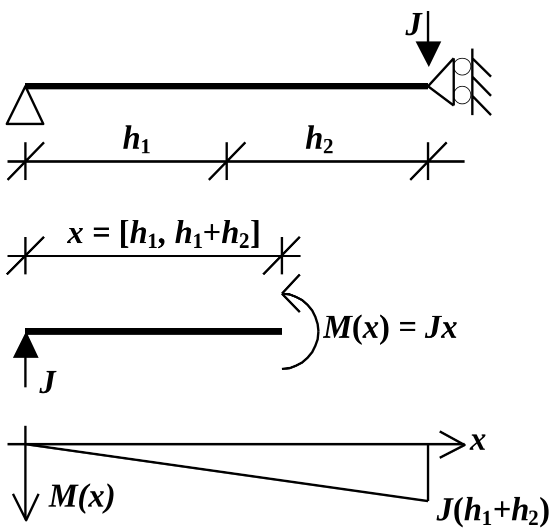
$$M_1(x) = \begin{cases} x, & x \in \{0..h_1\} \\ h_1, & x \in \{h_1..h_1 + h_2\} \end{cases} \tag{4}$$

Taivutusmomentti jarrukuormasta (kuva 4)

$$M_0(x) = Jx \tag{5}$$


1
h1
h2
x = [0, h1]
M(x) = Ax
A = 1
x = [h1, h1+h2]
h1
1
M(x) = Ax-(x-h1)
A = 1
x
M(x)
h1

Kuva 3.


J
h1
h2
x = [h1, h1+h2]
M(x) = Jx
J
x
M(x)
J(h1+h2)

Kuva 4.

Siirtymät

$$\delta_{10} = \int \frac{M_1 M_0}{D} dx \tag{6}$$

$$= \int_0^{h_1} \frac{Jx^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{h_1 Jx}{D_2} dx \tag{7}$$

$$= J\left[\frac{h_1^3}{3D_1} + \frac{h_1 h_2 (2h_1 + h_2)}{2D_2}\right] \tag{8}$$

$$= \quad 0{,}983 \text{ m}$$

$$\delta_{11} = \int \frac{M_1 M_1}{D} dx \tag{9}$$

$$= \int_0^{h_1} \frac{x^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{h_1^2}{D_2} dx \tag{10}$$

$$= \frac{h_1^3}{3D_1} + \frac{h_1^2 h_2}{D_2} \tag{11}$$

$$= \quad 1{,}435 \text{ m/MN}$$

Yhtälöistä 2, 3, 8 ja 11 saadaan tukireaktioksi

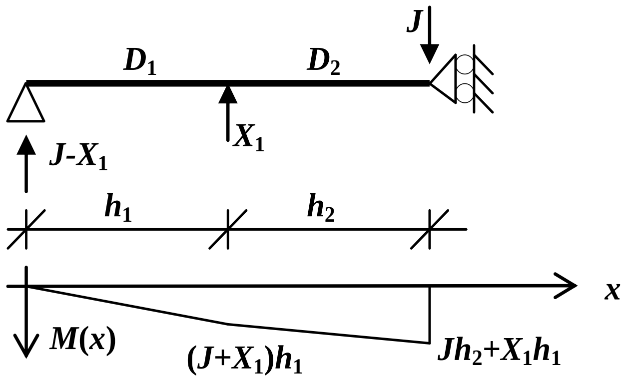
$$X_1 = \frac{\delta_{10}}{\frac{1}{k} + \delta_{11}} \tag{12}$$

$$= \quad 0{,}681 \text{ MN}$$

Rakenteen taivutusmomentti (kuva 5)

$$M = M_0 - X_1 M_1 \tag{13}$$

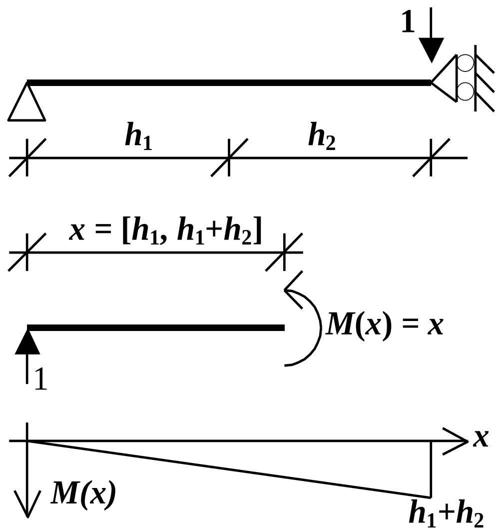
$$= \begin{cases} (J - X_1)x, & x \in \{0 \ldots h_1\} \\ Jx - X_1 h_1, & x \in \{h_1 \ldots h_1 + h_2\} \end{cases} \tag{14}$$



Kuva 5.

Taivutusmomentti yksikkövoimasta jarruvoiman kohdalla (kuva 6)

$$M_C(x) = x \tag{15}$$



Kuva 6.

Pilarin yläpään siirtymä

$$a = \delta_1 \tag{16}$$

$$= \int \frac{MM_C}{D} dx \tag{17}$$

$$= \int_0^{h_1} \frac{(J - X_1)x^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{Jx^2 - X_1 h_1 x}{D_2} dx \tag{18}$$

$$= \frac{(J - X_1)h_1^3}{3D_1} + \frac{Jh_2\left(3h_1^2 + 3h_1h_2 + h_2^2\right)}{3D_2} - \frac{X_1 h_1 h_2 (2h_1 + h_2)}{2D_2} \tag{19}$$

= **0,051 m**

<u>VASTAUS</u>: **Pilarin yläpään siirtymä**

a = **0,051 m**

34. TEHTÄVÄ

Sillan välitukeen kohdistuu tieliikennekuormien mukainen suurin jarrukuorma

J = 0,500 MN

Välituki koostuu kuvan 1 mukaisesti kaivinpaalusta (1) ja pilarista (2).
Kuinka paljon pilarin yläpää siirtyy jarrukuorman johdosta (*a* -mitta)?

Ratkaise tehtävä käyttäen hyväksi ulokkeen siirtymien lausekkeita!

Maan kimmoisuus approksimoidaan (kuvan 1 mukaisesti) yhdellä jousella, jonka jousivakio on

k = 120 MN/m

Kaivinpaalun ja pilarin poikkileikkaus on ympyrä, jonka halkaisija on

d_1 = 1,200 m

d_2 = 0,800 m

Korkeus

h_1 = 10 m

h_2 = 8 m

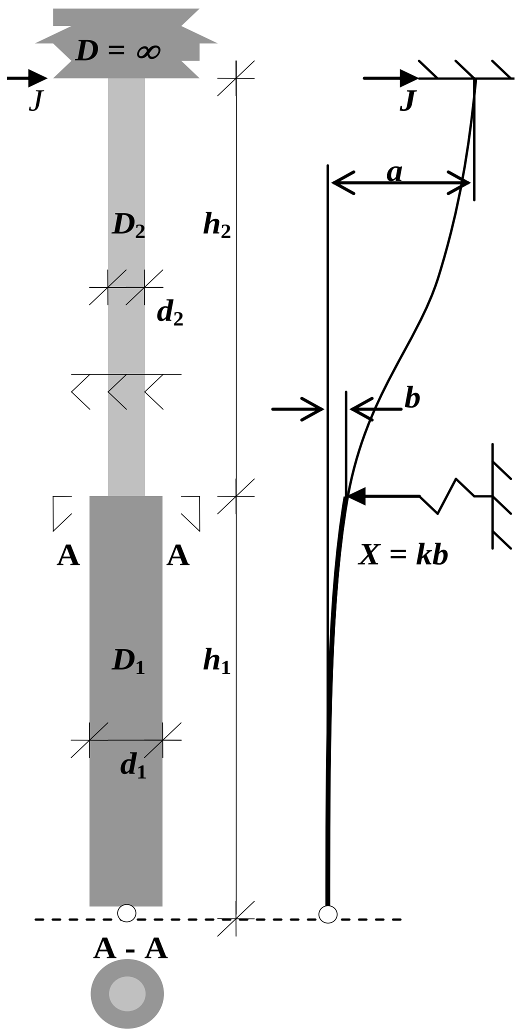
Teräsbetonin kimmokerroin

E = 30 000 MN/m^2

Sillan päällysrakennetta voidaan pitää taivutukseen nähden äärettömän jäykkänä.

D = ∞

Paalun alapään kiinnitystä voidaan pitää nivelellisenä.



Kuva 1.

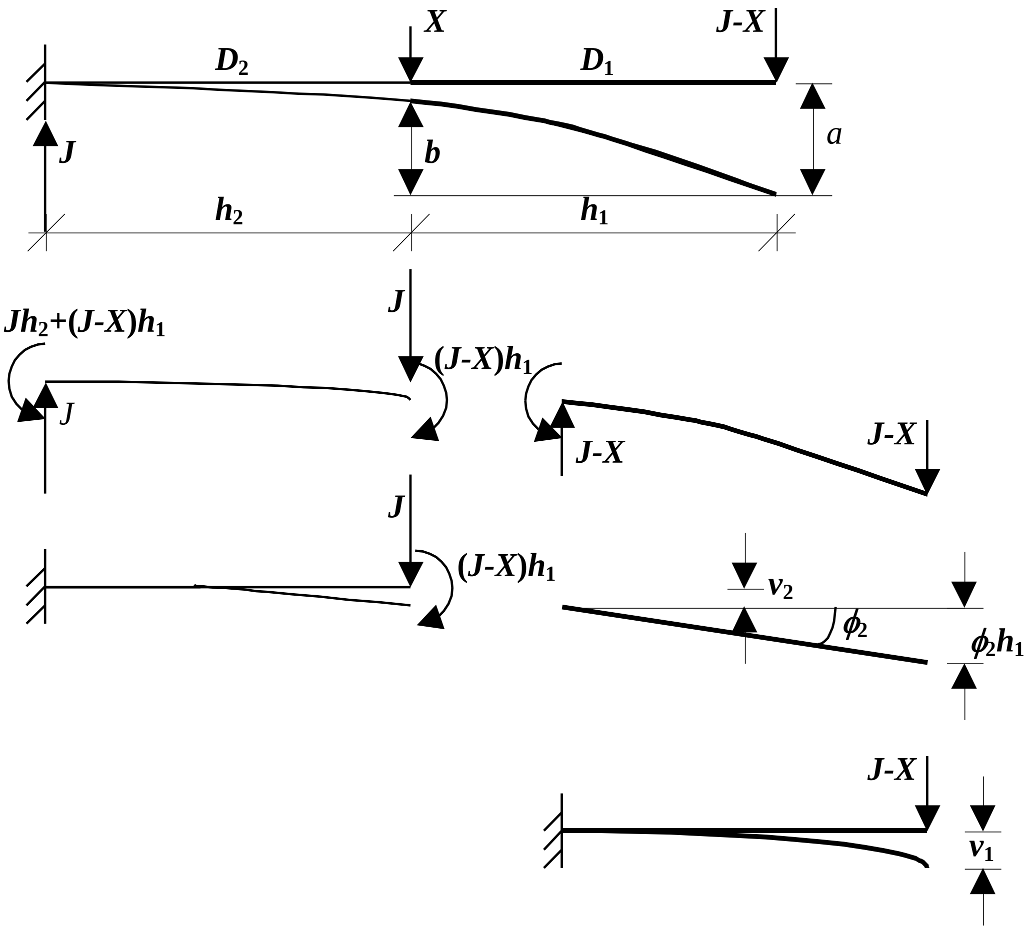
Kaivinpaalun ja pilarin jäykkyys

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$D_1 =$ 3 054 MNm^2

$D_2 =$ 603 MNm^2

Sovellettu *Kaavakokoelman* taipumaviivan yhtälöitä.



Kuva 2.

Jousen painuma ulokkeen (1) siirtymien avulla lausuttuna (kuva 2)

$$b = v_1 + \phi_2 h_1 \tag{2}$$

$$= \frac{(J-X)h_1^3}{3D_1} + \left[\frac{Jh_2^2}{2D_2} + \frac{(J-X)h_1h_2}{D_2}\right]h_1 \tag{3}$$

jossa jousivoima

$$X = kb \tag{4}$$

Ratkaisemalla näistä jousen painuma, saadaan

$$b = J\,\frac{\dfrac{h_1^3}{3D_1} + \dfrac{h_1h_2^2}{2D_2} + \dfrac{h_1^2h_2}{D_2}}{1 + \dfrac{kh_1^3}{3D_1} + \dfrac{kh_1^2h_2}{D_2}} \tag{5}$$

$$= \quad 0{,}006 \text{ m}$$

Pilarin yläpään siirtymä on jousen painuman b ja ulokkeen (2) taipuman v_2 summa

$$a = b + v_2 \tag{6}$$

$$= b + \frac{Jh_2^3}{3D_2} + \frac{(J-X)h_1h_2^2}{2D_2} \tag{7}$$

$$= \quad 0{,}051 \text{ m}$$

<u>VASTAUS</u>: **Pilarin yläpään siirtymä**

$$a = \quad 0{,}051 \text{ m}$$

L i s ä t a r k a s t e l u

Taipumaviivan muoto, kun origo on suurpaalun alaosassa, eli ulokkeen päässä (katkoviiva kuvassa 3)

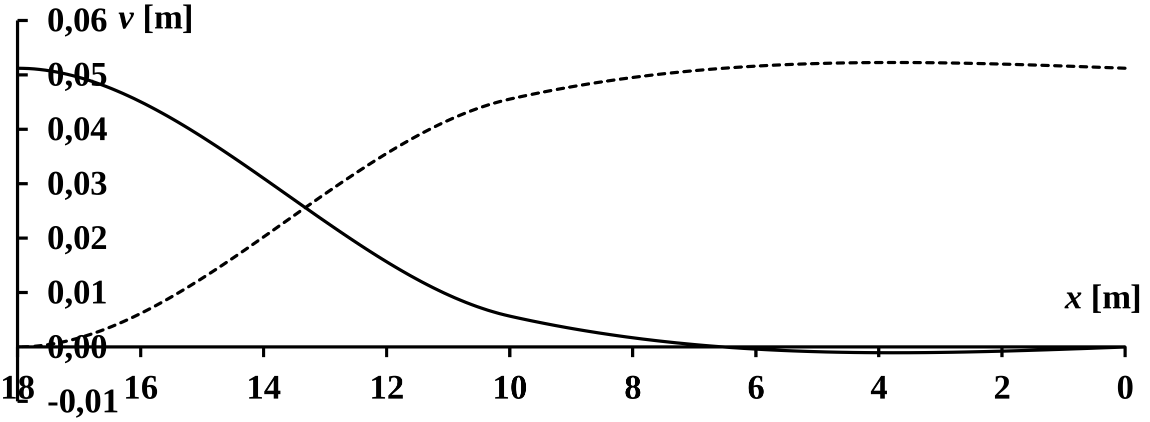
$$v(x)=\begin{cases}\dfrac{(J-X)h_1^3}{6D_1}\left[2-3\dfrac{x}{h_1}+\left(\dfrac{x}{h_1}\right)^3\right]+ \\ \left[\dfrac{Jh_2^2}{2D_2}+\dfrac{(J-X)h_1h_2}{D_2}\right](h_1-x)+a-b, \quad x=[0, \quad h_1] \\ \dfrac{Jh_2^3}{6D_2}\left[2-3\dfrac{x-h_1}{h_2}+\left(\dfrac{x-h_1}{h_2}\right)^3\right]+\dfrac{(J-X)h_1}{2D_2}(h_2-x+h_1)^2, \\ x=[h_1, \quad h_2]\end{cases} \tag{8}$$

jossa jousivoima yhtälöstä 22

X = 0,681 MN

Peilataan käyrä niin, että taipuma origossa on nolla (yhtenäinen viiva kuvassa 8)

$$v_p(x) = a\text{-}v(x) \tag{9}$$



Kuva 3.

35. TEHTÄVÄ

Sillan välitukeen kohdistuu tieliikennekuormien mukainen suurin jarrukuorma

J = 0,500 MN

Välituki koostuu kuvan 1 mukaisesti kaivinpaalusta (1) ja pilarista (2).
Kuinka paljon pilarin yläpää siirtyy jarrukuorman johdosta (*a* -mitta)?

Ratkaise tehtävä Mohrin menetelmällä!

Maan kimmoisuus approksimoidaan (kuvan 1 mukaisesti) yhdellä jousella, jonka jousivakio on

k = 120 MN/m

Kaivinpaalun ja pilarin poikkileikkaus on ymp

d_1 = 1,200 m
d_2 = 0,800 m

Korkeus

h_1 = 10 m
h_2 = 8 m

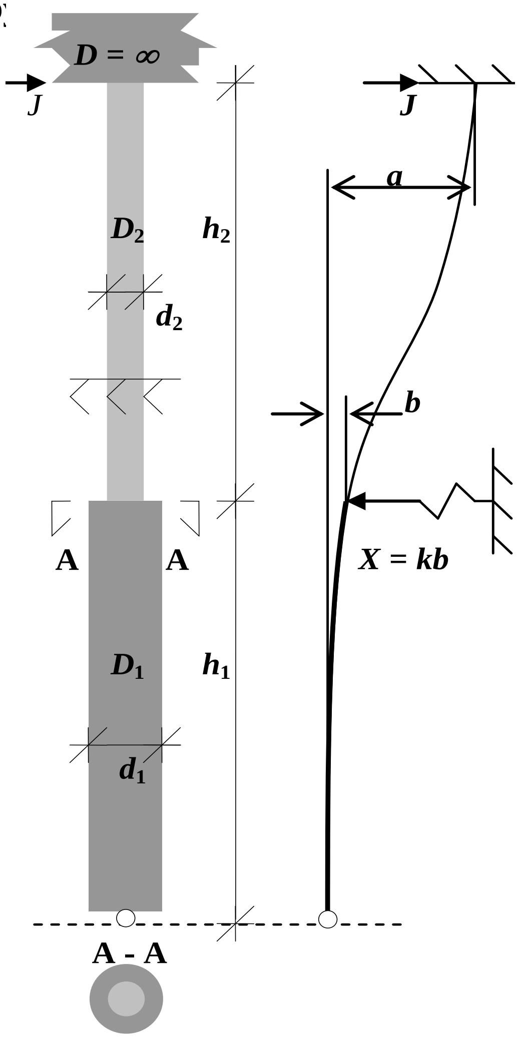
Teräsbetonin kimmokerroin

E = 30 000 MN/m^2

Sillan päällysrakennetta voidaan pitää taivutukseen nähden äärettömän jäykkänä.

$D = \infty$

Paalun alapään kiinnitystä voidaan pitää nivelellisenä.



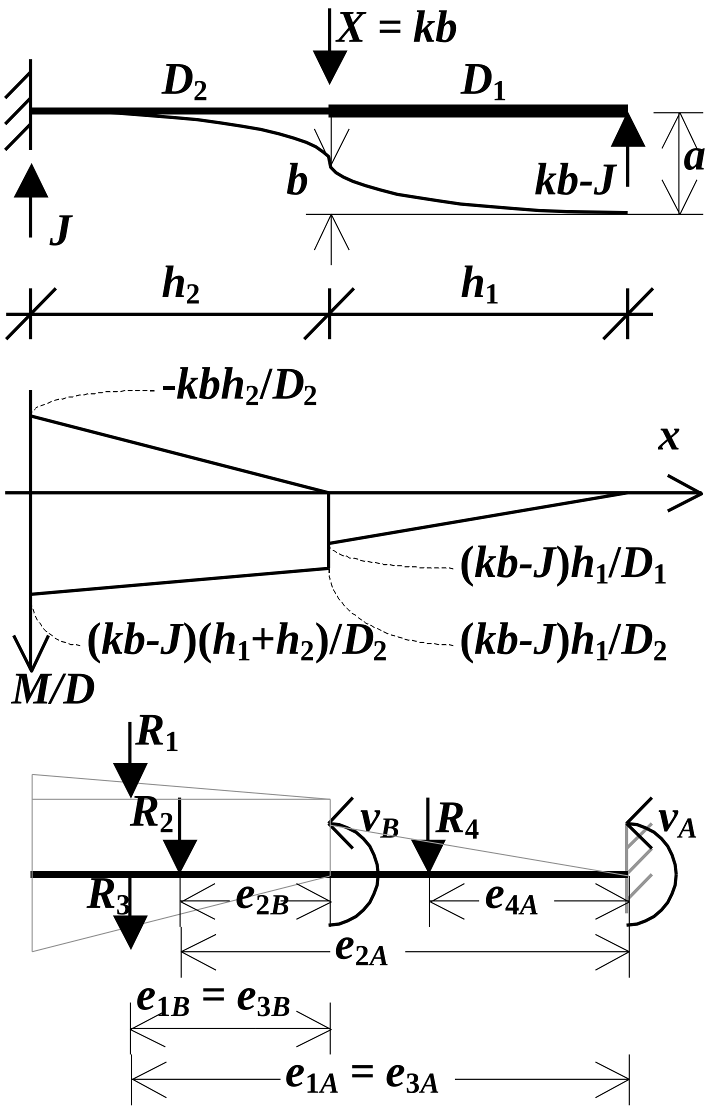
Kuva 1.

Kaivinpaalun ja pilarin jäykkyys

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$D_1 =$ 3 054 $\mathrm{MNm^2}$

$D_2 =$ 603 $\mathrm{MNm^2}$



Kuva 2.

Taipuma v jousen kohdalla ($_B$) ja kaivinpaalun alapäässä ($_A$) (kuva 2)

$$\begin{cases} v_B = a - b = -R_1 e_{1B} - R_2 e_{2B} - R_3 e_{3B} \\ v_A = \quad a = -R_1 e_{1A} - R_2 e_{2A} - R_3 e_{3A} - R_4 e_{4A} \end{cases} \quad \text{(28a, b)}$$

Laskemalla yhteen saadaan

$$-b = R_1(e_{1A} - e_{1B}) + R_2(e_{2A} - e_{2B}) + R_3(e_{3A} - e_{3B}) + R_4 e_{4A} \quad (29)$$

Kuormaresultantit ovat

$$R_1 = \frac{kb - J}{2D_2} h_2^2 \quad (30)$$

$$R_2 = \frac{kb - J}{D_2} h_1 h_2 \quad (31)$$

$$R_3 = \frac{-kb}{2D_2} h_2^2 \quad (32)$$

$$R_4 = \frac{kb - J}{2D_1} h_1^2 \quad (33)$$

Etäisyydet ovat

$$e_{iA} - e_{iB} = h_1 \quad (34)$$

$$e_{4A} = 2h_1/3 \quad (35)$$

Jousen painuma yhtälöstä 29

$$b = J \frac{\dfrac{h_1 h_2^2}{2D_2} + \dfrac{h_1^2 h_2}{D_2} + \dfrac{h_1^3}{3D_1}}{1 + \dfrac{k h_1^2 h_2}{D_2} + \dfrac{k h_1^3}{3D_1}} \quad (36)$$

$$= \quad 0{,}006 \text{ m}$$

Pilarin yläpään siirtymä yhtälöstä 28a

$$a = b + \frac{J h_2^3}{3D_2} - \frac{kb - J}{2D_2} h_1 h_2^2 \quad (37)$$

$$= \quad 0{,}051 \text{ m}$$

VASTAUS:

Pilarin yläpään siirtymä

a = **0,051 m**

36. TEHTÄVÄ

Sillan välitukeen kohdistuu tieliikennekuormien mukainen suurin jarrukuorma

J = 0,500 MN

Välituki koostuu kuvan 1 mukaisesti kaivinpaalusta (1) ja pilarista (2).
Kuinka paljon pilarin yläpää siirtyy jarrukuorman johdosta (*a* -mitta)?

Ratkaise tehtävä elementtimenetelmällä!

Maan kimmoisuus approksimoidaan (kuvan 1 mukaisesti) yhdellä jousella, jonka jousivakio on

k = 120 MN/m

Kaivinpaalun ja pilarin poikkileikkaus on ymp

d_1 = 1,200 m

d_2 = 0,800 m

Korkeus

h_1 = 10 m

h_2 = 8 m

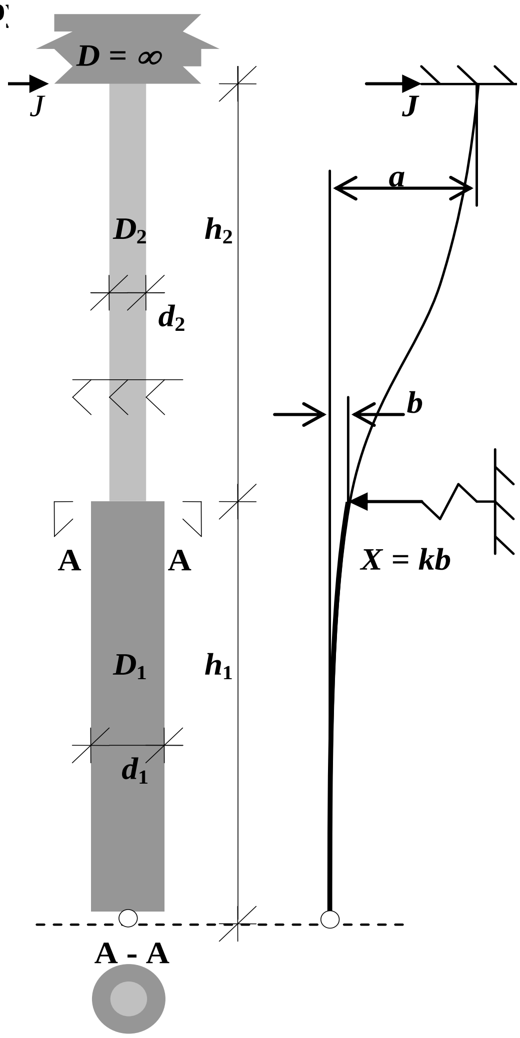
Teräsbetonin kimmokerroin

E = 30 000 MN/m^2

Sillan päällysrakennetta voidaan pitää taivutukseen nähden äärettömän jäykkänä.

$D = \infty$

Paalun alapään kiinnitystä voidaan pitää nivelellisenä.



Kuva 1.

Kaivinpaalun ja pilarin jäykkyys

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$$D_1 = 3\,054 \text{ MNm}^2$$

$$D_2 = 603 \text{ MNm}^2$$

Jousivakio

$$k_{01} = \infty \text{ MN/m}$$

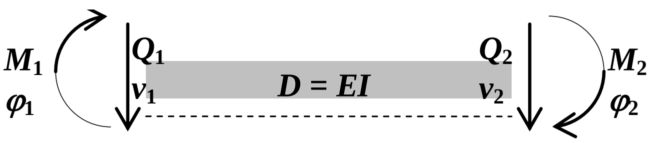
$$= 1{,}0\text{E}+100 \text{ MN/m} \quad \text{(laskennassa)}$$

$$k_1 = 120 \text{ MN/m}$$

$$k_2 = 0 \text{ MN/m}$$

Palkkielementin tasapainoehto (kuva 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} \tag{2}$$



Kuva 2.

Elementin 1 jäykkyysmatriisi

$$K_i^e = \begin{bmatrix} 36{,}644 & 183{,}218 & -36{,}644 & 183{,}218 \\ 183{,}218 & 1\,221{,}451 & -183{,}218 & 610{,}726 \\ -36{,}644 & -183{,}218 & 36{,}644 & -183{,}218 \\ 183{,}218 & 610{,}726 & -183{,}218 & 1\,221{,}451 \end{bmatrix}$$

Elementin 2 jäykkyysmatriisi

$$K_i^e = \begin{bmatrix} 14{,}137 & 56{,}549 & -14{,}137 & 56{,}549 \\ 56{,}549 & 301{,}593 & -56{,}549 & 150{,}796 \\ -14{,}137 & -56{,}549 & 14{,}137 & -56{,}549 \\ 56{,}549 & 150{,}796 & -56{,}549 & 301{,}593 \end{bmatrix}$$

Systeemin jäykkyysmatriisi (kuva 3)

$$K = \begin{bmatrix} k_{11}^1 + k_{01} & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 \\ k_{21} = k_{12} & k_{22}^1 & k_{23}^1 & k_{24}^1 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^2 & k_{34}^2 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^2 \end{bmatrix}$$

(3)

$$= \begin{bmatrix} 1{,}0E+100 & 183{,}218 & -36{,}644 & 183{,}218 & 0{,}000 & 0{,}000 \\ 183{,}218 & 1221{,}451 & -183{,}218 & 610{,}726 & 0{,}000 & 0{,}000 \\ -36{,}644 & -183{,}218 & 170{,}781 & -126{,}669 & -14{,}137 & 56{,}549 \\ 183{,}218 & 610{,}726 & -126{,}669 & 1523{,}044 & -56{,}549 & 150{,}796 \\ 0{,}000 & 0{,}000 & -14{,}137 & -56{,}549 & 14{,}137 & -56{,}549 \\ 0{,}000 & 0{,}000 & 56{,}549 & 150{,}796 & -56{,}549 & 301{,}593 \end{bmatrix}$$

Systeemin kuormavektorin alkiot elementin *i* yläpäässä

$F_1 =$ 0,000 MN

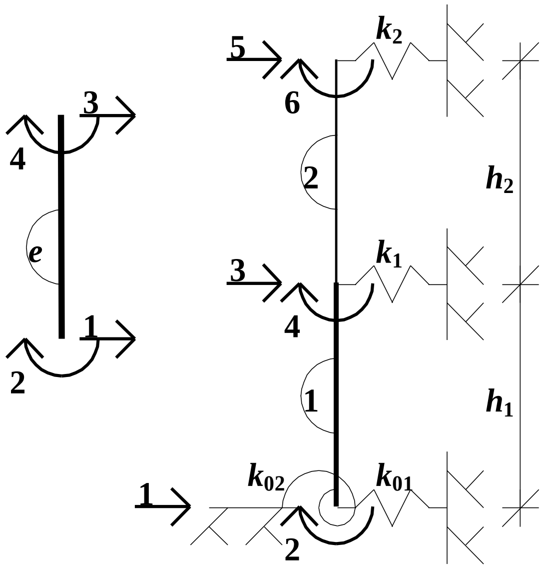
$M_1 =$ 0,000 MNm

$F_2 =$ 0,000 MN

$M_2 =$ 0,000 MNm

$F_3 =$ 0,500 MN

$M_3 =$ -2,192 MNm



Kuva 3.

Systeemin siirtymävektorin alkiot elementin *i* yläpäässä

$$w_1 = 0{,}000 \text{ m}$$
$$\varphi_1 = 0{,}000 \text{ rad}$$
$$w_2 = 0{,}006 \text{ m}$$
$$\varphi_2 = 0{,}003 \text{ rad}$$
$$w_3 = 0{,}051 \text{ m}$$
$$\varphi_3 = 0{,}000 \text{ rad}$$

<u>VASTAUS</u>:

Etsimällä sellainen M_3:n arvo, jolla kiertymä

$$\varphi_3 = 0$$

saadaan yläpään siirtymä

$$a = w_3 \qquad (4)$$
$$= 0{,}051 \text{ m}$$

L i s ä t a r k a s t e l u

Elementin voimasuureet

$$F_i^{\,e} = K_i^{\,e}\,\delta_i^{\,e} \tag{5}$$

1. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0{,}000 \text{ m} \\ 0{,}000 \\ 0{,}006 \text{ m} \\ 0{,}003 \end{bmatrix} = \begin{bmatrix} 0{,}181 \text{ MN} \\ 0{,}000 \text{ MNm} \\ -0{,}181 \text{ MN} \\ 1{,}808 \text{ MNm} \end{bmatrix}$$

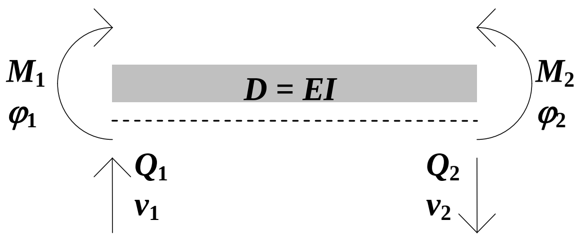
2. elementti

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} 0{,}006 \text{ m} \\ 0{,}003 \\ 0{,}051 \text{ m} \\ 0{,}000 \end{bmatrix} = \begin{bmatrix} -0{,}500 \text{ MN} \\ -1{,}808 \text{ MNm} \\ 0{,}500 \text{ MN} \\ -2{,}192 \text{ MNm} \end{bmatrix}$$

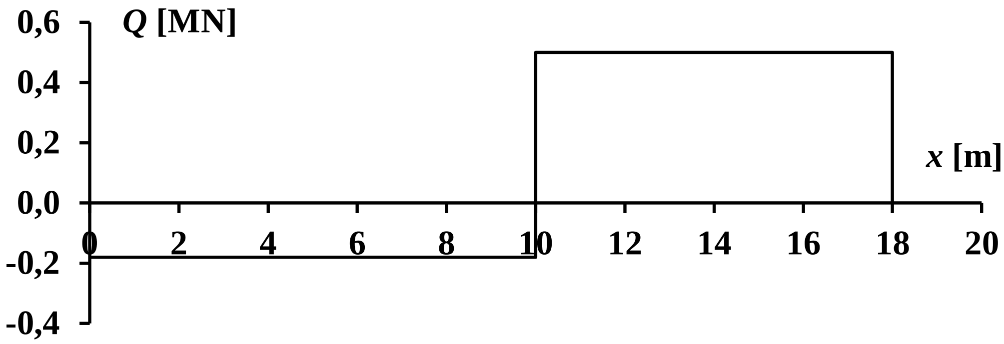
Systeemin leikkausvoima-, taivutusmomentti- ja taipumajakauma on esitetty taulukossa 1 sekä kuvissa 5, 6 ja 7. Positiiviset suunnat ilmenevät kuvasta 4.

Taulukko 1.

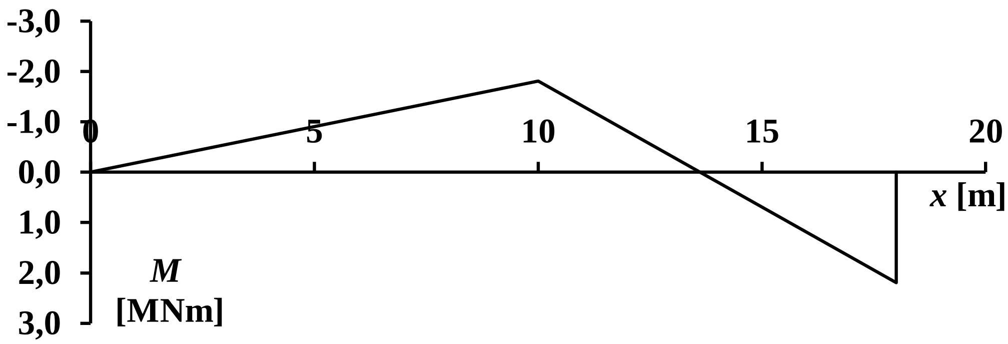
x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,181	0,000	0,000
10,000	-0,181	-1,808	0,006
10,000	0,500	-1,808	0,006
18,000	0,500	2,192	0,051
18,000	0	0	



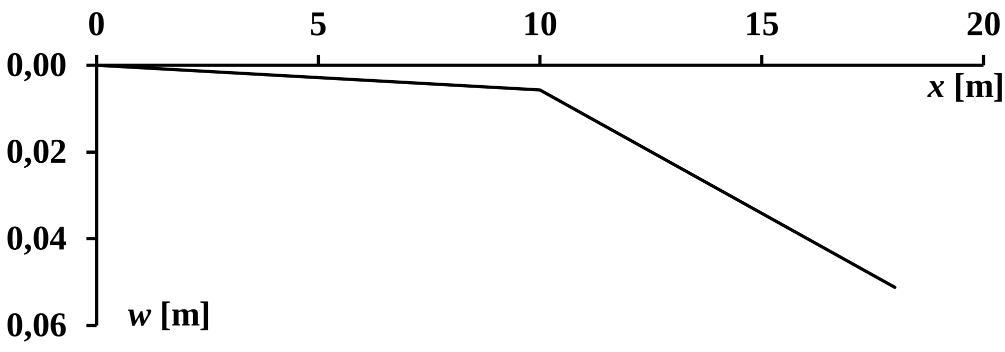
Kuva 4. Positiiviset suunnat (vrt. kuva 2).



Kuva 5. Leikkausvoimakuvaaja.



Kuva 6. Taivutusmomenttikuvaaja.



Kuva 7. Taipumakuvaaja.

37. TEHTÄVÄ

Jäykän peruslaatan (kuva 1) oikea reuna on painumattomalla kalliolla (tuki B). Muulta osin alusta on kimmoinen (Winkler) ja sen alustavakio

c = 10 MN/m^3

Laatan pituus

L = 6 m

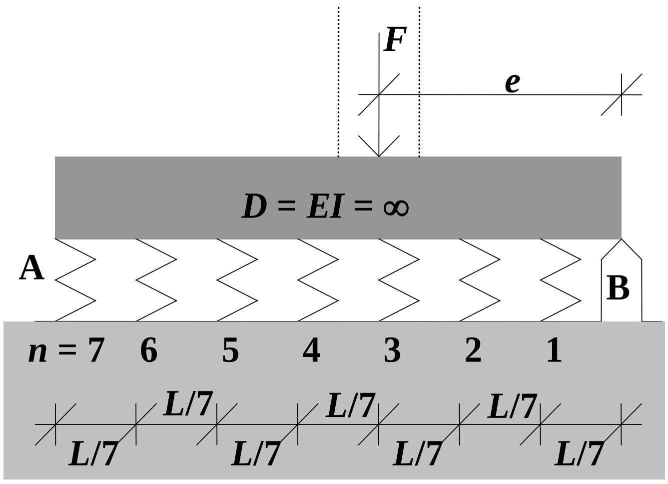
A) **Mille etäisyydelle (e) seinä voidaan sijoittaa niin, ettei tuelle B tule vetoa? Seinän aiheuttama pystyvoima juoksumetriä kohden on**

F = 1 MN/m

B) **Määritä vasemman reunan (piste A) painuma A-kohdan vastauksen maksimiarvolla!**

C) **Laske jousivoimat A-kohdan vastauksen maksimiarvolla, jos samanlaisia jousia on kuvan 1 mukaisesti**

n = 7 kpl

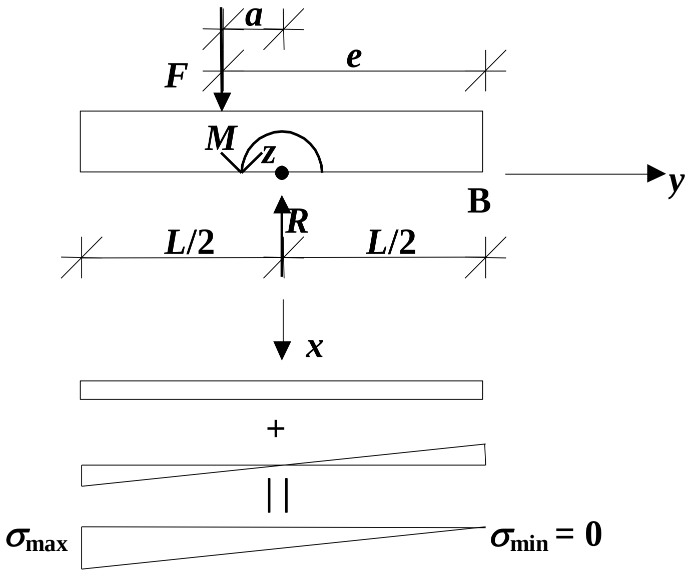


Kuva 1.

A)

Jännitys (kuva 2)

$$\sigma(y) = \frac{F}{A} + \frac{M}{I_z} y \tag{1}$$



Kuva 2.

Pinta-ala

$$A = bL \tag{2}$$

jossa b on laatan leveys kuvatasoa vastaan kohtisuorassa suunnassa.

Jäyhyysmomentti z -akselin suhteen

$$I_z = \frac{bL^3}{12} \tag{3}$$

Jännitys σ_B tuella B on nolla eli

$$\sigma_B = \frac{F}{A} - \frac{aF}{I_z}\frac{L}{2} = 0 \tag{4}$$

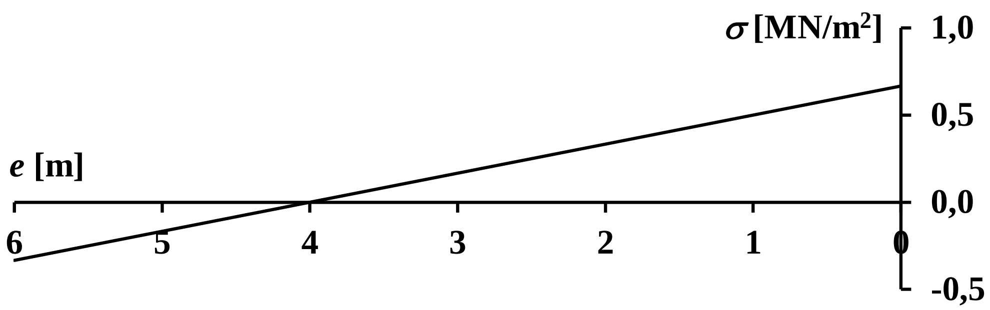
$$\Rightarrow \quad 0 = \frac{F}{bL}\left(1 - \frac{6a}{L}\right) \tag{5}$$

$$\Rightarrow a = \frac{L}{6} \tag{6}$$
$$= \quad 1{,}000 \ \text{m}$$

Etäisyys

$$e \leq \frac{L}{2} + a \tag{7}$$
$$\leq \quad 4{,}000 \ \text{m}$$

Pohjapaine tuella B kuormituskohdan *e* funktiona on esitetty kuvassa 3.



Kuva 3.

<u>VASTAUS</u>:

Mitta

$$e \ \leq \quad 4{,}000 \ \text{m}$$

B)

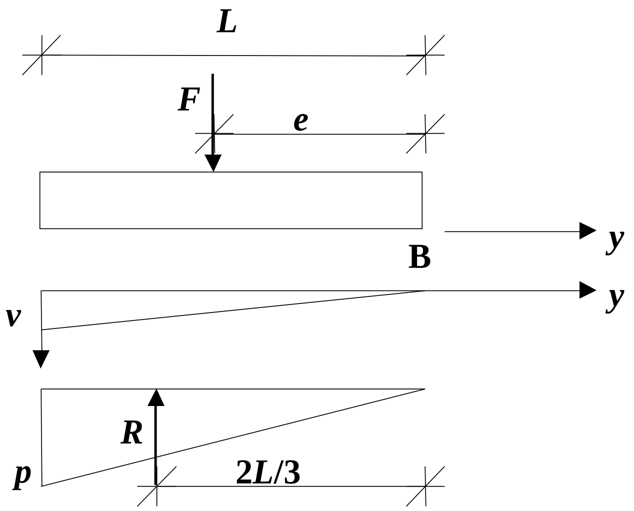
Pohjapaine on alustavakion *c* ja taipuman *v* tulo

$$p \ = cv \tag{8}$$

Pohjapaineen resultantti (kuva 4)

$$R = \frac{1}{2} pL \tag{9}$$

$$\Rightarrow R = \frac{1}{2} cvL \tag{10}$$



Kuva 4.

Momenttitasapainoehto tuen B suhteen

$$M_B = 0 \tag{11}$$

$$\Rightarrow \quad 0 = eF - \frac{2}{3} LR \tag{12}$$

$$\Rightarrow \quad 0 = eF - \frac{2}{3} L \frac{1}{2} cvL \tag{13}$$

$$\Rightarrow v(e) = \frac{3F}{cL^2} e \tag{14}$$

$$= \quad 0{,}008\ e$$

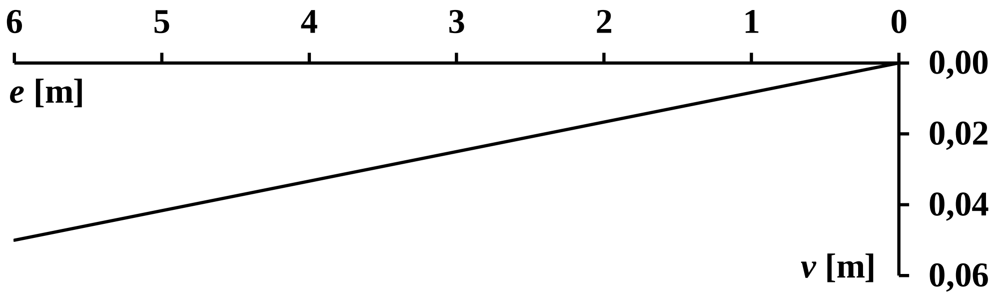
Vasemman reunan taipuman suuruus kuormituskohdan e funktiona on esitetty kuvassa 5.

Kun

$$e = 4{,}000 \text{ m}$$

niin taipuma

$$v = 0{,}033 \text{ m}$$



Kuva 5.

<u>VASTAUS</u>:

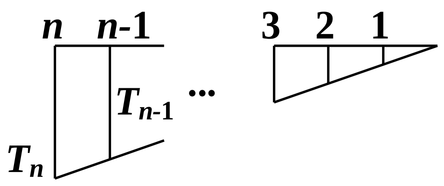
Taipuma vasemmassa reunassa

$$v(e) = 0{,}008\, e$$

C)

Jos jousivoima viimeisessä jousessa on T_n, niin jousen i jousivoima on (kuva 6)

$$T_i = \frac{i}{n} T_n \qquad (15)$$



Kuva 6.

Tasapainoyhtälö

$$\sum_{i=1}^{n} \frac{i}{n} T_n = F \tag{16}$$

$$\Rightarrow \qquad T_n = \frac{nF}{\sum_{i=1}^{n} i} \tag{17}$$

Jousivoima jousessa *i*

$$T_i = \frac{iF}{\sum_{i=1}^{n} i} \tag{18}$$

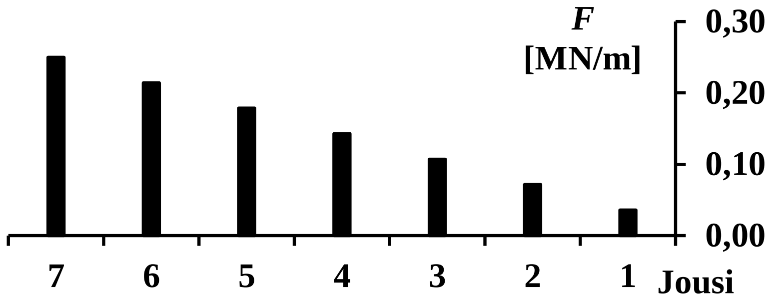
<u>VASTAUS</u>:

Jousivoimat on esitetty kuvassa 7.

F_1	=	0,036	MN/m
F_2	=	0,071	MN/m
F_3	=	0,107	MN/m
F_4	=	0,143	MN/m
F_5	=	0,179	MN/m
F_6	=	0,214	MN/m
F_7	=	0,250	MN/m

Tarkistus

ΣF_i	=	1,000	MN/m



Kuva 7.

38. TEHTÄVÄ

Laske kuvan 1 lattian painuma-, pohjapaine-, momentti- ja leikkausvoimakuvio!

Johda tarvittavat taivutusmomentin lausekkeet toisaalta taipumaviivan differentiaaliyhtälön ja toisaalta maanpaineen resultanttien avulla!

Mitta

$a = 2{,}500$ m

Betonilaatan lujuus

$K = 30$ MN/m^2

Polystyreenin kimmokerroin

$E_2 = 15$ MN/m^2

Soran kimmokerroin

$E_3 = 40$ MN/m^2

Maapohjan alustaluku

$c_4 = 5$ MN/m^3

Kerrosten paksuusmitat

$h_1 = 0{,}100$ m

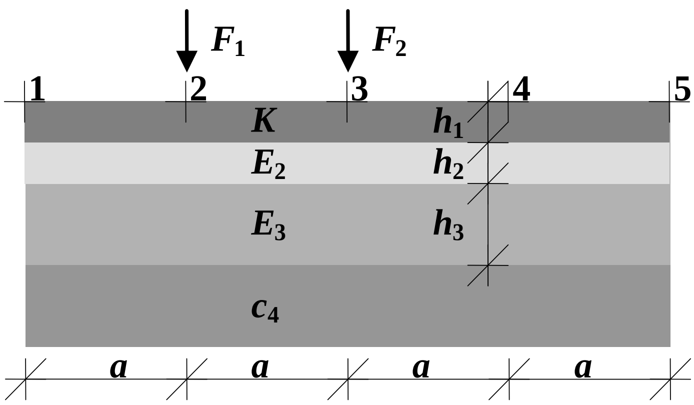
$h_2 = 0{,}100$ m

$h_3 = 0{,}200$ m

Kuormitus pituusyksikköä kohti

$F_1 = 0{,}050$ MN/m

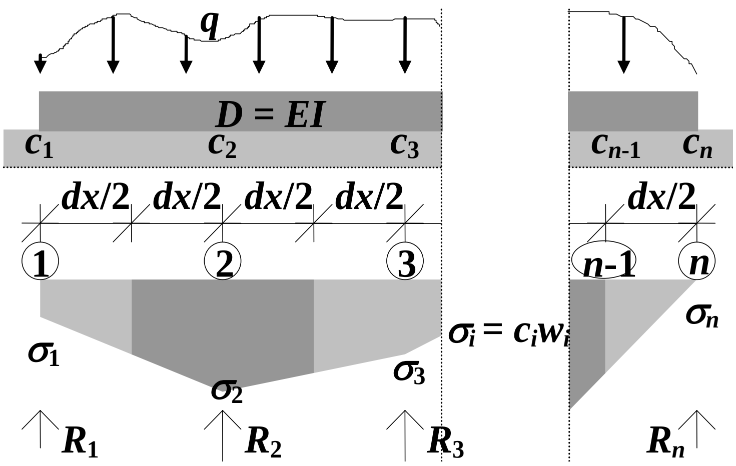
$F_2 = 0{,}025$ MN/m



Kuva 1.

Maanpaineen resultantit R_i (tukireaktiot) pituusyksikköä kohti yleisesti pisteissä $1\ldots n$ saadaan laskemalla oheiseen kuvaan merkityn jännityskuvion osien pinta-alat (kuva 2).

$$R_i = \begin{cases} \sigma_i \dfrac{dx}{2} + \dfrac{1}{2}\dfrac{\sigma_{i+1}-\sigma_i}{2}\dfrac{dx}{2}, & i=1 \\ \sigma_i \dfrac{dx}{2} - \dfrac{1}{2}\dfrac{\sigma_i-\sigma_{i-1}}{2}\dfrac{dx}{2} + \sigma_i \dfrac{dx}{2} + \dfrac{1}{2}\dfrac{\sigma_{i+1}-\sigma_i}{2}\dfrac{dx}{2}, & \begin{cases} i \in \{2\ldots n-1\} \\ i \in N \end{cases} \\ \sigma_i \dfrac{dx}{2} - \dfrac{1}{2}\dfrac{\sigma_i-\sigma_{i-1}}{2}\dfrac{dx}{2}, & i=n \end{cases} \tag{1}$$



Kuva 2.

Kun jännitys σ_i on alustaluvun c_i ja painuman w_i tulo

$$\sigma_i = c_i w_i \tag{2}$$

saadaan, kun alustaluku c on vakio,

$$R_i = \begin{cases} \dfrac{cdx}{8}(3w_i + w_{i+1}), & i=1 \\ \dfrac{cdx}{8}(w_{i-1} + 6w_i + w_{i+1}), & i \in \{2\ldots n-1\}, \quad i \in N \\ \dfrac{cdx}{8}(w_{i-1} + 3w_i), & i=n \end{cases} \tag{3}$$

Maanpaineen resultantit (tukireaktiot) tehtävän solmupisteissä (kuva 3)

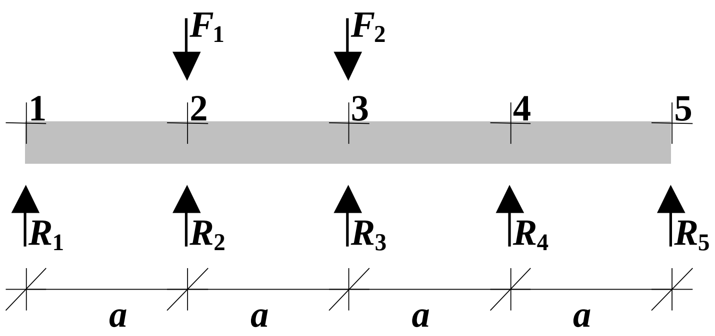
$$R_1 = \frac{ca}{8}(3w_1 + w_2) \tag{4}$$

$$R_2 = \frac{ca}{8}(w_1 + 6w_2 + w_3) \tag{5}$$

$$R_3 = \frac{ca}{8}(w_2 + 6w_3 + w_4) \tag{6}$$

$$R_4 = \frac{ca}{8}(w_3 + 6w_4 + w_5) \tag{7}$$

$$R_5 = \frac{ca}{8}(w_4 + 3w_5) \tag{8}$$



Kuva 3.

Taipuman w derivaatta (l. muutosnopeuden raja-arvo) pisteen i ja sen naapuripisteiden puolivälissä

$$w'_{i-1/2} = \frac{w_i - w_{i-1}}{dx} \tag{9}$$

$$w'_{i+1/2} = \frac{w_{i+1} - w_i}{dx} \tag{10}$$

Taipuman II derivaatta pisteessä i

$$w''_i = \frac{w'_{i+1/2} - w'_{i-1/2}}{dx} \tag{11}$$

Taivutusmomentti

$$M_i = -EI_i w_i'' \tag{12}$$

$$\Rightarrow M_i = -EI_i \frac{w'_{i+1/2} - w'_{i-1/2}}{dx} \tag{13}$$

$$\Rightarrow M_i = -EI_i \frac{w_{i+1} - w_i - w_i + w_{i-1}}{dx^2} \tag{14}$$

$$\Rightarrow M_i = \frac{EI_i}{dx^2}\left(-w_{i-1} + 2w_i - w_{i+1}\right) \tag{15}$$

Taivutusmomentti pisteissä 2, 3 ja 4

$$M_2 = R_1 a = -\frac{EI}{a^2}(w_1 - 2w_2 + w_3) \tag{16}$$

$$M_3 = 2R_5 a + R_4 a = -\frac{EI}{a^2}(w_2 - 2w_3 + w_4) \tag{17}$$

$$M_4 = R_5 a = -\frac{EI}{a^2}(w_3 - 2w_4 + w_5) \tag{18}$$

Pystysuorien voimien tasapainoehto (kuva 3)

$$F_1 + F_2 = R_1 + R_2 + R_3 + R_4 + R_5 \tag{19}$$

Momenttitasapaino solmupisteen 3 suhteen

$$2aR_1 + aR_2 = aR_4 + 2aR_5 + aF_1 \tag{20}$$

Yhtälöryhmä yhtälöistä 16…20

$$\begin{cases}
\frac{ca^2}{8}(3w_1 + w_2) = -\frac{EI}{a^2}w_1 + 2\frac{EI}{a^2}w_2 - \frac{EI}{a^2}w_3 \\
2\frac{ca^2}{8}(w_4 + 3w_5) + \frac{ca^2}{8}(w_3 + 6w_4 + w_5) = -\frac{EI}{a^2}w_2 + 2\frac{EI}{a^2}w_3 - \frac{EI}{a^2}w_4 \\
\frac{ca^2}{8}(w_4 + 3w_5) = -\frac{EI}{a^2}w_3 + 2\frac{EI}{a^2}w_4 - \frac{EI}{a^2}w_5 \\
F_1 + F_2 = \frac{ca}{8}\big(3w_1 + w_2 + w_1 + 6w_2 + w_3 + w_2 + 6w_3 + w_4 + \\
\qquad + w_3 + 6w_4 + w_5 + w_4 + 3w_5\big) \\
0 = 2\frac{ca^2}{8}(3w_1 + w_2) + \frac{ca^2}{8}(w_1 + 6w_2 + w_3) - \frac{ca^2}{8}(w_3 + 6w_4 + w_5) + \\
\quad - 2\frac{ca^2}{8}(w_4 + 3w_5) - aF_1
\end{cases} \tag{21a…e}$$

$$\Rightarrow \begin{cases} \left(3\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_1+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_2+\dfrac{EI}{a^2}w_3 = 0 \\ \dfrac{EI}{a^2}w_2+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_3+\left(8\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_4+7\dfrac{ca^2}{8}w_5 = 0 \\ \dfrac{EI}{a^2}w_3+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_4+\left(3\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_5 = 0 \\ 4\dfrac{ca}{8}w_1+8\dfrac{ca}{8}w_2+8\dfrac{ca}{8}w_3+8\dfrac{ca}{8}w_4+4\dfrac{ca}{8}w_5 = F_1+F_2 \\ 7\dfrac{ca}{8}w_1+8\dfrac{ca}{8}w_2-8\dfrac{ca}{8}w_4-7\dfrac{ca}{8}w_5 = F_1 \end{cases} \qquad (22a\ldots e)$$

Betonilaatan jäyhyysmomentti pituusyksikköä kohti

$$I = h_1^{\,3}/12 \qquad (23)$$

$$= 0{,}000083 \text{ m}^4/\text{m}$$

Betonin kimmokerroin

$$E = k\sqrt{KK_0} \qquad (24)$$

jossa k = 1 ja $K_0 = 25 \cdot 10^6$ MN/m^2.

$$E = 27\,386 \text{ MN/m}^2$$

Taivutusjäykkyys

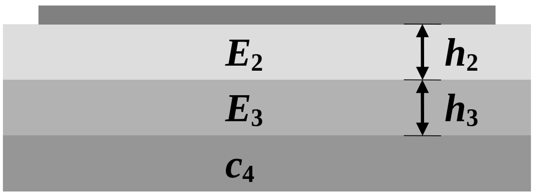
$$D = EI \qquad (25)$$

$$= 2{,}282 \text{ MNm}^2$$

Alustaluku (kuva 4; BY 31, Betonilattiat, s. 39)

$$c = \frac{1}{\dfrac{h_2}{E_2}+\dfrac{h_3}{E_3}+\dfrac{1}{c_4}} \qquad (26)$$

$$= 4{,}724 \text{ MN/m}^3$$



Kuva 4.

Kerroinmatriisi

$$[K] = \begin{bmatrix} 11,438 & 2,961 & 0,365 & 0,000 & 0,000 \\ 0,000 & 0,365 & 2,961 & 29,893 & 25,837 \\ 0,000 & 0,000 & 0,365 & 2,961 & 11,438 \\ 5,906 & 11,811 & 11,811 & 11,811 & 5,906 \\ 10,335 & 11,811 & 0,000 & -11,811 & -10,335 \end{bmatrix}$$

Kerroinmatriisin käänteismatriisi

$$[K]^{-1} = \begin{bmatrix} 0,112 & -0,010 & -0,001 & -0,001 & -0,027 \\ -0,102 & 0,051 & -0,003 & -0,009 & 0,118 \\ 0,049 & -0,091 & 0,049 & 0,104 & -0,114 \\ -0,003 & 0,051 & -0,102 & -0,009 & 0,009 \\ -0,001 & -0,010 & 0,112 & -0,001 & 0,001 \end{bmatrix}$$

Voimavektorin alkiot (alaindeksi viittaa järjestykseen yhtälöryhmässä)

F_a = 0,000 MNm/m

F_b = 0,000 MNm/m

F_c = 0,000 MNm/m

F_d = 0,075 MN/m

F_e = 0,050 MN/m

Siirtymävektori saadaan kerroinmatriisin käänteismatriisin ja voimavektorin tulona

$$\{w\} = [K]^{-1}\{f\} \tag{27}$$

Siirtymävektorin alkiot

w_1 = -0,001413 m
w_2 = 0,005194 m
w_3 = 0,002139 m
w_4 = -0,000279 m
w_5 = 0,000004 m

Leikkausvoima

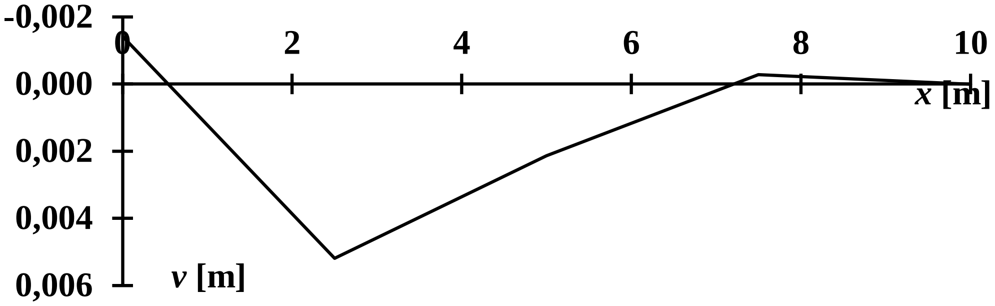
$$Q_i = \frac{M_{i+1} - M_i}{a} \tag{28}$$

<u>VASTAUS</u>:

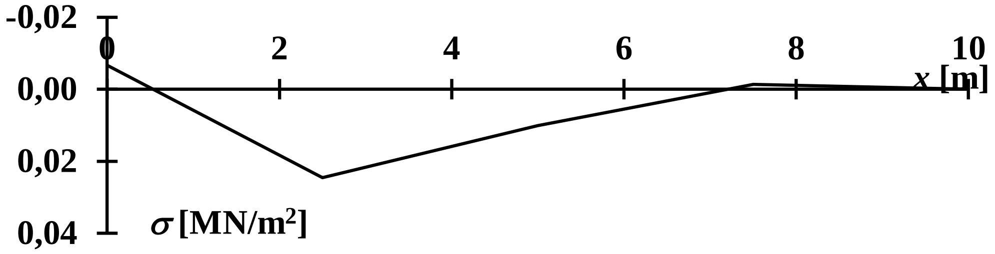
Laatan taipuma-, pohjapaine-, taivutusmomentti- ja leikkausvoimajakauma on esitetty taulukossa 1 sekä kuvissa 5…8.

Taulukko 1.

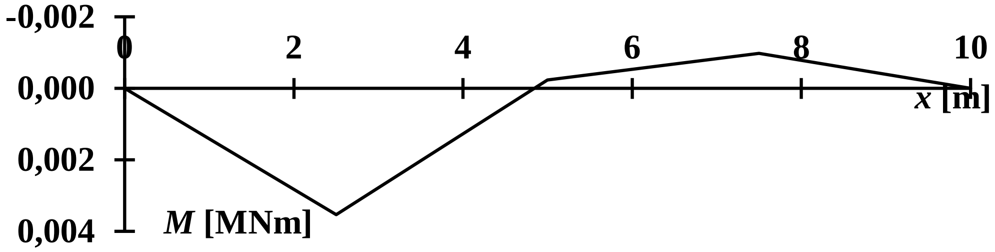
x_i [m]	w_i [m]	σ_i (2) [MN/m²]	R_i (3) [MN]	M_i (15) [MNm]	Q_i (28) [MN]
0,000	-0,00141	-0,00667	0,00141	0,00000	0,00141
2,500	0,00519	0,02454	0,04708	0,00353	0,00141
2,500	0,00519	0,02454	0,04708	0,00353	-0,00150
5,000	0,00214	0,01010	0,02620	-0,00023	-0,00150
5,000	0,00214	0,01010	0,02620	-0,00023	-0,00030
7,500	-0,00028	-0,00132	0,00070	-0,00099	-0,00030
7,500	-0,00028	-0,00132	0,00070	-0,00099	0,00039
10,000	0,00000	0,00002	-0,00039	0,00000	0,00039
10,000		0		0	0



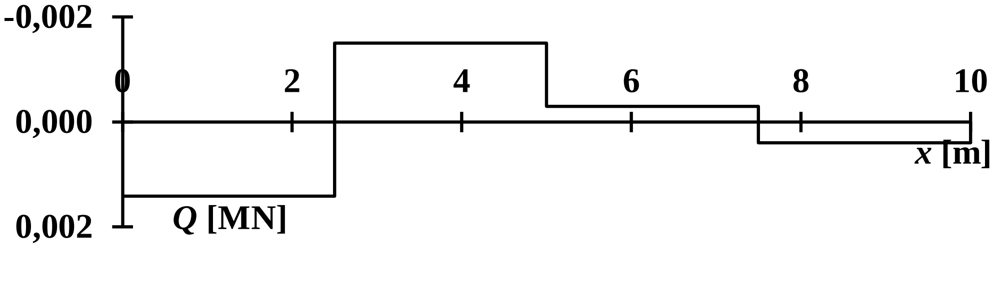
Kuva 5. Taipumakuvaaja.



Kuva 6. Pohjapainekuvaaja.



Kuva 7. Taivutusmomenttikuvaaja.



Kuva 8. Leikkausvoimakuvaaja.

39. TEHTÄVÄ

Rakennuksen lämpötilan muutoksesta johtuen sokkelimuurin (kuva 1) yläreuna siirtyy sivusuuntaan matkan

δ = 0,020 m

Määritä pohjapaineen jakautuma jäykän peruslaatan alla maan alustaluvun funktiona, kun alustaluku vaihtelee välillä

c_{min} = 10 MN/m^3

c_{max} = 300 MN/m^3

ja kun alustaluku on

c_{∞} = ∞ MN/m^3

Pystyvoima

F_x = 0,300 MN/m

Muurin kimmokerroin

E = 15 000 MN/m^2

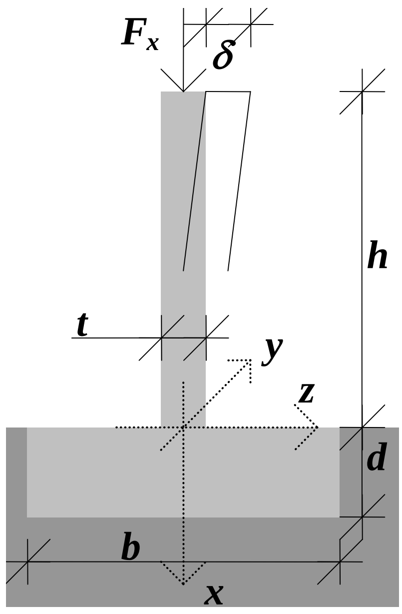
Mitta

d = 0,500 m

h = 3,000 m

b = 1,000 m

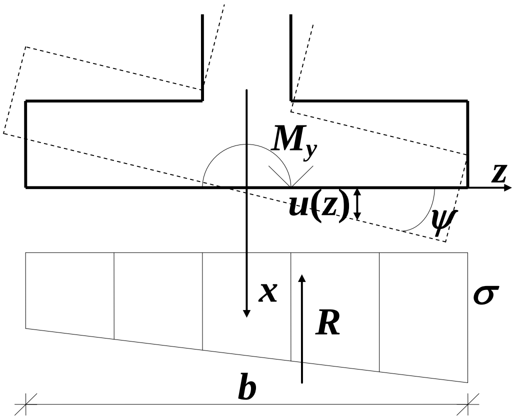
t = 0,200 m



Kuva 1.

Pohjapaineen resultantti (kuva 2)

$$R = \int \sigma(z)dz \tag{1}$$



Kuva 2.

Pohjapaine

$$\sigma(z) = cu(z) \tag{2}$$

Peruslaatan painuma

$$u(z) = \psi z \tag{3}$$

jossa ψ on laatan kiertymäkulma.

Momentti y -akselin ympäri

$$M_y = \int \sigma(z)zdz \tag{4}$$

$$= c\psi \int z^2 dz \tag{5}$$

Laatan (l) pohjan jäyhyysmomentti y -akselin suhteen

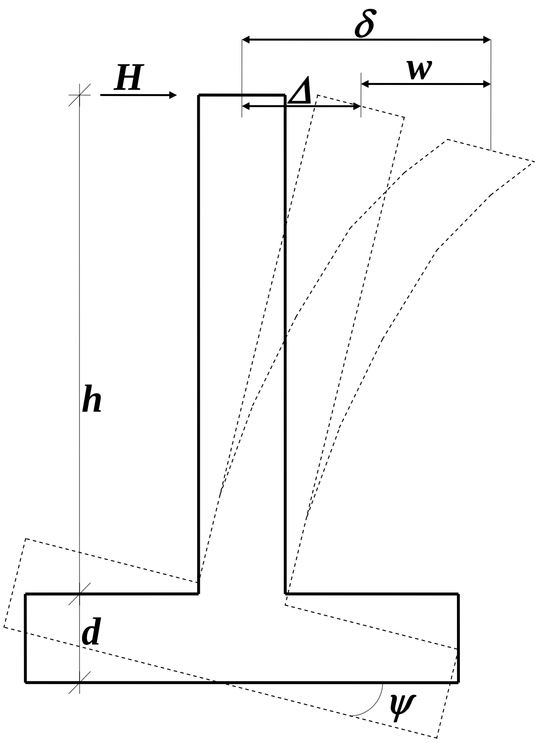
$$I_{y,l} = \int z^2 dz \tag{6}$$

Täten momentti

$$M_y(c) = c\,\psi I_{y,l} \tag{7}$$

Toisaalta momentti oletetun vaakavoiman H funktiona (kuva 3)

$$M_y = H(h+d) \tag{8}$$



Kuva 3.

Momentin lausekkeista kiertymäkulmaksi seuraa

$$\psi = \frac{H(h+d)}{cI_{y,l}} \tag{9}$$

Kiertymäkulmaa vastaava siirtymä muurin yläpäässä

$$\Delta = (h+d)\psi \tag{10}$$

$$= \frac{H(h+d)^2}{cI_{y,l}} \tag{11}$$

Oletetun vaakavoiman aiheuttama taipuma muurin (m) yläpäässä

$$w = \frac{Hh^3}{3D_{y,m}} \tag{12}$$

Horisontaalinen kokonaissiirtymä muurin yläpäässä

$$\delta(c) = \Delta + w \tag{13}$$

$$= \frac{H(h+d)^2}{cI_{y,l}} + \frac{Hh^3}{3D_{y,m}} \tag{14}$$

Horisontaalivoima

$$H(c) = \frac{\delta}{\dfrac{(h+d)^2}{cI_{y,l}} + \dfrac{h^3}{3D_{y,m}}} \tag{15}$$

Pohjapaine

$$\sigma(z) = \frac{F_x}{A_l} + \frac{M_y}{I_{y,l}} z \tag{16}$$

$$= \frac{F_x}{A_l} + \frac{H(h+d)}{I_{y,l}} z \tag{17}$$

$$\Rightarrow \sigma(c,z) = \frac{F_x}{A_l} + \frac{\delta(h+d)}{I_{y,l}\left(\dfrac{(h+d)^2}{cI_{y,l}} + \dfrac{h^3}{3EI_{y,m}}\right)} z \tag{18}$$

$$= \frac{F_x}{A_l} + \frac{\delta(h+d)}{\dfrac{(h+d)^2}{c} + \dfrac{h^3 I_{y,l}}{3EI_{y,m}}} z \tag{19}$$

Peruslaatan pohjan ala pituusyksikköä kohti

$$A_l = b \tag{20}$$

Laatan ja muurin jäyhyysmomentit y -akselin suhteen pituusyksikköä kohti

$$I_{y,l} = \frac{b^3}{12} \tag{21}$$

$$I_{y,m} = \frac{t^3}{12} \tag{22}$$

Pohjapaine

$$\sigma(c,z)=\frac{F_x}{b}+\frac{\delta(h+d)}{\frac{(h+d)^2}{c}+\frac{h^3b^3}{3Et^3}}z \tag{23}$$

Kun alustaluku c lähenee ääretöntä

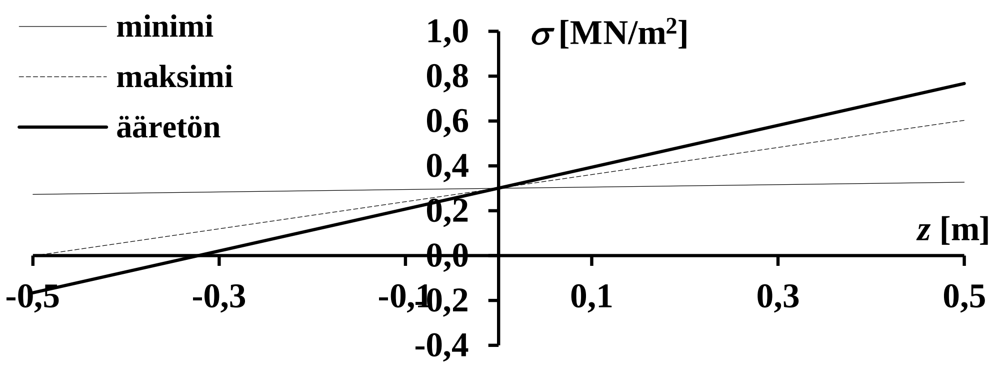
$$\sigma(z)=\frac{F_x}{b}+\frac{3Et^3\delta(h+d)}{h^3b^3}z \tag{24}$$

<u>VASTAUS</u>:

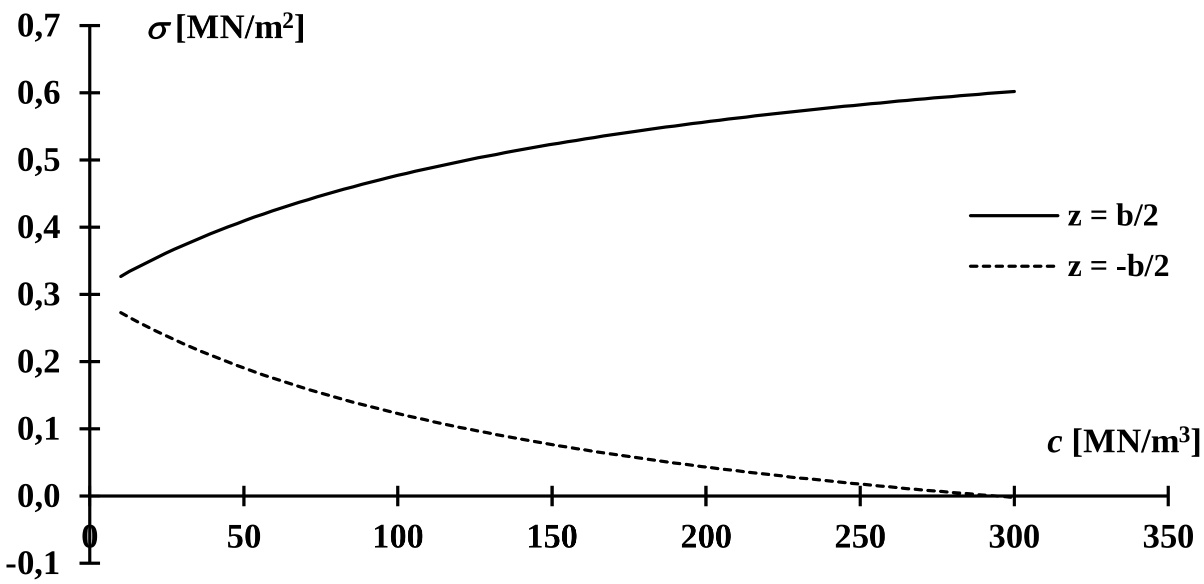
Pohjapaineen ääriarvot on esitetty taulukossa 1. Pohjapaineet kysytyillä alustaluvun arvoilla z-koordinaatin funktiona on esitetty kuvassa 4. Pohjapaineet laatan reunojen kohdalla alustaluvun funktiona on esitetty kuvassa 5.

Taulukko 1. Pohjapaineen ääriarvot.

		σ_{min} $z = -b/2$ [MN/m^2]	σ_{max} $z = b/2$ [MN/m^2]
c_{min} =	10 MN/m^3	0,273	0,327
c_{max} =	300 MN/m^3	-0,002	0,602
c_{∞} =	∞ MN/m^3	-0,167	0,767



Kuva 4.

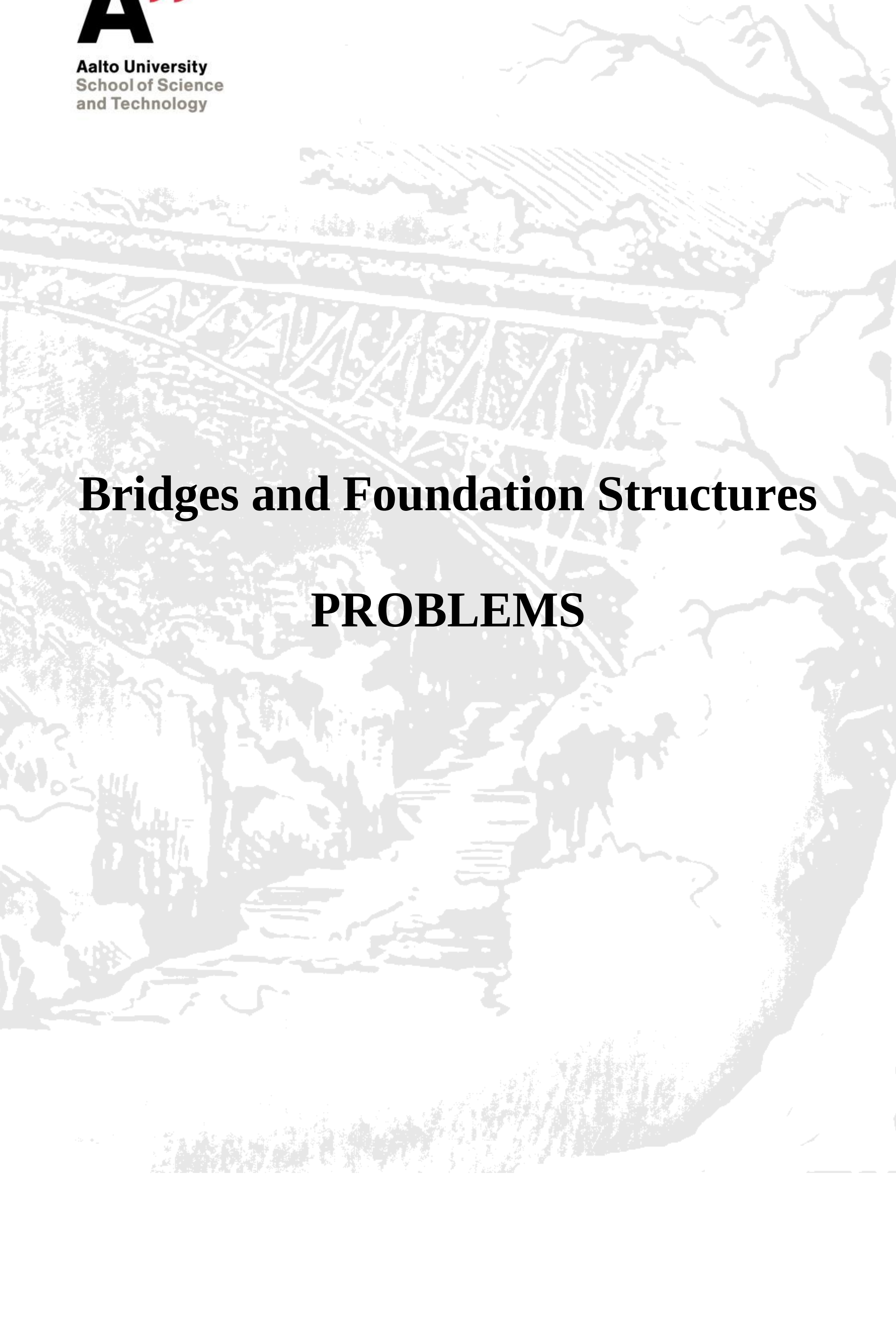


Kuva 5. Pohjapaineet laatan reunojen kohdalla alustaluvun funktiona.

Rak-11.2107 Sillat ja Perustukset

PROBLEMS

Aalto University
School of Science
and Technology



Bridges and Foundation Structures

PROBLEMS

Authors	**Jutila, A., Syrjä, R.**
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Place	**Espoo, Finland**
Year	**2011**

FOREWORD

This educational material includes problems of Bridges and Foundation Structures -course (Rak-11.2107). The course is part of the degree programme of structural engineering and building technology in Aalto University.

The problems are grouped to the subject matters. Each problem has a description of the problem and the model solution.

This material has prepared since the end of 1990 decade. Two design practices, Eurocode and Finnish standard, so-called B-set preceding Eurocodes, are represented on the material.

This material is also published in Finnish (*Sillat ja perustukset - Esimerkkitehtävät*).

Otaniemi, 20th of December 2011
Authors

CONTENTS

1. PROBLEM

Determine the horizontal component of the support reaction force of the crane frame (Figure 1) by using force, slope-deflection and moment methods, respectively!

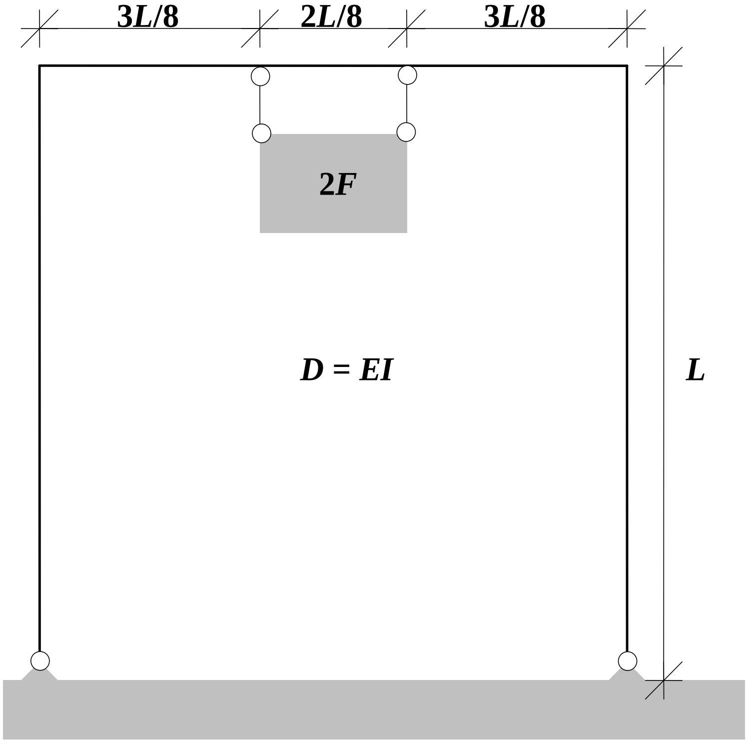


Figure 1.

FORCE METHOD

At the statically determinate basic form the sum of the deflection (δ_{10}) from the external load ($2F$) and the deflection ($X_1\delta_{11}$) from the unknown horizontal force (X_1) is zero. Here δ_{11} is the displacement from force 1. See Figure 2.

$$\delta_{10}+X_1\delta_{11} = 0 \tag{1}$$

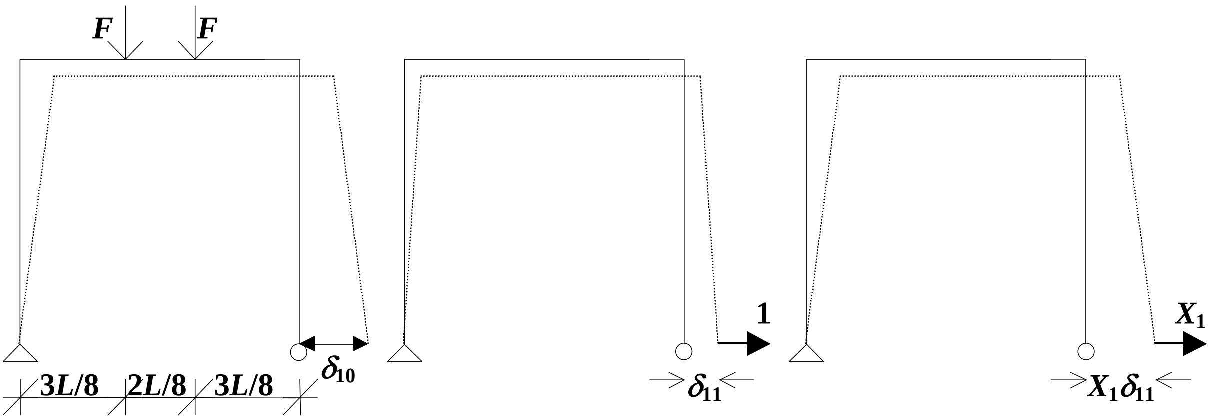


Figure 2.

Bending moment distribution curve due to external load is shown in Figure 3a and due to force 1 in Figure 3b.

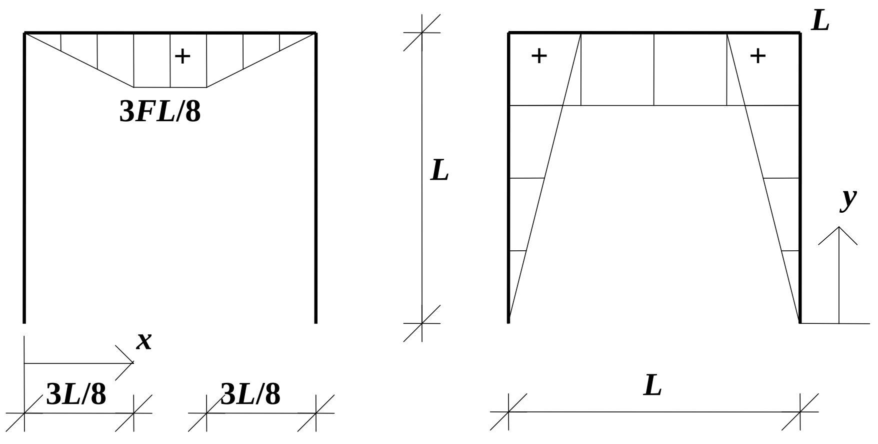
Bending moment in the horizontal beam due to external load

$$M_0 = \begin{cases} Fx, & x = \left[0,\ \dfrac{3L}{8}\right] \\ \dfrac{3FL}{8}, & x = \left[\dfrac{3L}{8},\ \dfrac{5L}{8}\right] \\ F(L-x), & x = \left[\dfrac{5L}{8},\ L\right] \end{cases} \tag{2a...c}$$

There is no moment at the vertical columns.

Bending moment in the horizontal beam due to force $F = 1$ (Fig. 3b)

$$M_1 = \begin{cases} L, & \text{palkki} \\ y, & \text{pilari} \end{cases} \tag{3a, b}$$



a) b)

Figure 3.

Displacements

$$\begin{cases} \delta_{10} = \int \dfrac{M_1 M_0}{D} ds \\ \delta_{11} = \int \dfrac{M_1 M_1}{D} ds \end{cases} \tag{4a, b}$$

When inserting the bending moments we get

$$\begin{cases} \delta_{10} = \dfrac{15FL^3}{64D} \\ \delta_{11} = \dfrac{5L^3}{3D} \end{cases} \tag{5a, b}$$

<u>ANSWER</u>:

Horizontal force

$$X_1 = \frac{-\delta_{10}}{\delta_{11}} \tag{6}$$

$$= \frac{-9F}{64} \tag{7}$$

Negative sign means that the force direction is opposite to the supposed original one.

DISPLACEMENT METHOD (especially slope-deflection method)

Corner moments (Figure 4)

$$\begin{cases} M_{12} = 0 \\ M_{21} = a_{21}^{o}\phi_{21} - c_{21}^{o}\psi_{21} + M_{K21}^{o} \\ M_{23} = a_{23}\phi_{23} + b_{23}\phi_{32} - c_{23}\psi_{23} + M_{K23} \\ M_{32} = a_{32}\phi_{32} + b_{32}\phi_{23} - c_{32}\psi_{32} + M_{K32} \\ M_{34} = a_{34}^{o}\phi_{34} - c_{34}^{o}\psi_{34} + M_{K34}^{o} \\ M_{43} = 0 \end{cases} \qquad (8a\ldots f)$$

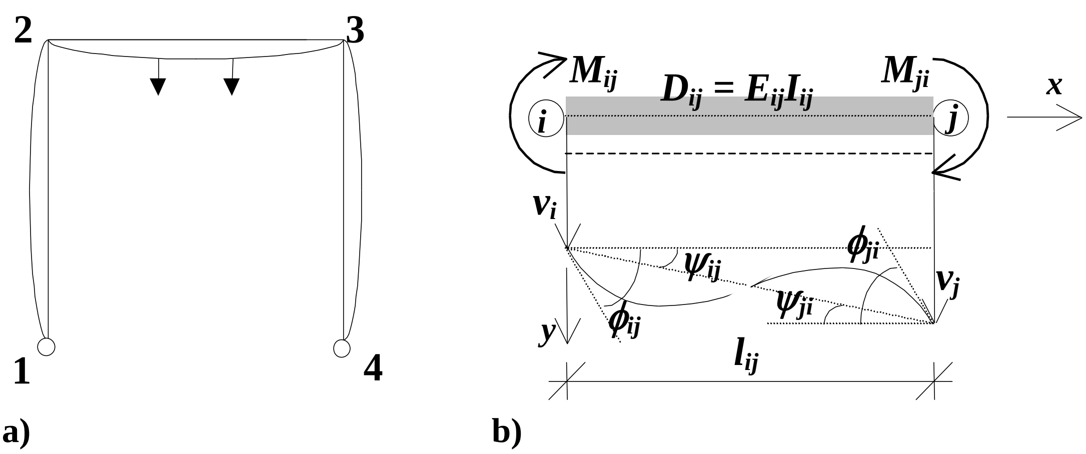


Figure 4.

Compatibility conditions

$$\phi_{21} = \phi_{23} = \phi_2 \qquad (9)$$

$$\phi_{32} = \phi_{34} = \phi_3 = -\phi_2 \qquad (10)$$

Equilibrium conditions

$$\begin{cases} 0 = M_{21} + M_{23} \Rightarrow M_{23} = -M_{21} = M_2 = M \\ 0 = M_{32} + M_{34} \Rightarrow M_{34} = -M_{32} = M_3 = M_2 = M \end{cases} \qquad (11a, b)$$

Slope rotation in basic form (*Formulary*)

$$\psi = \frac{v_2 - v_1}{l} \tag{12}$$

Member coefficients in basic form

$$a = \frac{4EI}{l} = \frac{4D}{l} \tag{13}$$

$$b = \frac{2EI}{l} = \frac{2D}{l} \tag{14}$$

$$c = \frac{6EI}{l} = \frac{6D}{l} \tag{15}$$

$$a^o = c^o = \frac{3EI}{l} = \frac{3D}{l} \tag{16}$$

Corner moments caused by loading (Figure 5) in basic form

$$M_{Ki} = -M_{Kj} = \frac{-Fa_1b_1^2}{l^2} + \frac{-Fa_2b_2^2}{l^2} \tag{17}$$

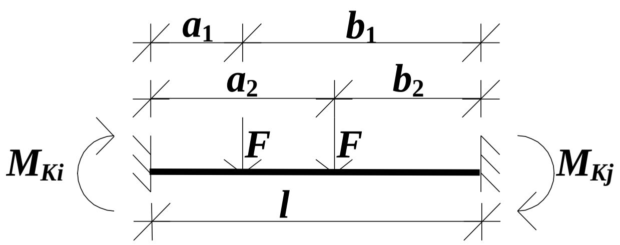


Figure 5.

Slope rotations in this case

$$\psi_{21} = \psi_{34} = \psi \tag{18}$$

$$\psi_{23} = \psi_{32} = 0 \tag{19}$$

Member coefficients in this case

$$a_{21} = a_{23} = a_{32} = a_{34} = \frac{4EI}{L} = \frac{4D}{L} = a \qquad (20)$$

$$b_{23} = b_{32} = \frac{2EI}{L} = \frac{2D}{L} = b \qquad (21)$$

$$c_{23} = c_{32} = \frac{6EI}{L} = \frac{6D}{L} = c \qquad (22)$$

$$a_{21}^{o} = a_{34}^{o} = c_{21}^{o} = c_{34}^{o} = \frac{3EI}{L} = \frac{3D}{L} = a^{o} \qquad (23)$$

Corner moments caused by loading in this case

$$M_{K23} = -M_{K32} = \frac{-15FL}{64} \qquad (24)$$

Corner moments after simplification

$$\begin{cases} -M_2 = a^o\,\phi - a^o\,\psi \\ M_2 = a\,\phi - b\,\phi + M_K \\ -M_3 = -a\,\phi + b\,\phi - M_K \\ M_3 = -a^o\,\phi - a^o\,\psi \end{cases} \qquad (25a\ldots d)$$

From Equations 11

$$\begin{cases} -M_K = +(a^o + a - b)\phi - a^o\psi \\ +M_K = -(a^o + a - b)\phi - a^o\psi \end{cases} \qquad (26a,\ b)$$

Slope

$$\psi = 0 \qquad (27)$$

From Equations 26a and 27

$$\phi = \frac{-M_K}{a^o + a - b} \qquad (28)$$

$$\Rightarrow \phi = \frac{3FL^2}{64D} \qquad (29)$$

From Equation 25a

$$M = -a^o \phi \tag{30}$$

$$\Rightarrow M = \frac{-9FL}{64} \tag{31}$$

Horizontal force at the support (Figure 6)

$$HL = -M_{34} \tag{32}$$

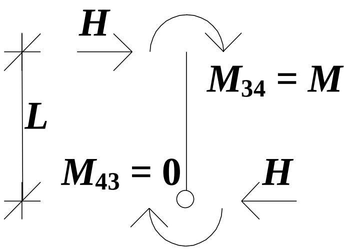


Figure 6.

<u>ANSWER</u>: Horizontal force

$$H = \frac{9F}{64} \tag{33}$$

FORCE METHOD (especially moment method)

Rotations at corners (Figure 4)

$$\begin{cases} \phi_{21} = \alpha_{21} M_{21} - \beta_{21} M_{12} + \psi_{21} + \alpha_{21}^{o} \\ \phi_{23} = \alpha_{23} M_{23} - \beta_{23} M_{32} + \psi_{23} + \alpha_{23}^{o} \\ \phi_{32} = \alpha_{32} M_{32} - \beta_{32} M_{23} + \psi_{32} + \alpha_{32}^{o} \\ \phi_{34} = \alpha_{34} M_{34} - \beta_{34} M_{43} + \psi_{34} + \alpha_{34}^{o} \end{cases} \quad \text{(34a…d)}$$

Compatibility conditions

$$\phi_{21} = \phi_{23} \quad \text{(35)}$$
$$\phi_{32} = \phi_{34} \quad \text{(36)}$$

Corner moments

$$\begin{cases} 0 = M_{21} + M_{23} \Rightarrow M_{23} = -M_{21} = M_2 = M \\ 0 = M_{32} + M_{34} \Rightarrow M_{34} = -M_{32} = M_3 = M_2 = M \\ 0 = M_{12} \\ 0 = M_{43} \end{cases} \quad \text{(37a…d)}$$

Slope rotation in basic form

$$\psi = \frac{v_2 - v_1}{l} \quad \text{(38)}$$

Member coefficients in basic form

$$\alpha = \frac{l}{3EI} = \frac{l}{3D} \quad \text{(39)}$$

$$\beta = \frac{l}{6EI} = \frac{l}{6D} \quad \text{(40)}$$

Rotations caused by loading (Figure 7) in basic form

$$\alpha_i^o = \frac{Fa_1b_1}{6Dl}(b_1 + l) + \frac{Fa_2b_2}{6Dl}(b_2 + l) \tag{41}$$

$$\alpha_j^o = \frac{-Fa_1b_1}{6Dl}(a_1 + l) + \frac{-Fa_2b_2}{6Dl}(a_2 + l) \tag{42}$$

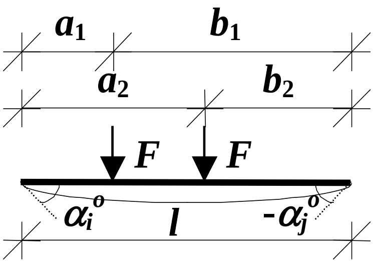


Figure 7.

Slope rotations in this case

$$\psi_{21} = \psi_{34} = \psi \tag{43}$$

$$\psi_{23} = \psi_{32} = 0 \tag{44}$$

Member coefficients in this case

$$\alpha_{21} = \alpha_{23} = \alpha_{32} = \alpha_{34} = \frac{L}{3D} = \alpha \tag{45}$$

$$\beta_{21} = \beta_{23} = \beta_{32} = \beta_{34} = \frac{L}{6D} = \beta \tag{46}$$

Rotations caused by loading in this case

$$\alpha_{21}^o = \alpha_{34}^o = 0 \tag{47}$$

$$\alpha_{23}^o = -\alpha_{32}^o = \frac{15FL^2}{128D} = \alpha^o \tag{48}$$

Rotations after simplification

$$\begin{cases} \phi_{21} = -\alpha M + \psi \\ \phi_{23} = (\alpha + \beta) M + \alpha^o \\ \phi_{32} = -(\alpha + \beta) M - \alpha^o \\ \phi_{34} = \alpha M + \psi \end{cases} \qquad (49a\ldots d)$$

From Equations 35 and 36

$$+\alpha^o = -(2\alpha + \beta) M + \psi \qquad (50)$$

$$-\alpha^o = +(2\alpha + \beta) M + \psi \qquad (51)$$

Slope

$$\psi = 0 \qquad (52)$$

From Equations 50 and 52

$$M = \frac{-\alpha^o}{2\alpha + \beta} \qquad (53)$$

When inserting member coefficients we get

$$M = \frac{-9FL}{64} \qquad (54)$$

Horizontal force at the support (Figure 6)

$$HL = -M_{34} \qquad (55)$$

<u>ANSWER</u>: Horizontal force

$$H = \frac{9F}{64} \qquad (56)$$

2. PROBLEM

A four-storey, steel-framed residential building with flat roof is shown in Figure 1. Determine all load combinations acting on the base slab of the outside column (grey colour)! Use ultimate limit state (STR/GEO) according to Eurocode and National Annex of Finland [*NA SFS-EN 1990* , Table A1.2(B)].
What is the maximum overturning moment at the top of the base slab, when wind is acting in the direction of the shorter side? The wind load is assumed to divide equally to all piles. The building is situated in Kemijärvi (Lapland).

Number of columns

n_1 = 3

n_2 = 5

Dimension

a = 5 m

Number of stories and storey height

n_k = 4

h_i = 3 m

The vertical forces are caused by the are shown by the dotted lines.
Self weight of roof and floors

p = 0,005 MN/m^2

Self weight of the main wall and column

v = 0,020 MN/m

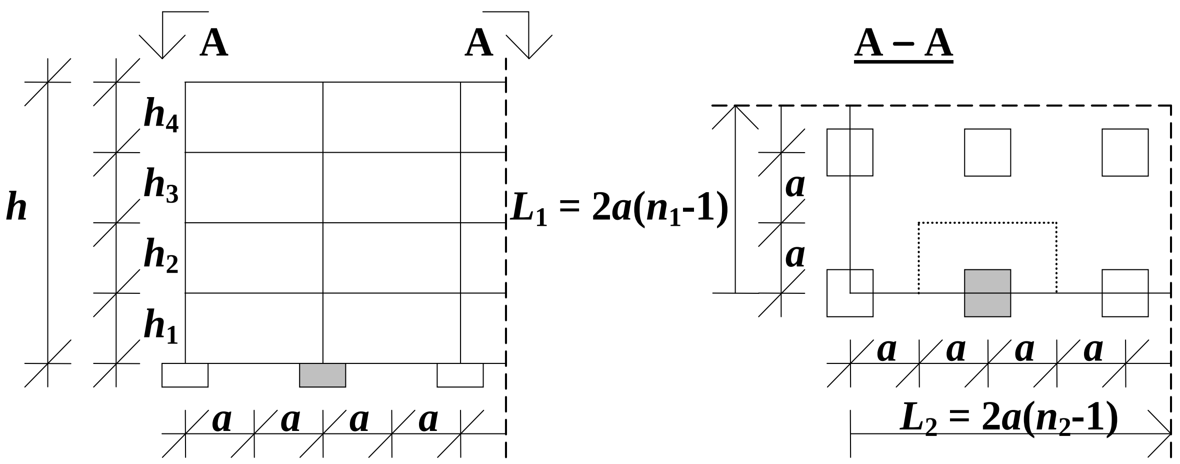


Figure 1.

COMBINATIONS OF ACTIONS

Design equation (*NA* SFS-EN 1990*, Table A1.2(B))

$$F_d = \left.\begin{matrix}\xi\gamma_{Gj,\sup} \\ \gamma_{Gj,\sup}\end{matrix}\right\} K_{FI} G_{kj,\sup} + \gamma_{Gj,\inf} G_{kj,\inf} + \left.\begin{matrix}\gamma_{Q1} \\ 0\end{matrix}\right\} K_{FI} Q_{k1} + \left.\begin{matrix}\gamma_{Qi} \\ 0\end{matrix}\right\} K_{FI} \sum \psi_{0i} Q_{ki} \tag{1}$$

where $G_{kj,\sup}$ is upper characteristic value of permanent action j and corresponding partial factor is

$\gamma_{Gj,\sup} =$ **1,35**

$G_{kj,\inf}$ is lower characteristic value of permanent action j and corresponding partial factor is

$\gamma_{Gj,\inf} =$ **0,9**

Q_{k1} is characteristic value of the leading variable action 1 and corresponding partial factor is

$\gamma_{Q1} =$ **1,5**

Q_{ki} is characteristic value of accompanying variable action i and corresponding partial factor is

$\gamma_{Qi} =$ **1,5**

reduction factor is

$\xi =$ **0,85**

Four-storey building belongs to reliability class RC2 (*NA SIS-EN 1990*, Table B1), thus factor of actions (*SFS-EN 1990*, Table B3)

$K_{FI} =$ **1,0**

Factor for combination value of a variable actions: imposed, snow and wind loads (*NA SFS-EN 1990*, Table A1.1, Class A)

$\psi_{0,imposed} =$ **0,7**

$\psi_{0,snow} =$ **0,7**

$\psi_{0,wind} =$ **0,6**

Products

$\xi\gamma_{Gj,\sup} \approx$ **1,15**

$\gamma_{Qi} K_{FI} \psi_{0,imposed} =$ **1,05**

$\gamma_{Qi} K_{FI} \psi_{0,snow} =$ **1,05**

$\gamma_{Qi} K_{FI} \psi_{0,wind} =$ **0,90**

HORIZONTAL FORCE DUE TO SWAY IMPERFECTIONS

Basic value of sway imperfection (*SFS-EN 1993-1-1* , Chapter 5.3.2)

$$\phi_0 = 0{,}005$$

Reduction factor for height *h* applicable to columns

$$\alpha_h = \min\begin{cases} \max\begin{cases} \dfrac{2}{\sqrt{h_0 h}} \\ \dfrac{2}{3} \end{cases} \\ 1 \end{cases} \tag{2}$$

where

$$h_0 = 1 \text{ m}^{-1}$$

It is obtained

$$\alpha_h = 0{,}667$$

Number of columns in a row

$$m = n_i - 1 \tag{3}$$
$$= 2$$

Reduction factor for the number of column in a row

$$\alpha_m = \sqrt{\frac{1}{2}\left(1+\frac{1}{m}\right)} \tag{4}$$
$$= 0{,}866$$

Global initial sway imperfection (*SFS-EN 1993-1-1* , Equation 5.5)

$$\phi = \phi_0 \alpha_h \alpha_m \tag{5}$$
$$= 0{,}002887$$

Horizontal force is acting in the level of storey floor and is

$$H = \phi N \tag{6}$$

where *N* is the corresponding axial force (*SFS-EN 1993-1-1* , Figure 5.4).

SELF WEIGHT OF STRUCTURE

Roof or floor

$$G_{pi} = 2a^2 p \tag{7}$$

$= 0{,}250$ MN

Wall and column

$$G_{vi} = 2a\,v \tag{8}$$

$= 0{,}200$ MN

Characteristic value of vertical force at one floor

$$G_i = G_{pi} + G_{vi} \tag{9}$$

$= 0{,}450$ MN

Height of the building

$$h = n_k h_i \tag{10}$$

$= 12{,}000$ m

Moment due to self weigh of roof and sway imperfection

$$M_{G1} = \phi G_{pi} h$$

$= 0{,}009$ MNm (11)

Moment due to self weigh of three upper floors and sway imperfection

$$M_{G2} = 3\phi G_i h/2 \tag{12}$$

$= 0{,}023$ MNm

Sum of moments

$$M_G = M_{G1} + M_{G2} \tag{13}$$

$= 0{,}032$ MNm

IMPOSED LOAD

Building belongs to category A (*NA SFS-EN 1990*, Table A1.1).
Load per unit area (*NA SFS-EN 1991-1-1*, Table 6.2)

$q_{ki} = 0{,}002$ MN/m^2

Characteristic value of vertical force

$$Q_{ii} = 2a^2 q_{ki} \tag{14}$$

$= 0{,}100$ MN

Number of stories above the second floor

$$n = n_k - 1 \tag{15}$$
$$= 3$$

Reduction factor (*NA SFS-EN 1991-1-1*, Chapter 6.3.1.2(11))

$$\alpha_n = \frac{2+(n-2)\psi_{0,imposed}}{n} \tag{16}$$
$$= 0{,}900$$

Moment due to imposed floor loads and sway imperfection

$$M_i = \phi(\alpha_n h_i + 2h_i + 3h_i)Q_{ii} \tag{17}$$
$$= 0{,}005 \text{ MNm}$$

SNOW LOAD

Characteristic value of snow load on the ground in Kemijärvi (*NA SFS-EN 1991-1-3*, Figure 4.1)

$$s_k = 0{,}002750 \text{ MN/m}^2$$

Snow load shape coefficient for the flat roof (*SFS-EN 1991-1-3*, Table 5.2)

$$\mu_i = 0{,}8$$

Exposure coefficient C_e (normal topography) and thermal coefficient C_t (*SFS-EN 1991-1-3*, Chapter 5.2(7) and 5.2(8))

$$C_e = 1{,}0$$
$$C_t = 1{,}0$$

Snow load (*SFS-EN 1991-1-3*, Equation 5.1)

$$s = \mu_i C_e C_t s_k \tag{18}$$
$$= 0{,}002200 \text{ MN/m}^2$$

Characteristic vertical force

$$Q_{ks} = 2a^2 s \tag{19}$$
$$= 0{,}110 \text{ MN}$$

Moment due to snow load and sway imperfection

$$M_s = \phi Q_{ks} h \tag{20}$$
$$= 0{,}004 \text{ MNm}$$

WIND LOAD

Shorter side length

$$L_1 = 2a(n_1 - 1) \tag{21}$$
$$= 20 \text{ m}$$

Longer side length

$$L_2 = 2a(n_2 - 1) \tag{22}$$
$$= 40 \text{ m}$$

Reference area

$$A_{ref} = hL_2 \tag{23}$$
$$= 480 \text{ m}^2$$

Fundamental value of the basic wind velocity (*NA SFS-EN 1994-1-4*, Chapter 4.2)

$$v_{b,0} = 21 \text{ m/s}$$

Directional factor (*SFS-EN 1991-1-4*, Chapter 4.2)

$$c_{dir} = 1{,}0$$

Season factor (*SFS-EN 1991-1-4*, Chapter 4.2)

$$c_{season} = 1{,}0$$

Basic wind velocity (*SFS-EN 1991-1-4*, Equation 4.1)

$$v_b = c_{dir} c_{season} v_{b,0} \tag{24}$$
$$= 21 \text{ m/s}$$

Air density (*SFS-EN 1991-1-4*, Chapter 4.5)

$$\rho = 1{,}250 \text{ kg/m}^3$$

Basic velocity pressure (*SFS-EN 1991-1-4*, Equation 4.10)

$$q_b = \frac{1}{2}\rho v_b^2 \tag{25}$$
$$= 276 \text{ N/m}^2$$

Terrain category is selected to be II (*SFS-EN 1991-1-4*, Appendix A).

Exposure factory at the top of the building (*SFS-EN 1991-1-4*, Figure 4.2)

$$C_{e,wind} = 2{,}5$$

Peak velocity pressure (*SFS-EN 1991-1-4*, Equation 4.8)

$$q_p = C_{e,wind} q_b \tag{26}$$
$$= 689 \text{ N/m}^2$$

Because the height h is less than the shorter side length, the wind pressure should be considered to be one part (*SFS-EN 1991-1-4*, Chapter 7.2.2, Figure 7.4).
Ratio (*SFS-EN 1991-1-4*, Figure 7.5)

$$\kappa = h/d = L_1/h \tag{27}$$
$$= 0{,}600$$

Pressure coefficient for the external pressure for zones D and E, when the loaded area is more than 10 m^2, is obtained by linear interpolation (*SFS-EN 1991-1-4*, Table 7.1)

$$C_{pe,10,D} = \frac{0{,}8-0{,}7}{1-0{,}25}(\kappa-0{,}25)+0{,}7 \tag{28}$$
$$= 0{,}747$$

$$C_{pe,10,E} = -\left[\frac{0{,}5-0{,}3}{1-0{,}25}(\kappa-0{,}25)+0{,}3\right] \tag{29}$$
$$= -0{,}393$$

Wind pressure acting on the external surfaces (*SFS-EN 1991-1-4*, Equation 5.1)

$$w_{e,i} = q_p C_{pe,\,10,\,i} \tag{30}$$
$$w_{e,D} = 515 \text{ N/m}^2$$
$$w_{e,E} = -271 \text{ N/m}^2$$

Structural factor (*SFS-EN 1991-1-4*, Chapter 6.2)

$$c_s c_d = 1$$

Wind force (*SFS-EN 1991-1-4*, Equation 5.5)

$$F_{w,e} = c_s c_d (w_{e,D} - w_{e,E}) A_{ref} \tag{31}$$
$$= 0{,}377 \text{ MN}$$

For one column

$$F_w = \frac{F_{w,e}}{n_1 n_2} \tag{32}$$
$$= 0{,}025 \text{ MN}$$

Moment

$$M_w = F_w h/2 \tag{33}$$
$$= 0{,}151 \text{ MNm}$$

<u>ANSWER</u>:

Combinations of actions

$$F_d = \left.\begin{matrix}1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}15\\1{,}35\end{matrix}\right\} G_{k,\sup} + 0{,}9 G_{k,\inf} + \left.\begin{matrix}1{,}5\\1{,}5\\1{,}5\\1{,}5\\1{,}05\\1{,}05\\0\\0\\1{,}05\\1{,}05\\0\\0\\0\end{matrix}\right\} Q_{ki} + \left.\begin{matrix}1{,}05\\1{,}05\\0\\0\\1{,}5\\1{,}5\\1{,}5\\1{,}5\\1{,}05\\0\\1{,}05\\0\\0\end{matrix}\right\} Q_{ks} + \left.\begin{matrix}0{,}9\\0\\0{,}9\\0\\0{,}9\\0\\0{,}9\\0\\1{,}5\\1{,}5\\1{,}5\\1{,}5\\0\end{matrix}\right\} Q_{kw} \quad \begin{matrix}(34a)\\(34b)\\(34c)\\(34d)\\(34e)\\(34f)\\(34g)\\(34h)\\(34i)\\(34j)\\(34k)\\(34l)\\(34m)\end{matrix}$$

Wind is caused the leading moment effect. Thus the unfavourable case is i.

Maximum moment

$$\begin{aligned} M &= 1{,}15 M_G + 1{,}05 M_i + 1{,}05 M_s + 1{,}5 M_w \\ &= 0{,}272 \text{ MNm} \end{aligned} \quad (35)$$

3. PROBLEM

A four-storey building with flat roof is shown in Figure 1. Determine the extreme values of vertical and horizontal forces acting on the base slab of the outside column (grey colour) - horizontal ones in the direction of the shorter side!
A load combination is determined in ultimate limit state (design of structural member) by using Finnish standard *RIL 144-2002* . The wind load is assumed to divide equally to all piles. The building is situated in Kemijärvi (Lapland).

Number of columns

n_1 = 3

n_2 = 5

Dimension

a = 5 m

Number of stories and storey height

n_k = 4

h_i = 3 m

The vertical forces are caused by the are shown by the dotted lines.
Self weight of roof and floors

p = 0,005 MN/m^2

Self weight of the main wall and column

v = 0,020 MN/m

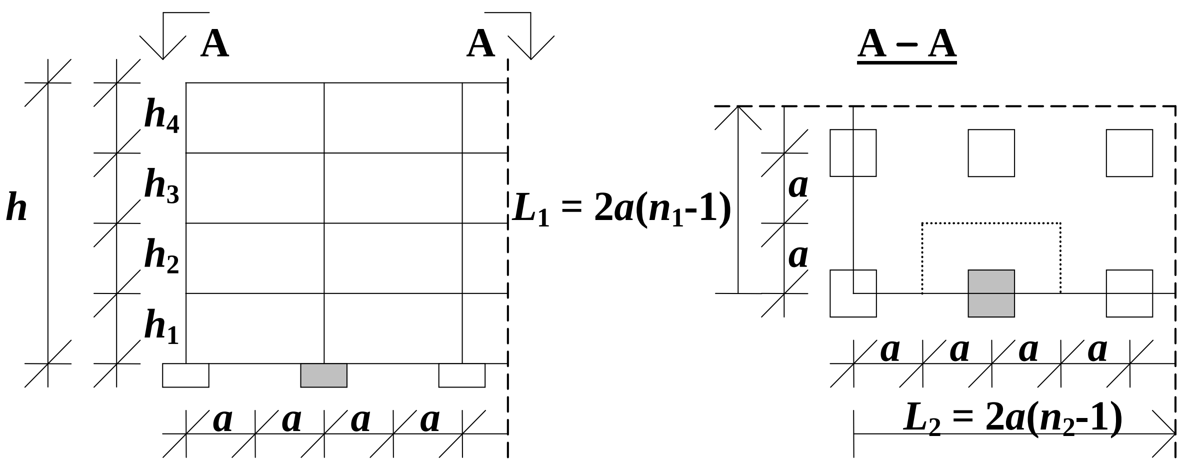


Figure 1.

SELF WEIGHT OF THE STRUCTURE

Roof or floor

$$G_{pi} = 2a^2 p \tag{1}$$
$$= \quad 0{,}250 \text{ MN}$$

Wall and column

$$G_{vi} = 2a\,v \tag{2}$$
$$= \quad 0{,}200 \text{ MN}$$

Characteristic value of vertical force at one floor

$$G_i = G_{pi} + G_{vi} \tag{3}$$
$$= \quad 0{,}450 \text{ MN}$$

Sum of loads

$$G = G_{pi} + \sum_{n=1}^{4} G_i \tag{4}$$
$$= \quad 2{,}050 \text{ MN}$$

SNOW LOAD

Characteristic value of snow load on the ground in Kemijärvi (*RIL 144-2002*, Figure 4.121, p. 24)

$$s_k = \quad 0{,}002 \text{ MN/m}^2$$

Snow load shape coefficient for flat roof (*RIL 144-2002*, Figure 4.122d, p. 27)

$$\mu = \quad 1{,}0$$

Characteristic vertical force (*RIL 144-2002*, Chapter 4.1, p. 23)

$$Q_k = 2a^2 \mu s_k \tag{5}$$
$$= \quad 0{,}100 \text{ MN}$$

IMPOSED LOAD

Load per unit area (*RIL 144-2002*, Table 5.12, p. 78)

$$q_{ik} = 0{,}001500 \ \mathrm{MN/m^2}$$

Characteristic value of vertical force

$$Q_{ii} = 2a^2 q_{ik} \qquad (6)$$

$$= 0{,}075 \ \mathrm{MN}$$

Reduction factor (*RIL 144-2002*, Table 8.21b, p. 150)

$$k = 0{,}75$$

Characteristic value of vertical force

$$Q_i = k\sum_{i=1}^{4} Q_{ii} \qquad (7)$$

$$= 0{,}225 \ \mathrm{MN}$$

HORIZONTAL FORCE DUE TO SWAY IMPERFECTIONS

Shorter side length

$$L_1 = 2a(n_1 - 1) \qquad (8)$$

$$= 20 \ \mathrm{m}$$

Longer side length

$$L_2 = 2a(n_2 - 1) \qquad (9)$$

$$= 40 \ \mathrm{m}$$

Horizontal force is acting in the level of storey floor and is

$$H = \phi N \qquad (10)$$

where ϕ is in the direction of the shorter side (*RIL 144-2002*, Chapter 6.5, p. 134)

$$\phi = 1/150$$

$$= 0{,}006667$$

and N is the corresponding axial force.

WIND LOAD

Height of the building

$$h = \sum_{n=1}^{4} h_i \tag{11}$$

$= \quad 12 \text{ m}$

Terrain category is selected to be II.

Wind pressure (*RIL 144-2002*, Table 4.22a, p. 31)

$$q_{wk} = q_0 \left(\frac{h}{h_0}\right)^{0,24} \tag{12}$$

where

$q_0 = 0,000650 \text{ MN/m}^2$

$h_0 = 10 \text{ m}$

Thus

$q_{wk} = 0,000679 \text{ MN/m}^2$

Pressure coefficient for the windward external pressure (*RIL 144-2002*, Table 4.231a, p. 38)

$C_{p,t} = 0,7$

Ratio between the side lengths, when wind acting in the direction of the shorter side

$L_1/L_2 = 0,500$

Pressure coefficient for the leeward external pressure

$C_{p,s} = -0,5$

Wind load for one column (*RIL 144-2002*, Equation 4.212b, p. 30)

$$Q_w = \frac{hL_2(C_{p,t} - C_{p,s})q_{wk}}{n_1 n_2} \tag{13}$$

$= 0,026 \text{ MN}$

COMBINATION OF ACTIONS

Design equation (*RIL 144-2002*, Chapter 8.2, p. 149)

$$q_d = \sum_{i=1}^{m} \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} + \sum_{i=3}^{n} \gamma_{qi} q_{ki} \tag{14}$$

can be written

$$q_d = \left.\begin{matrix} 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{matrix}\right\} g + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right\} q_i + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 0{,}8 \\ 0 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0{,}8 \\ 0 \\ 0 \end{matrix}\right\} q_k + \left.\begin{matrix} 0{,}8 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0{,}8 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \end{matrix}\right\} q_w \qquad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \\ (9) \\ (10) \end{matrix} \tag{15a…j}$$

Sub index d refers to design value.
Below the sub indexes are referred to the load combination cases (row number).
(Amount of load cases is double, if $\gamma_{q1} = 0$ when $\gamma_g = 1{,}2$, and $\gamma_{q1} = 1{,}6$ when $\gamma_g = 0{,}9$.)

Horizontal forces

Maximum snow load

$$V_{d,1,2} = 1{,}2G + 1{,}6Q_i + 1{,}6Q_k \tag{16}$$
$$= 2{,}980 \text{ MN}$$

Maximum wind load

$$V_{d,3} = 1{,}2G + 1{,}6Q_i + 0{,}8Q_k \tag{17}$$
$$= 2{,}900 \text{ MN}$$

Minimum self weight

$$V_{d,10} = 0{,}9G \tag{18}$$
$$= 1{,}845 \text{ MN}$$

Horizontal forces and corresponding distances from the base slab

Maximum snow load

$$\pm H_1 = 1{,}2\phi(G_{p5} + 4G_i) + 1{,}6\phi \cdot 4Q_{ii} + 1{,}6\phi Q_k + 0{,}8Q_w \tag{19}$$

= **0,042 MN**

$$e_1 = \frac{1{,}2\phi\left(G_{p5}h + 4G_i \frac{3}{8}h\right) + 1{,}6\phi \cdot 4Q_{ii}\frac{3}{8}h + 1{,}6\phi Q_k h + 0{,}8Q_w \frac{h}{2}}{H_1} \tag{20}$$

= **5,807 m**

Maximum wind load

$$\pm H_3 = 1{,}2\phi(G_{p5} + 4G_i) + 1{,}6\phi \cdot 4Q_{ii} + 0{,}8\phi Q_k + 1{,}6Q_w \tag{21}$$

= **0,062 MN**

$$e_3 = \frac{1{,}2\phi\left(G_{p5}h + 4G_i \frac{3}{8}h\right) + 1{,}6\phi \cdot 4Q_{ii}\frac{3}{8}h + 0{,}8\phi Q_k h + 1{,}6Q_w \frac{h}{2}}{H_3} \tag{22}$$

= **5,819 m**

Minimum loads

H_{10} = **0 MN**

<u>ANSWER</u>:

Vertical forces

$V_{d,max}$ = **2,980 MN**

$V_{d,min}$ = **1,845 MN**

Horizontal forces and corresponding distances from the base slab

$H_{d,max}$ = **0,062 MN**

$e_{d,max}$ = **5,819 m**

$H_{d,min}$ = **0,000 MN**

4. PROBLEM

A massive reinforced concrete dam with a inspection manway is shown in Figure 1. Determine water pressures and buoyancy (uplift force) resultants and corresponding locations in *x*-*y*- coordinate system, when the water deep of the lower pond is

$h_{a,1}$ = 3,000 m

Determine stability against tilting and sliding, when the water deep of the lower pond is

$h_{a,2}$ = 0 m

Use the equations given in Finnish standard *Vesirakenteiden suunnittelu RIL 123-1979* [*Design of Waterway Structures*].

Friction coefficient between rock and concrete (slightly fissured and varved rock, *RIL 123-1979*, p. 64)

μ = 0,700

Water deep of the upper pond

h_y = 9,000 m

Dimensions

h = 9,500 m
h_t = 2,000 m
a = 1,000 m
d = 2,500 m
c = 0,500 m
α = 30 °

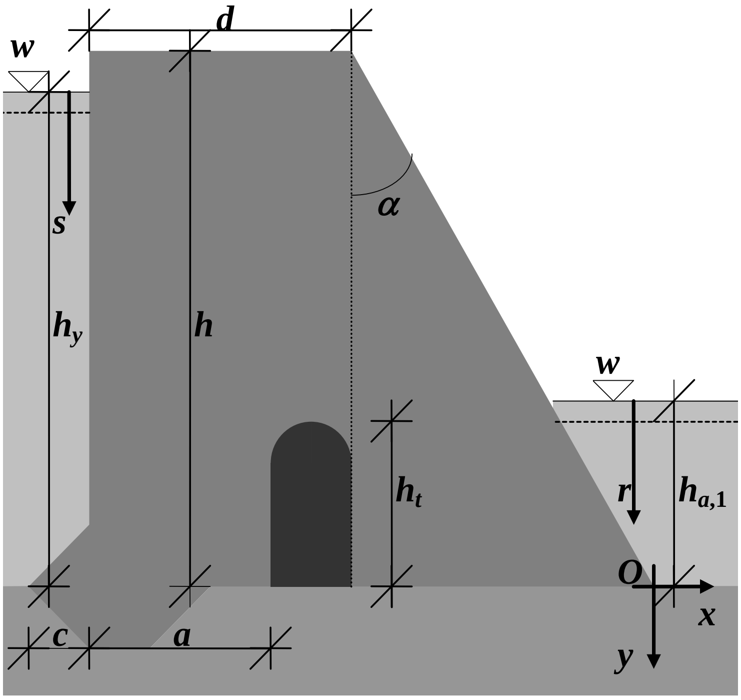


Figure 1.

Unit weight of reinforced concrete

$$\gamma_c = 25\,000 \text{ N/m}^3$$

Unit weight of water

$$\gamma_w = 9\,810 \text{ N/m}^3$$

Width of the dam as function of "deep" t (Figure 2)

$$b(t) = d + \frac{t}{\sqrt{3}} \tag{1}$$

Width at bottom surface

$$b_p = b(t = h) \tag{2}$$

$$= 7{,}985 \text{ m}$$

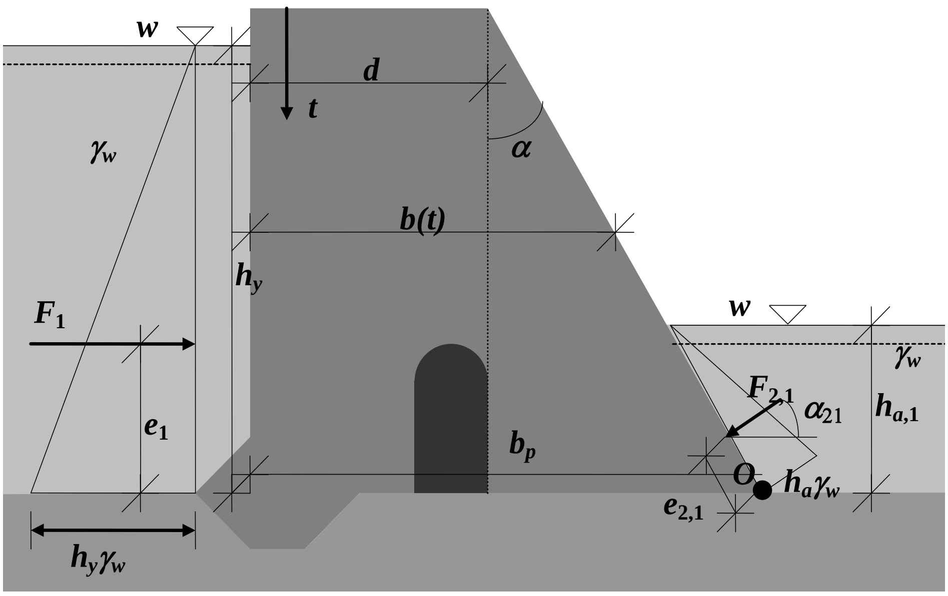


Figure 2.

WATER PRESSURE OF THE UPPER POND

Resultant (Figure 2)

$$F_1 = \frac{1}{2}\gamma_w h_y^2 \tag{3}$$

$$= 397\,305 \text{ N/m}$$

and corresponding distance from the bottom surface

$$e_1 = h_y/3 \tag{4}$$

$$= 3{,}000 \text{ m}$$

WATER PRESSURE OF THE LOWER POND

Horizontal component of the resultant (Figure 2)

$$F_{2,1h} = \gamma_w h_{a,1}^2 \cos\alpha/2 \tag{5}$$

$$= 38\,231 \text{ N/m}$$

and corresponding distance from point *O*

$$e_{2,1h} = h_{a,1}/3 \tag{6}$$

$$= 1{,}000 \text{ m}$$

Vertical component of the resultant

$$F_{2,1v} = \gamma_w h_{a,1}^2 \sin\alpha/2 \tag{7}$$

$$= 22\,073 \text{ N/m}$$

and corresponding distance from point *O*

$$e_{2,1v} = e_{2,1h} \tan\alpha \tag{8}$$

$$= 0{,}577 \text{ m}$$

BUOYANCY AT THE SIDE OF THE UPPER POND

Resultant (*RIL 123-1979.* Chapter 2. Figure 7.) (Figure 3)

$$F_{3,i} = \left[0{,}4\left(h_y - h_{a,i}\right) + \frac{1}{2}\left(h_y - 0{,}4\left(h_y - h_{a,i}\right)\right)\right]\gamma_w\left(a + c\right) \tag{9}$$

$$F_{3,1} = 83\,876 \text{ N/m}$$

$$F_{3,2} = 92\,705 \text{ N/m}$$

Corresponding distance from the surface of upper pond water

$$e_{3,i} = \frac{0{,}4\left(h_y - h_{a,i}\right)\frac{a+c}{2} + \frac{1}{2}\left[h_y - 0{,}4\left(h_y - h_{a,i}\right)\right]\frac{a+c}{3}}{0{,}4\left(h_y - h_{a,i}\right) + \frac{1}{2}\left[h_y - 0{,}4\left(h_y - h_{a,i}\right)\right]} - c \qquad (10)$$

$e_{3,1} =$ 0,105 m

$e_{3,2} =$ 0,143 m

***x*-coordinate**

$x_{3,i} = e_{3,I} - b_p$ **(11)**

$x_{3,1} =$ -7,880 m

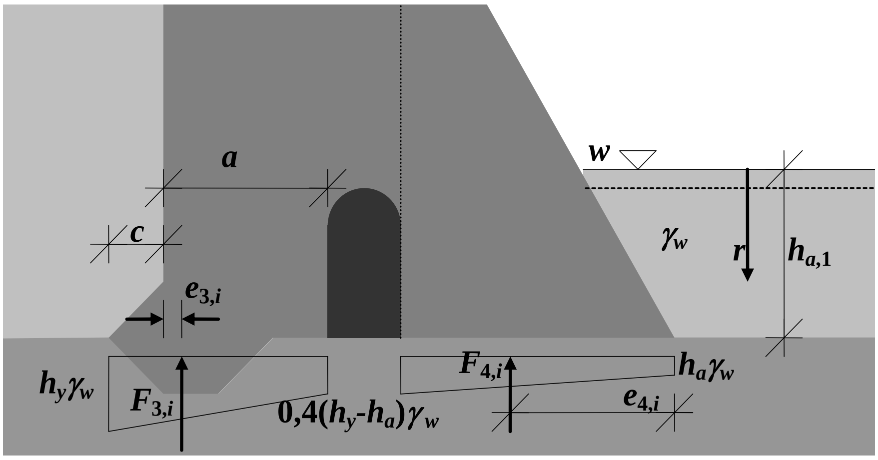


Figure 3.

BUOYANCY AT THE SIDE OF THE LOWER POND

Resultant (*RIL 123-1979.* Chapter 2. Figure 7.) (Figure 3)

$$F_{4,i} = \left\{h_{a,i} + \frac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]\right\}\gamma_w\left(b_p - d\right) \qquad (12)$$

$F_{4,1} =$ 145 277 N/m

$F_{4,2} =$ 96 851 N/m

Corresponding distance from the point *O*

$$e_{4,i} = \frac{h_{a,i}\dfrac{b_p - d}{2} + \dfrac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]\dfrac{2}{3}\left(b_p - d\right)}{h_{a,i} + \dfrac{1}{2}\left[0{,}4\left(h_y - h_{a,i}\right) - h_{a,i}\right]} \qquad (13)$$

$e_{4,1}$ = 2,641 m

$e_{4,2}$ = 3,657 m

SELF WEIGHT OF THE DAM

Resultant (Figure 4)

$$G = [hd + h(b_p - d)/2]\gamma_c \qquad (14)$$

= 1 245 073 N/m

Corresponding distance from the point *O*

$$e_G = \frac{\gamma_c}{G}\left[hd\left(b_p - \frac{d}{2}\right) + \frac{1}{2}h\left(b_p - d\right)\frac{2}{3}\left(b_p - d\right)\right] \qquad (15)$$

= 5,125 m

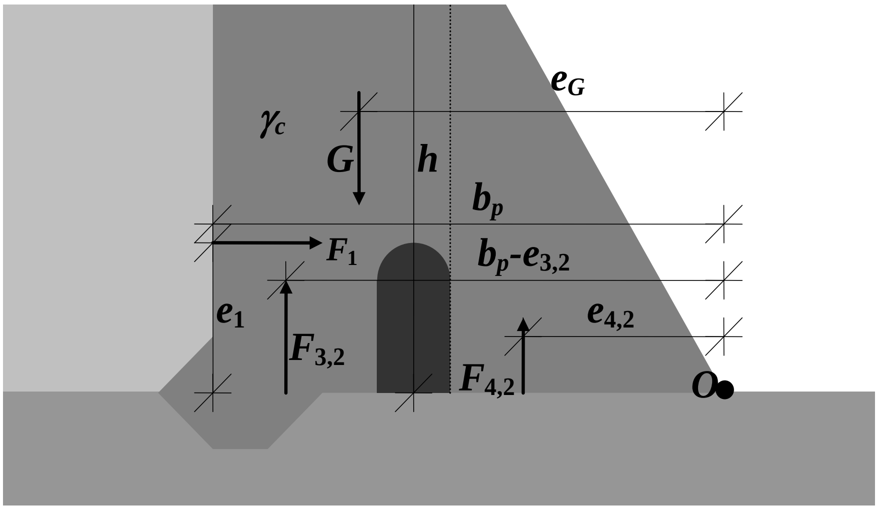


Figure 4.

ANSWER: **Resultants and x - and y -coordinates are given in Table 1.**

Table 1.

		F_x [N/m]	y [m]	F_y [N/m]	x [m]
Water pressure	Upper pond	397 305	-3,000		
	Lower pond	-38 231	1,000	22 073	-0,577
Buoyancy	Upper pond side			-83 876	-7,880
	Lower pond side			-145 277	-2,641

Safety against sliding (*RIL 123-1979*, Chapter 2.42, p. 63)

$$n = \frac{\sum M_p}{\sum M_k} = \frac{Ge_G}{F_1 e_1 + F_{3,2}(b_p - e_{3,2}) + F_{4,2} e_{4,2}} \quad (16)$$

= 2,807 > 1,5 **O.K.**

Safety against tilting (sill of dam is ignored)

$$n = \frac{\mu\left(\sum V - N\right)}{\sum H} \quad (17)$$

$$\Rightarrow n = \frac{\mu\left[G - (F_{3,2} + F_{4,2})\right]}{F_1} \quad (18)$$

= 1,860 < 2 **Sliding!**

5. PROBLEM

The end supports of the bridge shown in Figure 1 are founded on rock and the middle support on sand. Due to loading, the middle support sinks

δ = 0,010 m.

Determine the bending moment and shear force distribution curve of the bridge due to dead load before and after sinking.

Total length of the bridge

L = 16 m

Bending stiffness of the bridge

D = 120 MNm^2

Dead load (self weight)

g = 0,010 MN/m

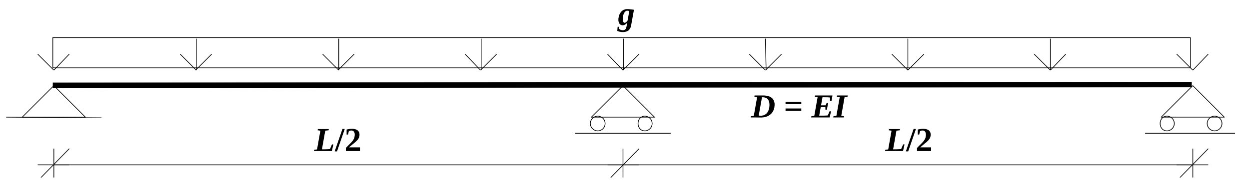


Figure 1.

In the statically determinate basic structure, the sum of the deflection (δ_{10}) due to the external load (g) and deflection ($-X_1\delta_{11}$) due to the unknown reaction force (X_1) has to be equal to the total deflection at the middle point. Here, δ_{11} is deflection due to force 1 acting at the middle point. See Figure 2.

$$\delta_{10}-X_1\delta_{11} = \delta \tag{1}$$

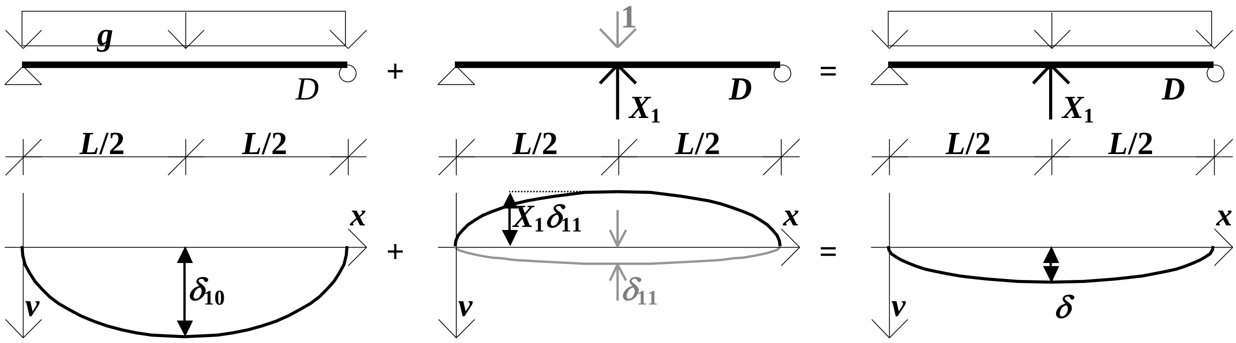


Figure 2.

Bending moment due to force 1 acting at the middle point is determined in Figure 3.

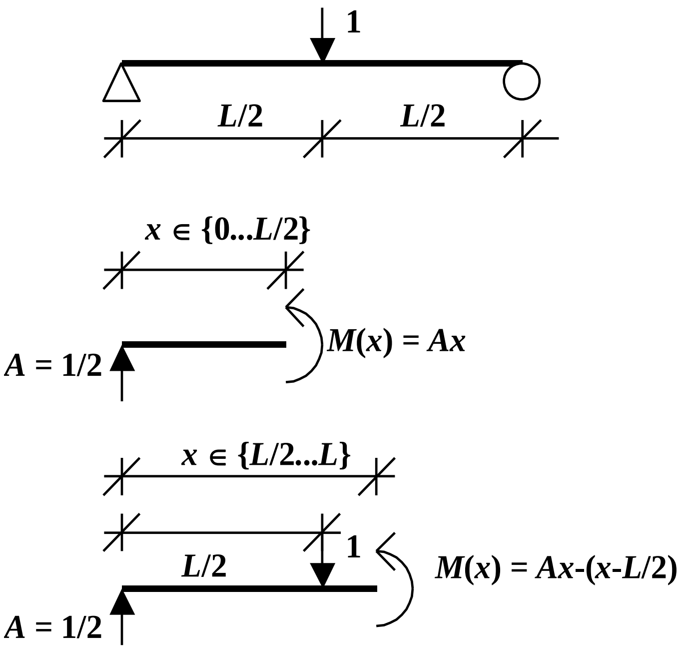


Figure 3.

Bending moment is (Figure 4)

$$M_1 = \begin{cases} \dfrac{x}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{L-x}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \tag{2a, b}$$

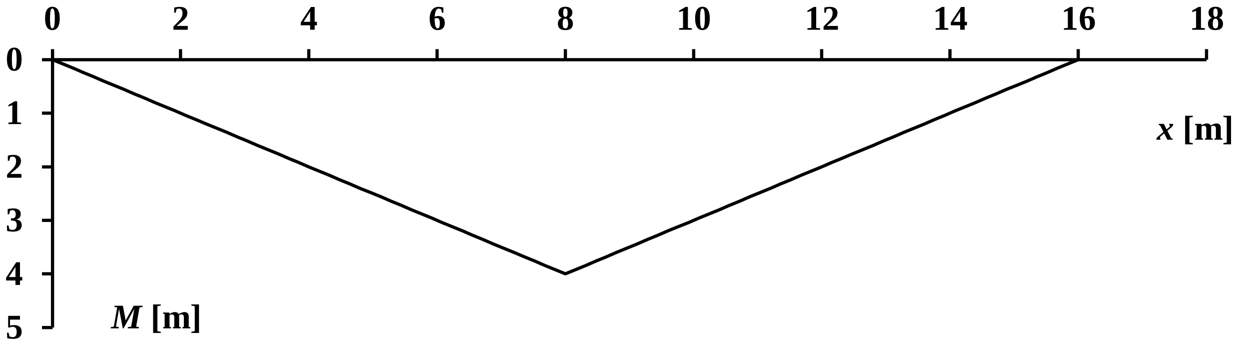


Figure 4.

Bending moment due to the dead load is determined in Figure 5.

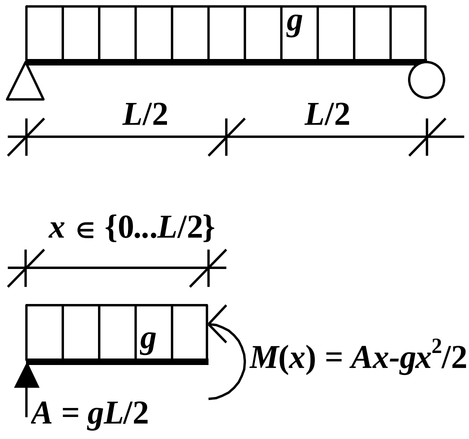


Figure 5.

Bending moment is (Figure 6)

$$M_0 = \frac{g}{2}\left(Lx - x^2\right), \quad x = [0, L] \tag{3}$$

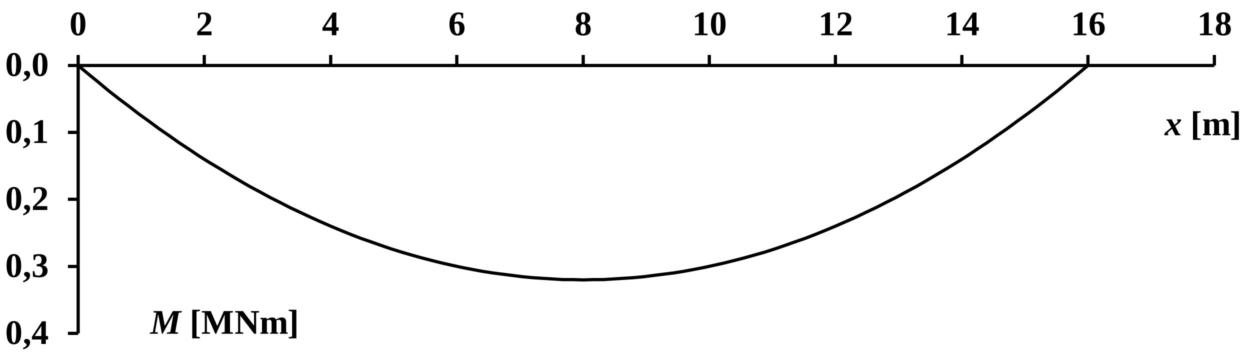


Figure 6.

Deflections

$$\begin{cases} \delta_{10} = \int_0^L \frac{M_1 M_0}{D} dx \\ \delta_{11} = \int_0^L \frac{M_1 M_1}{D} dx \end{cases} \tag{4a, b}$$

When inserting the bending moments we get

$$\begin{cases} \delta_{10} = \frac{5gL^4}{384D} \\ \delta_{11} = \frac{L^3}{48D} \end{cases} \tag{5a, b}$$

$$\Rightarrow \begin{cases} \delta_{10} = & 0{,}071 \ \text{m} \\ \delta_{11} = & 0{,}711 \ \text{m/MN} \end{cases}$$

Reaction force

$$X_{1,i} = \frac{\delta_{10} - \delta_i}{\delta_{11}} \tag{6}$$

Before sinking ($i = b$)

$$\delta_b = 0{,}000 \ \text{m}$$

$$X_{1,b} = 0{,}100 \ \text{MN}$$

After sinking ($i = a$)

$$X_{1,a} = 0{,}086 \ \text{MN}$$

Bending moment

$$M_i(x) = M_0 - X_{1,i}M_1 \tag{7}$$

$$= \begin{cases} \dfrac{g}{2}\left(Lx - x^2\right) - X_{1,i}\dfrac{x}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{g}{2}\left(Lx - x^2\right) - X_{1,i}\dfrac{L-x}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \tag{8a, b}$$

Shear force

$$Q_i(x) = \frac{dM}{dx} \tag{9}$$

$$= \begin{cases} \dfrac{g}{2}(L - 2x) - \dfrac{X_{1,i}}{2}, & x = \left[0, \dfrac{L}{2}\right] \\ \dfrac{g}{2}(L - 2x) + \dfrac{X_{1,i}}{2}, & x = \left[\dfrac{L}{2}, L\right] \end{cases} \tag{10a, b}$$

<u>ANSWER</u>:

The bending moment distribution curve before (b) and after (a) sinking, respectively, is shown in Figure 7.

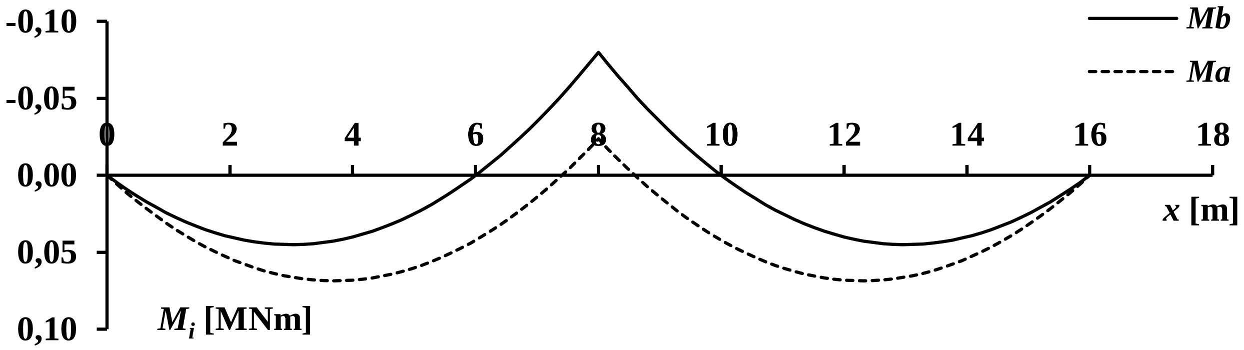


Figure 7.

The shear force distribution curve before (*b*) and after (*a*) sinking, respectively, is shown in Figure 8.

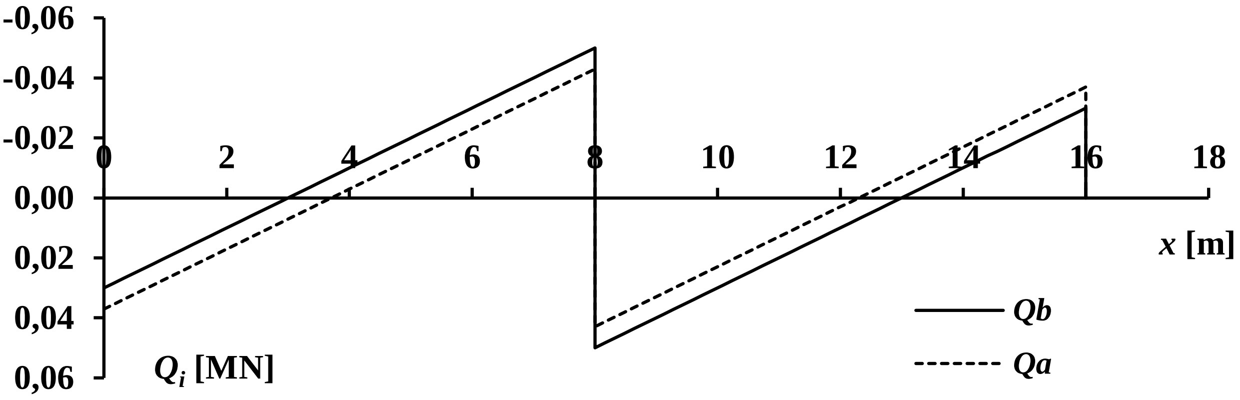


Figure 8.

6. PROBLEM

Determine the deflection curve of the simply supported bridge due to the train load shown in Figure 1! Compare Eurocode: *EN 1991-2 Traffic Loads on Bridges*, Chapter 6.3.2, Load Model 71, when factor

α = **1,46**

Distance

a_0 = **3 m**

Length of the bridge

L = **16 m**

Bending stiffness of the bridge

D = **12 000 MNm^2**

Axel load of the engine

F = **0,350 MN**

Cars (arbitrary length)

q = **0,120 MN/m**

Distance

b = **0,800 m**

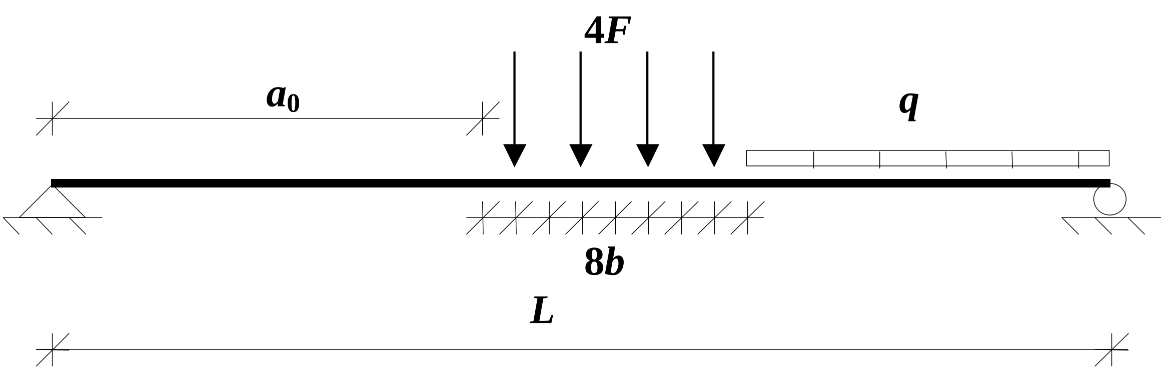


Figure 1.

When integrated, the differential equation of the deflection is

$$v''(x) = \frac{-M(x)}{EI} \tag{1}$$

$$\Rightarrow v'(x) = \frac{-1}{D}\int_0^x M(x)dx + A = \phi(x) \tag{2}$$

$$\Rightarrow \quad v(x) = \frac{-1}{D}\int_0^x\int_0^x M(x)(dx)^2 + Ax + B \tag{3}$$

POINT LOAD (P = 1) ACTING AT POINT a

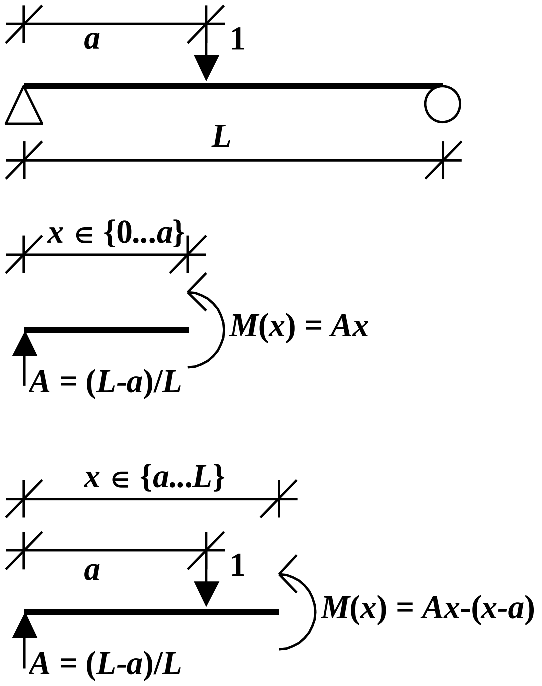


Figure 2.

Bending moment (Figure 2)

$$M(x) = \begin{cases} \frac{x}{L}(L-a), & x \in \{0 \ldots a\} \\ \frac{a}{L}(L-x), & x \in \{a \ldots L\} \end{cases} \tag{4a, b}$$

Deflection as a function of *x*

$$v'(x) = \begin{cases} v_1'(x), & x \in \{0..a\} \\ v_2'(x), & x \in \{a...L\} \end{cases} \tag{5a, b}$$

$$= \begin{cases} \dfrac{x^2}{2DL}(a-L) + A_1, & x \in \{0..a\} \\ \dfrac{ax}{2DL}(x-2L) + A_2, & x \in \{a...L\} \end{cases} \tag{6a, b}$$

$$\Rightarrow v(x) = \begin{cases} v_1(x), & x \in \{0..a\} \\ v_2(x), & x \in \{a...L\} \end{cases} \tag{7a, b}$$

$$= \begin{cases} \dfrac{x^3}{6DL}(a-L) + A_1 x + B_1, & x \in \{0..a\} \\ \dfrac{a}{6DL}\left(x^3 - 3Lx^2\right) + A_2 x + B_2, & x \in \{a...L\} \end{cases} \tag{8a, b}$$

Boundary conditions

$$v_1'(a) = v_2'(a) \tag{9}$$

$$v_1(0) = 0 \tag{10}$$

$$v_2(L) = 0 \tag{11}$$

$$v_1(a) = v_2(a) \tag{12}$$

Integration constants from boundary conditions 9, 10, 11 and 12

$$\begin{cases} A_1 = \dfrac{a}{6DL}\left(2L^2 - 3La + a^2\right) & (13) \\ B_1 = 0 & (14) \\ A_2 = \dfrac{a}{6DL}\left(2L^2 + a^2\right) & (15) \\ B_2 = \dfrac{-a^3}{6D} & (16) \end{cases}$$

Deflection curve from point load F acting at point $x = a$

$$v_F(x) = \begin{cases} \dfrac{F}{6DL}\left[(a-L)x^3 + a(2L^2 - 3La + a^2)x\right], & x \in \{0...a\} \\ \dfrac{F}{6DL}\left[ax^3 - 3Lax^2 + a(2L^2 + a^2)x - La^3\right], & x \in \{a...L\} \end{cases} \tag{17a, b}$$

UNIFORMLY DISTRIBUTED LOAD (q = 1) BETWEEN x = a AND x = L

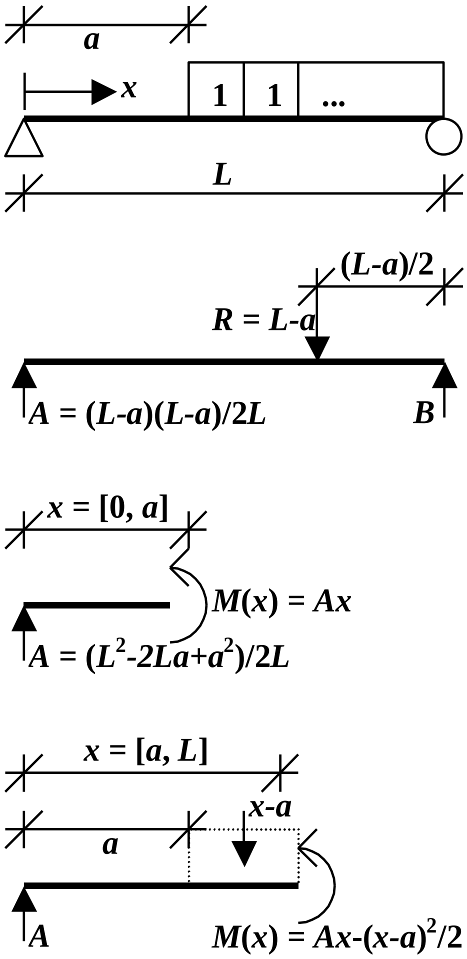


Figure 3.

Bending moment (Figure 3)

$$M(x)=\begin{cases}\dfrac{x}{2L}\left(L^2-2La+a^2\right), & x\in\{0...a\}\\ \dfrac{1}{2L}\left[-Lx^2+\left(L^2+a^2\right)x-La^2\right], & x\in\{a...L\}\end{cases} \tag{18a, b}$$

Deflection as a function of x

$$v'(x)=\begin{cases} v_1'(x), & x\in\{0...a\} \\ v_2'(x), & x\in\{a...L\}\end{cases} \tag{5a, b}$$

$$=\begin{cases} \dfrac{x^2}{4DL}\left(-L^2+2La-a^2\right)+A_1, & x\in\{0...a\} \\ \dfrac{1}{12DL}\left[2Lx^3+3\left(L^2+a^2\right)x^2-6La^2x\right]+A_2, & x\in\{a...L\}\end{cases} \tag{19a, b}$$

$$\Rightarrow v(x)=\begin{cases} v_1(x), & x\in\{0...a\} \\ v_2(x), & x\in\{a...L\}\end{cases} \tag{7a, b}$$

$$=\begin{cases} \dfrac{x^3}{12DL}\left(-L^2+2La-a^2\right)+A_1x+B_1, & x\in\{0...a\} \\ \\ \dfrac{1}{24DL}\left[Lx^4-2\left(L^2+a^2\right)x^3+6La^2x^2\right]+A_2x+B_2, \\ \quad x\in\{a...L\}\end{cases} \tag{20a, b}$$

Integration constants from boundary conditions 9, 10, 11 and 12

$$\begin{cases} A_1=\dfrac{1}{24DL}\left(L^4-4L^2a^2+4La^3-a^4\right) & (21) \\ B_1=0 & (22) \\ A_2=\dfrac{1}{24DL}\left(L^4-4L^2a^2-a^4\right) & (23) \\ B_2=\dfrac{a^4}{24D} & (24)\end{cases}$$

Deflection curve due to uniformly distributed load q

$$v_q(x)=\begin{cases} \dfrac{q}{24DL}\left[2\left(-L^2+2La-a^2\right)x^3+\right. \\ \left.+\left(L^4-4L^2a^2+4La^3-a^4\right)x\right], & x\in\{0...a\} \\ \\ \dfrac{q}{24DL}\left[Lx^4-2\left(L^2+a^2\right)x^3+6La^2x^2+\right. \\ \left.+\left(L^4-4L^2a^2-a^4\right)x+La^4\right], & x\in\{a...L\}\end{cases} \tag{25a, b}$$

SUPERPOSITION

Table 1. **Location of the point loads and the uniformly distributed load, respectively.**

		a_i m
Point load F_1	$a_1 = a_0 + b$ =	3,800
Point load F_2	$a_2 = a_0 + 3b$ =	5,400
Point load F_3	$a_3 = a_0 + 5b$ =	7,000
Point load F_4	$a_4 = a_0 + 7b$ =	8,600
Uniform load q	$a_5 = a_0 + 8b$ =	9,400

Inserting $a_1 \ldots a_4$ (Table 1) in Equation 17 and a_5 in Equation 25 and by using superposition principle, the total deflection curve is obtained (Figure 4).

ANSWER: **Deflection curve is shown in Figure 4.**

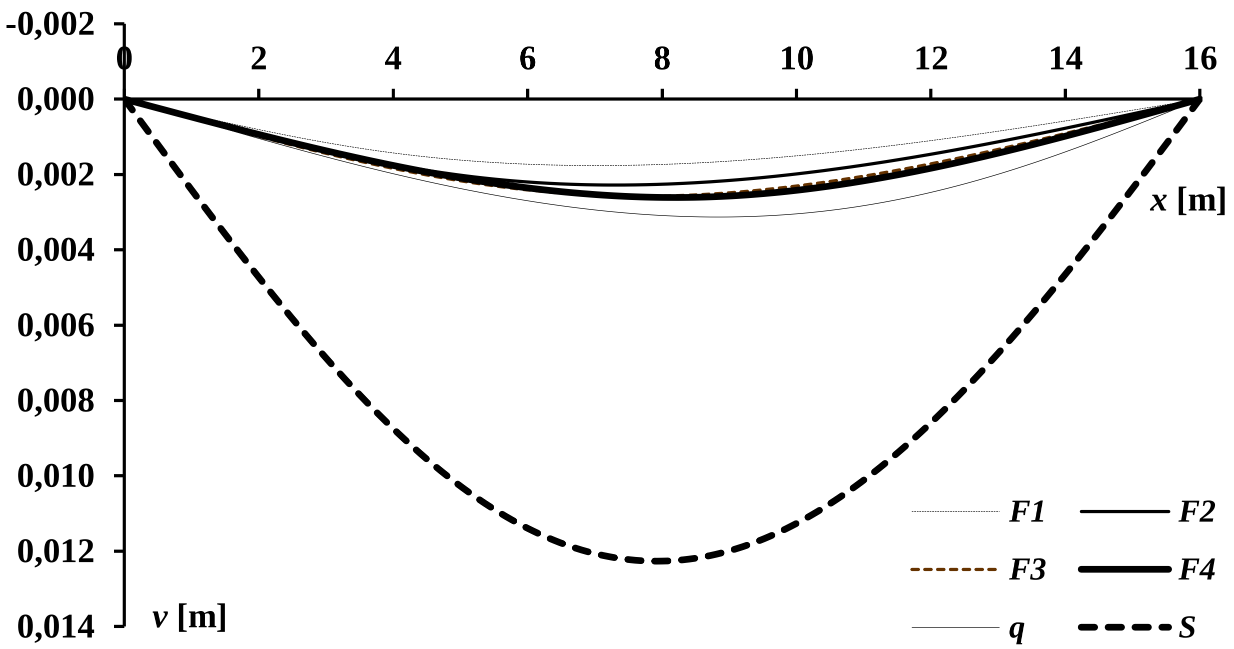


Figure 4. Deflection curve.

7. PROBLEM

Determine the position of the service vehicle (distance a) so that the mid-support reaction force of the double-span bridge, shown in Figure 1, is maximized!

Traffic load representing service vehicle is accidental loading model for footbridges given in *Eurocode* (*EN 1991-2* § 5.6.3).

Length of the bridge

L = 16 m

Bending stiffness of the bridge

D = 120 MNm^2

Uniformly distributed load on the whole bridge due to dead weight of bridge

p = 0,009 MN/m

Axel loads of the service vehicle

F_1 = 0,080 MN

F_2 = 0,040 MN

Distance between axels

b = 3,000 m

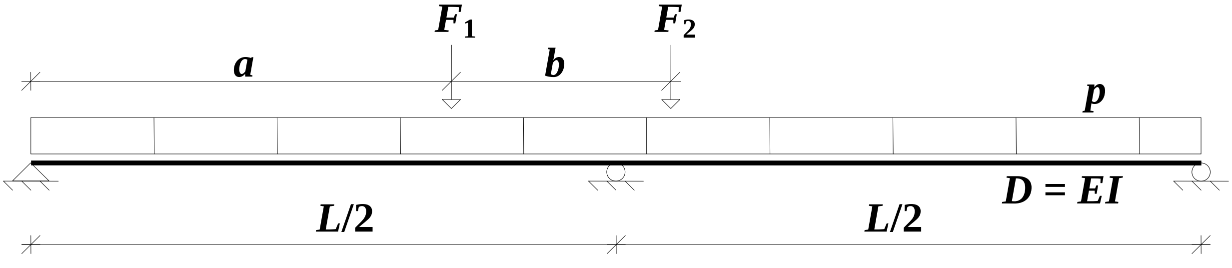


Figure 1.

By removing the middle support, the statically determinate basic form is obtained (Figure 2).

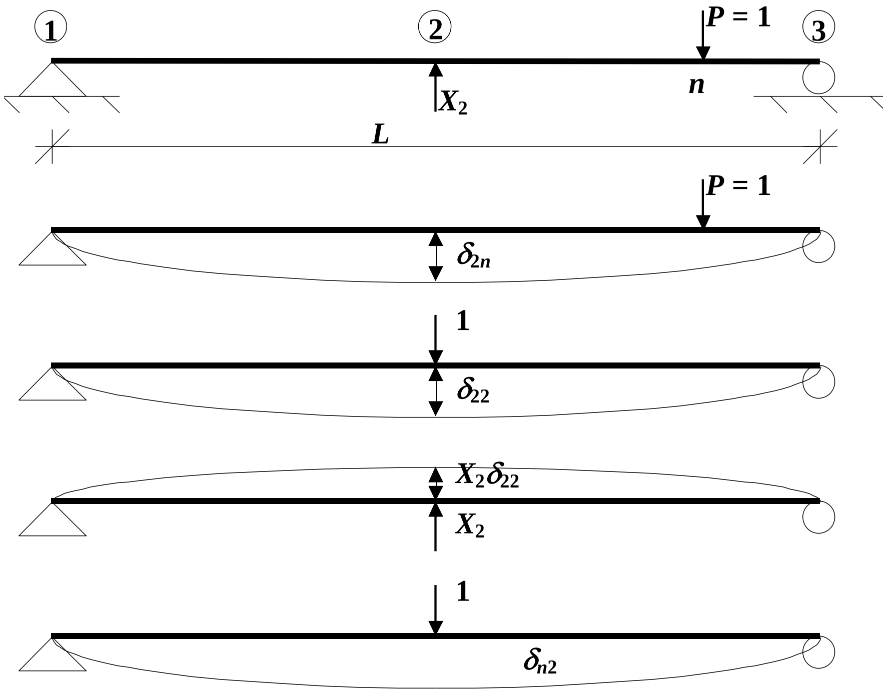


Figure 2.

At the mid-point (point 2), the sum of deflection δ_{2n} caused by load $P = 1$ at point n (value of x-coordinate) and deflection $X_2\delta_{22}$ caused by point load X_2 at point 2 has to be equal to zero. Here, δ_{22} is the mid-point deflection due to point load $P = 1$ acting at the same point. Thus,

$$0 = \delta_{2n} - X_2\delta_{22} \quad (1)$$

$$\Rightarrow X_2 = \frac{\delta_{2n}}{\delta_{22}} \quad (2)$$

Minus-sign in Equation 1 means, that the reaction force is acting upwards. According to Equation 2, reaction force X_2 is maximized, when deflection δ_{2n} is maximized.

Following from Maxwell's rule: mid-point deflection δ_{2n} caused by point load $P = 1$ at point n is equal to deflection δ_{n2} at point n caused by point load $P = 1$ at the mid-point. Thus,

$$\delta_{2n} = \delta_{n2} \quad (3)$$

Influence line of the reaction force (due to point load $P = 1$ moving over the bridge) is obtained by using deflection curve δ_{n2} (due to point load $P = 1$ acting at the mid-point).

$$X_2 = \frac{\delta_{n2}}{\delta_{22}} \tag{4}$$

Deflection $\delta_{n2} = v(x)$ is determined by Mohr's method.

Dividing bending moment by bending stiffness we get (Figure 3)

$$\frac{M}{D}(x) = \begin{cases} \dfrac{x}{2D}, & x \in \{0 \ldots L/2\} \\ \dfrac{L-x}{2D}, & x \in \{L/2 \ldots L\} \end{cases} \tag{5a, b}$$

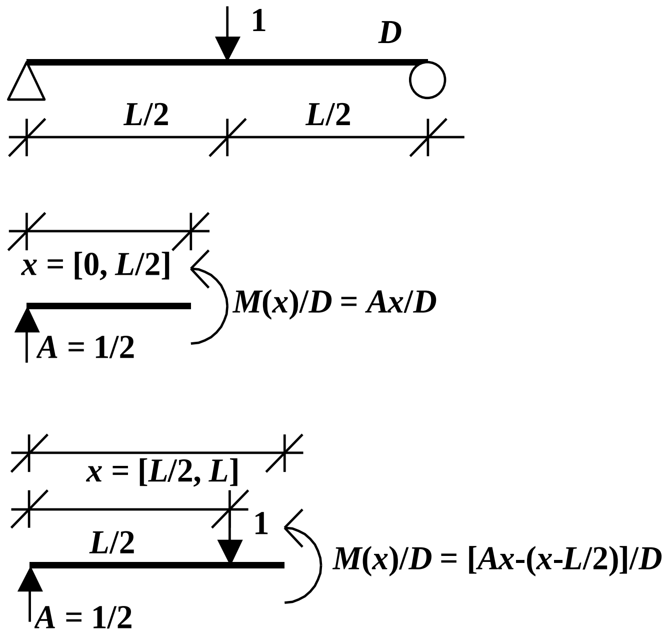


Figure 3.

Deflection curve is equal to the bending moment distribution curve $M_{M/D}$ caused by load M/D (Figure 4)

$$v(x) = M_{M/D}(x) \tag{6}$$

$$T = \frac{L^2}{16D}$$

$$e_1 = \frac{x}{3}$$

$$R_1 = \frac{x^2}{4D}$$

$$e_2 = x - \frac{L}{3}$$

$$R_2 = \frac{L^2}{16D}$$

$$e_3 = \frac{2}{3}\left(x - \frac{L}{2}\right)$$

$$R_3 = \frac{2x - L}{8D}\left(x - \frac{L}{2}\right)$$

$$e_4 = \frac{1}{2}\left(x - \frac{L}{2}\right)$$

$$R_4 = \frac{L - x}{2D}\left(x - \frac{L}{2}\right)$$

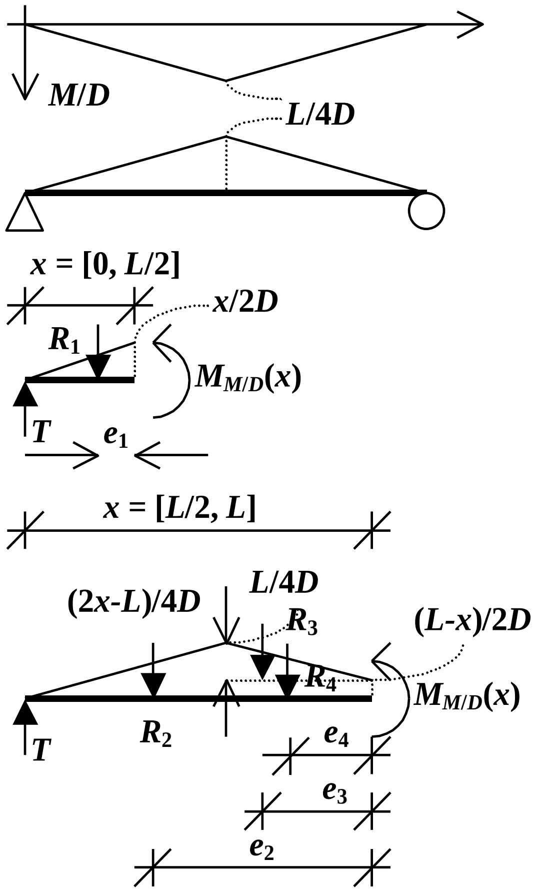


Figure 4.

Deflection (as function of x) due to point load $P = 1$ acting at the mid-point

$$v(x) = \begin{cases} Tx - R_1 e_1, & x \in \{0 \ldots L/2\} \\ Tx - R_2 e_2 - R_3 e_3 - R_4 e_4, & x \in \{L/2 \ldots L\} \end{cases} \qquad \text{(7a, b)}$$

$$= \begin{cases} \dfrac{1}{48D}\left(-4x^3 + 3L^2 x\right), & x \in \{0 \ldots L/2\} \\ \dfrac{1}{48D}\left(4x^3 - 12Lx^2 + 9L^2 x - L^3\right), & x \in \{L/2 \ldots L\} \end{cases} \qquad \text{(8a, b)}$$

Deflection (as function of x and F) due to point load F_i acting at the mid-point

$$v(x,F)=\begin{cases}\dfrac{F}{48D}\left(-4x^3+3L^2x\right), & x\in\{0\ldots L/2\}\\ \dfrac{F}{48D}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2\ldots L\}\end{cases} \quad \text{(9a, b)}$$

Deflection at the mid-point due to point load $P = 1$ acting at the same point is obtained from Equation 8

$$\delta_{22}=v\left(x=\frac{L}{2}\right) \quad \text{(10)}$$

$$=\frac{L^3}{48D} \quad \text{(11)}$$

Influence line of reaction force is obtained by inserting Equations 8 and 11 in Equation 4. See Figure 5.

$$X_2(x)=\begin{cases}\dfrac{1}{L^3}\left(-4x^3+3L^2x\right), & x\in\{0\ldots L/2\}\\ \dfrac{1}{L^3}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2\ldots L\}\end{cases} \quad \text{(12a, b)}$$

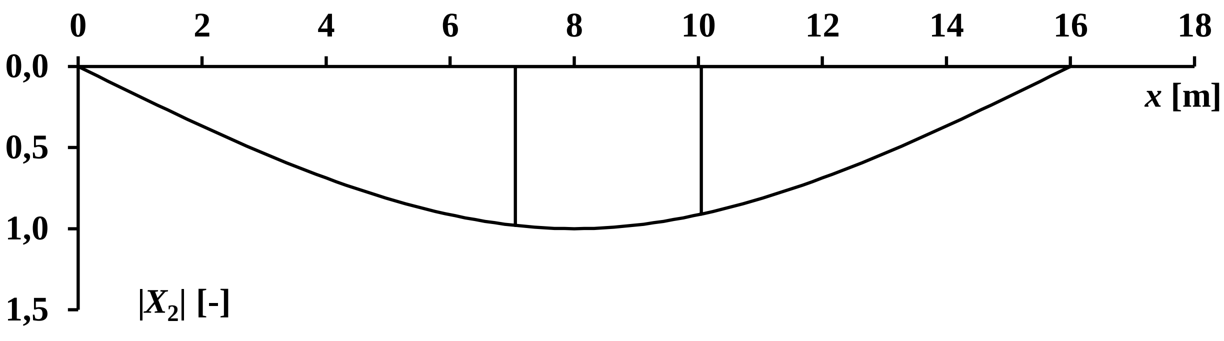


Figure 5.

Influence line of the reaction force due to axel load F_i is

$$X_{2,F_i}(x)=\begin{cases}\dfrac{F_i}{L^3}\left(-4x^3+3L^2x\right), & x\in\{0\ldots L/2\}\\ \dfrac{F_i}{L^3}\left(4x^3-12Lx^2+9L^2x-L^3\right), & x\in\{L/2\ldots L\}\end{cases} \quad \text{(13a, b)}$$

Reaction force due to uniformly distributed load p is obtained by multiplying Equation 12 by load p and integrating over the bridge.

$$X_{2,p} = \int_0^{L/2} \frac{p}{L^3}\left(-4x^3 + 3L^2 x\right)dx + \tag{14}$$
$$+ \int_{L/2}^{L} \frac{p}{L^3}\left(4x^3 - 12Lx^2 + 9L^2 x - L^3\right)dx$$
$$= \frac{5}{8}pL \tag{15}$$

By using iteration the maximum reaction force and the corresponding load position is found (Equations 13 and 15, Table 1)

a = 7,050 m

Table 1.

	x_i [m]		$X_{2,i}$ [MN]
Point load F_1	x_1 = a =	7,050	0,078
Point load F_2	$x_2 = a + b$ =	10,050	0,036
Uniform load p			0,090
		Σ	0,205

ANSWER: **Position of the service vehicle: distance**

a = 7,050 m

Reaction force

X_2 = 0,205 MN

8. PROBLEM

Determine distribution of earth pressure acting against a basement wall and the corresponding resultant force, when the angle of earth surface with respect to horizontal plane is

A) β_A = **0 °**

B) β_B = **20 °**

The filling is compressed followingly:

Compression machine	**Vibrating slab, 400 kg**
Compression times	**4**
Layer height	**0,350 m**

Height of the wall

h = **2,500 m**

Angle of internal friction of the earth

φ = **34 °**

Effective unit weight of soil

γ = **0,019 MN/m^3**

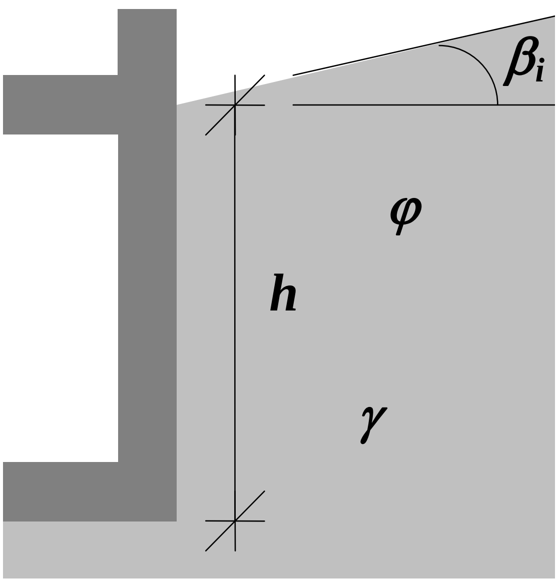


Figure 1.

Table 1. Permanent earth pressure caused by filling behind the non-transferable supporting structure (Figure 2)[1].

Compressing machine	Weight of machine [kg]	Number of compressing times	Layer height h [m]	Fold depth z [m]	Earth pressure p [MN/m^2]
Vibrating roller	3 000	6	0,400	0,500	0,019
Vibrating slab	400	4	0,350	0,500	0,016
Vibrating slab	100	4	0,200	0,500	0,012

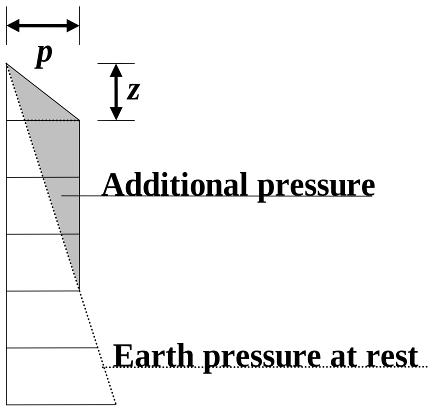


Figure 2. Earth pressure distribution curve.

1) **Finnish planning instructions for Eurocode EN 1997-1, *Geotekninen suunnittelu RIL 207-2009*, Chapter 9.5.5.**

Earth pressure acting on the non-transferable structure corresponds to pressure at rest. It acts in horizontal direction.

A) Angle of earth surface is zero

Coefficient of earth pressure at rest

$$K_o = 1\text{-}\sin\varphi \tag{1}$$

$$= 0{,}441$$

Pressure at the bottom level of the wall without compressing (Figure 3)

$$p_o = K_o \gamma h \tag{2}$$

$$= 0{,}021 \text{ MN/m}^2$$

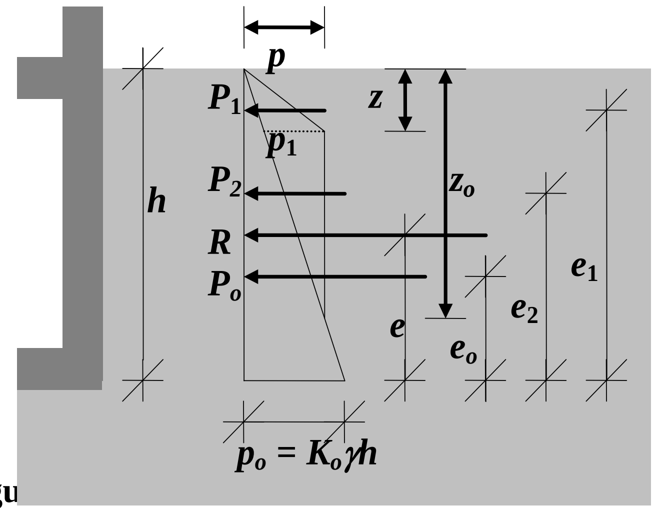
Resultant of earth pressure

$$P_o = \frac{1}{2} p_o h \quad 0{,}026 \text{ MN/m} \tag{3}$$

Distance of the resultant measured from the bottom of the wall

$$e_o = h/3 \tag{4}$$

$$= 0{,}833 \text{ m}$$



Figu

Earth pressure at depth *z* after compressing (Figures 2 and 3)

p = **0,016 MN/m^2**

Depth of the fold of the earth pressure distribution curve

z = **0,500 m**

Depth of the lower edge of the additional pressure distribution curve (compare with Equation 2)

$$z_o = \frac{p}{K_o \gamma}$$ **1,910 m** **(5)**

Additional earth pressure at the fold, i.e. at depth *z*,

$p_l = p - K_o \gamma z$ **(6)**

= **0,012 MN/m^2**

Resultant force due to additional pressure above *z*

$P_1 = p_l z/2$ **(7)**

= **0,003 MN/m**

Distance of the resultant measured from the bottom of the wall

$$e_1 = h - \frac{2}{3}z$$ **,167 m** **(8)**

Resultant force due to additional pressure under *z*

$P_2 = p_l (z_o - z)/2$ **(9)**

= **0,008 MN/m**

Distance of the resultant measured from the bottom of the wall

$$e_2 = h - z - \frac{1}{3}(z_o - z) \tag{10}$$

<u>ANSWER</u>: Earth pressure distribution curve and its resultant is shown in Figure 4.

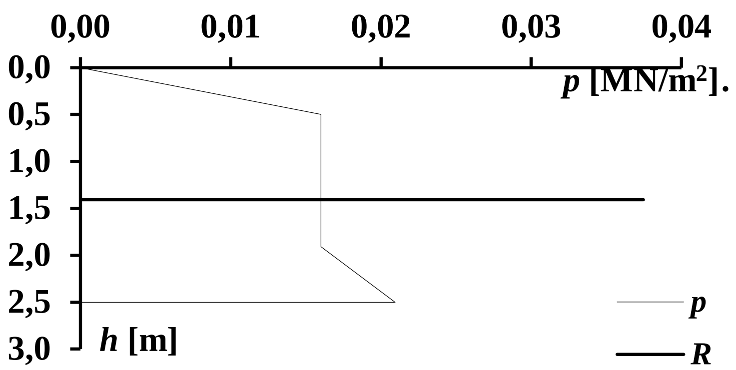


Figure 4.

Resultant of the total earth pressure

$$R = P_o + P_1 + P_2 \tag{11}$$
$$= 0{,}0375 \text{ MN/m}$$

Distance of the resultant measured from the bottom of the wall

$$e = \frac{P_o e_o + P_1 e_1 + P_2 e_2}{R} \tag{12}$$

B) Angle of earth surface differs from zero

Coefficient of earth pressure at rest (compare with Equation 1)

$$K_{oB} = (1-\sin\varphi)(1+\sin\beta) \quad (13)$$

$$= \quad 0{,}592$$

Earth pressure at the base level (Figure 5)

$$p_{oB} = K_{oB}\,\gamma h \quad (14)$$

$$= \quad 0{,}028 \ \text{MN/m}^2$$

Resultant of earth pressure

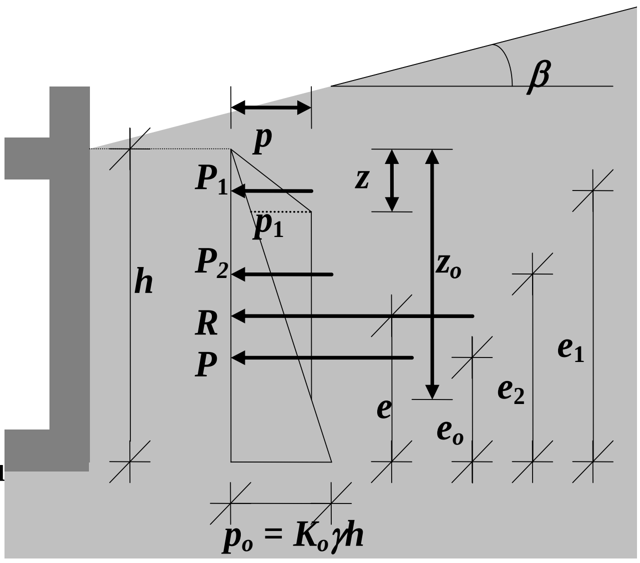
$$P_{oB} = \frac{1}{2} p_{oB} h \quad (15)$$

$$= \quad 0{,}035 \ \text{MN/m}$$

Distance of the resultant measured from the bottom of the wall

$$e_{oB} = h/3 \quad (16)$$

$$= \quad 0{,}833 \ \text{m}$$



Figu

Depth of the lower edge of the additional pressure distribution curve (compare with Equation 2)

(17)

$$= \quad 1{,}424 \text{ m}$$

$$z_{oB} = \frac{p}{K_{oB}\gamma}$$

Additional earth pressure at the fold, i.e. at depth z,

$$p_{lB} = p - K_{oB}\gamma z \qquad (18)$$

$$= \quad 0{,}010 \text{ MN/m}^2$$

Resultant force due to additional pressure above z

$$P_{1B} = p_{lB}\, z/2 \qquad (19)$$

$$= \quad 0{,}003 \text{ MN/m}$$

Distance of the resultant measured from the bottom of the wall

(20)

$$= \quad 2{,}167 \text{ m}$$

$$e_{1B} = h - \frac{2}{3}z$$

Resultant force due to additional pressure under z

$$P_{2B} = p_{lB}(z_{oB} - z)/2 \qquad (21)$$

$$= \quad 0{,}005 \text{ MN/m}$$

Distance of the resultant measured from the bottom of the wall

(22)

$$= \quad 1{,}692 \text{ m}$$

$$e_{2B} = h - z - \frac{1}{3}(z_{oB} - z)$$

ANSWER: Earth pressure distribution curve and its resultant is shown in Figure 6.

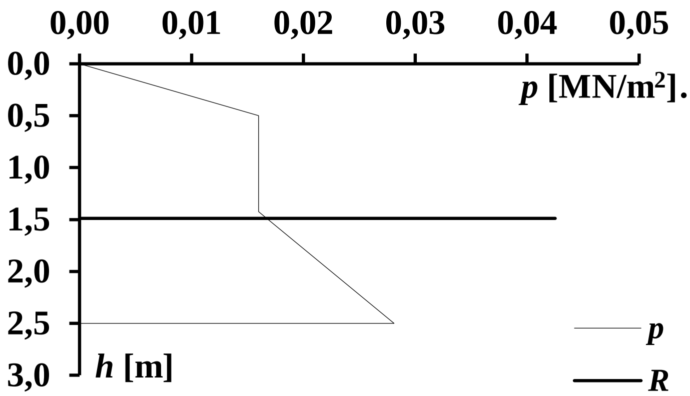


Figure 6.

Resultant of the total earth pressure

$$R = P_{oB} + P_{1B} + P_{2B} \tag{23}$$

$$= \quad 0{,}0425 \ \text{MN/m}$$

Distance of the resultant measured from the bottom of the wall

(24)

$$= \quad 1{,}012 \ \text{m}$$

$$e = \frac{P_{oB}e_{oB} + P_{1B}e_{1B} + P_{2B}e_{2B}}{R}$$

9. PROBLEM

Determine distribution of active earth pressure (horizontal component) and the corresponding resultant acting on the concrete caisson shown in Figure 1, when the filling is see-sand!

Dimensions

a =	1	m
h_1 =	2	m
h_2 =	1	m
h_3 =	6	m
h_4 =	1	m

Inverse of tangent of the angle of earth surface respective to the horizontal plane

k = 10

Angle of internal friction of the earth

φ = 38 °

Unit weight of soil

γ = 0,018 MN/m^3

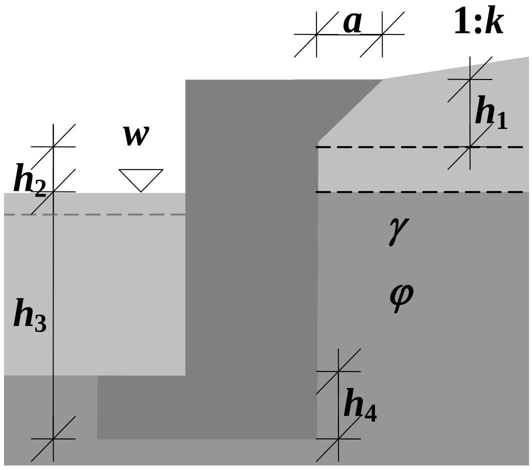


Figure 1.

Direction: To calculate earth pressure, use three layers separated by the dashed lines shown in Figure 1.

Angle of earth surface with respect to the horizontal plane (Figure 2)

$$\beta = \arctan\left(\frac{1}{k}\right) \tag{1}$$

$$= \quad 5{,}711\ °$$

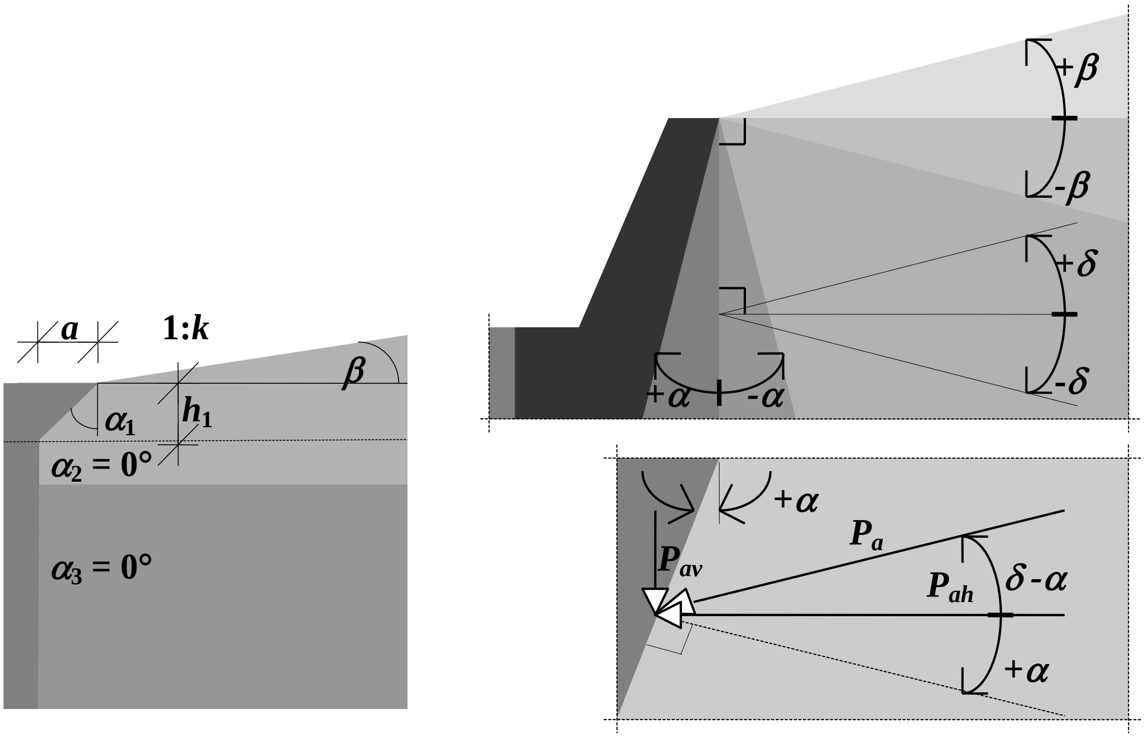
Angle of structure at any layer with respect to the vertical plane (Figure 2)

$$\alpha_1 = \arctan\left(\frac{a}{h_1}\right) \tag{2}$$

$$= \quad 26{,}565\ °$$

$$\alpha_2 = \quad 0{,}000\ °$$

$$\alpha_3 = \quad 0{,}000\ °$$



a) b)

Figure 2.

Angle of wall friction, when the sliding surface is between the concrete strucure and soil

$$\delta = \frac{3}{4}\varphi \tag{3}$$

$$= \quad 28{,}500\ °$$

Coefficient of the horizontal component of active earth pressure

$$K_{ahi} = \frac{\cos^2(\varphi+\alpha_i)}{\cos^2\alpha_i\left(1+\sqrt{\frac{\sin(\varphi+\delta)\sin(\varphi-\beta)}{\cos(\alpha_i-\delta)\cos(\alpha_i+\beta)}}\right)^2} \qquad (4)$$

$K_{ah\,1}$ = 0,074
$K_{ah\,2}$ = 0,203
$K_{ah\,3}$ = 0,203

Unit weight of soil above the ground water level

$\gamma_1 = \gamma_2 =$ 0,018 MN/m^3

Unit weight of soil under the ground water level (Table 1 at the end of this problem)

$\gamma_3 =$ 0,012 MN/m^3

Active earth pressure at the top (y) and bottom (a) of layer n, respectively,

$$\begin{cases} p_{ny} = \begin{cases} 0, & n=1 \\ K_n \sum_{i=1}^{n-1}(\gamma_i h_i), & n \geq 2 \end{cases}, \quad n \in N \\ p_{na} = K_n \sum_{i=1}^{n}(\gamma_i h_i), \quad n \geq 1 \end{cases} \qquad (5a, b)$$

p_{1y} = 0 MN/m^2

$p_{1a} = K_{ah\,1}(\gamma_1 h_1)$ (6)
= 0,002675 MN/m^2

$p_{2y} = K_{ah\,2}(\gamma_1 h_1)$ (7)
= 0,007312 MN/m^2

$p_{2a} = K_{ah\,2}(\gamma_1 h_1 + \gamma_2 h_2)$ (8)
0,010968 MN/m^2

$p_{3y} = K_{ah\,3}(\gamma_1 h_1 + \gamma_2 h_2)$ (9)
0,010968 MN/m^2

$p_{3a} = K_{ah\,3}(\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3)$ (10)
= 0,025593 MN/m^2

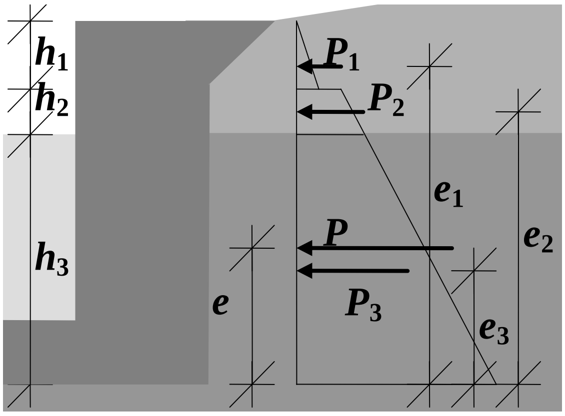


Figure 3.

Resultant of active earth pressure at layer n (Figure 3)

$$P_n = p_{ny} h_n + \frac{1}{2}(p_{na} - p_{ny}) h_n \tag{11}$$

$P_1 =$ 0,003 MN/m

$P_2 =$ 0,009 MN/m

$P_3 =$ 0,110 MN/m

and corresponding distances from the base of the caisson

$$e_n = \frac{p_{ny}\frac{h_n}{2} + \frac{1}{2}\left(p_{na} - p_{ny}\right)\frac{h_n}{3}}{p_{ny} + \frac{1}{2}\left(p_{na} - p_{ny}\right)} + \begin{cases} \sum_{i=n+1}^{n_{\max}} h_i, & n = [1, \quad n_{\max}] \\ 0, & n = n_{\max} \end{cases} \tag{12}$$

$e_1 =$ 7,667 m

$e_2 =$ 6,467 m

$e_3 =$ 2,600 m

<u>ANSWER</u>:

Resultant of active earth pressure

$$P = P_1 + P_2 + P_3 \tag{13}$$

$=$ 0,121 MN/m

Distance from the bottom of the caisson

$$e = \frac{\sum_{n=1}^{n_{max}} P_n e_n}{\sum_{n=1}^{n_{max}} P_n} \tag{14}$$

$$= \quad 3{,}002 \text{ m}$$

Earth pressure distribution curve and resultant is shown in Figure 4.

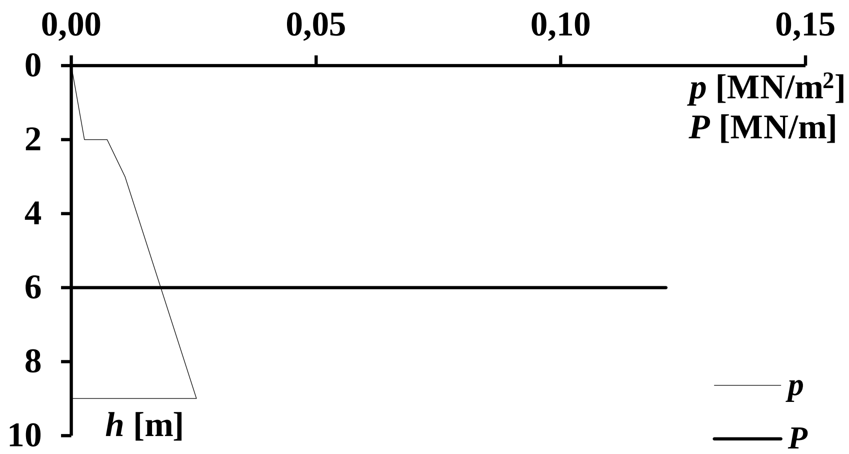


Figure 4.

Table 1. Estimation of coarse soils based on grain size.[1)]

Soil		Unit weight of soil above the ground water level [kN/m³]	Unit weight of soil under the ground water level [kN/m³]	Internal friction angle ϕ [°]
Fine sand $d_{10} \leq 0{,}06$	Loose	15…17	9…	30
	Normal			33
	Tight	16…18	11	36
Sand $d_{10} > 0{,}06$	Loose	16…18	10…	32
	Normal			35
	Tight	17…19	12	38
Gravel	Loose	17…19	10…	34
	Normal			37
	Tight	18…20	12	40
Moraine	Very loose	16…19	10…12	…34
	Loose	17…20	10…12	…36
	Normal	18…21	11…13	…38
	Tight	19…23	11…14	…40
Compressed filling under the foundation[2)]	Blast stones	15…18	9…11	45
	Crushed stone	19…22	11…13	42
	Gravel	18…21	11…13	40

1) Finnish Road Administration: Finnish standard of bridge foundation engineering. TIEL 2172068-99. Helsinki 1999. 71 s. ISBN 951-726-583-2. Table 1. p. 9.

2) To use these values it is required, that the work and materials fulfil the requirements of the Finnish specifications for bridges (2.7.1.2/24/).

10. PROBLEM

Determine distribution of earth pressure against the concrete retaining wall shown in Figure 1 and the corresponding resultant in two cases: active earth pressure and earth pressure at rest, respectively. The filling material is sand.

Dimensions

a	=	2,200	m
b	=	5,200	m
c	=	1,000	m
d	=	1,000	m
h_1	=	6,200	m
h_2	=	2,100	m

Loading

q	=	0,012	MN/m^2

Effective unit weight of soil

γ	=	0,018	MN/m^3

Angle of internal friction of the earth

φ_1	=	32	°
φ_2	=	34	°

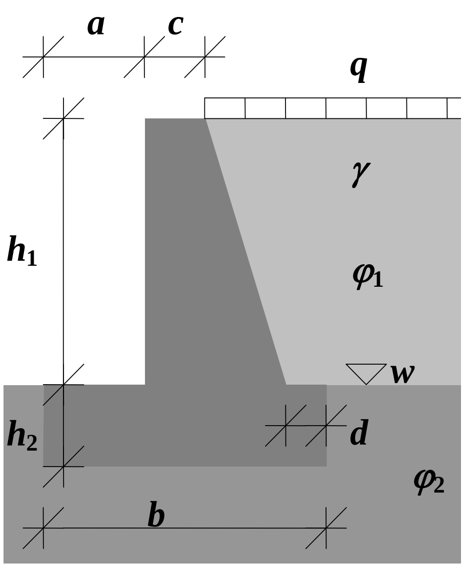


Figure 1.

Unit weight of soil above the ground water level

$\gamma_1 =$ **0,018** MN/m^3

Unit weight of soil under the ground water level (Table 1 of Problem 9)

$\gamma_2 =$ **0,011** MN/m^3

EARTH PRESSURE AT REST

Coefficient of earth pressure at rest

$$K_{oi} = 1\text{-}\sin\varphi_i \tag{1}$$

$K_{o1} =$ **0,470**

$K_{o2} =$ **0,441**

Earth pressure at rest at the top (y) and bottom (a) of layer n, respectively, (Figure 2)

$$\begin{cases} p_{ony} = \begin{cases} K_{on}q\,, & n=1 \\ K_{on}\left(\sum_{i=1}^{n-1}\gamma_i h_i + q\right), & n \ge 2 \end{cases} & , \quad n \in N \\ p_{ona} = K_{on}\left(\sum_{i=1}^{n}\gamma_i h_i + q\right), \quad n \ge 1 \end{cases} \tag{2a, b}$$

$$p_{o1y} = K_{o1}q \tag{3}$$

$= $ **0,005641** MN/m^2

$$p_{o1a} = K_{o1}(\gamma_1 h_1 + q) \tag{4}$$

$= $ **0,058102** MN/m^2

$$p_{o2y} = K_{o2}(\gamma_1 h_1 + q) \tag{5}$$

$= $ **0,054484** MN/m^2

$$p_{o2a} = K_{o2}(\gamma_1 h_1 + \gamma_2 h_2 + q) \tag{6}$$

$= $ **0,064666** MN/m^2

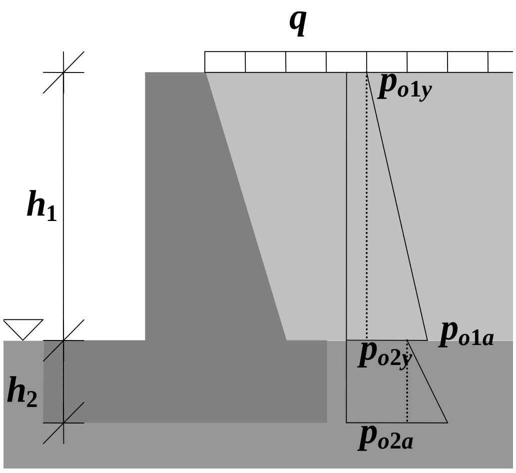


Figure 2.

Resultant of earth pressure at rest at layer *n* and the total sum of resultants (Figure 3)

$$P_{on} = p_{ony}h_n + \frac{1}{2}(p_{ona} - p_{ony})h_n \tag{7}$$

$P_{o\,1} =$ 0,198 MN/m

$P_{o\,2} =$ 0,125 MN/m

$$P_o = P_{o\,1} + P_{o\,2} \tag{8}$$

$=$ 0,323 MN/m

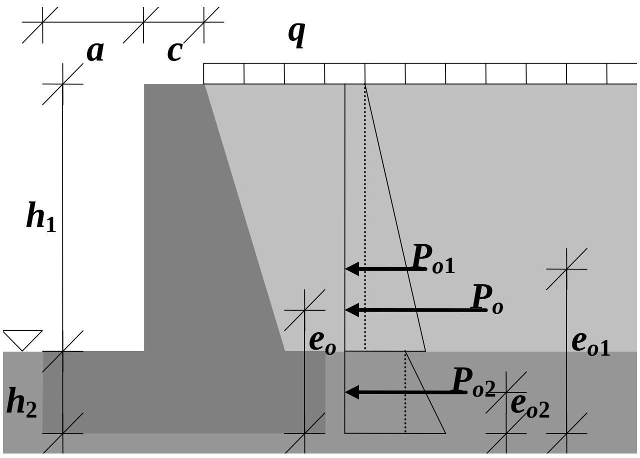


Figure 3.

Corresponding distances from the bottom of the retaining wall (Figure 3)

$$e_{on} = \frac{p_{ony}\frac{h_n}{2} + \frac{1}{2}\left(p_{ona} - p_{ony}\right)\frac{h_n}{3}}{p_{ony} + \frac{1}{2}\left(p_{ona} - p_{ony}\right)} + \begin{cases} \sum_{i=n+1}^{n_{\max}} h_i, & n = [1, \quad n_{\max} - 1] \\ 0, & n = n_{\max} \end{cases} \tag{9}$$

e_{o1} = 4,350 m

e_{o2} = 1,020 m

$$e_o = \frac{\sum_{n=1}^{n_{\max}} P_{on} e_{on}}{\sum_{n=1}^{n_{\max}} P_{on}} \tag{10}$$

= 3,059 m

ACTIVE EARTH PRESSURE

Angle of earth surface with respect to the horizontal plane (Figure 4)

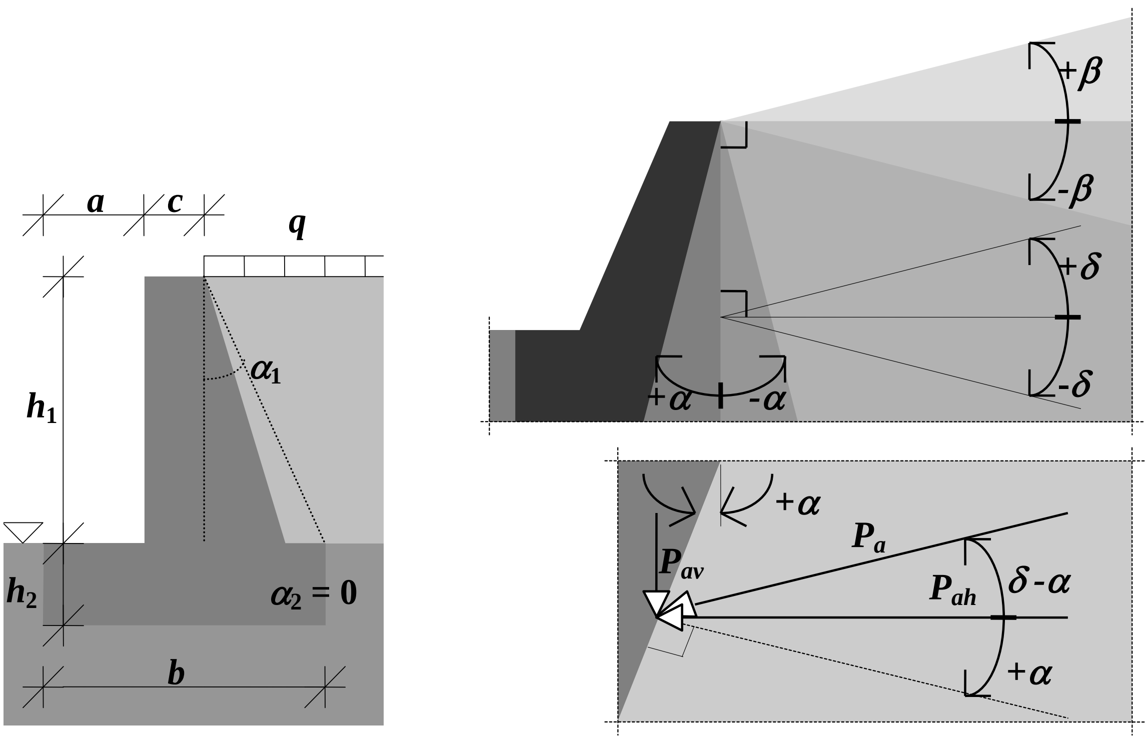
$$\beta = 0\ °$$

Angle of sliding surface with respect to the vertical plane (Figure 4)

$$\alpha_1 = -\arctan\left(\frac{b-a-c}{h_1}\right) \tag{11}$$

$$= -17{,}879\ °$$

$$\alpha_2 = 0{,}0000\ °$$



a) b)

Figure 4.

Angle of wall friction, when the sliding surface is not between the steel structure and soil

$$\delta_1 = \varphi_1 \tag{12}$$

$$= 32{,}000\ °$$

Angle of wall friction, when the sliding surface is between the concrete structure and soil

$$\delta_2 = \frac{3}{4}\varphi_2 \qquad (13)$$

$$= \quad 25{,}500\ °$$

Coefficient of the horizontal component of the active earth pressure

$$K_{ahi} = \frac{\cos^2(\varphi_i + \alpha_i)}{\cos^2\alpha_i \left(1 + \sqrt{\dfrac{\sin(\varphi_i+\delta_i)\sin(\varphi_i-\beta)}{\cos(\alpha_i-\delta_i)\cos(\alpha_i+\beta)}}\right)^2} \qquad (14)$$

$$K_{ah\,1} = \quad 0{,}293$$

$$K_{ah\,2} = \quad 0{,}229$$

Active earth pressure at the top (y) and bottom (a) of layer n, respectively,

$$\begin{cases} p_{ny} = \begin{cases} K_n q, & n = 1 \\ K_n\left(\sum_{i=1}^{n-1}\gamma_i h_i + q\right), & n \geq 2 \end{cases}, & n \in N \\ p_{na} = K_n\left(\sum_{i=1}^{n}\gamma_i h_i + q\right), & n \geq 1 \end{cases} \qquad (15a, b)$$

$$p_{1y} = K_{ah\,1} q \qquad (16)$$

$$= \quad 0{,}003521 \text{ MN/m}^2$$

$$p_{1a} = K_{ah\,1}(\gamma_1 h_1 + q) \qquad (17)$$

$$= \quad 0{,}036263 \text{ MN/m}^2$$

$$p_{2y} = K_{ah\,2}(\gamma_1 h_1 + q) \qquad (18)$$

$$= \quad 0{,}028363 \text{ MN/m}^2$$

$$p_{2a} = K_{ah\,2}(\gamma_1 h_1 + \gamma_2 h_2 + q) \qquad (19)$$

$$= \quad 0{,}033664 \text{ MN/m}^2$$

Resultant of active earth pressure at layer n and the total sum of resultants (Figure 5)

$$P_n = p_{ny}h_n + \frac{1}{2}(p_{na} - p_{ny})h_n \tag{20}$$

$P_1 =$ 0,123 MN/m

$P_2 =$ 0,065 MN/m

$$P = P_1 + P_2 \tag{21}$$

$=$ 0,188 MN/m

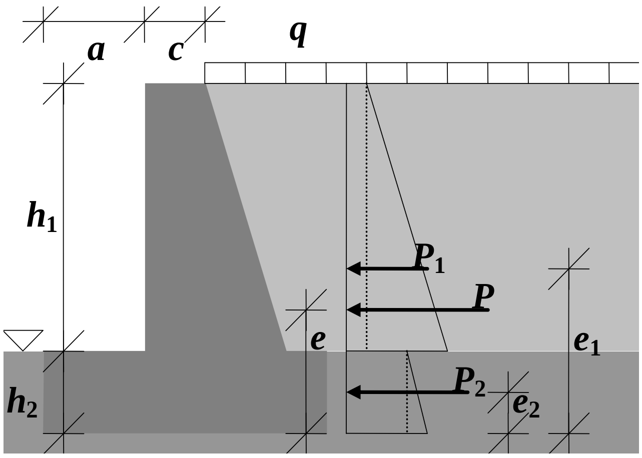


Figure 5.

Corresponding distances from the bottom of the retaining wall (Figure 5)

$$e_n = \frac{p_{ny}\dfrac{h_n}{2} + \dfrac{1}{2}\left(p_{na} - p_{ny}\right)\dfrac{h_n}{3}}{p_{ny} + \dfrac{1}{2}\left(p_{na} - p_{ny}\right)} + \begin{cases} \sum\limits_{i=n+1}^{n_{\max}} h_i, & n = [1, \quad n_{\max}] \\ 0, & n = n_{\max} \end{cases} \tag{22}$$

$e_1 =$ 4,350 m

$e_2 =$ 1,020 m

$$e = \frac{\sum\limits_{n=1}^{n_{\max}} P_n e_n}{\sum\limits_{n=1}^{n_{\max}} P_n} \tag{23}$$

$=$ 3,199 m

ANSWER:

Earth pressure resultant and the corresponding distance from the bottom of the retaining wall

Earth pressure at rest

P_o = 0,323 MN/m

e_o = 3,059 m

Active earth pressure

P = 0,188 MN/m

e = 3,199 m

Earth pressure distribution curve is shown in Figure 6.

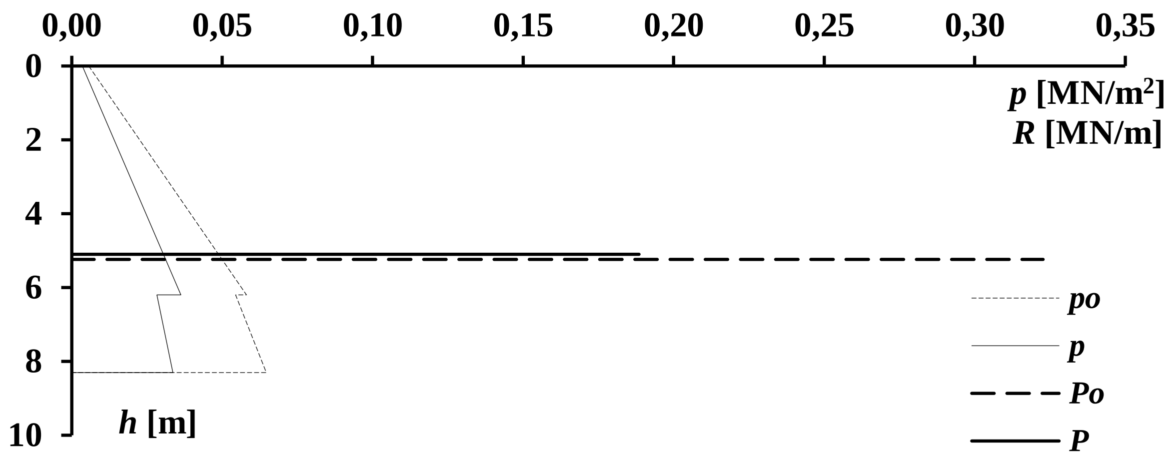


Figure 6.

11. PROBLEM

Determine the core figure of the homogenous isosceles triangle shown in Figure 1.

Dimensions

b	=	2 m
h	=	3 m

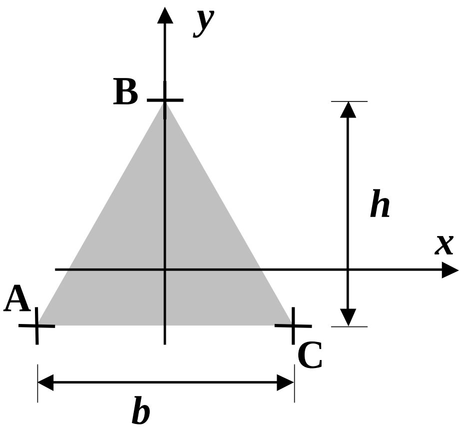


Figure 1.

The main coordinate system and the principal coordinate system directions are combined.

Cross-sectional area

$$A = bh/2 \tag{1}$$

$$= 3{,}000 \text{ m}^2$$

Moment of inertia with respect to x - and y -axis

$$I_x = \frac{bh^3}{36} \tag{2}$$

$$= 1{,}500 \text{ m}^4$$

$$I_y = \frac{hb^3}{48} \tag{3}$$

$$= 0{,}500 \text{ m}^4$$

Radius of gyration with respect to s -axis for homogenous surface area

$$i_s = \sqrt{\frac{I_s}{A}} \tag{4}$$

Thus

$$i_x^2 = \frac{I_x}{A} \tag{5}$$

$$= 0{,}500 \text{ m}^2$$

$$i_y^2 = \frac{I_y}{A} \tag{6}$$

$$= 0{,}167 \text{ m}^2$$

Coordinates of "convex corner points"

$$x_{\text{Ap}} = -b/2 \tag{7}$$

$$= -1{,}000 \text{ m}$$

$$y_{\text{Ap}} = -h/3 \tag{8}$$

$$= -1{,}000 \text{ m}$$

$$x_{\text{Bp}} = 0{,}000 \text{ m}$$

$$y_{\text{Bp}} = 2h/3 \tag{9}$$

$$= 2{,}000 \text{ m}$$

Boundary line of the core figure

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \tag{10}$$

$$\Rightarrow y(x) = \alpha x + \beta \tag{11}$$

where

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \tag{12}$$

$$\beta = -\frac{i_x^2}{y_{ip}} \tag{13}$$

Hence,

	α_ι			β_ι		
$y_A(x) =$	**-3,000**	x	+	**0,500**	**[m]**	**[m]**
$y_B(x) =$	**0,000**	x	+	**-0,250**	**[m]**	**[m]**

The x -coordinate of the intersection point of lines i and j

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \tag{14}$$

Numerical values of the intersection points are presented in Table 1.

Table 1.

ij	x_{ij} m	y_{ij} m
AB	4,000	-1,000
Ax	6,000	0,000
Ay	0,000	-3,000

ANSWER:

Boundary lines of the core figure are shown in Figure 2.

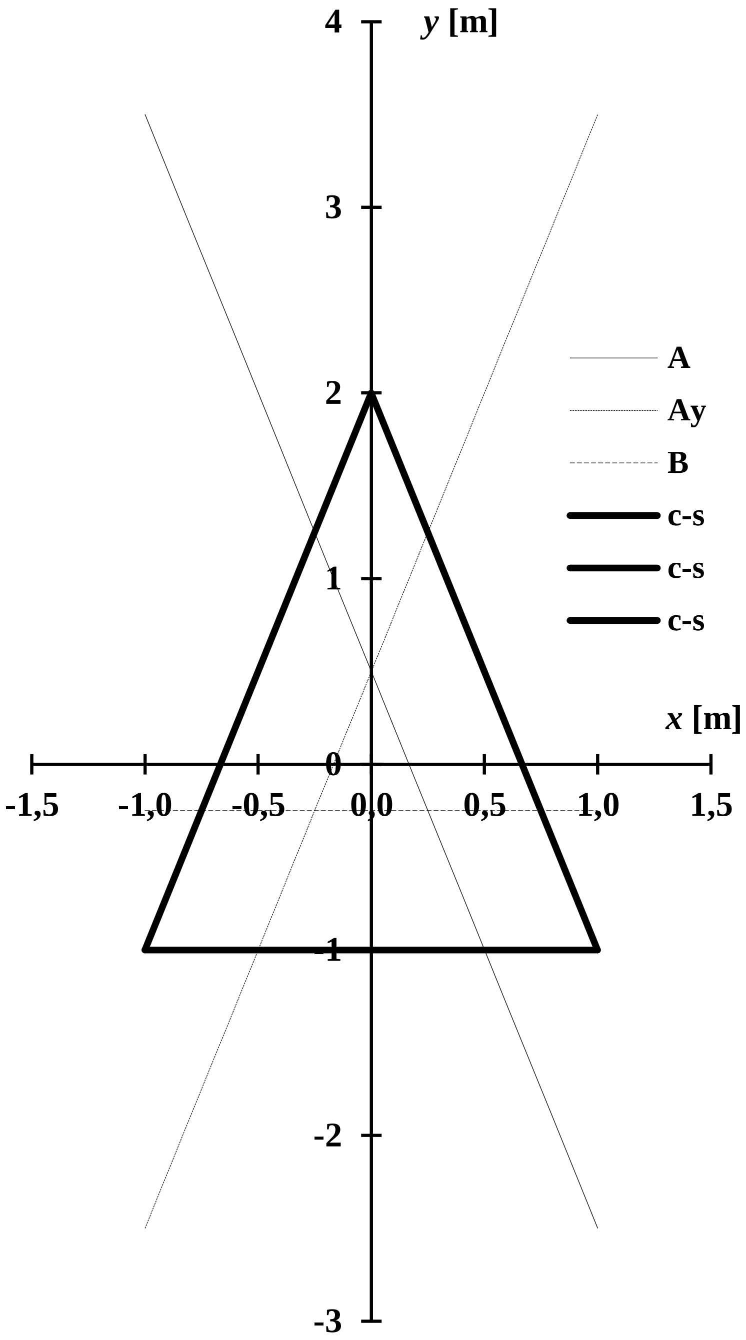


Figure 2.

12. PROBLEM

A composite base slab lying on the rock consists of a steel band, which inner and outer edges are elliptical, and casted concrete inside the steel band (Figure 1). Determine the core figure of the slab before casting and after hardening!

Dimensions

$$a = \pi \text{ m} \approx 3{,}142 \text{ m}$$
$$b = \pi/2 \text{ m} \approx 1{,}571 \text{ m}$$
$$t = 1/\pi^{\pi} \text{ m} \approx 0{,}027 \text{ m}$$

Modulus of elasticity of concrete (c) and steel (s)

$$E_c = 30\,000 \text{ MN/m}^2$$
$$E_s = 210\,000 \text{ MN/m}^2$$

Advice: Estimate the shape of the core figure by determine the core figure boundary lines corresponding to the points A, B and C shown in Figure 1. Angle

$$\alpha = \pi/8$$

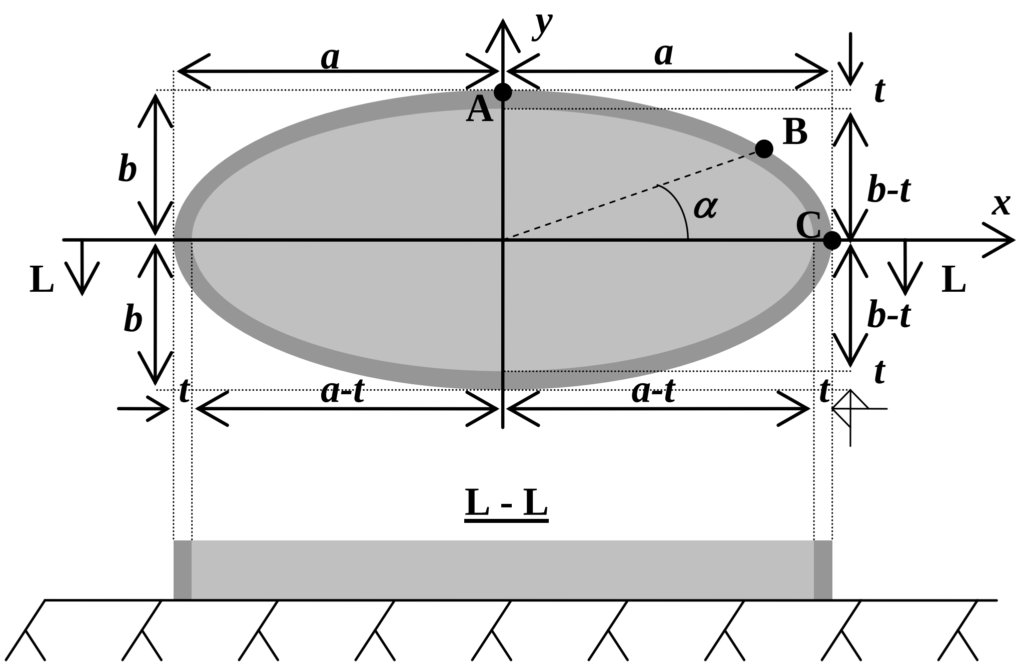


Figure 1.

Area

Concrete

$$A_c = \pi(a\text{-}t)(b\text{-}t) \quad (1)$$

$$= 15{,}099 \ \mathrm{m}^2$$

Steel

$$A_s = \pi ab\text{-}A_c \quad (2)$$

$$= 0{,}404 \ \mathrm{m}^2$$

Composite slab

$$A = A_c + A_s \quad (3)$$

$$= 15{,}503 \ \mathrm{m}^2$$

Moment of inertia with respect to *x*- and *y*- axis

Concrete

$$I_{cx} = \pi(a\text{-}t)(b\text{-}t)^3/4 \quad (4)$$

$$= 8{,}992 \ \mathrm{m}^4$$

$$I_{cy} = \pi(b\text{-}t)(a\text{-}t)^3/4 \quad (5)$$

$$= 36{,}609 \ \mathrm{m}^4$$

Steel

$$I_{sx} = \pi ab^3/4\text{-}I_{cx} \quad (6)$$

$$= 0{,}571 \ \mathrm{m}^4$$

$$I_{sy} = \pi ba^3/4\text{-}I_{cy} \quad (7)$$

$$= 1{,}644 \ \mathrm{m}^4$$

STEEL BAND

Radius of gyration with respect *x* -axis and *y*- axis power two

$$i_{sx}^2 = I_{sx}/A_s \quad (8)$$

$$= 1{,}416 \ \mathrm{m}^2$$

$$i_{sy}^2 = I_{sy}/A_s \quad (9)$$

$$= 4{,}072 \ \mathrm{m}^2$$

Slope

$$k = \tan(\alpha) \approx 0{,}414 \tag{10}$$

Coordinates of "convex corner points"

$$x_{\mathrm{Ap}} = 0{,}000 \text{ m}$$

$$y_{\mathrm{Ap}} = b = 1{,}571 \text{ m}$$

$$\begin{cases} \dfrac{x_{Bp}^2}{a^2} + \dfrac{y_{Bp}^2}{b^2} = 1 \\ y_{Bp} = k x_{Bp} \end{cases} \tag{11a, b}$$

$$\Rightarrow \quad x_{Bp} = \overset{+}{(-)}\sqrt{\frac{a^2 b^2}{k^2 a^2 + b^2}} \tag{12}$$

$$= 2{,}419 \text{ m}$$

$$y_{\mathrm{Bp}} = 1{,}002 \text{ m}$$

$$x_{\mathrm{Cp}} = a = 3{,}142 \text{ m}$$

$$y_{\mathrm{Cp}} = 0{,}000 \text{ m}$$

Boundary line of the core figure

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \tag{13}$$

$$\Rightarrow y(x) = \alpha x + \beta \tag{14}$$

where

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \tag{15}$$

$$\beta = -\frac{i_x^2}{y_{ip}} \tag{16}$$

Hence,

	α_ι			β_ι	
$y_{As}(x)$ =	0,000	x	+	-0,901 [m]	[m]
$y_{Bs}(x)$ =	-0,839	x	+	-1,413 [m]	[m]

When $y_{ip} = 0$,

$$x = -i_y^{\,2}/x_{ip} \tag{17}$$

$$x_{Cs} = -1{,}296 \text{ m}$$

The x **-coordinate of the intersection point of lines** i **and** j

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \tag{18}$$

Numerical values of the intersection points are presented in Table 1.

Table 1.

ij	x_{ij} m	y_{ij} m
AB	-1,641	1,479
BC	-1,296	0,992

Let's assume, that the core figure is an ellipse, which half rays are

$$n_s = -x_{Cs}$$

$$= 1{,}296 \text{ m}$$

$$m_s = -\beta_{As}$$

$$= 0{,}901 \text{ m}$$

An intersection of the core figure boundary line corresponding to point B and the ellipse mentioned above

$$\begin{cases} \dfrac{x_o^2}{n_s^2} + \dfrac{y_o^2}{m_s^2} = 1 \\ y_o = \alpha_B x_o + \beta_B \end{cases} \tag{19a, b}$$

$$\Rightarrow x_o = \frac{-n_s^2 \alpha_B \beta_B \pm n_s m_s \sqrt{n_s^2 \alpha_B^2 + m_s^2 - \beta_B^2}}{m_s^2 + n_s^2 \alpha_B^2} \tag{20}$$

x_o = -0,998 m
y_o = -0,575 m

Tangent of the ellipse at this point is

$$\frac{x_o}{n_s^2}x+\frac{y_o}{m_s^2}y=1 \tag{21}$$

$$\Rightarrow y=-\frac{m_s^2 x_o}{n_s^2 y_o}x+\frac{m_s^2}{y_o} \tag{22}$$

= -0,839 x + -1,413 [m]

which is same as core figure boundary line $y_B(x)$ obtained above.

<u>ANSWER</u>: Boundary lines of the core figure are shown in Figure 2.

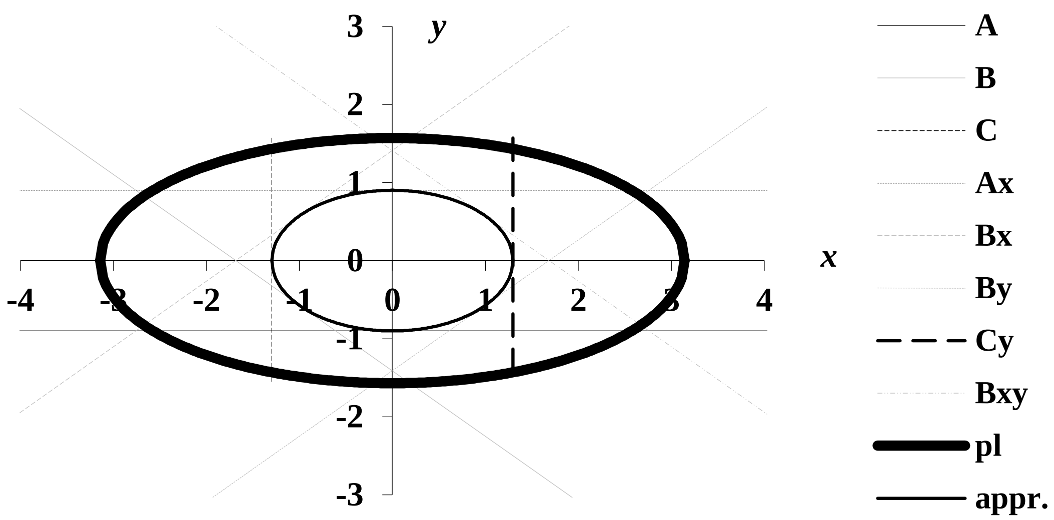


Figure 2.

Note! The core figure is not similar with inner or outer edge of the steel band:

a/b = 2,000
$(a\text{-}t)/(b\text{-}t)$ = 2,018
n_s/m_s = 1,438

COMPOSITE SLAB

Axial stiffness

Concrete

$$C_c = E_c A_c \tag{23}$$

$$= 452\,984 \text{ MN}$$

Steel

$$C_s = E_s A_s \tag{24}$$

$$= 84\,768 \text{ MN}$$

Composite slab

$$C = C_c + C_s \tag{25}$$

$$= 537\,753 \text{ MN}$$

Bending stiffness

Concrete

$$D_{cx} = E_c I_{cx} \tag{26}$$

$$= 269\,751 \text{ MNm}^2$$

$$D_{cy} = E_c I_{cy} \tag{27}$$

$$= 1\,098\,265 \text{ MNm}^2$$

Steel

$$D_{sx} = E_s I_{sx} \tag{28}$$

$$= 119\,994 \text{ MNm}^2$$

$$D_{sy} = E_s I_{sy} \tag{29}$$

$$= 3{,}45\text{E}{+}05 \text{ MNm}^2$$

Composite slab

$$D_x = D_{cx} + D_{sx} \tag{30}$$

$$= 389\,746 \text{ MNm}^2$$

$$D_y = D_{cy} + D_{sy} \tag{31}$$

$$= 1{,}44\text{E}{+}06 \text{ MNm}^2$$

Radius of gyration with respect *x* -axis and *y*- axis power two

$$i_x^{\,2} = D_x / C \tag{32}$$

$$= 0{,}725 \text{ m}^2$$

$$i_y^{\,2} = D_y / C \qquad (33)$$
$$= \quad 2{,}684 \text{ m}^2$$

Boundary lines of the core figure

	α_ι		β_ι	
$y_A(x) =$	$0{,}000\ x$	+	$-0{,}461$ [m]	[m]
$y_B(x) =$	$-0{,}652\ x$	+	$-0{,}723$ [m]	[m]

When $y_{ip} = 0$,

$$x_C = \quad -0{,}854 \text{ m}$$

Numerical values of the intersection points are presented in Table 2.

Table 2.

ij	x_{ij} m	y_{ij} m
AB	-2,489	1,149
BC	-0,854	-0,034

Let's assume, that the core figure is an ellipse, which half rays are

$$n = -x_C$$
$$= \quad 0{,}854 \text{ m}$$
$$m = -\beta_A$$
$$= \quad 0{,}461 \text{ m}$$

An intersection of the core figure boundary line corresponding to point B and the ellipse mentioned above

$$\begin{cases} x_o = \dfrac{-n^2\alpha_B\beta_B \pm nm\sqrt{n^2\alpha_B^2 + m^2 - \beta_B^2}}{m^2 + n^2\alpha_B^2} \\ y_o = \alpha_B x_o + \beta_B \end{cases} \qquad (34a, b)$$

$$\Rightarrow \begin{cases} x_o = & -0{,}658 \text{ m} \\ y_o = & -0{,}294 \text{ m} \end{cases}$$

Tangent of the ellipse at this point is

$$y = \frac{-m^2 x_o}{n^2 y_o} x + \frac{m^2}{y_o} \tag{35}$$

$= \quad -0{,}652\ x \quad + \quad -0{,}723\ [\mathrm{m}]$

which is same as core figure boundary line $y_B(x)$ obtained above.

ANSWER: **Boundary lines of the core figure are shown in Figure 3.**

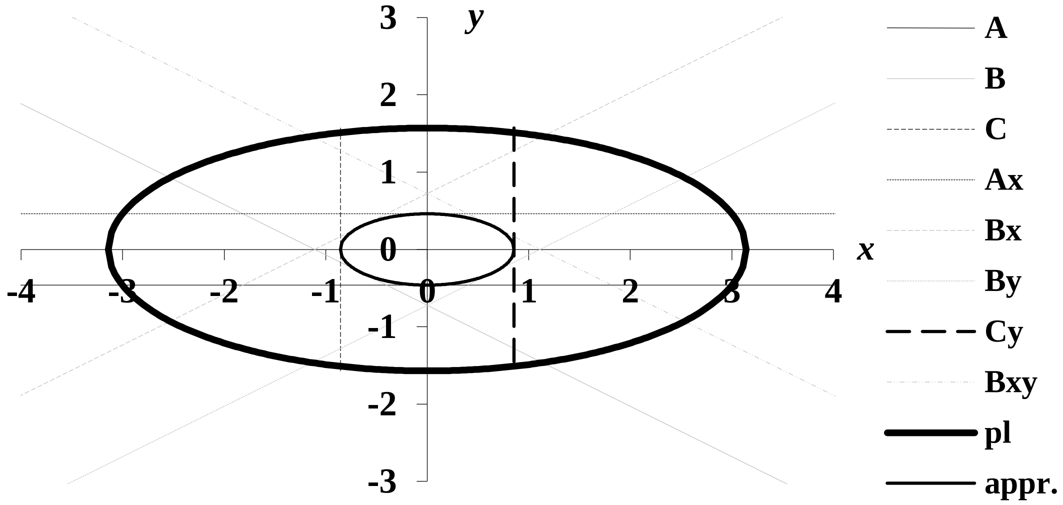


Figure 3.

Note! The core figure is not similar with inner or outer edge of the steel band:

$a/b \;=\; 2{,}000$

$(a\text{-}t)/(b\text{-}t) \;=\; 2{,}018$

$n/m \;=\; 1{,}852$

13. PROBLEM

Determine the area, where vertical force can be acting so that all piles are compressed (core figure)! The piles are in vertical direction and they all have equal cross-section, material and length. See Figure 1 and Table 1.

In the answer use principal coordinate system (ψ, ζ) or center-of-mass coordinate system (y, z) parallel to the given coordinate system (y_0, z_0)!

Dimensions

a = 1,5 m

d = 1,5 m

Table 1. Coordinates of the top of the piles.

Symbol	i	z_{0i}	y_{0i}
Unit	-	m	m
Numerical value	1	-1,500	0,750
	2	1,500	0,750
	3	0,000	-0,750
	4	-1,500	-0,750

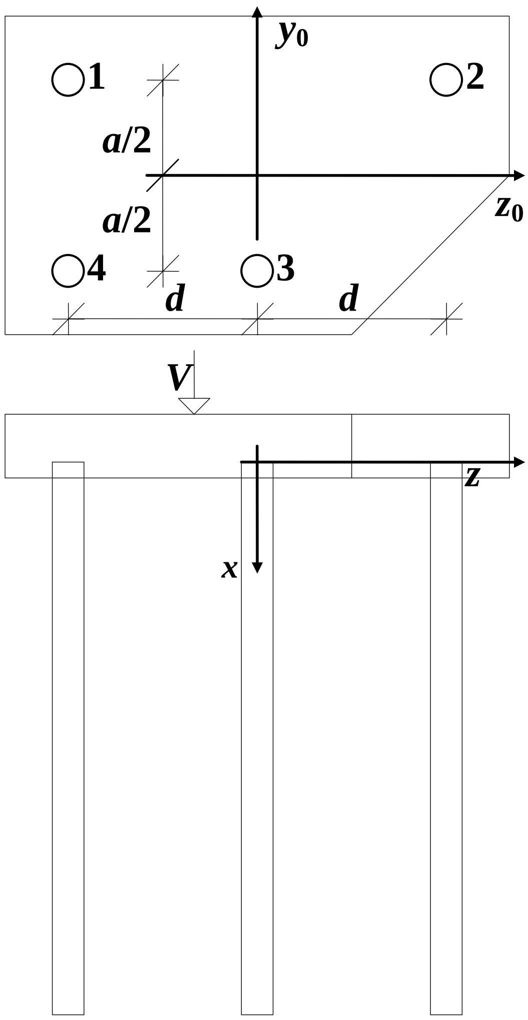


Figure 1.

It is supposed, that moments and product of inertia with respect to centroid line of any pile are small and therefore can be neglected.

$$I_{z'i} = 0 \tag{1}$$

$$I_{y'i} = 0 \tag{2}$$

$$I_{y'z'i} = 0 \tag{3}$$

Cross-section area of pile *i* (without dimension)

$$A_i = 1 \tag{4}$$

Cross-section area of the pile group

$$A = 4A_i \tag{5}$$

$$= 4$$

Center of gravity (centroid)

$$z_{0p} = \Sigma z_i A_i / A \tag{6}$$

$$= -0{,}375 \text{ m}$$

$$y_{0p} = \Sigma y_i A_i / A \tag{7}$$

$$= 0{,}000 \text{ m}$$

***z*-coordinate of piles in the center-of-mass coordinate system (*y*-*z*)**

$$z_i = z_{0i} - z_{0p} \tag{8}$$

$$z_1 = -1{,}125 \text{ m}$$

$$z_2 = 1{,}875 \text{ m}$$

$$z_3 = 0{,}375 \text{ m}$$

$$z_4 = -1{,}125 \text{ m}$$

***y*-coordinate of piles in the center-of-mass coordinate system (*y*-*z*)**

$$y_i = y_{0i} - y_{0p} \tag{9}$$

$$y_1 = 0{,}750 \text{ m}$$

$$y_2 = 0{,}750 \text{ m}$$

$$y_3 = -0{,}750 \text{ m}$$

$$y_4 = -0{,}750 \text{ m}$$

Modulus of inertia of the piles with respect to *z*-axis (Steiner's rule)

$$I_{zi} = I_{z'i} + A_i y_{pi}^{\ 2} \tag{10}$$

$$I_{z1} = 0{,}563 \text{ m}^2$$

$$I_{z2} = 0{,}563 \text{ m}^2$$

$$I_{z3} = 0{,}563 \text{ m}^2$$

$$I_{z4} = 0{,}563 \text{ m}^2$$

Modulus of inertia of the piles with respect to y-axis

$$I_{yi} = I_{y'i} + A_i z_{pi}^2 \tag{11}$$

$I_{y1} =$ 1,266 m^2

$I_{y2} =$ 3,516 m^2

$I_{y3} =$ 0,141 m^2

$I_{y4} =$ 1,266 m^2

Product of inertia of the piles in the center-of-mass coordinate system (y, z)

$$I_{yzi} = I_{y'z'i} + A_i y_{pi} z_{pi} \tag{12}$$

$I_{yz1} =$ -0,844 m^2

$I_{yz2} =$ 1,406 m^2

$I_{yz3} =$ -0,281 m^2

$I_{yz4} =$ 0,844 m^2

Moment and product of inertia for the pile group

$$I_z = \Sigma I_{zi} \tag{13}$$

$=$ 2,250 m^2

$$I_y = \Sigma I_{yi} \tag{14}$$

$=$ 6,188 m^2

$$I_{yz} = \Sigma I_{yzi} \tag{15}$$

$=$ 1,125 m^2

Angle of principal direction

$$\alpha = \frac{1}{2}\arctan\frac{2I_{yz}}{I_y - I_z} \tag{16}$$

$=$ 0,260 rad

$=$ 14,872 °

Principal moments of inertia

$$I_\zeta = I_z \cos^2\alpha + I_y \sin^2\alpha - I_{yz}\sin(2\alpha) \tag{17}$$

$=$ 1,951 m^2

$$I_\psi = I_z \sin^2\alpha + I_y \cos^2\alpha + I_{yz}\sin(2\alpha) \tag{18}$$

$=$ 6,486 m^2

Square of radius of gyration with respect to ζ- and ψ-axis

$$i_\zeta^2 = I_\zeta / A \tag{19}$$

$=$ 0,488 m^2

$$i_\psi^{\ 2} = I_\psi / A \tag{20}$$

$$= 1{,}622 \text{ m}^2$$

ζ-coordinates of piles

$$\zeta_i = y_i \sin\alpha + z_i \cos\alpha \tag{21}$$

$\zeta_1 =$ -0,895 m

$\zeta_2 =$ 2,005 m

$\zeta_3 =$ 0,170 m

$\zeta_4 =$ -1,280 m

ψ-coordinates of piles

$$\psi_i = y_i \cos\alpha - z_i \sin\alpha \tag{22}$$

$\psi_1 =$ 1,014 m

$\psi_2 =$ 0,244 m

$\psi_3 =$ -0,821 m

$\psi_4 =$ -0,436 m

Equation of core figure boundary line in the principal coordinate system

$$0 = 1 + \frac{\varsigma_{ip}}{i_\psi^2}\varsigma + \frac{\psi_{ip}}{i_\varsigma^2}\psi \tag{23}$$

$$\Rightarrow \psi(\varsigma) = A\varsigma + B \tag{24}$$

where

$$A = -\frac{\varsigma_{ip} i_\varsigma^2}{\psi_{ip} i_\psi^2} \tag{25}$$

$$B = -\frac{i_\varsigma^2}{\psi_{ip}} \tag{26}$$

By inserting numerical values

	B_ι			A_ι	
$\psi_1(\zeta) =$	0,266	ζ	+	-0,481	[m]
$\psi_2(\zeta) =$	-2,475	ζ	+	-2,002	[m]
$\psi_3(\zeta) =$	0,062	ζ	+	0,594	[m]
$\psi_4(\zeta) =$	-0,883	ζ	+	1,119	[m]

ζ-coordinate of the intersection point of lines i and j is

$$\zeta_{ij} = \frac{B_i - B_j}{A_j - A_i} \tag{27}$$

Numerical values of the intersection points are presented in Table 2.

Table 2.

ij	ζ_{ij} m	ψ_{ij} m	z_{ij} m	y_{ij} m
12	-0,555	-0,629	-0,375	-0,750
23	-1,023	0,530	-1,125	0,250
34	0,555	0,629	0,375	0,750
41	1,393	-0,111	1,375	0,250

Equation of core figure boundary line in the center-of-mass coordinate system

$$y(z) = Cz + D \tag{28}$$

where

$$C = \tan(\alpha + \arctan A) \tag{29}$$

$$D = B(\cos\alpha + C\sin\alpha) \tag{30}$$

By inserting numerical values

	C_i			D_i	
$y_1(z) =$	0,571	z	+	-0,536	[m]
$y_2(z) =$	-1,333	z	+	-1,250	[m]
$y_3(z) =$	0,333	z	+	0,625	[m]
$y_4(z) =$	-0,500	z	+	0,938	[m]

The z-coordinate of intersection point of lines i and j

$$z_{ij} = \frac{D_i - D_j}{C_j - C_i} \tag{31}$$

Numerical values of intersection points are expressed in Table 2.

ANSWER: **Boundary lines 1, 2, 3 and 4 of the core figure are shown in Figures 2 and 3, respectively.**

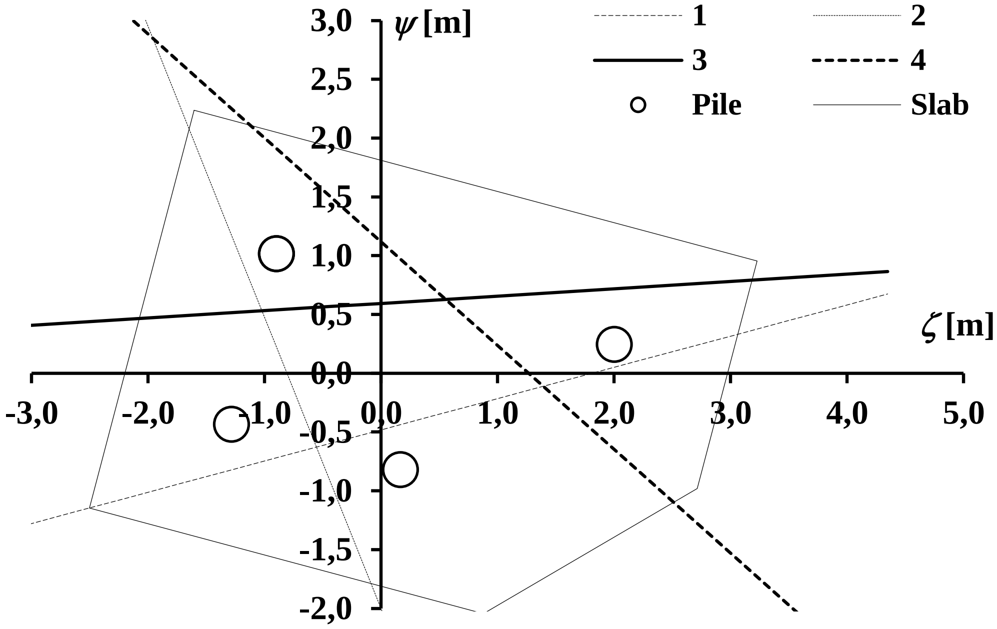


Figure 2. Principal coordinate system.

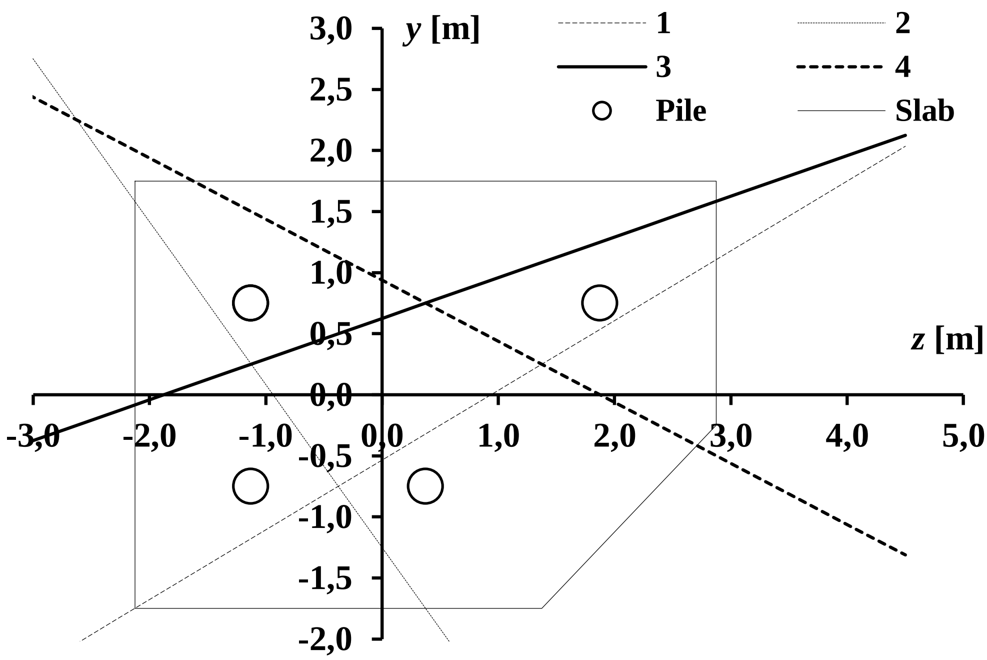


Figure 3. Center-of-mass coordinate system.

ALTERNATIVE WAY (Compare Problem number 29.)

Centroid is calculated as above

z_{0p} = -0,375 m

y_{0p} = 0,000 m

Table 3.

Variable	Pile	Coordinates			Angle	Stiffness
Symbol	i	x_i	y_i	z_i	α_i	k_i
Unit		m	m	m	°	-
Numerical value	1	0	0,750	-1,125	0	1
	2	0	0,750	1,875	0	1
	3	0	-0,750	0,375	0	1
	4	0	-0,750	-1,125	0	1

⋮

Variable	Pile	Cosine of directional angles			Moment arms		
Symbol	i	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Unit		-	-	-	m	m	m
Numerical value	1	1,000	0,000	0,000	0,000	-1,125	-0,750
	2	1,000	0,000	0,000	0,000	1,875	-0,750
	3	1,000	0,000	0,000	0,000	0,375	0,750
	4	1,000	0,000	0,000	0,000	-1,125	0,750

⋮

Variable	Pile	Elements of stiffness matrix of the piles					
Symbol	i	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Unit	1	1,000	-1,125	-0,750	1,266	0,844	0,563
Numerical	2	1,000	1,875	-0,750	3,516	-1,406	0,563
value	3	1,000	0,375	0,750	0,141	0,281	0,563
	4	1,000	-1,125	0,750	1,266	-0,844	0,563
Unit		MN/m	MN	MN	MNm	MNm	MNm
Numerical value		4,000	0,000	0,000	6,188	-1,125	2,250
Symbol		k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Variable		Elements of stiffness matrix of pile group					

Elements of the stiffness matrix are calculated in Table 3.

$$[K]=\begin{bmatrix} 4{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 6{,}188 & -1{,}125 \\ 0{,}000 & -1{,}125 & 2{,}250 \end{bmatrix} \quad \begin{bmatrix} \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN} & \text{MN} & \text{MNm} \end{bmatrix}$$

Inverse matrix

$$[K]^{-1}=\begin{bmatrix} 0{,}250 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}178 & 0{,}089 \\ 0{,}000 & 0{,}089 & 0{,}489 \end{bmatrix} \quad \begin{bmatrix} \text{m/N} & \text{m/N} & \text{1/N} \\ \text{m/N} & \text{m/N} & \text{1/N} \\ \text{1/N} & \text{1/N} & \text{1/Nm} \end{bmatrix}$$

From equilibrium condition

$$\{F\} = [K]\{\delta\} \tag{32}$$

the displacement vector

$$\{\delta\} = [K]^{-1}\{F\} \tag{33}$$

$$\Rightarrow \begin{Bmatrix} u \\ \varphi \\ \theta \end{Bmatrix} = [K]^{-1} \begin{Bmatrix} 1 \\ z \\ -y \end{Bmatrix} \quad \begin{matrix} (x\text{-axial vertical force [MN]}) \\ (\text{Moment with respect to } y\text{-axis [MNm]}) \\ (\text{Moment with respect to } z\text{-axis [MNm]}) \end{matrix} \tag{34}$$

$$\Rightarrow \begin{cases} u = & 0{,}000\, y & + & 0{,}000\, z & + & 0{,}250 \\ \varphi = & -0{,}089\, y & + & 0{,}178\, z & + & 0{,}000 \\ \theta = & -0{,}489\, y & + & 0{,}089\, z & + & 0{,}000 \end{cases}$$

is obtained.

Pile force

$$N_i = k_i \Delta_i \tag{35}$$

$$= k_i (p_{x,i}\, u + r_{y,i}\, \varphi + r_{z,i}\, \theta) \tag{36}$$

$$\begin{matrix} N_1 = & 0{,}467\, y & + & -0{,}267\, z & + & 0{,}250 & = 0 \\ N_2 = & 0{,}200\, y & + & 0{,}267\, z & + & 0{,}250 & = 0 \\ N_3 = & -0{,}400\, y & + & 0{,}133\, z & + & 0{,}250 & = 0 \\ N_4 = & -0{,}267\, y & + & -0{,}133\, z & + & 0{,}250 & = 0 \end{matrix}$$

Equations of core figure boundary line in the center-of-mass coordinate system

$$\begin{matrix} y_1(z) = & 0{,}571\, z & + & -0{,}536 \text{ [m]} \\ y_2(z) = & -1{,}333\, z & + & -1{,}250 \text{ [m]} \\ y_3(z) = & 0{,}333\, z & + & 0{,}625 \text{ [m]} \\ y_4(z) = & -0{,}500\, z & + & 0{,}938 \text{ [m]} \end{matrix}$$

ANSWER: As above (Figure 3).

14. PROBLEM

Determine the core figure of the two-component base slab shown in Figure 1.

Strength of concrete

$K_1 =$ 60 MN/m^2

$K_2 =$ 20 MN/m^2

Dimensions are

$a =$ 1 m

$b =$ 3 m

$c =$ 1 m

$d =$ 3 m

$e =$ 4 m

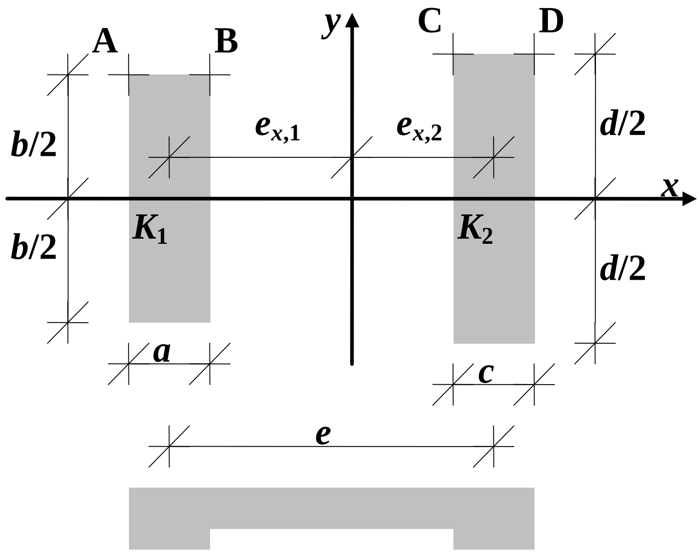


Figure 1.

Modulus of elasticity of concrete

$$E_i = k\sqrt{K_i K_0} \tag{1}$$

where k = 1 and $K_0 = 25 \cdot 10^6$ MN/m^2. Modus of elasticity of part i

$$E_1 = 38730 \text{ MN/m}^2$$

$$E_2 = 22361 \text{ MN/m}^2$$

Cross-sectional area of part i

$$A_1 = ab \tag{2}$$

$$= 3{,}000 \text{ m}^2$$

$$A_2 = cd \tag{3}$$

$$= 3{,}000 \text{ m}^2$$

Axial stiffness of part i

$$C_i = E_i A_i \tag{4}$$

$$C_1 = 116\,190 \text{ MN}$$

$$C_2 = 67\,082 \text{ MN}$$

Axial stiffness for the whole cross-section

$$C = \sum_{i=1}^{n} C_i \tag{5}$$

$$= 183\,272 \text{ MN}$$

The main coordinate system and the principal coordinate system directions are combined. Origin of the coordinate system is located at the center of gravity weighted by modulus of elasticity.

Moment of inertia of a sub-section with respect to its own x'-axis going through the the sub-section's center of gravity and being parallel with the principle axis x of the whole cross-section

$$I_{x',1} = \frac{ab^3}{12} \tag{6}$$

$$= 2{,}250 \text{ m}^4$$

$$I_{x',2} = \frac{cd^3}{12} \tag{7}$$

$$= 2{,}250 \text{ m}^4$$

Correspondingly, with respect to y'-axis,

$$I_{y',1} = \frac{ba^3}{12} \tag{8}$$

$$= \quad 0{,}250 \ \mathrm{m}^4$$

$$I_{y',2} = \frac{dc^3}{12} \tag{9}$$

$$= \quad 0{,}250 \ \mathrm{m}^4$$

Steiner's rule: moment of inertia with respect to an arbitrary r-axis for sub-section i is

$$I_{r,i} = I_{r',i} + A_i e_{s,i}^{\,2} \tag{10}$$

where $I_{r',i}$ is moment of inertia with respect to r'-axis, which is parallel to the x-axis going through the center of gravity, and $e_{s,i}$ is perpendicular distance between r- and r'-axis.

Correspondingly, for a composite cross-section, the bending stiffness is

$$D_{r,i} = D_{r',i} + C_i e_{s,i}^{\,2} \tag{11}$$

Bending stiffness of the sub-sections with respect to x'- and y'-axis, respectively,

$$D_{r',i} = E_i I_{r',i} \tag{12}$$

$$D_{x',1} = \quad 87\,142 \ \mathrm{MNm}^2$$

$$D_{x',2} = \quad 50\,312 \ \mathrm{MNm}^2$$

$$D_{y',1} = \quad 9\,682 \ \mathrm{MNm}^2$$

$$D_{y',2} = \quad 5\,590 \ \mathrm{MNm}^2$$

Bending moment equilibrium condition with respect to y-axis

$$C_1 e_{x,1} = C_2 e_{x,2} \tag{13}$$

$$\Rightarrow C_1 e_{x,1} = C_2 (e - e_{x,1}) \tag{14}$$

Distances between y-axis and center of gravity of sub-section i

$$e_{x,1} = \frac{eC_2}{C_1 + C_2} \tag{15}$$

$$= \quad 1{,}464 \ \mathrm{m}$$

$$e_{x,2} = e - e_{x,1} \tag{16}$$

$$= \quad 2{,}536 \ \mathrm{m}$$

Bending stiffness of the sub-sections with respect to x - and y -axis, respectively, by using Equation 11

$D_{x,1}$ = 87 142 MNm^2
$D_{x,2}$ = 50 312 MNm^2
$D_{y,1}$ = 258 746 MNm^2
$D_{y,2}$ = 436 980 MNm^2

Bending stiffness for the whole cross-section

$$D_r = \sum_{i=1}^{n} D_{r,i} \tag{17}$$

D_x = 137 454 MNm^2
D_y = 695 726 MNm^2

For a homogenous cross-section, the radius of gyration is

$$i_s = \sqrt{\frac{I_s}{A}} \tag{18}$$

Correspondingly, for a composite cross-section,

$$i_s = \sqrt{\frac{D_s}{C}} \tag{19}$$

Thus

$$i_x^2 = \frac{D_x}{C} \tag{20}$$

= 0,750 m^2

$$i_y^2 = \frac{D_y}{C} \tag{21}$$

= 3,796 m^2

Coordinates of the "convex corner points"

$$x_{Ap} = -e_{x,1} - a/2 \tag{22}$$

= -1,964 m

$$y_{Ap} = b/2 \tag{23}$$

= 1,500 m

$$x_{Bp} = -e_{x,1} + a/2 \tag{24}$$

= -0,964 m

$$y_{Bp} = b/2 \tag{25}$$

= 1,500 m

$$x_{Cp} = e_{x,2} - c/2 \tag{26}$$
$$= 2{,}036 \text{ m}$$
$$y_{Cp} = d/2 \tag{27}$$
$$= 1{,}500 \text{ m}$$
$$x_{Dp} = e_{x,2} + c/2 \tag{28}$$
$$= 3{,}036 \text{ m}$$
$$y_{Dp} = d/2 \tag{29}$$
$$= 1{,}500 \text{ m}$$

Equation of core figure boundary line

$$0 = 1 + \frac{x_{ip}}{i_y^2} x + \frac{y_{ip}}{i_x^2} y \tag{30}$$

$$\Rightarrow y(x) = \alpha x + \beta \tag{31}$$

where

$$\alpha = -\frac{x_{ip}}{y_{ip}} \frac{i_x^2}{i_y^2} \tag{32}$$

$$\beta = -\frac{i_x^2}{y_{ip}} \tag{33}$$

By inserting numerical values

	α_i			β_i	
$y_A(x) =$	0,259	x	+	-0,500	[m]
$y_B(x) =$	0,127	x	+	-0,500	[m]
$y_C(x) =$	-0,268	x	+	-0,500	[m]
$y_D(x) =$	-0,400	x	+	-0,500	[m]

The x-coordinate of the intersection point of lines i and j

$$x_{ij} = \frac{\beta_i - \beta_j}{\alpha_j - \alpha_i} \tag{34}$$

Numerical values of the intersection points are presented in Table 1.

Table 1.

ij	x_{ij} m	y_{ij} m
A*x*	1,933	0,000
B*x*	3,937	0,000
C*x*	-1,865	0,000
D*x*	-1,250	0,000
AB	0,000	-0,500
BC	0,000	-0,500
CD	0,000	-0,500

<u>ANSWER</u>:

Boundary lines of the core figure are shown in Figure 2.

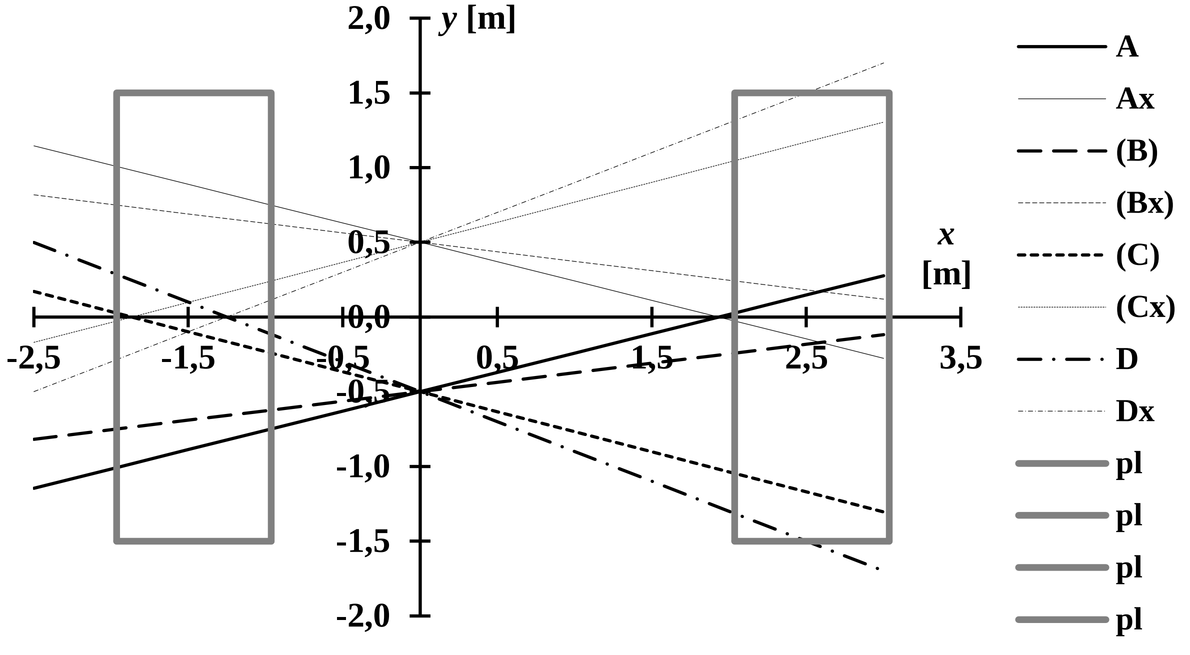


Figure 2.

15. PROBLEM

A multi-component bar consists of n nested pipes. At cross-section (x-y -plane) inner and outer edges of each bands are ellipses and the principal coordinate system directions in any part are congruent (Figure 1). The innermost area is solid.

Half ray, in direction of x - and y- axis, respectively, of the outer edge of band i is

$$a_i > 0 \wedge r_i \in R$$
$$b_i > 0 \wedge r_i \in R$$

The corresponding modulus of elasticity valid in that area is

$$E_i \geq 0 \wedge E_i \in R$$

If $E_i = 0$, no material exists. Here $i \in N \wedge i \in \{1 \ldots n\}$.

Determine the core figure of the cross-section and apply the solution to a homogenous bar and to an empty circular tube with infinitely thin wall thickness!

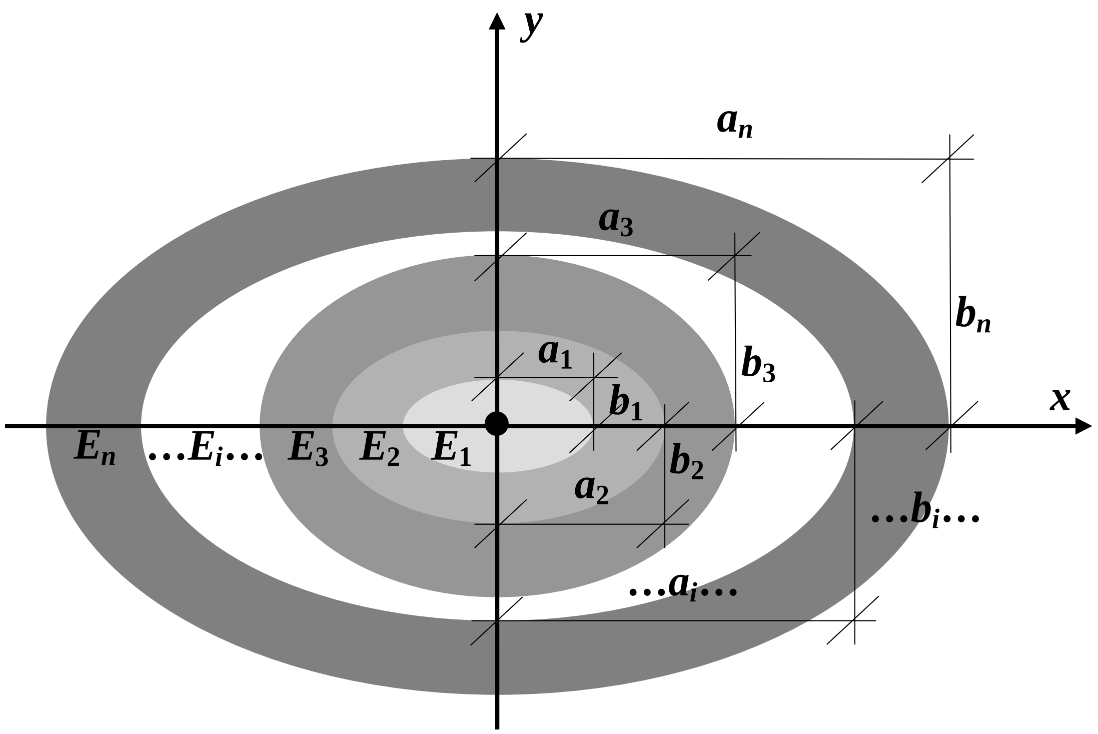


Figure 1.

Equation of the ellipse is (Figure 2)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{1}$$

Equation of core figure boundary line, or neutral axis corresponding to load acting at point (x_p, y_p) is

$$1+\frac{x_p}{i_y^2}x+\frac{y_p}{i_x^2}y=0 \tag{2}$$

$$\Rightarrow \qquad y=\alpha x+\beta \tag{3}$$

where

$$\alpha=-\frac{x_p}{y_p}\frac{i_x^2}{i_y^2} \tag{4}$$

$$\beta=-\frac{i_x^2}{y_p} \tag{5}$$

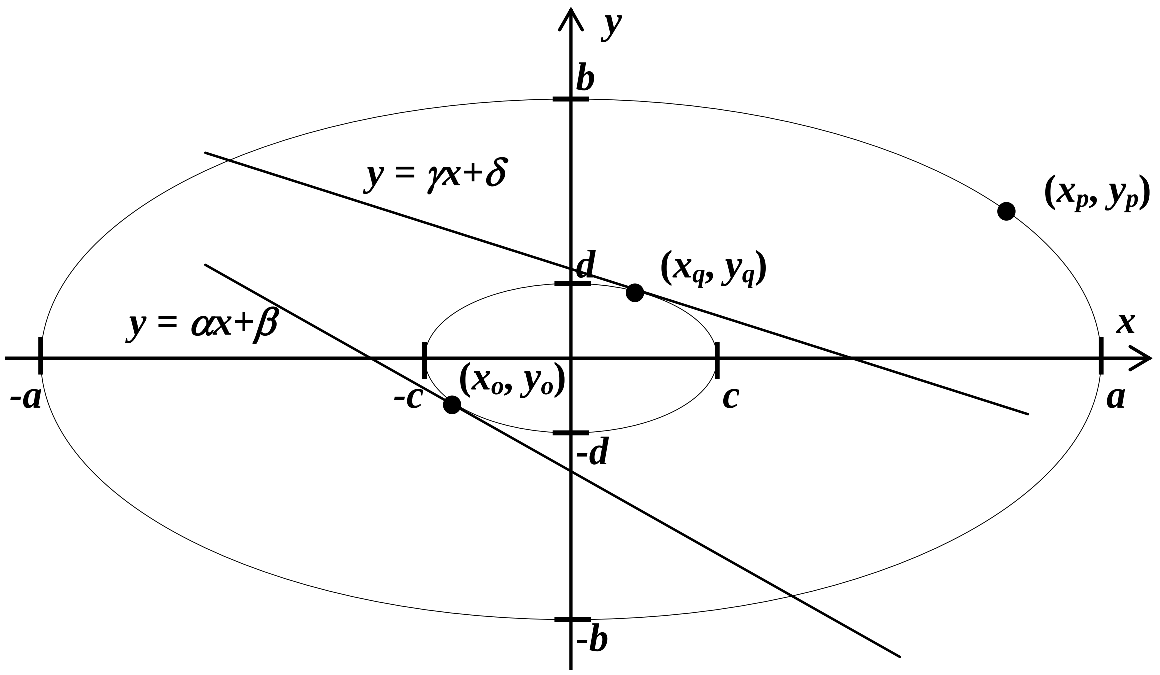


Figure 2.

Let's assume, that the shape of the core figure of ellipse area (boundary given in Equation 1) is ellipse

$$\frac{x^2}{c^2}+\frac{y^2}{d^2}=1 \tag{6}$$

Tangent of that core figure ellipse at point (x_q, y_q) is

$$\frac{x_q}{c^2}x + \frac{y_q}{d^2}y = 1 \tag{7}$$

$$\Rightarrow \qquad y = \gamma x + \delta \tag{8}$$

where

$$\gamma = -\frac{x_q}{y_q}\frac{d^2}{c^2} \tag{9}$$

$$\delta = \frac{d^2}{y_q} \tag{10}$$

Equation of core figure boundary line of ellipse (Equation 1) corresponding to point (a, 0) is

$$1 + \frac{a}{i_y^2}x = 0 \tag{11}$$

where from half ray of the core figure ellipse, parallel to x- axis, at point (-c, 0) is obtained

$$c = \frac{i_y^2}{a} \tag{12}$$

Respectively, core figure boundary line corresponding to point (0, b) is

$$1 + \frac{b}{i_x^2}y = 0 \tag{13}$$

where from half ray of the core figure ellipse, parallel to y- axis, at point (0, -d) is obtained

$$d = \frac{i_x^2}{b} \tag{14}$$

By using half ray c and d the Equations 4 and 5 can be written in form

$$\alpha = -\frac{x_p}{y_p}\frac{bd}{ac} \tag{15}$$

$$\beta = -\frac{bd}{y_p} \tag{16}$$

Equation of core figure boundary line (Equation 3) of ellipse (Equation 1) corresponding to point (x_p, y_p) and core figure ellipse (Equation 6) have an intersection (x_o, y_o)

$$\begin{cases} \dfrac{x_o^2}{c^2} + \dfrac{y_o^2}{d^2} = 1 \\ y_o = \alpha x_o + \beta \end{cases} \qquad (17a, b)$$

$$\Rightarrow \begin{cases} x_o = \dfrac{-c^2\alpha\beta \pm cd\sqrt{q}}{d^2 + c^2\alpha^2} \\ y_o = \dfrac{d^2\beta \pm cd\sqrt{q}}{d^2 + c^2\alpha^2} \end{cases} \qquad (18a, b)$$

where

$$q = d^2 + c^2\alpha^2 - \beta^2 \qquad (19)$$

$$\Rightarrow q = d^2 + c^2\left(\frac{x_p}{y_p}\frac{bd}{ac}\right)^2 - \left(\frac{bd}{y_p}\right)^2 \quad \Bigg| \cdot \frac{y_p^2}{b^2 d^2} \qquad (20)$$

$$\Rightarrow \frac{y_p^2}{b^2 d^2} q = \frac{y_p^2}{b^2} + \frac{x_p^2}{a^2} - 1 \qquad (21)$$

$$\Rightarrow q = 0 \qquad (22)$$

Or intersection (from Equation 18 and 22) is

$$\begin{cases} x_o = \dfrac{-c^2\alpha\beta}{d^2 + c^2\alpha^2} \\ y_o = \dfrac{d^2\beta}{d^2 + c^2\alpha^2} \end{cases} \qquad (23a, b)$$

Because point (x_q, y_q) is an arbitrary point of the core figure ellipse (Equation 6), we can substitute

$$\begin{cases} x_q = x_o \\ y_q = y_o \end{cases} \qquad (24a, b)$$

Gamma (from Equations 9, 24 and 23) is

$$\gamma = -\frac{-c^2\alpha\beta}{d^2+c^2\alpha^2}\frac{d^2+c^2\alpha^2}{d^2\beta}\frac{d^2}{c^2} \tag{25}$$

$$\Rightarrow \gamma = \alpha \tag{26}$$

Delta (from Equations 10, 24b, 23b and 15) is

$$\delta = d^2\frac{d^2+c^2\alpha^2}{d^2\beta} \tag{27}$$

$$\Rightarrow \delta = \frac{d^2}{\beta}\left[1+\frac{c^2}{d^2}\left(\frac{x_p}{y_p}\frac{bd}{ac}\right)^2\right] \tag{28}$$

$$\Rightarrow \delta = \frac{d^2}{\beta}\left[1+\frac{x_p^2}{y_p^2}\frac{b^2}{a^2}\right] \tag{29}$$

Because from Equation 1 we obtain

$$\frac{x_p^2}{y_p^2}\frac{b^2}{a^2}+1=\frac{b^2}{y_p^2} \tag{30}$$

so (from Equation 16)

$$\delta = \frac{d^2}{\beta}\frac{b^2}{y_p^2} \tag{31}$$

$$\Rightarrow \delta = \beta \tag{32}$$

Thus Equation of core figure boundary line (Equation 3) of ellipse (Equation 1) corresponding to point (x_p, y_p) and the tangent of core figure ellipse (Equation 8) are congruent. So, the core figure of ellipse area is an ellipse area.

Half ray (in direction of *x*- axis) of the core figure ellipse of the multi-component bar is (Equation 12)

$$c = \frac{D_y}{a_n C} \tag{33}$$

$$\Rightarrow c = \frac{\sum_{i=1}^{n} E_i I_{yi}}{a_n \sum_{i=1}^{n} E_i A_i} \tag{34}$$

$$\Rightarrow c = \frac{\sum_{i=1}^{n} E_i \left(b_i a_i^3 - b_{i-1} a_{i-1}^3\right)}{4 a_n \sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1}\right)} \tag{35}$$

Here D_y is bending stiffness with respect to y- axis and C is axial stiffness of the whole cross-section. Respectively I_{yi} is moment of inertia and A_i area of band i.

Respectively half ray in direction of y-axis is (Equation 14)

$$d = \frac{D_x}{b_n C} \tag{36}$$

$$\Rightarrow d = \frac{\sum_{i=1}^{n} E_i I_{xi}}{b_n \sum_{i=1}^{n} E_i A_i} \tag{37}$$

$$\Rightarrow d = \frac{\sum_{i=1}^{n} E_i \left(a_i b_i^3 - a_{i-1} b_{i-1}^3\right)}{4 b_n \sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1}\right)} \tag{38}$$

In the case of homogenous solid cross-section $n = 1$ and

$$c = \frac{a_1}{4} \tag{39}$$

$$d = \frac{b_1}{4} \tag{40}$$

Or ellipses are equiform

$$\frac{a_1}{c} = \frac{b_1}{d} = 4 \tag{41}$$

In the case of homogenous band $n = 2$ and $E_1 = 0$ and

$$c = \frac{b_2 a_2^3 - b_1 a_1^3}{4a_2 \left(a_2 b_2 - a_1 b_1\right)} \tag{42}$$

$$d = \frac{a_2 b_2^3 - a_1 b_1^3}{4b_2 \left(a_2 b_2 - a_1 b_1\right)} \tag{43}$$

In the case of very thin circular band $b_i = a_i$ and $a_1 \to a_2$ and

$$\left.\begin{matrix} c \\ d \end{matrix}\right\} = \frac{a_2}{2} = r \tag{44}$$

where r is radius of the core figure circle.

<u>ANSWER</u>:

The core figure of the multi-component bar is ellipse area, which equation in the principal coordinate system is

$$16a_n^2 \frac{\left[\sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1}\right)\right]^2}{\left[\sum_{i=1}^{n} E_i \left(b_i a_i^3 - b_{i-1} a_{i-1}^3\right)\right]^2} x^2 + 16b_n^2 \frac{\left[\sum_{i=1}^{n} E_i \left(a_i b_i - a_{i-1} b_{i-1}\right)\right]^2}{\left[\sum_{i=1}^{n} E_i \left(a_i b_i^3 - a_{i-1} b_{i-1}^3\right)\right]^2} y^2 \le 1 \tag{45}$$

In the case of homogenous solid cross-section the equation of the core figure is (Figure 3)

$$\frac{16}{a_1^2} x^2 + \frac{16}{b_1^2} y^2 \le 1 \tag{46}$$

In the case of very thin circular band the equation of the core figure is (Figure 4)

$$x^2 + y^2 \le \frac{a_2^2}{4} \qquad \textbf{(47)}$$

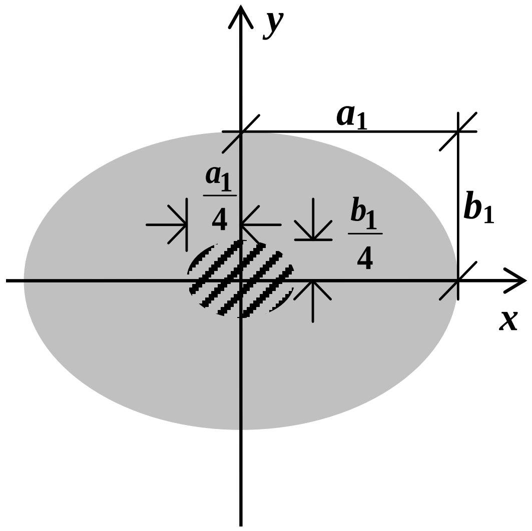


Figure 3.

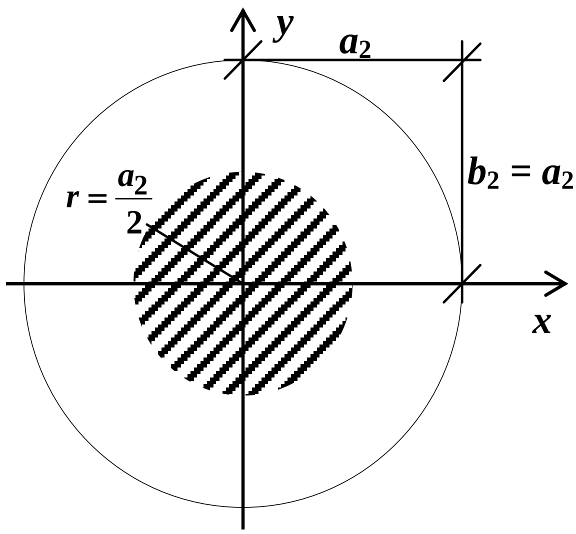


Figure 4.

16. PROBLEM

Vertical force and moment

$$F_{zo} = 2 \text{ MN}$$

$$M_{xo} = 0{,}350 \text{ MNm}$$

act on a base slab lying on rock.

A) Find the minimum side width *a* so that the whole base slab is compressed! What is the corresponding maximum stress?

B) Find the minimum side width *a* so that the maximum stress in rock is (*RIL 121-2004*, Chapter 5.5.3.1, p. 82)

$$\sigma_{max} = 10 \text{ MN/m}^2$$

C) Find the minimum side width *a* in case when the base slab is anchored by using vertical rock anchors located as shown in Figure 1, where distance

$$e = 0{,}10 \text{ m}$$

and the maximum stress in rock is

$$\sigma_{max} = 10 \text{ MN/m}^2$$

How big is the corresponding anchor force?

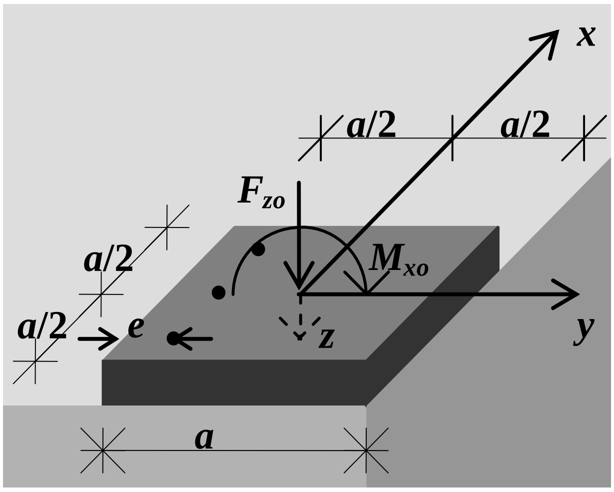


Figure 1.

A)

Normal stress (Figure 2)

$$\sigma(y) = \frac{F_z}{A} + \frac{M_x}{I_x} y \tag{1}$$

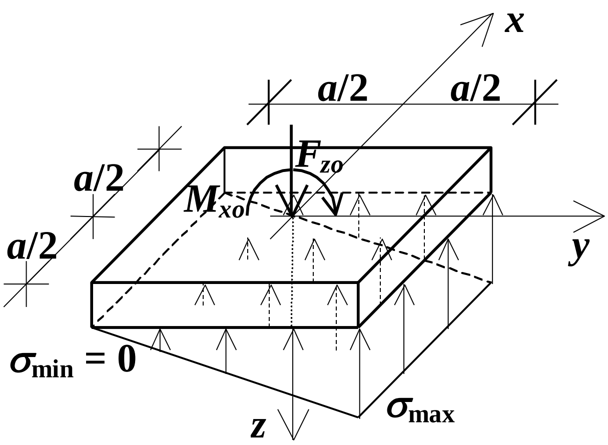


Figure 2. Stress distribution, a three-side prism.

Cross-section area

$$A = a^2 \tag{2}$$

Moment of inertia

$$I_x = a^4/12 \tag{3}$$

From Equations 1, 2 and 3

$$\sigma(y) = \frac{F_{zo}}{a^2} + 12\frac{M_{xo}}{a^4} y \tag{4}$$

Normal stress at the edge

$$\sigma_{\min} = \sigma\left(y = \frac{-a}{2}\right) = 0 \tag{5}$$

$$\Rightarrow \quad 0 = \frac{F_{zo}}{a^2} - 6\frac{M_{xo}}{a^3} \tag{6}$$

Side width

$$a = 6\frac{M_{xo}}{F_{zo}} \tag{7}$$

$$= \quad 1{,}050 \text{ m}$$

Maximum stress

$$\sigma_{\max} = \sigma\left(\frac{a}{2}\right) \tag{8}$$

$$= \frac{F_{zo}}{a^2} + 6\frac{M_{xo}}{a^3} \tag{9}$$

$$= \quad 3{,}628\ \mathrm{MN/m^2}$$

Normal stresses at the edge as a function of side width is shown in Figure 3.

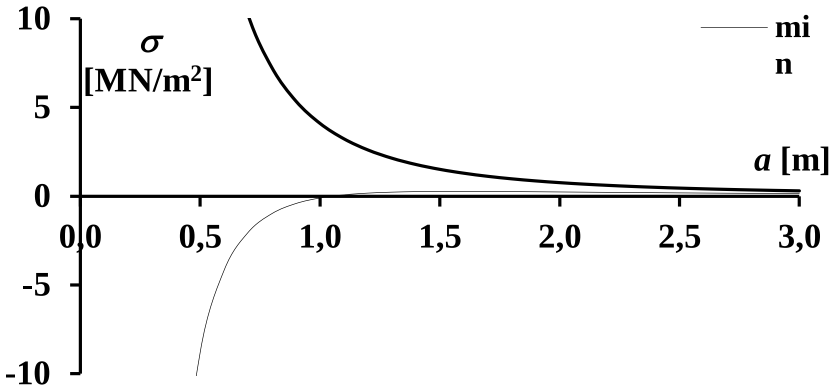


Figure 3.

B)

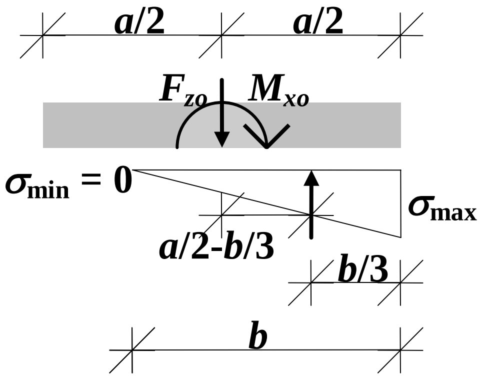


Figure 4.

Equilibrium conditions (Figure 4)

$$\begin{cases}\sum F_z = 0 \\ \sum M_x = 0\end{cases} \tag{10a, b}$$

$$\Rightarrow \quad \begin{cases} 0 = F_{zo} - \dfrac{ab}{2}\sigma_{\max} \\ 0 = M_{xo} - \left(\dfrac{a}{2} - \dfrac{b}{3}\right)\dfrac{ab}{2}\sigma_{\max} \end{cases} \tag{11a, b}$$

$$\Rightarrow \quad \begin{cases} b = \dfrac{2F_{zo}}{a\sigma_{\max}} \\ a = \dfrac{-M_{xo} \overset{(+)}{-} \sqrt{M_{xo}^2 + \dfrac{4}{3}\dfrac{F_{zo}^3}{\sigma_{\max}}}}{-F_{zo}} \end{cases} \tag{12a, b}$$

Side width a **and effective width** b

a = 0,720 m

b = 0,555 m

Maximum stress as a function of side width is shown in Figure 5.

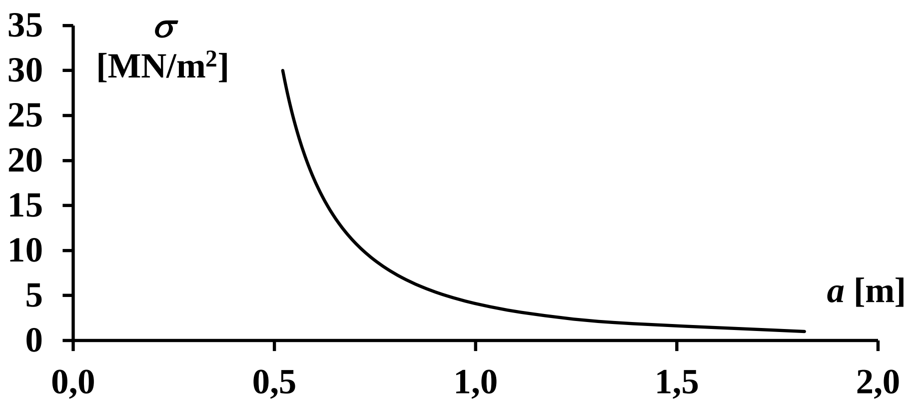


Figure 5.

C)

Equilibrium conditions (Figure 6)

$$\begin{cases} R = F_{zo} + P \\ M_{xo} = P\left(\frac{a}{2} - e\right) \end{cases} \tag{13a, b}$$

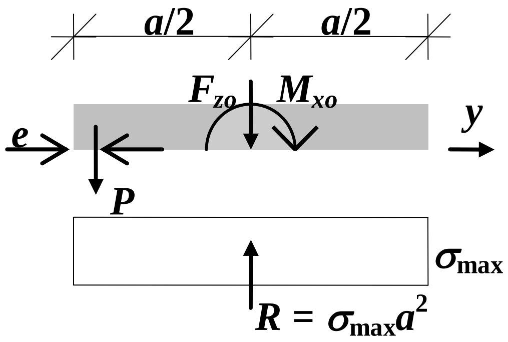


Figure 6.

Anchor force P from Equation 13b

$$P = \frac{M_{xo}}{\frac{a}{2} - e} \tag{14}$$

Anchor force P is inserted in Equation 13a

$$\sigma_{max}a^2 = F_{zo} + \frac{M_{xo}}{\frac{a}{2} - e} \quad \Big| \cdot (a - 2e) \tag{15}$$

$$\Rightarrow \qquad \sigma_{max}a^2(a - 2e) = F_{zo}(a - 2e) + 2M_{xo} \tag{16}$$

$$\Rightarrow \sigma_{max}a^3 - 2e\sigma_{max}a^2 - F_{zo}a + 2eF_{zo} - 2M_{xo} = 0 \tag{17}$$

Expression at the left hand side of Equation 17 has value

0,000 MNm

when

a = 0,609 m

Anchor force from Equation 14

P = 1,711 MN

The value of the left hand side of Equation 17 as function of side width a is shown in Figure 7.

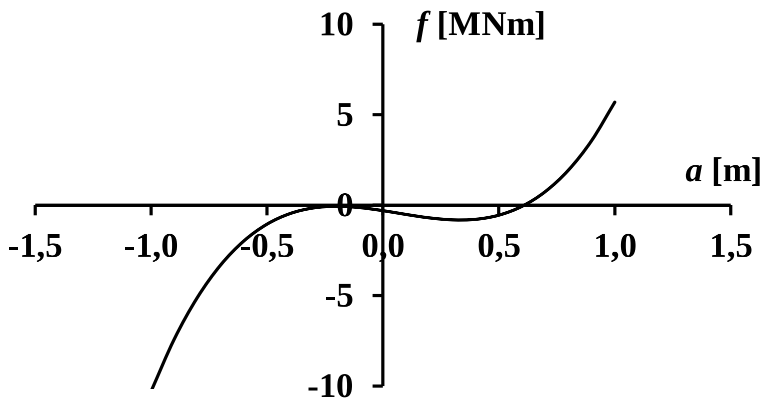


Figure 7.

<u>ANSWER</u>:

A) The minimum side width, when the whole slab is compressed, is

a = 1,050 m.

The corresponding maximum stress is

σ_{max} = 3,628 MN/m^2.

B) The minimum side width, when the maximum stress in rock is

σ_{max} = 10 MN/m^2,

is

a = 0,720 m.

C) The minimum side width, when anchors are used, is

a = 0,609 m.

The corresponding anchor force is

P = 1,711 MN

17. PROBLEM

Vertical force

N = 0,750 MN

acts on a base slab anchored by using prestressed vertical rock anchors (Figure 1).

A) **Determine the stress distribution, when the prestressing force is**

P = 0 MN?

B) **How big prestressing force is needed to obtain a uniform stress distribution under the slab?**

Dimensions of the base slab

a_1 = 0,400 m
a_2 = 0,800 m
a_3 = 0,600 m
b = 2,000 m
L = 6,000 m

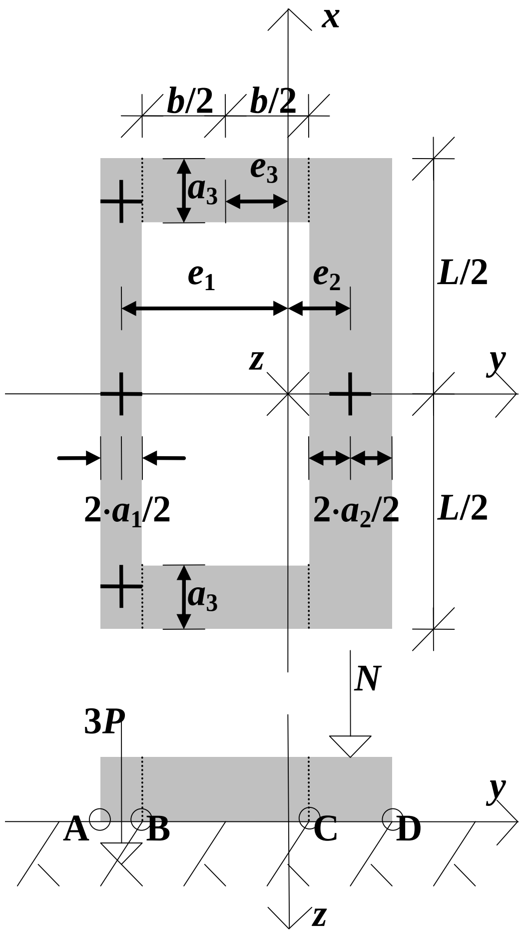


Figure 1.

Cross-section areas

$$A_1 = a_1 L \quad (1)$$
$$= 2{,}400 \text{ m}^2$$
$$A_2 = a_2 L \quad (2)$$
$$= 4{,}800 \text{ m}^2$$
$$A_3 = a_3 b \quad (3)$$
$$= 1{,}200 \text{ m}^2$$
$$A = A_1 + A_2 + 2A_3 \quad (4)$$
$$= 9{,}600 \text{ m}^2$$

Origin is situated to the centroid.

Distances between x-axis and the centroid of the sub-section 1

$$e_1 = \frac{A_1 \cdot 0 + A_2(e_1 + e_2) + 2A_3(e_1 - e_3)}{A} \quad (5)$$

$$= \frac{A_2\left(\frac{a_1}{2} + b + \frac{a_2}{2}\right) + 2A_3\left(\frac{a_1}{2} + \frac{b}{2}\right)}{A} \quad (6)$$

$$= 1{,}600 \text{ m}$$

Respectively

$$e_2 = \frac{a_1}{2} + b + \frac{a_2}{2} - e_1 \quad (7)$$
$$= 1{,}000 \text{ m}$$

$$e_3 = e_1 - \frac{a_1}{2} - \frac{b}{2} \quad (8)$$
$$= 0{,}400 \text{ m}$$

A)

Moment of inertia with respect to x-axis (Steiner's rule)

$$I_x = \frac{La_1^3}{12} + A_1 e_1^2 + \frac{La_2^3}{12} + A_2 e_2^2 + 2\left(\frac{a_3 b^3}{12} + A_3 e_3^2\right) \tag{9}$$

$$= \quad 12{,}416 \ \mathrm{m}^4$$

Bending moment with respect to x-axis

$$M_x = Ne_2 \tag{10}$$

$$= \quad 0{,}750 \ \mathrm{MNm}$$

Stress at point A

$$\sigma(y) = \frac{M_x}{I_x} y + \frac{N}{A} \tag{11}$$

$$\Rightarrow \quad \sigma_A = \frac{M_x}{I_x}\left(\frac{-a_1}{2} - e_1\right) + \frac{N}{A} \tag{12}$$

$$= \quad -0{,}031 \ \mathrm{MN/m}^2 \qquad < 0, \text{ tension}$$

Tension is not possible. Stress at point B is studied. Part 1 is neglected.

$$A_B = A_2 + 2A_3 \tag{13}$$

$$= \quad 7{,}200 \ \mathrm{m}^2$$

$$e_{2B} = \frac{2A_3\left(\frac{a_2}{2} + \frac{b}{2}\right)}{A_B} \tag{14}$$

$$= \quad 0{,}467 \ \mathrm{m}$$

$$e_{3B} = \frac{a_2}{2} + \frac{b}{2} - e_{2B} \tag{15}$$

$$= \quad 0{,}933 \ \mathrm{m}$$

$$I_{xB} = \frac{La_2^3}{12} + A_2 e_{2B}^2 + 2\left(\frac{a_3 b^3}{12} + A_3 e_{3B}^2\right) \tag{16}$$

$$= \quad 4{,}192 \ \mathrm{m}^4$$

$$M_{xB} = Ne_{2B} \quad (17)$$
$$= \quad 0{,}350 \text{ MNm}$$
$$\sigma_B = \frac{M_{xB}}{I_{xB}}\left(\frac{-b}{2} - e_{3B}\right) + \frac{N}{A_B} \quad (18)$$
$$= \quad -0{,}057 \text{ MN/m}^2 \quad < 0, \text{ vetoa} \quad (a)$$

Position of the neutral axis

$$c_n = 3a_2/2 \quad (19)$$
$$= \quad 1{,}200 \text{ m} \quad > \quad a_2 \quad (b)$$

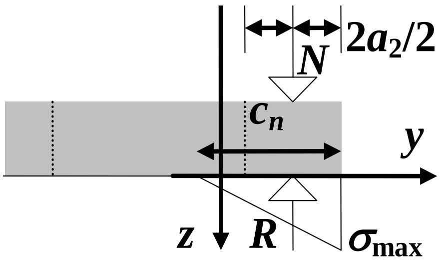


Figure 2.

According to notes (a) and (b), the neutral axis is located between points B and C.

Stress distribution is shown in Figure 3.

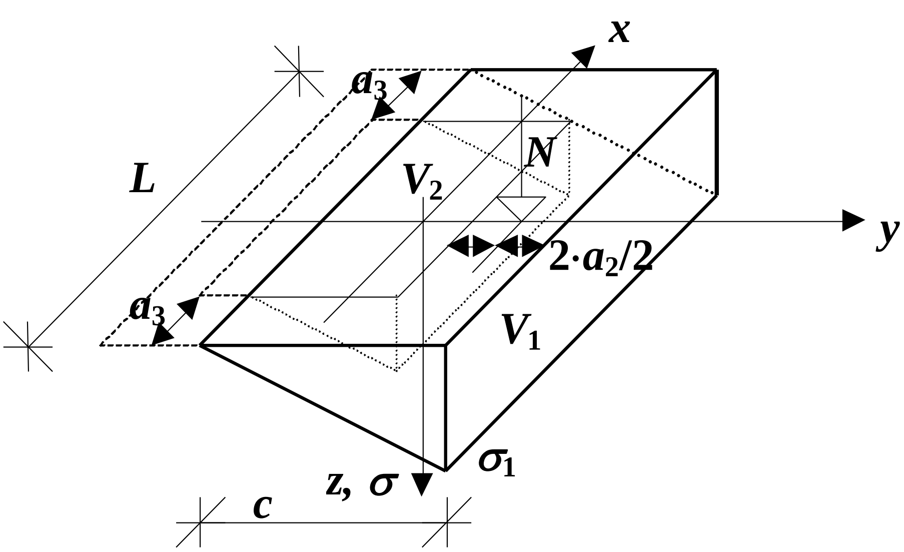


Figure 3.

Equilibrium condition of vertical forces

$$\sum V = 0 \tag{20}$$

$$\Rightarrow N = V_1 - V_2 \tag{21}$$

$$\Rightarrow N = \frac{1}{2} L c \sigma_1 - \frac{1}{2}(L - 2a_3)(c - a_2)\left(\frac{c - a_2}{c}\right)\sigma_1 \tag{22}$$

$$\Rightarrow \sigma_1 = \frac{2Nc}{Lc^2 - (L - 2a_3)(c - a_2)^2} \tag{23}$$

Equilibrium condition of moment with respect to point D

$$\sum M = 0 \tag{24}$$

$$\Rightarrow N\frac{a_2}{2} = V_1 \frac{c}{3} - V_2\left(\frac{c - a_2}{3} + a_2\right) \tag{25}$$

$$\Rightarrow N\frac{a_2}{2} = \frac{1}{6} L c^2 \sigma_1 - \frac{1}{6c}(L - 2a_3)(c - a_2)^2 (c + 2a_2)\sigma_1 \tag{26}$$

$$\Rightarrow N\frac{a_2}{2} = \frac{\sigma_1}{6c}\left[Lc^3 - (L - 2a_3)(c - a_2)^2 (c + 2a_2)\right] \tag{27}$$

Following from Equations 23 and 27

$$\frac{a_2}{2} = \frac{Lc^3 - (L - 2a_3)(c - a_2)^2 (c + 2a_2)}{3Lc^2 - 3(L - 2a_3)(c - a_2)^2} \tag{28}$$

From an iteration process (compare to the solution of cubic equation at the end of this problem)

$a_2/2 =$ **0,400 m**

when

$c =$ **1,600 m**

From Equation 23

$\sigma_1 =$ **0,195 MN/m²**

ANSWER:

Stress distribution is similar to Figure 3, where

$$c = 1{,}600 \text{ m}$$

$$\sigma_1 = 0{,}195 \text{ MN/m}^2$$

B)

Stress distribution is uniform, when the force resultant is acting at the centroid (Figure 4).

$$Ne_2 = 3Pe_1 \tag{29}$$

$$\Rightarrow \quad P = \frac{Ne_2}{3e_1} \tag{30}$$

$$= 0{,}156 \text{ MN}$$

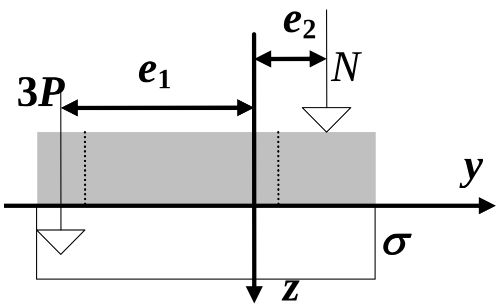


Figure 4.

ANSWER:

Prestressing force needed is

$$P = 0{,}156 \text{ MN}$$

SOLUTION OF CUBIC EQUATION

Cubic equation is

$$\alpha c^3 + \beta c^2 + \gamma c + \delta = 0 \tag{I}$$

Exploiting

$$\mu = \frac{\lambda^2}{4} + \frac{\kappa^3}{27} \tag{II}$$

where

$$\kappa = \frac{3\alpha\gamma - \beta^2}{3\alpha^2} \tag{III}$$

$$\lambda = \frac{2\beta^3 - 9\alpha\beta\gamma + 27\alpha^2\delta}{27\alpha^3} \tag{IV}$$

If μ < 0, the cubic equation has three real roots

$$c_n = 2\sqrt{\frac{-\kappa}{3}}\cos\frac{\phi + 2n\pi}{3} - \frac{\beta}{3\alpha}, \quad n \in \{1, 2, 3\} \tag{Va, b, c}$$

where

$$\phi = \arccos\left(\frac{-\lambda}{2}\sqrt{\frac{-27}{\kappa^3}}\right) \tag{VI}$$

If μ = 0, the cubic equation has two inequal real roots

$$\begin{aligned} c_1 &= 2\cdot\sqrt[3]{\frac{-\lambda}{2}} - \frac{\beta}{3\alpha} \\ \left.\begin{matrix} c_2 \\ c_3 \end{matrix}\right\} &= \sqrt[3]{\frac{\lambda}{2}} - \frac{\beta}{3\alpha} \end{aligned} \tag{VIIa, b}$$

where the last one is a double root.
Furthermore, if λ = 0, the equation has one real triple root.

If μ > 0, the cubic equation has one real root

$$c = \sqrt[3]{\frac{-\lambda}{2} + \sqrt{\mu}} + \sqrt[3]{\frac{-\lambda}{2} - \sqrt{\mu}} - \frac{\beta}{3\alpha} \tag{VIII}$$

and two complex roots.

Equation 20 can be written in the form of Equation I, where

$$\alpha = 4a_3 \qquad \text{(IX)}$$
$$= 2{,}400 \text{ m}$$
$$\beta = -6a_2 a_3 \qquad \text{(X)}$$
$$= -2{,}880 \text{ m}^2$$
$$\gamma = 0{,}000 \text{ m}^3$$
$$\delta = a_2^{\,3}(2a_3 - L) \qquad \text{(XI)}$$
$$= -2{,}458 \text{ m}^4$$

Hence

$$\kappa = -0{,}480 \text{ m}^2$$
$$\lambda = -1{,}152 \text{ m}^3$$
$$\mu = 0{,}328 \text{ m}^6$$
$$c = 1{,}600 \text{ m}$$

See Figure I.

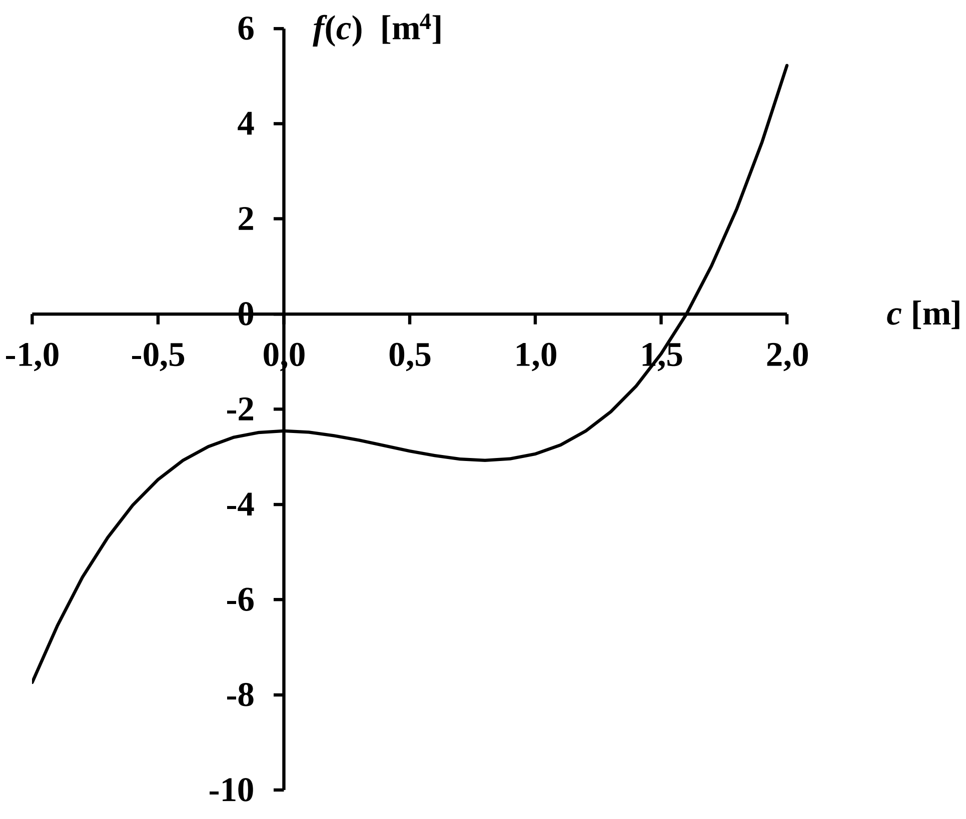


Figure I.

18. PROBLEM

Determine stability against tilting and sliding of the retaining wall shown below (Figure 1). The structure is lying on rock. Use safety factory analysis!

Dimensions

a = 1,000 m

b = 2,000 m

h = 5,000 m

t = 0,500 m

Superimposed load

q = 0,010 MN/m^2

Angle of internal friction of the soil

φ = 36,000 °

Unit weight of soil

γ = 0,018 MN/m^3

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Friction coefficient between rock and concrete

μ = 0,750

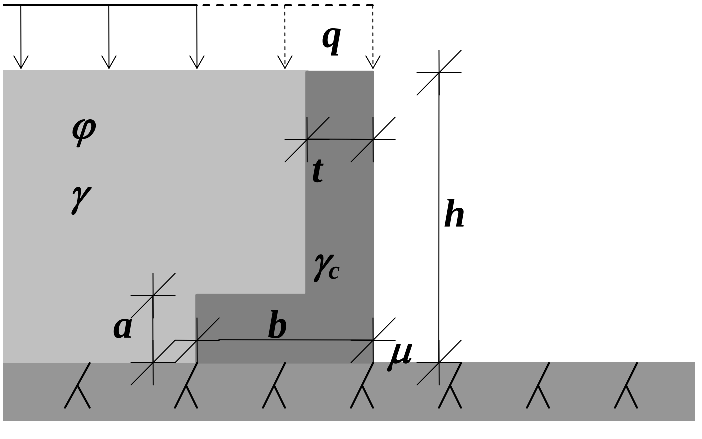


Figure 1.

When the structure is lying on rock, the minimum safety factory against tilting and sliding is (Finnish standards of foundation engineering, *RIL 121-2004*, Chapter 5.1.2.2, p. 58)

$$n = 1{,}500$$

Coefficient of earth pressure at rest

$$K_o = 1\text{-}\sin\varphi \tag{1}$$

$$= 0{,}412$$

Resultant of earth pressure due to soil mass and the corresponding distance from the base of the retaining wall, respectively, (Figure 2)

$$P_\gamma = \frac{1}{2}K_0\gamma h^2 \tag{2}$$

$$= 0{,}093 \text{ MN/m}$$

$$e_\gamma = h/3 \tag{3}$$

$$= 1{,}667 \text{ m}$$

Resultant of earth pressure due to superimposed load and the corresponding distance from the base of the retaining wall, respectively, (Figure 2)

$$P_q = K_0 q h \tag{4}$$

$$= 0{,}021 \text{ MN/m}$$

$$e_q = h/2 \tag{5}$$

$$= 2{,}500 \text{ m}$$

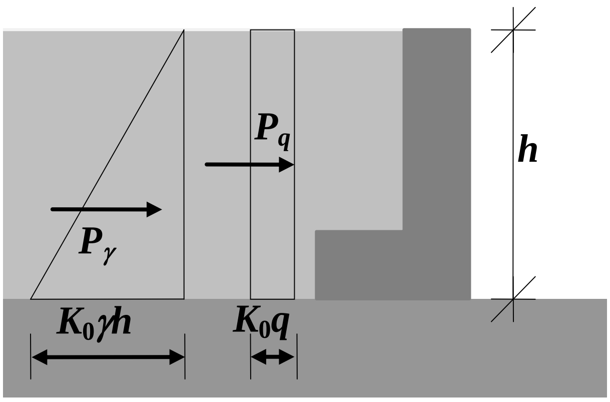


Figure 2.

Overturning moment (Figure 4)

$$M_k = P_\gamma e_\gamma + P_q e_q \quad (6)$$

$$= 0{,}206 \text{ MNm/m}$$

Weight resultant of the retaining wall and the corresponding distance from the front corner point, respectively,

$$G_c = \gamma_c [t(h-a)+ab] \quad (7)$$

$$= 0{,}100 \text{ MN/m}$$

$$e_c = \frac{\gamma_c \left[\frac{t}{2} t(h-a) + \frac{b}{2} ab \right]}{G_c} \quad (8)$$

$$= 0{,}625 \text{ m}$$

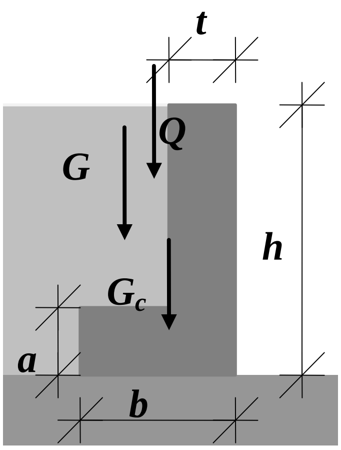


Figure 3.

Weight resultant of soil and the corresponding distance from the front corner point, respectively, (Figure 3)

$$G = \gamma(b-t)(h-a) \quad (9)$$

$$= 0{,}108 \text{ MN/m}$$

$$e = t + \frac{b-t}{2} \quad (10)$$

$$= 1{,}250 \text{ m}$$

It is assumed, that the superimposed load q is not acting above the retaining wall (dashed line in Figure 1). This is the main loading case. The superimposed load resultant and the corresponding distance from the front corner point, respectively, (Figure 3)

$$Q = 0 \cdot qb \tag{11}$$
$$= 0{,}000 \text{ MN/m}$$
$$e_Q = b/2 \tag{12}$$
$$= 1{,}000 \text{ m}$$

Balancing moment (Figure 4)

$$M_p = G_c e_c + Ge + Qe_Q \tag{13}$$
$$= 0{,}198 \text{ MNm/m}$$

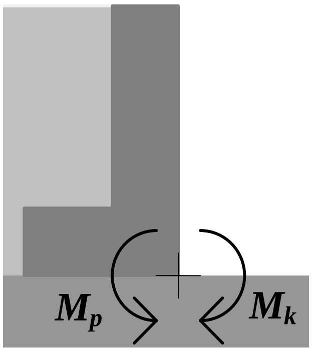


Figure 4.

Safety against tilting

$$n_t = \frac{M_p}{M_k} \tag{14}$$
$$= 0{,}958 \quad < n = 1{,}500 \qquad \text{Turning over!}$$

The retaining wall is anchored by using vertical rock anchors located as shown in Figure 5, where distance

$$c = 0{,}100 \text{ m}$$

Vertical force capacity needed is (from Equation 14)

$$\frac{P(b-c)+M_p}{M_k} \geq n \tag{15}$$

$$\Rightarrow \qquad P \geq \frac{nM_k - M_p}{b-c} \tag{16}$$

$$\geq \quad 0{,}059 \ \text{MN/m}$$

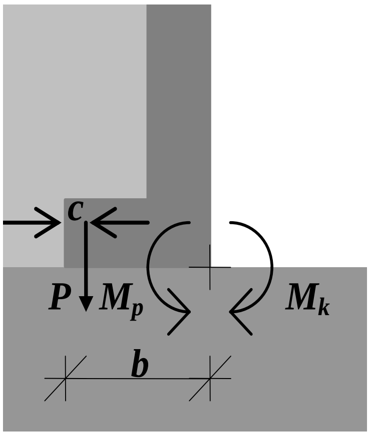


Figure 5.

Safety against sliding

$$n_s = \frac{\mu V}{H} \tag{17}$$

$$= \frac{\mu(G_c + G + Q)}{P_\gamma + P_q} \tag{18}$$

$$= \quad 1{,}376 \quad < n = \quad 1{,}500 \qquad \text{Sliding!}$$

The retaining wall has to be anchored by using vertical prestressing force. From safety against sliding

$$\frac{\mu(V+P)}{H - Q_p} \geq n \tag{19}$$

Here P is anchor force and Q is shear capacity of the anchor.

If it is assumed that

$$Q_p = 0$$

then the anchor force required is

$$P \geq \frac{nH - \mu V}{\mu} \tag{20}$$

$$\Rightarrow P \geq \frac{n(P_\gamma + P_q) - \mu(G_c + G + Q)}{\mu} \tag{21}$$

$$\Rightarrow P \geq \quad 0{,}019 \ \text{MN/m}$$

<u>ANSWER</u>:

Conditions for tilting and sliding are not satisfied. That is why the retaining wall has to be anchored by using vertical prestressing force

$$P = \quad 0{,}060 \ \text{MN/m}$$

located at a distance

$$c = \quad 0{,}100 \ \text{m}$$

from the edge of the base slab.

19. PROBLEM

A base slab have a breadth a (Figure 1). Determine the smallest possible dimension a so that the bearing resistance is adequate! Use design approach 2 of ultimate limit state (GEO) (Chapter 2.4.7.3.4.3) and Annexes A (Chapter. A.3) and D of Eurocode *SFS-EN1997-1!*

Characteristic values of loading due to the overhead structure

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Proportion of the permanent load from the total load is

k_V = 0,400

k_H = 0,000

k_M = 0,400

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Unit weight of soil

γ_m = 0,020 MN/m^3

Angle of internal friction of the earth

φ = 30,000 °

Dimensions

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m

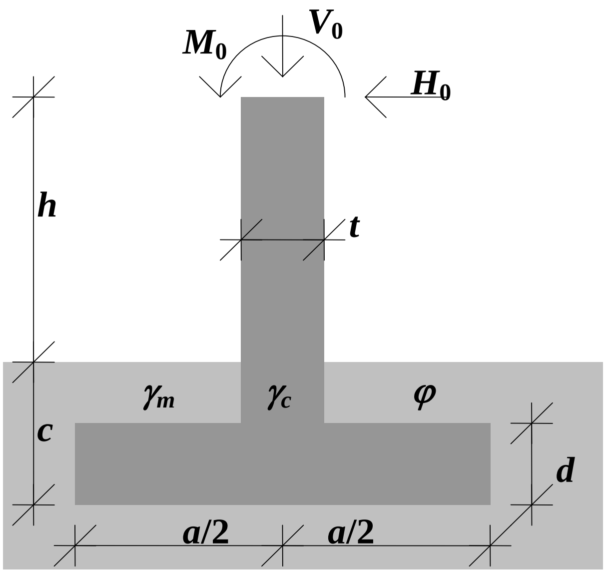


Figure 1.

Length of the base slab (perpendicular to the figure plane)

L = 4,000 m

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Approach given by Eurocode *SFS-EN 1997-1* is used.

Load combination and partial safety factors may be differs in national annex.
This is situation in Finnish national annex.

Combination of sets of partial factors of design approach 2

A1 "+" M1 "+" R2

where

- **A refers to loads or actions of loads (Table A.3 of Appendix A),**
- **M refers to soil parameters (Table A.4),**
- **R refers to resistance (Table A.5),**
- **numbers 1 and 2 refer to sets 1 and 2 and**
- **"+" implies "to be combinated with".**

Load combination

$$q_d = \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} \tag{1}$$

can be written

$$q_d = \left.\begin{matrix} 1{,}35 \\ 1{,}35 \\ 1{,}35 \\ 1{,}35 \\ 1{,}0 \\ 1{,}0 \\ 1{,}0 \\ 1{,}0 \end{matrix}\right\} (g_c + g_m) + \left.\begin{matrix} 1{,}5 \\ 1{,}5 \\ 0 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \\ 0 \end{matrix}\right\} q_{k1} + \left.\begin{matrix} 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \\ 1{,}5 \\ 0 \end{matrix}\right\} q_{k2} \quad \begin{matrix} (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f) \\ (g) \\ (h) \end{matrix} \tag{2a…h}$$

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select *a*- value.

$$a = 2{,}100 \text{ m}$$

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved.
The sub indexes are referred to that ones used in Equation 1.

Self weight of the base structure (as function of *a*)

$$V_{gc} = \gamma_c[(h+c-d)t+ad]L \tag{3}$$

$$= 0{,}250 \text{ MN}$$

Self weight of soil (as function of *a*)

$$V_{gm} = \gamma_m(a-t)(c-d)L \tag{4}$$

$$= 0{,}122 \text{ MN}$$

Permanent vertical load

$$V_g = k_V V_0 \tag{5}$$

$$= 0{,}600 \text{ MN}$$

Variable vertical imposed load

$$V_{q1} = (1-k_V)V_0 \tag{6}$$

$$= 0{,}900 \text{ MN}$$

Permanent horizontal load

$$H_g = k_H H_0 \tag{7}$$

$$= 0{,}000 \text{ MN}$$

Variable horizontal wind load

$$H_{q2} = (1-k_H)H_0 \tag{8}$$

$$= 0{,}130 \text{ MN}$$

Moment due to permanent loads

$$M_g = k_M M_0 + k_H H_0 (c + h) \quad (9)$$

$$= \quad 0,096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1-k_M) M_0 \quad (10)$$

$$= \quad 0,144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1-k_H) H_0 (c + h) \quad (11)$$

$$= \quad 0,481 \text{ MNm}$$

Design values of loads

Vertical load

$$V_d = 1,35(V_{gm} + V_g + V_{gc}) + 1,5 V_{q1} \quad (12)$$

$$= \quad 2,663 \text{ MN}$$

Horizontal load

$$H_d = 1,35 H_g + 1,5 H_{q2} \quad (13)$$

$$= \quad 0,195 \text{ MN}$$

Moment

$$M_d = 1,35 M_g + 1,5 M_{q1} + 1,5 M_{q2} \quad (14)$$

$$= \quad 1,067 \text{ MNm}$$

From equilibrium condition of the moments with respect to the pivot (Figure 2)

$$M_d - V_d e_o = 0 \quad (15)$$

the load eccentricity is obtained

$$e_o = \frac{M_d}{V_d} \quad (16)$$

$$= \quad 0,401 \text{ m}$$

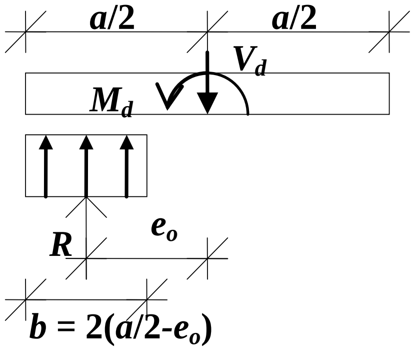


Figure 2.

Effective foundation length in the direction of *a*

$$b = 2(\frac{a}{2} - e_o) \quad (17)$$

$$= \quad 1{,}298 \text{ m}$$

Effective foundation width (Figure 3)

$$B' = \min\begin{cases} b \\ L \end{cases} \quad (18)$$

$$= \quad 1{,}298 \text{ m}$$

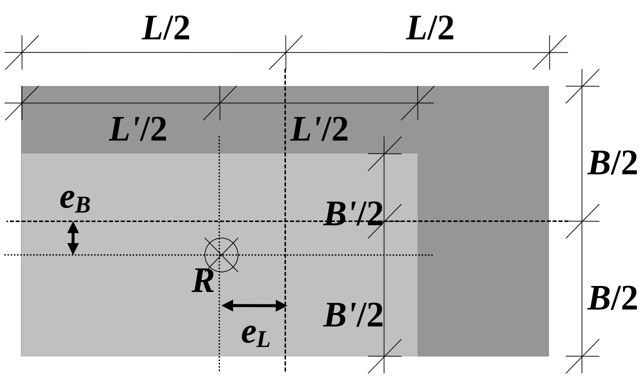


Figure 3.

Effective foundation length

$$L' = \max\begin{cases} b \\ L \end{cases} \quad (19)$$

$$= \quad 4{,}000 \text{ m}$$

Effective foundation area (Figure 3)

$$A' = B'L' \quad (20)$$

$$= \quad 5{,}194 \text{ m}^2$$

Design value of bottom pressure

$$q_d = \frac{V_d}{A'} \quad (21)$$

$$= \quad 0{,}513 \text{ MN/m}^2$$

Design value of cohesion

$$c' = \quad 0 \text{ MN/m}^2$$

Unit weight of soil above the foundation level

$$\gamma = \gamma_m \quad (22)$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Unit weight of soil below the foundation level

$$\gamma' = \gamma_m \quad (23)$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Effective overburden pressure at the level of the foundation area

$$q' = c\,\gamma \quad (24)$$

$$= \quad 0{,}030 \text{ MN/m}^2$$

Inclination of the foundation base

$$\alpha = \quad 0\ °$$

Partial factor for angle of shearing resistance of the soil (*EN 1997-1*, Annex A. Table A.4.)

$$\gamma_{\varphi} = 1{,}000$$

Design value of the internal friction angle of the earth

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_{\varphi}}\right) \tag{25}$$

$$= 30{,}000\ °$$

Bearing capacity factors

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} \tag{26}$$

$$= 18{,}401$$

$$N_{\gamma} = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi \tan\varphi'} - 1\right]\tan\varphi' \tag{27}$$

$$= 20{,}093$$

Factors for the inclination of the base

$$\left.\begin{matrix} b_q \\ b_{\gamma} \end{matrix}\right\} = (1 - \alpha \tan\varphi')^2 \tag{28a, b}$$

$$= 1{,}000$$

Shape factors of the base slab

$$s_q = 1 + \frac{B'}{L'}\sin\varphi' \tag{29}$$

$$= 1{,}162$$

$$s_{\gamma} = 1 - 0{,}3\frac{B'}{L'} \tag{30}$$

$$= 0{,}903$$

Parameter *m*

$$m = m_B \tag{31}$$

$$= \frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}} \tag{32}$$

$$= \quad 1{,}755$$

Inclination factors of the load resultant

$$i_q = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^m \tag{33}$$

$$= \quad 0{,}875$$

$$i_\gamma = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^{m+1} \tag{34}$$

$$= \quad 0{,}811$$

Partial resistance factor for bearing (*EN 1997-1* , Appendix A, Table A.5)

$$\gamma_R \;=\quad 1{,}4$$

Bearing capacity

$$q_m = \frac{R}{A'} \tag{35}$$

$$= c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{36}$$

$$= \quad 0{,}752 \ \mathrm{MN/m^2}$$

Design value of the bearing capacity

$$q_{md} = \frac{q_m}{\gamma_R} \tag{37}$$

$$= \quad 0{,}537 \ \mathrm{MN/m^2}$$

Ratio between bearing capacity and bottom pressure

$$n = q_{md}/q_d \tag{38}$$

$$= 1{,}048 > 1\text{, OK}$$

Breadth of the base slab is obtained by an iteration process.

$$a = 2{,}100 \text{ m}$$

The value is rounded up by precision of 0,1 m.

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

ANSWER: **Breadth of the base slab**

$$a = 2{,}400 \text{ m}$$

Table 1.

Quant.	Eq.	Loading Cases								Units
i		a	b	c	d	e	f	g	h	
a		2,055	1,347	2,124	0,784	2,042	1,250	2,358	0,660	m
ext(a)								Max	Min	
a_{round}		2,100	1,400	2,200	0,800	2,100	1,300	2,400	0,700	m
γ_g	2	1,35	1,35	1,35	1,35	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,50	1,50	0,00	0,00	1,50	1,50	0,00	0,00	
$\gamma_{qk\,2}$	2	1,50	0,00	1,50	0,00	1,50	0,00	1,50	0,00	
V_{gc}	3	0,247	0,205	0,251	0,171	0,247	0,199	0,265	0,164	MN
V_{gm}	4	0,119	0,068	0,124	0,028	0,118	0,061	0,141	0,019	MN
V_d	12	2,655	2,529	1,317	1,078	2,315	2,210	1,006	0,782	MN
H_d	13	0,195	0,000	0,195	0,000	0,195	0,000	0,195	0,000	MN
M_d	14	1,067	0,346	0,851	0,130	1,034	0,312	0,818	0,096	MNm
e_0	16	0,402	0,137	0,646	0,120	0,446	0,141	0,812	0,123	m
b	17	1,251	1,074	0,832	0,544	1,149	0,968	0,733	0,414	m
B'	18	1,251	1,074	0,832	0,544	1,149	0,968	0,733	0,414	m
L'	19	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	20	5,005	4,294	3,327	2,174	4,597	3,871	2,934	1,657	m^2
q_d	21	0,530	0,589	0,396	0,496	0,504	0,571	0,343	0,472	MN/m^2
s_q	29	1,156	1,134	1,104	1,068	1,144	1,121	1,092	1,052	
s_γ	30	0,906	0,919	0,938	0,959	0,914	0,927	0,945	0,969	
m	31	1,762	1,788	1,828	1,880	1,777	1,805	1,845	1,906	
i_q	32	0,874	1,000	0,746	1,000	0,855	1,000	0,672	1,000	
i_γ	33	0,810	1,000	0,636	1,000	0,783	1,000	0,542	1,000	
q_m	35	0,743	0,824	0,554	0,694	0,705	0,799	0,480	0,661	MN/m^2
q_{md}	36	0,530	0,589	0,396	0,496	0,504	0,571	0,343	0,472	MN/m^2
n	37	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

20. PROBLEM

A base slab have a breadth a (Figure 1). Determine the smallest possible dimension a so that the bearing resistance is adequate! Use design approach 2 of ultimate limit state (GEO) (Chapter 2.4.7.3.4.3) and Finnish Annexes A (Chapter. A.3) and D of Eurocode *SFS-EN1997-1* !

Characteristic values of loading due to the overhead structure

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Proportion of the permanent load from the total load is

k_V = 0,400

k_H = 0,000

k_M = 0,400

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Unit weight of soil

γ_m = 0,020 MN/m^3

Angle of internal friction of the earth

φ = 30,000 °

Dimensions

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m

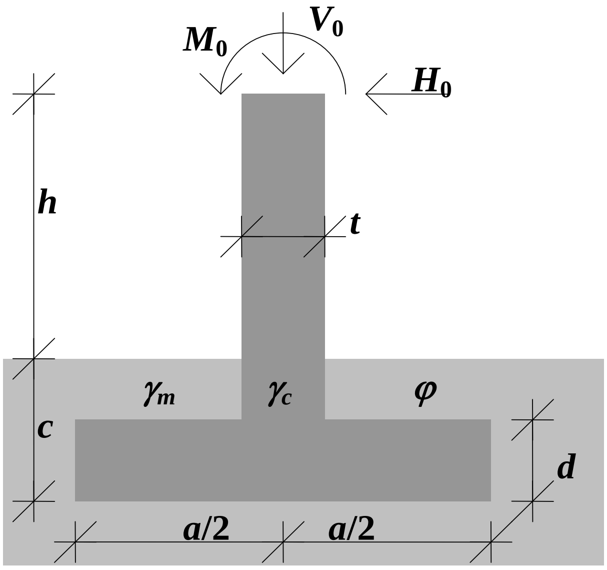


Figure 1.

Length of the base slab (perpendicular to the figure plane)

L = 4,000 m

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Approach given by Eurocode *SFS-EN 1997-1* is used.

Load combination and partial safety factors are from Finnish Annex. These are different than guide given in Appendix A of *SFS-EN 1997-1* .

Combination of sets of partial factors of design approach 2

A1 "+" M1 "+" R2

where

- **A refers to loads or actions of loads (Table A.3 of Appendix A),**
- **M refers to soil parameters (Table A.4),**
- **R refers to resistance (Table A.5),**
- **numbers 1 and 2 refer to sets 1 and 2 and**
- **"+" implies "to be combinated with".**

Factor for combination value of imposed load (*Finnish Annex for SFS-EN 1990* , Table A1.1, Category A)

$$\psi_{0,imposed} = 0,7$$

Factor for combination value of wind load

$$\psi_{0,wind} = 0,6$$

thus

$$1,5\psi_{0,imposed} = 1,05$$

$$1,5\psi_{0,wind} = 0,90$$

When load factor

$$K_{FI} = 1$$

then the load combination is

$$F_d = \left.\begin{matrix}1,15\\1,35\end{matrix}\right\} G_{kj,\sup} + 0,9G_{kj,\inf} + \left.\begin{matrix}1,5\\0\end{matrix}\right\} Q_{k1} + \left.\begin{matrix}1,5\\0\end{matrix}\right\} \sum \psi_{0i} Q_{ki} \qquad (1)$$

which can be written

$$q_d = g_m + \left.\begin{array}{l} 1{,}15 \\ 1{,}15 \\ 1{,}15 \\ 1{,}15 \\ 1{,}35 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{array}\right\} g_c + \left.\begin{array}{l} 1{,}5 \\ 1{,}5 \\ 1{,}05 \\ 0 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 1{,}05 \\ 0 \\ 0 \end{array}\right\} q_{k1} + \left.\begin{array}{l} 0{,}9 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \\ 0{,}9 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \end{array}\right\} q_{k2} \quad \begin{array}{c} (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f) \\ (g) \\ (h) \\ (i) \\ (j) \end{array} \qquad (2a\ldots j)$$

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select

$$a \ = \ 1{,}900 \text{ m}$$

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved. The sub indexes are referred to that ones used in Equation 1.

Self weight of the base structure and soil (as function of a)

$$V_{gc,m} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \qquad (3)$$

$$= \ 0{,}346 \text{ MN}$$

Permanent vertical load

$$V_g = k_V V_0 \qquad (4)$$

$$= \ 0{,}600 \text{ MN}$$

Variable vertical imposed load

$$V_{q1} = (1\text{-}k_V)V_0 \tag{5}$$

$$= 0{,}900 \text{ MN}$$

Permanent horizontal load

$$H_g = k_H H_0 \tag{6}$$

$$= 0{,}000 \text{ MN}$$

Variable horizontal wind load

$$H_{q2} = (1\text{-}k_H)H_0 \tag{7}$$

$$= 0{,}130 \text{ MN}$$

Moment due to permanent loads

$$M_g = k_M M_0 + k_H H_0(c+h) \tag{8}$$

$$= 0{,}096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1\text{-}k_M)M_0 \tag{9}$$

$$= 0{,}144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1\text{-}k_H)H_0(c+h) \tag{10}$$

$$= 0{,}481 \text{ MNm}$$

Design values of loads: vertical load, horizontal one and moment

$$V_d = 1{,}15(V_{gc,m} + V_g) + 1{,}5V_{q1} \tag{11}$$

$$= 2{,}438 \text{ MN}$$

$$H_d = 1{,}15H_g + 0{,}9H_{q2} \tag{12}$$

$$= 0{,}117 \text{ MN}$$

$$M_d = 1{,}15M_g + 1{,}5M_{q1} + 0{,}9M_{q2} \tag{13}$$

$$= 0{,}759 \text{ MNm}$$

From equilibrium condition of the moments with respect to the pivot (Figure 2)

$$M_d - V_d e_o = 0 \tag{14}$$

the load eccentricity is obtained

$$e_o = \frac{M_d}{V_d} \tag{15}$$

$$= \quad 0{,}311 \ \text{m}$$

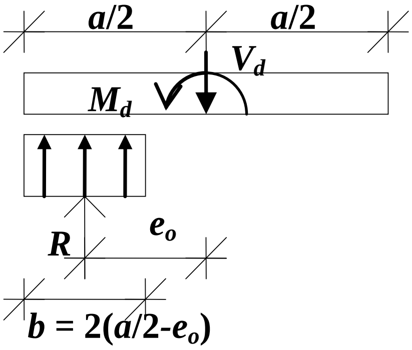


Figure 2.

Effective foundation length in the direction of *a*

$$b = 2\left(\frac{a}{2} - e_o\right) \tag{16}$$

$$= \quad 1{,}277 \ \text{m}$$

Effective foundation width (Figure 3)

$$B' = \min\begin{cases} b \\ L \end{cases} \tag{17}$$

$$= \quad 1{,}277 \ \text{m}$$

Effective foundation length

$$L' = \max\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= \quad 4{,}000 \ \text{m}$$

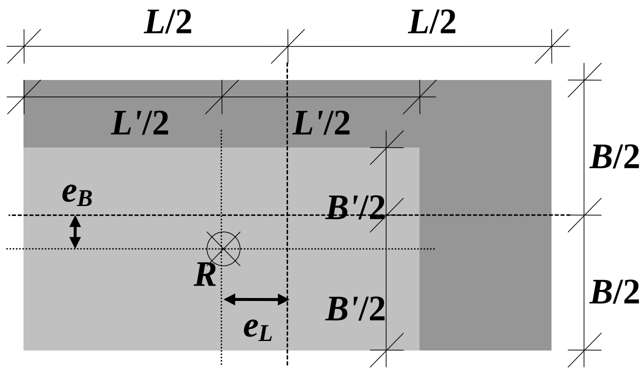


Figure 3.

Effective foundation area (Figure 3)

$$A' = B'L' \tag{19}$$

$$= 5{,}108 \text{ m}^2$$

Design value of bottom pressure

$$q_d = \frac{V_d}{A'} \tag{20}$$

$$= 0{,}477 \text{ MN/m}^2$$

Design value of cohesion

$$c' = 0 \text{ MN/m}^2$$

Unit weight of soil above the foundation level

$$\gamma = \gamma_m \tag{21}$$

$$= 0{,}020 \text{ MN/m}^3$$

Unit weight of soil below the foundation level

$$\gamma' = \gamma_m \tag{22}$$

$$= 0{,}020 \text{ MN/m}^3$$

Effective overburden pressure at the level of the foundation area

$$q' = c\,\gamma \tag{23}$$

$$= 0{,}030 \text{ MN/m}^2$$

Inclination of the foundation base

$$\alpha = 0\,°$$

Partial factor for angle of shearing resistance of the soil (*Finnish annex for EN 1997-1* , Application A. Table A.4.)

$$\gamma_{\varphi} = 1{,}000$$

Design value of the internal friction angle of the earth

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_{\varphi}}\right) \tag{24}$$

$$= 30{,}000\,°$$

Bearing capacity factors

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'} \tag{25}$$

$$= 18{,}401$$

$$N_{\gamma} = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'} - 1\right] \tan\varphi' \tag{26}$$

$$= 20{,}093$$

Factors for the inclination of the base

$$\left.\begin{matrix} b_q \\ b_{\gamma} \end{matrix}\right\} = (1 - \alpha \tan\varphi')^2 \tag{27a, b}$$

$$= 1{,}000$$

Shape factors of the base slab

$$s_q = 1 + \frac{B'}{L'} \sin\varphi' \tag{28}$$

$$= 1{,}160$$

$$s_{\gamma} = 1 - 0{,}3 \frac{B'}{L'} \tag{29}$$

$$= 0{,}904$$

Parameter *m*

$$m = m_B \tag{30}$$

$$= \frac{2+\frac{B'}{L'}}{1+\frac{B'}{L'}} \tag{31}$$

$$= \quad 1{,}758$$

Inclination factors of the load resultant

$$i_q = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^m \tag{32}$$

$$= \quad 0{,}917$$

$$i_\gamma = \left(1 - \frac{H_d}{V_d + A'c'\cot\varphi'}\right)^{m+1} \tag{33}$$

$$= \quad 0{,}873$$

Partial resistance factor for bearing (*Finnish annex for EN 1997-1* , Appendix A, Table A.5)

$$\gamma_R = \quad 1{,}55$$

Bearing capacity

$$q_m = \frac{R}{A'} \tag{34}$$

$$= c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{35}$$

$$= \quad 0{,}790 \ \text{MN/m}^2$$

Design value of the bearing capacity

$$q_{md} = \frac{q_m}{\gamma_R} \tag{36}$$

$$= \quad 0{,}510 \ \text{MN/m}^2$$

Ratio between bearing capacity and bottom pressure

$$n = q_{md} / q_d \tag{37}$$

$= 1,068 > 1$, **OK**

Breadth of the base slab is obtained by an iteration process.

$a = a_1 =$ **1,900 m**

The value is rounded up by precision of 0,1 m.

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

ANSWER: **Breadth of the base slab**

$a =$ **2,600 m**

Table 1.

Quant	Eq.	Loading Cases										Units
i		a	b	c	d	e	f	g	h	i	j	
a		1,839	1,377	2,110	2,292	0,834	1,803	1,302	2,128	2,517	0,658	m
ext(a)										Max	Min	
a_{round}		1,900	1,400	2,200	2,300	0,900	1,900	1,400	2,200	2,600	0,700	m
γ_g	2	1,15	1,15	1,15	1,15	1,35	0,90	0,90	0,90	0,90	0,90	
γ_{qk1}	2	1,50	1,50	1,05	0,00	0,00	1,50	1,50	1,05	0,00	0,00	
γ_{qk2}	2	0,90	0,00	1,50	1,50	0,00	0,90	0,00	1,50	1,50	0,00	
$V_{gc,m}$	3	0,338	0,277	0,374	0,398	0,205	0,333	0,267	0,376	0,427	0,182	MN
V_d	11	2,429	2,358	2,065	1,147	1,087	2,190	2,130	1,823	0,925	0,704	MN
H_d	12	0,117	0,000	0,195	0,195	0,000	0,117	0,000	0,195	0,195	0,000	MN
M_d	13	0,759	0,326	0,983	0,832	0,130	0,735	0,302	0,959	0,808	0,086	MNm
e_0	15	0,313	0,138	0,476	0,725	0,119	0,336	0,142	0,526	0,874	0,123	m
b	16	1,213	1,100	1,158	0,842	0,595	1,132	1,018	1,076	0,769	0,413	m
B'	17	1,213	1,100	1,158	0,842	0,595	1,132	1,018	1,076	0,769	0,413	m
L'	18	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	19	4,853	4,400	4,631	3,369	2,382	4,527	4,071	4,304	3,077	1,651	m^2
q_d	20	0,500	0,536	0,446	0,341	0,456	0,484	0,523	0,424	0,301	0,426	MN/m^2
s_q	28	1,152	1,138	1,145	1,105	1,074	1,141	1,127	1,135	1,096	1,052	
s_γ	29	0,909	0,917	0,913	0,937	0,955	0,915	0,924	0,919	0,942	0,969	
m	31	1,767	1,784	1,776	1,826	1,870	1,779	1,797	1,788	1,839	1,906	
i_q	32	0,916	1,000	0,838	0,712	1,000	0,907	1,000	0,817	0,647	1,000	
i_γ	33	0,872	1,000	0,759	0,591	1,000	0,858	1,000	0,730	0,511	1,000	
q_m	34	0,776	0,831	0,691	0,528	0,707	0,750	0,811	0,657	0,466	0,661	MN/m^2
q_{md}	36	0,501	0,536	0,446	0,341	0,456	0,484	0,523	0,424	0,301	0,426	MN/m^2
n	37	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

21. PROBLEM

A base slab have a breadth *a* (Figure 1). Determine the smallest possible dimension *a* so that the bearing resistance is adequate!

Use design approach DA2* of the ultimate limit state (GEO):

- ***Finnish Annex for Standard SFS-EN 1997-1*, Chapter 4 and Table A.3.**
- ***SFS-EN 1997-1+AC*, Annex D; or *Formulary* of the course, Chapter 2.1.**

Characteristic values of loading due to the overhead structure

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Proportion of the permanent load from the total load is

k_V = 0,400

k_H = 0,000

k_M = 0,400

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Unit weight of soil

γ_m = 0,018 MN/m^3

Angle of internal friction of the earth

φ = 34,000 °

Dimensions

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m

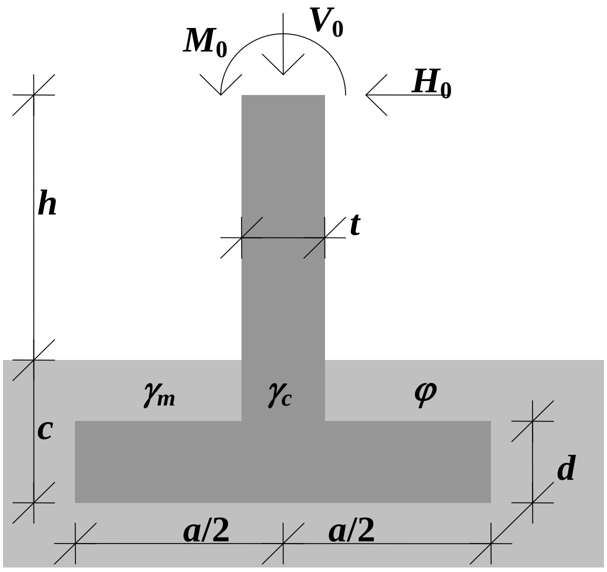


Figure 1.

Length of the base slab (perpendicular to the figure plane)

L = 4,000 m

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Approach given by Eurocode *SFS-EN 1997-1* is used.

Load combination and partial safety factors are from Finnish Annex.
These are different than guide given in Appendix A of *SFS-EN 1997-1* .

Combination of sets of partial factors of design approach 2

A1 "+" M1 "+" R2

where

- **A refers to loads or actions of loads (Table A.3 of Appendix A),**
- **M refers to soil parameters (Table A.4),**
- **R refers to resistance (Table A.5),**
- **numbers 1 and 2 refer to sets 1 and 2 and**
- **"+" implies "to be combinated with".**

Factor for combination value of imposed load (*Finnish Annex for SFS-EN 1990* , Table A1.1, Category A)

$$\psi_{0,imposed} = 0,7$$

Factor for combination value of wind load

$$\psi_{0,wind} = 0,6$$

thus

$$1,5\psi_{0,imposed} = 1,05$$

$$1,5\psi_{0,wind} = 0,90$$

When load factor

$$K_{FI} = 1$$

then the load combination is

$$F_d = \left.\begin{matrix}1,15\\1,35\end{matrix}\right\} G_{kj,\sup} + 0,9G_{kj,\inf} + \left.\begin{matrix}1,5\\0\end{matrix}\right\} Q_{k1} + \left.\begin{matrix}1,5\\0\end{matrix}\right\} \sum \psi_{0i} Q_{ki} \qquad (1)$$

which can be written

$$q_d = \begin{Bmatrix} 1{,}15 \\ 1{,}15 \\ 1{,}15 \\ 1{,}15 \\ 1{,}35 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{Bmatrix}(g_c + g_m) + \begin{Bmatrix} 1{,}5 \\ 1{,}5 \\ 1{,}05 \\ 0 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 1{,}05 \\ 0 \\ 0 \end{Bmatrix} q_{k1} + \begin{Bmatrix} 0{,}9 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \\ 0{,}9 \\ 0 \\ 1{,}5 \\ 1{,}5 \\ 0 \end{Bmatrix} q_{k2} \quad \begin{matrix} (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f) \\ (g) \\ (h) \\ (i) \\ (j) \end{matrix} \qquad (2a\ldots j)$$

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select

$$a = 1{,}800 \text{ m}$$

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved. The sub indexes are referred to that ones used in Equation 1.

Self weight of the base structure and soil (as function of a)

$$V_{gc,m} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \qquad (3)$$

$$= 0{,}323 \text{ MN}$$

Permanent vertical load

$$V_g = k_V V_0 \qquad (4)$$

$$= 0{,}600 \text{ MN}$$

Variable vertical imposed load

$$V_{q1} = (1-k_V)V_0 \tag{5}$$
$$= 0{,}900 \text{ MN}$$

Permanent horizontal load

$$H_g = k_H H_0 \tag{6}$$
$$= 0{,}000 \text{ MN}$$

Variable horizontal wind load

$$H_{q2} = (1-k_H)H_0 \tag{7}$$
$$= 0{,}130 \text{ MN}$$

Moment due to permanent loads

$$M_g = k_M M_0 + k_H H_0(c+h) \tag{8}$$
$$= 0{,}096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1-k_M)M_0 \tag{9}$$
$$= 0{,}144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1-k_H)H_0(c+h) \tag{10}$$
$$= 0{,}481 \text{ MNm}$$

Characteristic values of loads: vertical load, horizontal one and moment

$$V_k = V_{gc,m} + V_g + V_{q1} \tag{11}$$
$$= 1{,}823 \text{ MN}$$
$$H_k = H_g + H_{q2} \tag{12}$$
$$= 0{,}130 \text{ MN}$$
$$M_k = M_g + M_{q1} + M_{q2} \tag{13}$$
$$= 0{,}721 \text{ MNm}$$

Design value of the vertical load

$$V_d = 1{,}15(V_{gc,m} + V_g) + 1{,}5V_{q\,1} \tag{14}$$

$$= \quad 2{,}411 \text{ MN}$$

From equilibrium condition of the moments with respect to the pivot (Figure 2)

$$M_k - V_k e_o = 0 \tag{15}$$

the load eccentricity is obtained

$$e_o = \frac{M_k}{V_k} \tag{16}$$

$$= \quad 0{,}396 \text{ m}$$

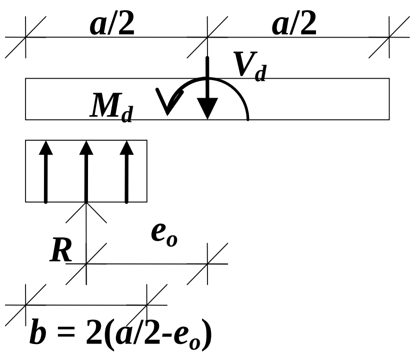


Figure 2.

Effective foundation length in the direction of *a*

$$b = 2\left(\frac{a}{2} - e_o\right) \tag{17}$$

$$= \quad 1{,}009 \text{ m}$$

Effective foundation width (Figure 3)

$$B' = \min\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= \quad 1{,}009 \text{ m}$$

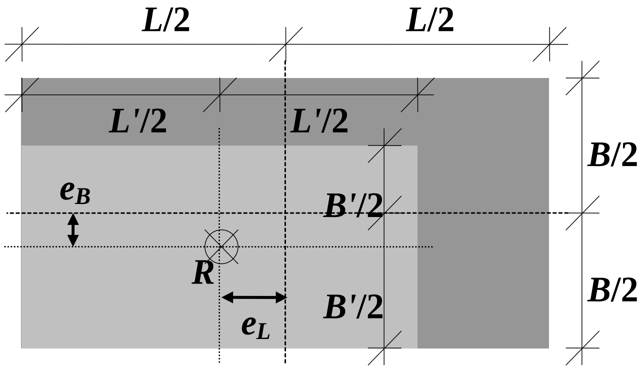


Figure 3.

Effective foundation length

$$L' = \max\begin{cases} b \\ L \end{cases} \tag{19}$$

$$= \quad 4{,}000 \text{ m}$$

Effective foundation area (Figure 3)

$$A' = B'L' \tag{20}$$

$$= \quad 4{,}035 \text{ m}^2$$

Design value of bottom pressure

$$q_d = \frac{V_d}{A'} \tag{21}$$

$$= \quad 0{,}597 \text{ MN/m}^2$$

Design value of cohesion

$$c' = \quad 0 \text{ MN/m}^2$$

Unit weight of soil above the foundation level

$$\gamma = \gamma_m \tag{22}$$

$$= \quad 0{,}018 \text{ MN/m}^3$$

Unit weight of soil below the foundation level

$$\gamma' = \gamma_m \tag{23}$$

$$= \quad 0{,}018 \ \mathrm{MN/m^3}$$

Effective overburden pressure at the level of the foundation area

$$q' = c\,\gamma \tag{24}$$

$$= \quad 0{,}027 \ \mathrm{MN/m^2}$$

Inclination of the foundation base

$$\alpha = \quad 0\ °$$

Partial factor for angle of shearing resistance of the soil (*Finnish annex for EN 1997-1* , Application A. Table A.4.)

$$\gamma_\varphi = \quad 1{,}000$$

Design value of the internal friction angle of the earth

$$\varphi' = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{25}$$

$$= \quad 34{,}000\ °$$

Bearing capacity factors

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan \varphi'} \tag{26}$$

$$= \quad 29{,}440$$

$$N_\gamma = 2\left[\tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan \varphi'} - 1\right] \tan \varphi' \tag{27}$$

$$= \quad 38{,}366$$

Factors for the inclination of the base

$$\left.\begin{matrix} b_q \\ b_\gamma \end{matrix}\right\} = (1 - \alpha \tan \varphi')^2 \tag{28a, b}$$

$$= \quad 1{,}000$$

Shape factors of the base slab

$$s_q = 1 + \frac{B'}{L'}\sin\varphi' \tag{29}$$

$$= \quad 1{,}141$$

$$s_\gamma = 1 - 0{,}3\frac{B'}{L'} \tag{30}$$

$$= \quad 0{,}924$$

Parameter *m*

$$m = m_B \tag{31}$$

$$= \frac{2 + \frac{B'}{L'}}{1 + \frac{B'}{L'}} \tag{32}$$

$$= \quad 1{,}799$$

Inclination factors of the load resultant

$$i_q = \left(1 - \frac{H_k}{V_k + A'c'\cot\varphi'}\right)^m \tag{33}$$

$$= \quad 0{,}875$$

$$i_\gamma = \left(1 - \frac{H_k}{V_k + A'c'\cot\varphi'}\right)^{m+1} \tag{34}$$

$$= \quad 0{,}813$$

Partial resistance factor for bearing (*Finnish annex for EN 1997-1* , Appendix A, Table A.5)

$$\gamma_R \; = \quad 1{,}55$$

Bearing capacity

$$q_m = \frac{R}{A'} \tag{35}$$

$$= c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{36}$$

$$= \quad 1{,}056 \ \text{MN/m}^2$$

Design value of the bearing capacity

$$q_{md} = \frac{q_m}{\gamma_R} \tag{37}$$

$$= \quad 0{,}681 \ \text{MN/m}^2$$

Ratio between bearing capacity and bottom pressure

$$n = q_{md} / q_d \tag{38}$$

$$= \quad 1{,}140 > 1, \text{ OK}$$

Breadth of the base slab is obtained by an iteration process.

$$a = \quad 1{,}800 \ \text{m}$$

The value is rounded up by precision of 0,1 m.

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

<u>ANSWER</u>: **Breadth of the base slab**

$$a = \quad 1{,}800 \ \text{m}$$

Table 1.

Quant	Eq.	Loading Cases										Units
i		a	b	c	d	e	f	g	h	i	j	
a		1,707	1,063	1,588	1,794	0,663	1,642	1,007	1,519	1,706	0,540	m
ext(a)					Max						Min	
a_{round}		1,800	1,100	1,600	1,800	0,700	1,700	1,100	1,600	1,800	0,600	m
γ_{cm}	2	1,15	1,15	1,15	1,15	1,35	0,90	0,90	0,90	0,90	0,90	
$\gamma_{qk\,1}$	2	1,50	1,50	1,05	0,00	0,00	1,50	1,50	1,05	0,00	0,00	
$\gamma_{qk\,2}$	2	0,90	0,00	1,50	1,50	0,00	0,90	0,00	1,50	1,50	0,00	
γ_{cmk}	2	1	1	1	1	1	1	1	1	1	1	
$\gamma_{qk\,1k}$	2	1	1	1	0	0	1	1	1	0	0	
$\gamma_{qk\,2k}$	2	1	0	1	1	0	1	0	1	1	0	
$V_{gc,m}$	3	0,311	0,231	0,296	0,322	0,181	0,303	0,224	0,288	0,311	0,165	MN
V_k	11	1,811	1,731	1,796	0,922	0,781	1,803	1,724	1,788	0,911	0,765	MN
H_k	12	0,130	0,000	0,130	0,130	0,000	0,130	0,000	0,130	0,130	0,000	MN
M_k	13	0,721	0,240	0,721	0,577	0,096	0,721	0,240	0,721	0,577	0,096	MNm
V_d	14	2,398	2,305	1,976	1,060	1,054	2,163	2,091	1,744	0,820	0,689	MN
e_0	16	0,398	0,139	0,401	0,626	0,123	0,400	0,139	0,403	0,633	0,125	m
b	17	0,911	0,786	0,786	0,542	0,417	0,842	0,728	0,713	0,439	0,289	m
B'	18	0,911	0,786	0,786	0,542	0,417	0,842	0,728	0,713	0,439	0,289	m
L'	19	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A'	20	3,643	3,143	3,142	2,167	1,666	3,368	2,913	2,851	1,757	1,155	m^2
q_d	21	0,658	0,734	0,629	0,489	0,633	0,642	0,718	0,612	0,467	0,596	MN/m^2
s_q	29	1,127	1,110	1,110	1,076	1,058	1,118	1,102	1,100	1,061	1,040	
s_γ	30	0,932	0,941	0,941	0,959	0,969	0,937	0,945	0,947	0,967	0,978	
m	32	1,815	1,836	1,836	1,881	1,906	1,826	1,846	1,849	1,901	1,933	
i_q	33	0,874	1,000	0,871	0,751	1,000	0,872	1,000	0,870	0,746	1,000	
i_γ	34	0,811	1,000	0,808	0,645	1,000	0,809	1,000	0,806	0,640	1,000	
q_m	36	1,020	1,137	0,975	0,758	0,981	0,995	1,113	0,948	0,723	0,925	MN/m^2
q_{md}	37	0,658	0,734	0,629	0,489	0,633	0,642	0,718	0,612	0,467	0,596	MN/m^2
n	38	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

22. PROBLEM

A base slab have a breadth a (Figure 1). Determine the smallest possible dimension a so that the bearing resistance is adequate! Use the equation of bearing capacity given in *RIL 121-2004* ! Use ultimate limit state method given in *RIL 144-2002* !

Characteristic values of loading due to the overhead structure

V_o = 1,500 MN

H_o = 0,130 MN

M_o = 0,240 MNm

Proportion of the permanent load from the total load is

k_V = 0,400

k_H = 0,000

k_M = 0,400

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Unit weight of soil

γ_m = 0,020 MN/m^3

Angle of internal friction of the earth

φ = 30,000 °

Dimensions

c = 1,500 m

h = 2,200 m

d = 0,600 m

t = 0,400 m

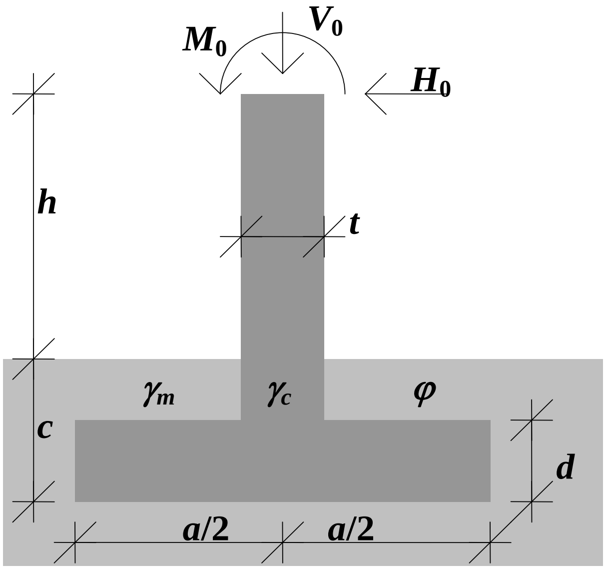


Figure 1.

Length of the base slab (perpendicular to the figure plane)

L = 4,000 m

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Load combination (*RIL 144-2002*, Chapter 8.21, p. 149)

$$q_d = \sum_{i=1}^{m} \gamma_{gi} g_i + \gamma_{q1} q_{k1} + \gamma_{q2} q_{k2} + \sum_{i=3}^{n} \gamma_{qi} q_{ki} \tag{1}$$

can be written

$$q_d = 1{,}0 g_c + \left.\begin{matrix} 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 1{,}2 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \\ 0{,}9 \end{matrix}\right\} g + \left.\begin{matrix} 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \\ 1{,}6 \\ 1{,}6 \\ 0 \\ 0 \end{matrix}\right\} q_{k1} + \left.\begin{matrix} 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \\ 1{,}6 \\ 0 \end{matrix}\right\} q_{k2} \qquad \begin{matrix} \text{(a)} \\ \text{(b)} \\ \text{(c)} \\ \text{(d)} \\ \text{(e)} \\ \text{(f)} \\ \text{(g)} \\ \text{(h)} \end{matrix} \tag{2a...h}$$

Partial safety factors are from books

- ***RIL 144-2002*, Table 8.21a, and**
- ***RIL 121-2004*, Chapter 5.1.2.3.**

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select

$$a = 2{,}700 \text{ m}$$

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved. The sub indexes are referred to that ones used in Equation 1.

Self weight of the base structure and soil (as function of *a*)

$$V_{gc} = \gamma_c[(h+c-d)t+ad]L+\gamma_m(a-t)(c-d)L \tag{3}$$

$$= 0{,}452 \text{ MN}$$

Permanent vertical load

$$V_g = k_V V_0 \tag{4}$$

$$= 0{,}600 \text{ MN}$$

Variable vertical imposed load

$$V_{q1} = (1-k_V)V_0 \tag{5}$$

$$= 0{,}900 \text{ MN}$$

Permanent horizontal load

$$H_g = k_H H_0 \tag{6}$$

$$= 0{,}000 \text{ MN}$$

Variable horizontal wind load

$$H_{q2} = (1-k_H)H_0 \tag{7}$$

$$= 0{,}130 \text{ MN}$$

Moment due to permanent loads

$$M_g = k_M M_0+H_g(c+h) \tag{8}$$

$$= 0{,}096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1-k_M)M_0 \tag{9}$$

$$= 0{,}144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1-k_H)H_0(c+h) \tag{10}$$

$$= 0{,}481 \text{ MNm}$$

Design values of loads: vertical load, horizontal one and moment

$$V_d = 1{,}0V_{gc} + 1{,}2V_g + 1{,}6V_{q1} \quad (11)$$

$$= 2{,}612 \text{ MN}$$

$$H_d = 1{,}2H_g + 1{,}6H_{q2} \quad (12)$$

$$= 0{,}208 \text{ MN}$$

$$M_d = 1{,}2M_g + 1{,}6M_{q1} + 1{,}6M_{q2} \quad (13)$$

$$= 1{,}115 \text{ MNm}$$

From equilibrium condition of the moments with respect to the pivot (Figure 2)

$$M_d - V_d e_o = 0 \quad (14)$$

the load eccentricity is obtained

$$e_o = \frac{M_d}{V_d} \quad (15)$$

$$= 0{,}427 \text{ m}$$

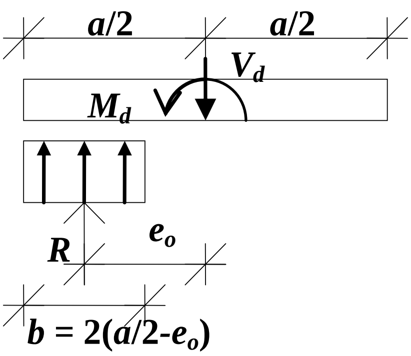


Figure 2.

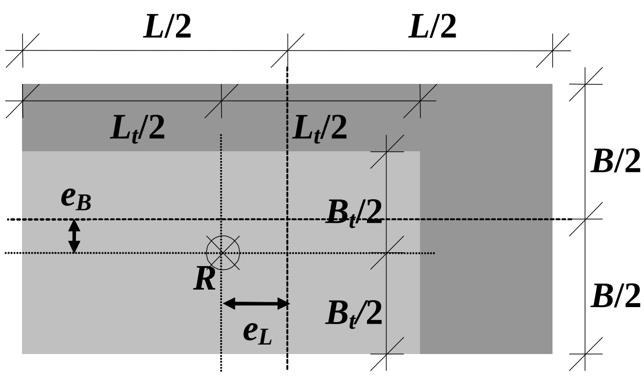


Figure 3.

Effective foundation length in the direction of *a*

$$b = 2\left(\frac{a}{2} - e_o\right) \tag{16}$$

$$= \quad 1{,}846 \text{ m}$$

Effective foundation width (Figure 3)

$$B_t = \min\begin{cases} b \\ L \end{cases} \tag{17}$$

$$= \quad 1{,}846 \text{ m}$$

Effective foundation length

$$L' = \max\begin{cases} b \\ L \end{cases} \tag{18}$$

$$= \quad 4{,}000 \text{ m}$$

Effective foundation area (Figure 3)

$$A_t = B_t L_t \tag{19}$$

$$= \quad 7{,}384 \text{ m}^2$$

Design value of bottom pressure

$$q_d = \frac{V_d}{A_t} \tag{20}$$

$$= \quad 0{,}354 \text{ MN/m}^2$$

Embedment depth

$$D = c \tag{21}$$

$$= \quad 1{,}500 \text{ m}$$

Design value of cohesion

$$c_d = \quad 0 \text{ MN/m}^2$$

Unit weight of soil above the foundation level

$$\gamma'_1 = \gamma_m \tag{22}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Unit weight of soil below the foundation level

$$\gamma'_2 = \gamma_m \tag{23}$$

$= \quad 0{,}020 \ \text{MN/m}^3$

Partial safety factor for angle of shearing resistance (*RIL 121-2004*, Table 7, p. 60)

$\gamma_\varphi = \quad 1{,}250$

Design value of the internal friction angle of the earth

$$\varphi_d = \arctan\left(\frac{\tan\varphi}{\gamma_\varphi}\right) \tag{24}$$

$= \quad 24{,}791\ °$

Bearing capacity factors

$$N_D = \tan^2(45° + \frac{\varphi_d}{2})e^{\pi \tan\varphi_d} \tag{25}$$

$= \quad 10{,}431$

$$N_B = 1{,}5\left[\tan^2\left(45° + \frac{\varphi_d}{2}\right)e^{\pi \tan\varphi_d} - 1\right]\tan\varphi_d \tag{26}$$

$= \quad 6{,}534$

Shape factors of the base slab

$$s_B = 1 - 0{,}4\left(\frac{B_t}{L_t}\right) \tag{27}$$

$= \quad 0{,}815$

$$s_D = 1 + 0{,}2\left(\frac{B_t}{L_t}\right) \tag{28}$$

$= \quad 1{,}092$

Inclination factors of the load resultant

$$i_D = \left(1 - \frac{H_d}{V_d + A_t c_d \cot\varphi_d}\right)^2 \tag{29}$$

$= \quad 0{,}847$

$$i_B = \left(1 - \frac{H_d}{V_d + A_t c_d \cot \varphi_d}\right)^4 \tag{30}$$

$= \quad 0,718$

Design value of the bearing capacity

$$q_{md} = c_d N_c s_c i_c + \gamma_1^{'} D N_D s_D i_D + \frac{1}{2} \gamma_2^{'} B_t N_B s_B i_B \tag{31}$$

$= \quad 0,360$ MN/m^2

Ratio between bearing capacity and bottom pressure

$$n = q_{md} / q_d \tag{31}$$

$= \quad 1,018 > 1$, OK

Breadth of the base slab is obtained by an iteration process.

$a = \quad 2,700$ m

The value is rounded up by precision of 0,1 m.

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

ANSWER: Breadth of the base slab

$a = \quad 2,800$ m

Table 1.

Quant.	Eq.	Loading Cases								Units
i		a	b	c	d	e	f	g	h	
a		2,673	1,763	2,666	0,890	2,630	1,666	2,787	0,755	
ext(a)								Max	Min	
a_{round}		2,700	1,800	2,700	0,900	2,700	1,700	2,800	0,800	
γ_m	2	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	
γ_g	2	1,20	1,20	1,20	1,20	0,90	0,90	0,90	0,90	
$\gamma_{qk\,1}$	2	1,60	1,60	0,00	0,00	1,60	1,60	0,00	0,00	
$\gamma_{qk\,2}$	2	1,60	0,00	1,60	0,00	1,60	0,00	1,60	0,00	
V_{gc}	3	0,448	0,328	0,447	0,213	0,442	0,315	0,463	0,195	MN
V_d	11	2,608	2,488	1,167	0,933	2,422	2,295	1,003	0,735	MN
H_d	12	0,208	0,000	0,208	0,000	0,208	0,000	0,208	0,000	MN
M_d	13	1,115	0,346	0,885	0,115	1,086	0,317	0,856	0,086	MNm
e_0	15	0,428	0,139	0,758	0,124	0,448	0,138	0,853	0,118	m
b	16	1,818	1,485	1,150	0,643	1,733	1,390	1,081	0,520	m
B_t	17	1,818	1,485	1,150	0,643	1,733	1,390	1,081	0,520	m
L_t	18	4,000	4,000	4,000	4,000	4,000	4,000	4,000	4,000	m
A_t	19	7,271	5,941	4,599	2,573	6,931	5,559	4,322	2,080	m^2
q_d	20	0,359	0,419	0,254	0,362	0,349	0,413	0,232	0,353	MN/m^2
s_B	27	0,818	0,851	0,885	0,936	0,827	0,861	0,892	0,948	
s_D	28	1,091	1,074	1,057	1,032	1,087	1,069	1,054	1,026	
i_D	29	0,847	1,000	0,675	1,000	0,836	1,000	0,628	1,000	
i_B	30	0,717	1,000	0,456	1,000	0,698	1,000	0,395	1,000	
q_{md}	31	0,359	0,419	0,254	0,362	0,350	0,413	0,232	0,353	MN/m^2
n	32	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	

23. PROBLEM

A base slab have a breadth a (Figure 1). Determine the smallest possible dimension a so that the bearing resistance is adequate! Use the equation of bearing capacity given in *RIL 121-2004* ! Use total safety method when phi factor is

n = 2

Characteristic values of loading due to the overhead structure

V_o = 1,500 MN
H_o = 0,130 MN
M_o = 0,240 MNm

Proportion of the permanent load from the total load is

k_V = 0,400
k_H = 0,000
k_M = 0,400

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

γ_c = 0,025 MN/m^3

Unit weight of soil

γ_m = 0,020 MN/m^3

Angle of internal friction of the earth

φ = 30,000 °

Dimensions

b = 1,500 m
h = 2,200 m
d = 0,600 m
t = 0,400 m

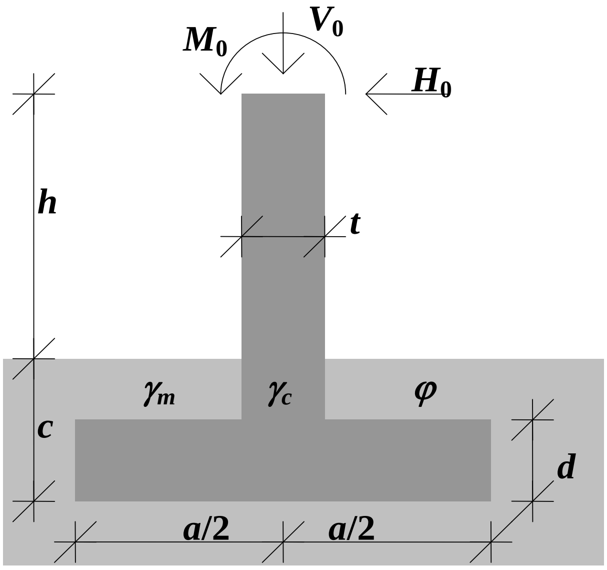


Figure 1.

Length of the base slab (perpendicular to the figure plane)

L = 4,000 m

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Load combination (*RIL 144-2002*, Chapter 8.21, p. 149)

$$q_d = \sum_{i=1}^{m} g_i + q_{k1} + q_{k2} + \sum_{i=3}^{n} q_{ki} \tag{1}$$

can be written

$$q_d = g + \left.\begin{matrix}1\\1\\0\\0\end{matrix}\right\} q_{k1} + \left.\begin{matrix}1\\0\\1\\0\end{matrix}\right\} q_{k2} \quad \begin{matrix}(a)\\(b)\\(c)\\(d)\end{matrix} \tag{2a…d}$$

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select

a = 2,300 m

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved. The sub indexes are referred to that ones used in Equation 1.

Self weight of the overhead structure, base structure and soil (as function of a)

$$V_g = k_V V_0 + \gamma_c[(h + c - d)t + ad]L + \gamma_m (a - t)(c - d)L \tag{3}$$

= 0,999 MN

Variable vertical imposed load

$$V_{q1} = (1 - k_V)V_0 \tag{4}$$

= 0,900 MN

Permanent horizontal load

$$H_g = k_H H_0 \tag{5}$$

$$= \quad 0{,}000 \text{ MN}$$

Variable horizontal wind load

$$H_{q2} = (1\text{-}k_H)H_0 \tag{6}$$

$$= \quad 0{,}130 \text{ MN}$$

Moment due to permanent loads

$$M_g = k_M M_0 + H_g(c+h) \tag{7}$$

$$= \quad 0{,}096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1\text{-}k_M)M_0 \tag{8}$$

$$= \quad 0{,}144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1\text{-}k_H)H_0(c+h) \tag{9}$$

$$= \quad 0{,}481 \text{ MNm}$$

Design values of loads: vertical load, horizontal one and moment

$$V_d = V_g + V_{q1} \tag{10}$$

$$= \quad 1{,}899 \text{ MN}$$

$$H_d = H_g + H_{q2} \tag{11}$$

$$= \quad 0{,}130 \text{ MN}$$

$$M_d = M_g + M_{q1} + M_{q2} \tag{12}$$

$$= \quad 0{,}721 \text{ MNm}$$

From equilibrium condition of the moments with respect to the pivot (Figure 2)

$$M_d - V_d e_o = 0 \tag{13}$$

the load eccentricity is obtained

$$e_o = \frac{M_d}{V_d} \tag{14}$$

= 0,380 m

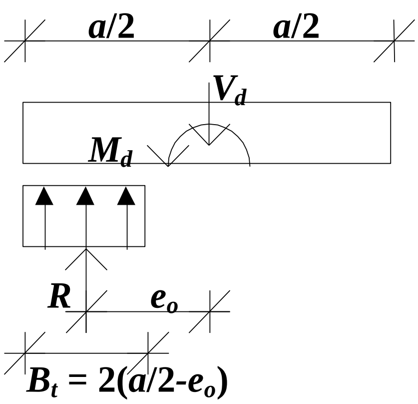


Figure 2.

Effective foundation length in the direction of *a*

$$b = 2(\frac{a}{2} - e_o) \tag{15}$$

= 1,541 m

Effective foundation width (Figure 3)

$$B_t = \min\begin{cases} b \\ L \end{cases} \tag{16}$$

= 1,541 m

Effective foundation length

$$L' = \max\begin{cases} b \\ L \end{cases} \tag{17}$$

= 4,000 m

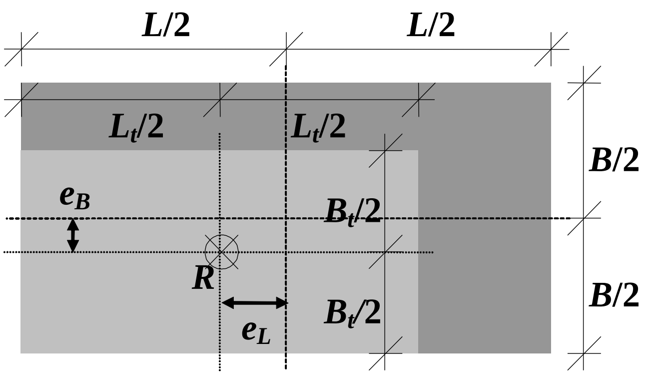


Figure 3.

Effective foundation area (Figure 3)

$$A_t = B_t L_t \tag{18}$$

$$= \quad 6{,}162 \text{ m}^2$$

Design value of bottom pressure

$$q_d = \frac{V_d}{A_t} \tag{19}$$

$$= \quad 0{,}308 \text{ MN/m}^2$$

Embedment depth

$$D = c \tag{20}$$

$$= \quad 1{,}500 \text{ m}$$

Design value of cohesion

$$c_d = \quad 0 \text{ MN/m}^2$$

Unit weight of soil above the foundation level

$$\gamma'_1 = \gamma_m \tag{21}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Unit weight of soil below the foundation level

$$\gamma'_2 = \gamma_m \tag{22}$$

$$= \quad 0{,}020 \text{ MN/m}^3$$

Design value of the internal friction angle of the earth

$$\varphi_d = \varphi \tag{22}$$
$$= \quad 30{,}000\ ^\circ$$

Bearing capacity factors

$$N_D = \tan^2(45° + \frac{\varphi_d}{2})e^{\pi \tan\varphi_d} \tag{23}$$
$$= \quad 18{,}401$$
$$N_B = 1{,}5\left[\tan^2\left(45° + \frac{\varphi_d}{2}\right)e^{\pi \tan\varphi_d} - 1\right]\tan\varphi_d \tag{24}$$
$$= \quad 15{,}070$$

Shape factors of the base slab

$$s_B = 1 - 0{,}4\left(\frac{B_t}{L_t}\right) \tag{25}$$
$$= \quad 0{,}846$$
$$s_D = 1 + 0{,}2\left(\frac{B_t}{L_t}\right) \tag{26}$$
$$= \quad 1{,}077$$

Inclination factors of the load resultant

$$i_D = \left(1 - \frac{H_d}{V_d + A_t c_d \cot\varphi_d}\right)^2 \tag{27}$$
$$= \quad 0{,}868$$
$$i_B = \left(1 - \frac{H_d}{V_d + A_t c_d \cot\varphi_d}\right)^4 \tag{28}$$
$$= \quad 0{,}753$$

Design value of the bearing capacity

$$q_{md} = c_d N_c s_c i_c + \gamma_1^{'} D N_D s_D i_D + \frac{1}{2}\gamma_2^{'} B_t N_B s_B i_B \tag{29}$$
$$= \quad 0{,}354\ \mathrm{MN/m^2}$$

Ratio between bearing capacity and bottom pressure

$$n = q_{md}/q_d \tag{30}$$

$= 2,154 > 2,000$ **OK**

Breadth of the base slab is obtained by an iteration process.

$a = 2,300$ **m**

The value is rounded up by precision of 0,1 m.

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

ANSWER: **Breadth of the base slab**

$a = 2,300$ **m**

Table 1.

Quant.	Eq.	Loading Cases				Unit
i		a	b	c	d	
a		2,209	1,470	2,141	0,853	m
ext(a)		Max			Min	
a_{round}		2,300	1,500	2,200	0,900	m
γ_g	2	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,00	1,00	0,00	0,00	
$\gamma_{qk\,2}$	2	1,00	0,00	1,00	0,00	
V_g	3	0,987	0,889	0,978	0,808	MN
V_d	10	1,887	1,789	0,978	0,808	MN
H_d	11	0,130	0,000	0,130	0,000	MN
M_d	12	0,721	0,240	0,577	0,096	MNm
e_0	14	0,382	0,134	0,590	0,119	m
b	15	1,445	1,202	0,961	0,616	m
B_t	16	1,445	1,202	0,961	0,616	m
L_t	17	4,000	4,000	4,000	4,000	m
A_t	18	5,779	4,807	3,843	2,463	m^2
q_d	19	0,327	0,372	0,254	0,328	MN/m^2
s_B	26	0,856	0,880	0,904	0,938	
s_D	27	1,072	1,060	1,048	1,031	
i_D	28	0,867	1,000	0,752	1,000	
i_B	29	0,752	1,000	0,565	1,000	
q_{md}	30	0,653	0,745	0,509	0,656	MN/m^2
n	31	2,000	2,000	2,000	2,000	

24. PROBLEM

A base slab have a breadth a (Figure 1). Determine the smallest possible dimension a so that the bearing resistance is adequate! (Compare to the prescriptive method of *Eurocode*, *EN 1997-1:2004*, 6.4(5)P.) The allowed bearing resistance is

$$\sigma_{sall} = 0{,}400 \ \mathrm{MN/m^2}$$

Characteristic values of loading due to the overhead structure

$$V_o = 1{,}500 \ \mathrm{MN}$$
$$H_o = 0{,}130 \ \mathrm{MN}$$
$$M_o = 0{,}240 \ \mathrm{MNm}$$

Proportion of the permanent load from the total load is

$$k_V = 0{,}400$$
$$k_H = 0{,}000$$
$$k_M = 0{,}400$$

and the rest is variable one. The proportion of the variable load of vertical load V_o and moment M_o are caused by effective load and horizontal load H_o by wind.

Unit weight of reinforced concrete

$$\gamma_c = 0{,}025 \ \mathrm{MN/m^3}$$

Unit weight of soil

$$\gamma_m = 0{,}020 \ \mathrm{MN/m^3}$$

Angle of internal friction of the earth

$$\varphi = 30{,}000\ ^\circ$$

Dimensions

$$c = 1{,}500 \ \mathrm{m}$$
$$h = 2{,}200 \ \mathrm{m}$$
$$d = 0{,}600 \ \mathrm{m}$$
$$t = 0{,}400 \ \mathrm{m}$$

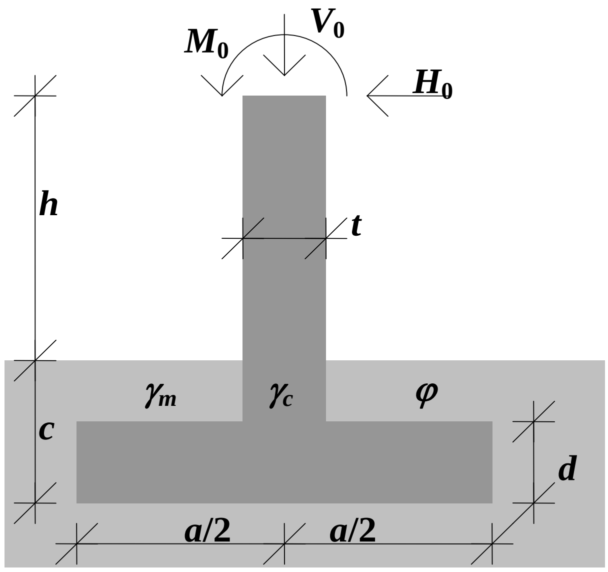


Figure 1.

Length of the base slab (perpendicular to the figure plane)

$$L = 4{,}000 \ \mathrm{m}$$

In this direction the loading is central.

COMBINATIONS OF ACTIONS

Load combination

$$q_d = \sum_{i=1}^{m} g_i + q_{k1} + q_{k2} + \sum_{i=3}^{n} q_{ki} \tag{1}$$

can be written

$$q_d = g + \left.\begin{matrix} 1 \\ 1 \\ 0 \\ 0 \end{matrix}\right\} q_{k1} + \left.\begin{matrix} 1 \\ 0 \\ 1 \\ 0 \end{matrix}\right\} q_{k2} \qquad \begin{matrix} (a) \\ (b) \\ (c) \\ (d) \end{matrix} \tag{2a…d}$$

First the load case "a" is studied and after that the others.

LOAD CASE a

Lets select

a = 2,400 m

The value given here is the final result after the iteration.

Characteristic load values at the surface of the base slab are solved. The sub indexes are referred to that ones used in Equation 1.

Self weight of the overhead structure, base structure and soil (as function of a)

$$V_g = k_V V_0 + \gamma_c [(h + c - d) t + ad] L + \gamma_m (a - t)(c - d) L \tag{3}$$

= 1,012 MN

Variable vertical imposed load

$$V_{q1} = (1 - k_V) V_0 \tag{4}$$

= 0,900 MN

Moment due to permanent loads

$$M_g = k_M M_0 + k_H H_0 (c+h) \tag{5}$$

$$= 0{,}096 \text{ MNm}$$

Moment due to imposed load

$$M_{q1} = (1-k_M) M_0 \tag{6}$$

$$= 0{,}144 \text{ MNm}$$

Moment due to wind load

$$M_{q2} = (1-k_H) H_0 (c+h) \tag{7}$$

$$= 0{,}481 \text{ MNm}$$

Design values of loads: vertical load and moment

$$V_d = V_g + V_{q1} \tag{8}$$

$$= 1{,}912 \text{ MN}$$

$$M_d = M_g + M_{q1} + M_{q2} \tag{9}$$

$$= 0{,}721 \text{ MNm}$$

Stress (Figure 2)

$$\sigma(y) = \frac{V_d}{A} + \frac{M_d}{I_x} y \tag{10}$$

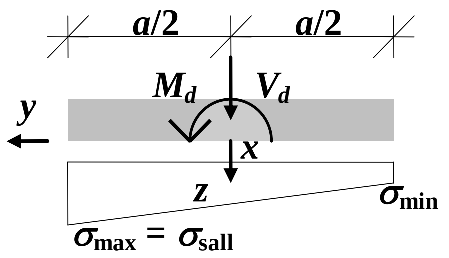


Figure 2.

Area

$$A = aL \tag{11}$$

$$= 9{,}600 \text{ m}^2$$

Moment of inertia with respect to x- axis

$$I_x = \frac{La^3}{12} \tag{12}$$

$$= \quad 4{,}608 \ \mathrm{m}^4$$

Maximum stress (Figure 2)

$$\sigma_{max} = \sigma(y = a/2) \tag{13}$$

$$= \quad 0{,}387 \ \mathrm{MN/m^2} \quad < \quad \sigma_{sall}$$

Breadth of the base slab is obtained by an iteration process.

$$a = \quad 2{,}400 \ \mathrm{m}$$

The value is rounded up by precision of 0,1 m.

Minimum stress (Figure 2)

$$\sigma_{min} = \sigma(y = -a/2) \tag{14}$$

$$= \quad 0{,}011 \ \mathrm{MN/m^2} \quad \text{Compression}$$

ALL LOAD CASES

Breadths of the base slab (a_i) in all loading cases are presented in Table 1. There are presented only the quantities, that are function of a and/or partial factors; all the other quantities are calculated above.

LOAD CASE c

In load case c, the tension is obtained as a minimum value of stress. Tension stress is not possible. Next is examined an effective breadth of cross-section (Figure 3).

Table 1.

Quantity	Eq.	Load Case				Unit
i		a	b	c	d	
a		2,343	1,673	1,791	0,907	m
ext(a)		Max			Min	
a_{round}		2,400	1,700	1,800	1,000	m
γ_g	2	1,00	1,00	1,00	1,00	
$\gamma_{qk\,1}$	2	1,00	1,00	0,00	0,00	
$\gamma_{qk\,2}$	2	1,00	0,00	1,00	0,00	
V_g	3	1,004	0,916	0,932	0,815	MN
V_d	8	1,904	1,816	0,932	0,815	MN
M_d	9	0,721	0,240	0,577	0,096	MNm
A	11	9,370	6,692	7,163	3,626	m^2
I_x	12	4,285	1,561	1,914	0,248	m^4
σ_{max}	13	0,400	0,400	0,400	0,400	MN/m^2
σ_{min}	14	0,006	0,143	-0,140	0,049	MN/m^2
ext(σ_{min})			Max	Min		

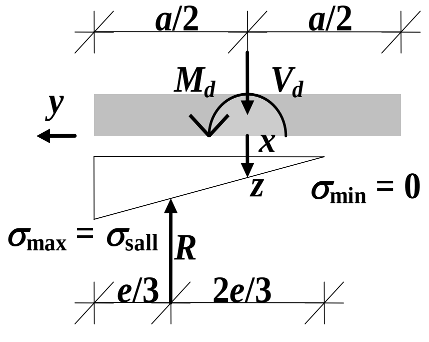


Figure 3.

Lets select

$$a = 2{,}100 \text{ m}$$

The value given here is the final result after the iteration.

Self weight of the overhead structure, base structure and soil (Equation 3, function of *a*)

$V_{gc} =$ **0,972 MN**

Design values of loads: vertical load and moment

$$V_{dc} = V_{gc} \tag{15}$$

$=$ **0,972 MN**

$$M_{dc} = M_g + M_{q2} \tag{16}$$

$=$ **0,577 MNm**

From the equilibrium condition of the vertical forces (Figure 3)

$$V_{dc} = \frac{1}{2} e L \sigma_{sall} \tag{17}$$

breadth of the compressed area is obtained

$$e = \frac{2V_{dc}}{L\sigma_{sall}} \tag{18}$$

$=$ **1,216 m**

Equilibrium condition of the moments with respect to origin

$$M_d = V_{dc}\left(\frac{a}{2} - \frac{e}{3}\right) \tag{19}$$

$=$ **0,627 MNm**

Ratio between the moment due to bottom stress and external moment

$$n = M_d / M_{3d} \tag{20}$$

$=$ **1,087** $\geq$ 1

Breadth of the base slab is obtained by an iteration process.

$a = a_c =$ **2,100 m**

<u>ANSWER</u>: **Breadth of the base slab**

$a =$ **2,400 m**

25. PROBLEM

Determine the pile forces due to point load

$$P = 6 \text{ MN}$$

of the pile group shown in Figure 1!

All piles have the same axial stiffness

$$C_i = E_i A_i$$

Length of the piles

$$L_1 = 14{,}000 \text{ m}$$

$$L_2 = 10{,}500 \text{ m}$$

$$L_3 = 7{,}000 \text{ m}$$

Number of piles in the rows

$$n_1 = 4$$

$$n_2 = 3$$

$$n_3 = 3$$

Distance between piles

$$d = 1{,}000 \text{ m}$$

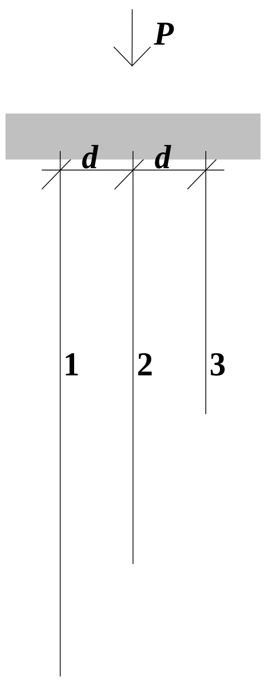


Figure 1.

Relative stiffness of the piles

$$k_i = \frac{C_i / L_i}{C_1 / L_1} = \frac{L_1}{L_i} \quad (1)$$

k_1 = 1,000
k_2 = 1,333
k_3 = 2,000

Distances of the rows from the centroid line, respectively, (Figure 2)

$$r_1 = \frac{\sum_{i=2}^{3} n_i k_i z_i}{\sum_{i=1}^{3} n_i k_i} \quad (2)$$

$$= \frac{n_2 k_2 d + n_3 k_3 \cdot 2d}{n_1 k_1 + n_2 k_2 + n_3 k_3} \quad (3)$$

= 1,143 m

$r_2 = r_1 - d$ (4)

= 0,143 m

$r_3 = r_1 - 2d$ (5)

= -0,857 m

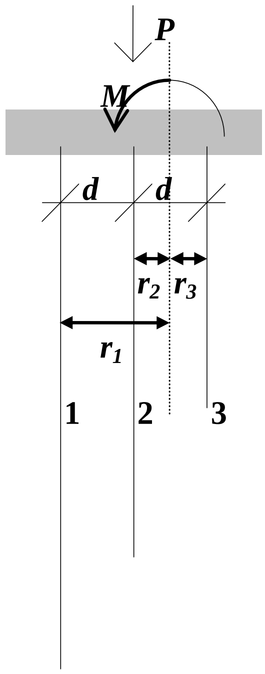


Figure 2.

Moment of the loads with respect to the centroid line

$M = Pr_2$ (6)

= 0,857 MNm

Pile force at row *i*

$$N_i = \frac{k_i}{\sum n_i k_i} P + \frac{k_i r_i}{\sum n_i k_i r_i^2} M \quad (7)$$

$$= \frac{k_i P}{n_1 k_1 + n_2 k_2 + n_3 k_3} + \frac{k_i r_i M}{n_1 k_1 r_1^2 + n_2 k_2 r_2^2 + n_3 k_3 r_3^2} \quad (8)$$

<u>ANSWER</u>: Pile forces are

N_1 = 0,529 MN
N_2 = 0,588 MN
N_3 = 0,706 MN

26. PROBLEM

Determine the minimum and maximum pile loads in the pile group shown below (Figure 1, Table 1 and 2)!

Loading

F_x = 8 MN

F_z = 2 MN

M_y = 1 MNm

Table 1. Vertical piles (z_{Vi} on the level of pile truncation).

Variable	Row	Number of piles	Horizontal location	Relative stiffness
Symbol	i_V	n_{Vi}	z_{Vi}	k_{Vi}
Unit	-		m	-
Numerical	1	10	-3,000	0,600
value	2	9	3,000	0,700

Table 2. Diagonal piles (z_{Di} on the level of pile truncation), inclination

$\kappa = 1/\tan\alpha$

$= 3{,}500$.

Variable	Row	Number of piles	Horizontal location	Relative stiffness
Symbol	i_D	n_{Di}	z_{Di}	k_{Di}
Unit	-		m	-
Numerical	1	8	-3,000	0,962
value	2	7	2,500	0,962

Relative stiffnesses of a pile

$$k_i = \frac{\frac{E_i A_i}{L_i}}{\frac{E_0 A_0}{L_0}}$$

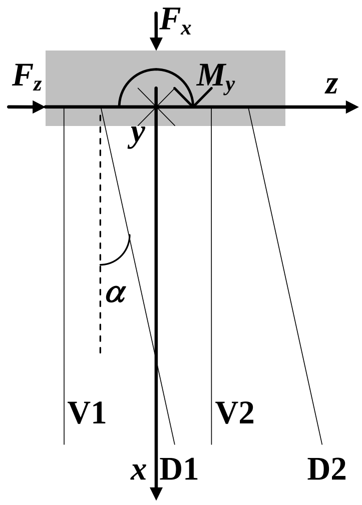


Figure 1.

Directional angle of diagonal piles

$$\alpha = \arctan(1/\kappa) \tag{1}$$
$$= 15{,}945\ °$$

Axial stiffnesses of the vertical and diagonal pile groups, respectively

$$A_V = \Sigma n_{Vi} k_{Vi} \tag{2}$$
$$= 12{,}300$$
$$A_D = \Sigma n_{Di} k_{Di} \tag{3}$$
$$= 14{,}423$$

***z*-coordinates of the centroid lines of vertical and horizontal piles on the level of pile truncation (Figure 2)**

$$z_V = \frac{\sum n_{Vi} k_{Vi} z_{Vi}}{A_V} \tag{4}$$
$$= 0{,}073\ \text{m}$$
$$z_D = \frac{\sum n_{Di} k_{Di} z_{Di}}{A_D} \tag{5}$$
$$= -0{,}433\ \text{m}$$

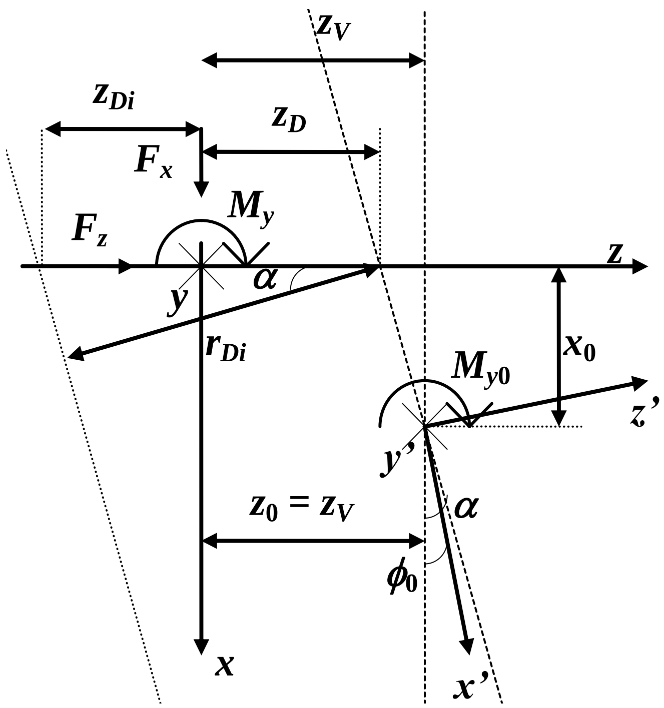


Figure 2.

Rotation center (Figure 2)

$$\begin{cases} x_0 = \dfrac{z_V - z_D}{\tan\alpha} \\ z_0 = z_V \end{cases} \qquad \text{(6a, b)}$$

$$\Rightarrow \begin{cases} x_0 = 1{,}773 \text{ m} \\ z_0 = 0{,}073 \text{ m} \end{cases}$$

Angle of principle direction clockwise (is not needed in this problem)

$$\phi_0 = \frac{1}{2}\arctan\left[\frac{-2A_D\cos\alpha\sin\alpha}{A_V + A_D\cos 2\alpha}\right] \qquad \text{(7)}$$

$$= -8{,}623\ °$$

$$= -0{,}150 \text{ rad}$$

Lever arms of vertical piles

$$r_{Vi} = z_{Vi} - z_V \qquad \text{(8)}$$

$$r_{V1} = -3{,}073 \text{ m}$$

$$r_{V2} = 2{,}927 \text{ m}$$

Lever arms of diagonal piles

$$r_{Di} = (z_{Di} - z_D)\cos\alpha \qquad \text{(9)}$$

$$r_{D1} = -2{,}468 \text{ m}$$

$$r_{D2} = 2{,}820 \text{ m}$$

Moment of inertia with respect to the rotation center

$$\begin{cases} I_V = \sum n_{Vi} k_{Vi} r_{Vi}^2 \\ I_D = \sum n_{Di} k_{Di} r_{Di}^2 \end{cases} \qquad \text{(10a, b)}$$

$$\Rightarrow \begin{cases} I_V = 110{,}634 \text{ m}^2 \\ I_D = 100{,}393 \text{ m}^2 \end{cases}$$

$$I = I_V + I_D \qquad \text{(11)}$$

$$= 211{,}027 \text{ m}^2$$

Loading at the rotation center

$$F_{x0} = F_x \tag{12}$$
$$= \quad 8{,}000 \ \text{MN}$$
$$F_{z0} = F_z \tag{13}$$
$$= \quad 2{,}000 \ \text{MN}$$
$$M_{y0} = M_y - F_x z_0 + F_z x_0 \tag{14}$$
$$= \quad 3{,}960 \ \text{MNm}$$

Load components (Figure 3)

$$F_{xD} = \frac{F_{z0}}{\sin\alpha} \tag{15}$$
$$= \quad 7{,}280 \ \text{MN} \qquad \text{Compression}$$
$$F_{xV} = F_{x0} - \frac{F_{z0}}{\tan\alpha} \tag{16}$$
$$= \quad 1{,}000 \ \text{MN} \qquad \text{Compression}$$

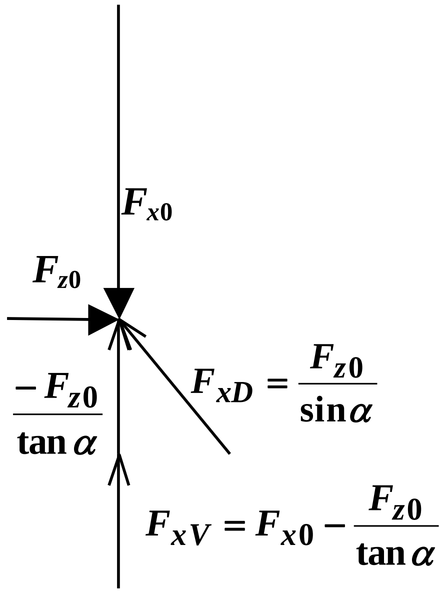


Figure 3.

Axial forces of vertical piles[1]

$$N_{Vi} = k_{Vi}\left(\frac{F_{xV}}{A_V} + \frac{M_{y0}}{I} r_{Vi}\right) \tag{17}$$

$N_{V1} =$ 0,014 MN Compression

$N_{V2} =$ 0,095 MN Compression

Axial forces of diagonal piles

$$N_{Di} = k_{Di}\left(\frac{F_{xD}}{A_D} + \frac{M_{y0}}{I} r_{Di}\right) \tag{18}$$

$N_{D1} =$ 0,441 MN Compression

$N_{D2} =$ 0,536 MN Compression

Cosines of directional angles with respect to x - and z -axis

$$p_{xVi} = \cos(0) \tag{19}$$

$= 1{,}000$

$$p_{zVi} = \sin(0) \tag{20}$$

$= 0{,}000$

$$p_{xDi} = \cos\alpha \tag{21}$$

$= 0{,}962$

$$p_{zDi} = \sin\alpha \tag{22}$$

$= 0{,}275$

Elements of stiffness matrix

$$k_{11} = \Sigma n_{Vi} k_{Vi} p_{xVi}^{\ 2} + \Sigma n_{Di} k_{Di} p_{xDi}^{\ 2} \tag{23}$$

$= 25{,}634$

$$k_{12} = \Sigma n_{Vi} k_{Vi} p_{xVi} p_{zVi} + \Sigma n_{Di} k_{Di} p_{xDi} p_{zDi} \tag{24}$$

$= 3{,}810$

$$k_{22} = \Sigma n_{Vi} k_{Vi} p_{zVi}^{\ 2} + \Sigma n_{Di} k_{Di} p_{zDi}^{\ 2} \tag{25}$$

$= 1{,}089$

---

1) Compare to

$$\sigma = \frac{F}{A} + \frac{M}{I} y$$

$$k_{33} = I$$
$$= \quad 211{,}027 \text{ m}^2$$

Stiffness matrix

$$\mathbf{K} = \begin{bmatrix} 25{,}634 & 3{,}810 & 0{,}000 \\ 3{,}810 & 1{,}089 & 0{,}000 \\ 0{,}000 & 0{,}000 & 211{,}027 \end{bmatrix}$$

The stiffness matrix is unitless except the element k_{33}.

Inverse matrix

$$\mathbf{K}^{-1} = \begin{bmatrix} 0{,}081 & -0{,}285 & 0{,}000 \\ -0{,}285 & 1{,}915 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}005 \end{bmatrix}$$

Elements of force vector $\{f\}$

$$F_{x0} = \quad 8{,}000 \text{ MN}$$
$$F_{z0} = \quad 2{,}000 \text{ MN}$$
$$M_{y0} = \quad 3{,}960 \text{ MNm}$$

Displacement vector, when coordinate system is situated in the rotation center

$$\{\delta\} = [K]^{-1}\{f\} \tag{27}$$

$$\Rightarrow \begin{cases} u_0 = & 0{,}081 \text{ MN} \\ w_0 = & 1{,}553 \text{ MN} \\ \varphi = & 0{,}019 \text{ MNm} \end{cases}$$

Relative displacements in the original coordinate system

$$u = u_0 - z_0 \sin\varphi \tag{28}$$
$$= \quad 0{,}080 \text{ MN}$$
$$w = w_0 + x_0 \sin\varphi \tag{29}$$
$$= \quad 1{,}586 \text{ MN}$$

The real displacements are obtained, when the real stiffnesses are used.

Axial forces of vertical piles can also be calculated by way of displacements

$$N_{Vi} = k_{Vi}(p_{xVi}u_0 + p_{zVi}w_0 + r_{yVi}\varphi) \quad (30)$$

$N_{V1} =$ **0,014 MN**

$N_{V2} =$ **0,095 MN**

Respectively for diagonal piles

$$N_{Di} = k_{Di}(p_{xDi}u_0 + p_{zDi}w_0 + r_{yDi}\varphi) \quad (31)$$

$N_{D1} =$ **0,441 MN**

$N_{D2} =$ **0,536 MN**

ANSWER:

Minimum and maximum pile forces

$N_{\min} =$	0,014	MN	Compression	V1
$N_{\max} =$	0,536	MN	Compression	D2

Relative displacements

$u =$ **0,080 MN**

$w =$ **1,586 MN**

$\varphi =$ **0,019 MNm**

Alternative Way to Solv the Pile Forces

Moment components

$$M_V = \frac{I_V}{I_V + I_D} M_{y0} \tag{32}$$

$= 2{,}076$ MNm

$$M_D = M_{y0} - M_V \tag{33}$$

$= 1{,}884$ MNm

Axial forces of vertical piles

$$N_{Vi} = k_{Vi}\left(\frac{F_{xV}}{A_V} + \frac{M_V r_{Vi}}{I_V}\right) \tag{34}$$

$N_{V1} =$ 0,014 MN Compression

$N_{V2} =$ 0,095 MN Compression

Axial forces of diagonal piles

$$N_{Di} = k_{Di}\left(\frac{F_{xD}}{A_D} + \frac{M_D r_{Di}}{I_D}\right) \tag{35}$$

$N_{D1} =$ 0,441 MN Compression

$N_{D2} =$ 0,536 MN Compression

27. PROBLEM

A pile group includes piles of wood, reinforced concrete and steel. See Figure 1 and Table 1.

Are the pile forces acceptable? Determine the principal coordinate system and the corresponding diagonal main elements of the stiffness matrix!

Table 1. Pile information.

Variable	Row	Number of piles	Slope	Cross-section area	Modulus of elasticity	Vertical location	Horizont. location
Symbol	i	j_i	$\tan\alpha_i$	A_i	E_i	x_i	z_i
Unit	-	-	-	m^2	MN/m^2	m	m
Numerical value	1	5	0,000	0,031	5 600	0	-2,000
	2	5	0,000	0,031	5 600	0	0,000
	3	3	0,200	0,090	31 600	0	0,000
	4	1	0,250	0,018	210 000	0	2,000

Acceptable stresses according to pile class III (Finnish standard) are as follows:

Wood[1]

$\sigma_{w,\ \text{allowable}}$ = 5 MN/m^2

Reinforced concrete[2]

$\sigma_{c,\ \text{allowable}}$ = 5 MN/m^2

Steel[3]

$\sigma_{s,\ \text{allowable}}$ = 40 MN/m^2

1) r = 100 mm, T40, Time Class A, Humidity Class 1.
2) a = 300 mm, K40.
3) r = 75 mm, Fe52C.

Loading

$$F_x = 2{,}000 \text{ MN}$$
$$F_z = 0{,}400 \text{ MN}$$
$$M_y = 1{,}000 \text{ MNm}$$

Depth

$$h = 8 \text{ m}$$

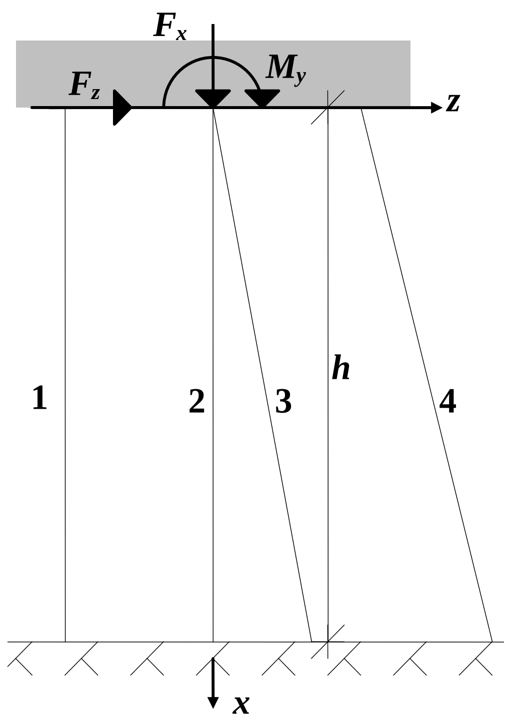


Figure 1.

Length (Figure 2)

$$L_i = h_i/\cos\alpha_i \tag{1}$$

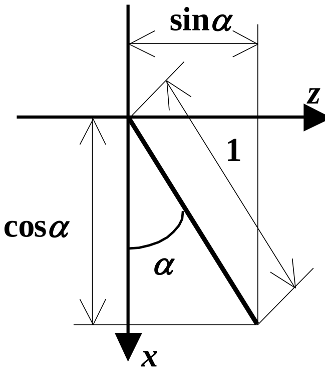


Figure 2.

Stiffness

$$k_i = \frac{C_i}{L_i} \tag{2}$$

$$\Rightarrow k_i = \frac{E_i A_i}{L_i} \tag{3}$$

Cosines of directional angles with respect to x - and z -axis

$$p_{x,i} = \cos\alpha_i \tag{4}$$

$$p_{z,i} = \cos(90° - \alpha_i) \tag{5}$$

$$= \sin\alpha_i \tag{6}$$

Lever arm with respect to origin

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{7}$$

Elements of stiffness matrix for pile row *i*

$$k_{11,i} = n_i k_i p_{x,i}^{\;2} \tag{8}$$

$$k_{12,i} = k_{21,i} \tag{9}$$

$$= n_i k_i p_{x,i} p_{z,i} \tag{10}$$

$$k_{13,i} = k_{31,i} \tag{11}$$

$$= n_i k_i p_{x,i} r_{y,i} \tag{12}$$

$$k_{22,i} = n_i k_i p_{z,i}^2 \quad (13)$$

$$k_{23,i} = k_{32,i} \quad (14)$$

$$= n_i k_i p_{z,i} r_{y,i} \quad (15)$$

$$k_{33,i} = n_i k_i r_{y,i}^2 \quad (16)$$

Elements of stiffness matrix for the pile group

$$k_{mn} = \sum_{i=0}^{max} k_{mn,i} \quad (17)$$

Table 2. Calculation of the stiffness matrix.

Variable	Row	Number of piles	Vertical location	Horizont. location	Slope	Directional angle	Cross-section area
Symbol	i	j_i	x_i	z_i	$\tan\alpha_i$	α_i	A_i
Unit	-	-	m	m	-	°	m^2
Numerical value	1	5	0,000	-2,000	0,000	0,000	0,031
	2	5	0,000	0,000	0,000	0,000	0,031
	3	3	0,000	0,000	0,200	11,310	0,090
	4	1	0,000	2,000	0,250	14,036	0,018
	⋮						

Variable	Row	Modulus of elasticity	Length	Stiffness	Cosines of directional angles		Lever arm
Symbol	i	E_i	L_i	k_i	$p_{x,i}$	$p_{z,i}$	$r_{y,i}$
Equation			(1)	(3)	(4)	(6)	(7)
Unit	-	MN/m^2	m	MN/m	-	-	m
Numerical value	1	5 600	8,000	21,700	1,000	0,000	-2,000
	2	5 600	8,000	21,700	1,000	0,000	0,000
	3	31 600	8,158	348,596	0,981	0,196	0,000
	4	210 000	8,246	458,392	0,970	0,243	1,940
	⋮						

… *Variable*	Row	Elements of the stiffness matrix of pile row *i*					
Symbol	*i*	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Equation		(8)	(10)	(12)	(13)	(15)	(16)
Unit	-	MN/m	MN/m	MN	MN/m	MN	MNm
Numerical	1	108,500	0,000	-217,000	0,000	0,000	434,000
value	2	108,500	0,000	0,000	0,000	0,000	0,000
	3	1005,567	201,113	0,000	40,223	0,000	0,000
	4	431,428	107,857	862,856	26,964	215,714	1725,712

⋮

Variable	Elements of the stiffness matrix of the pile group					
Symbol	k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Equation	(17)	(17)	(17)	(17)	(17)	(17)
Unit	MN/m	MN/m	MN	MN/m	MN	MNm
Numerical value	1653,995	308,970	645,856	67,187	215,714	2159,712

Elements of the stiffness matrix for the pile group are calculated in Table 2.

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{18}$$

$$=\begin{bmatrix} 1653,995 & 308,970 & 645,856 \\ 308,970 & 67,187 & 215,714 \\ 645,856 & 215,714 & 2159,712 \end{bmatrix} \quad + \begin{bmatrix} \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN} & \text{MN} & \text{MNm} \end{bmatrix}$$

Displacement vector is obtained from equilibrium condition

$$\{f\} = [K]\{\delta\} \tag{19}$$

$$\Rightarrow \{\delta\} = [K]^{-1}\{f\} \tag{20}$$

Element of the displacement vector (displacements and rotation of the base slab)

$$u = 0,001 \text{ m}$$

$$w = 0,003 \text{ m}$$

$$\varphi = 0,000 \text{ rad}$$

Pile forces are calculated in Table 3.

$$N_i = k_i \Delta_i \tag{21}$$

$$= k_i (p_{x,i}\, u + p_{z,i}\, w + r_{y,i}\, \varphi) \tag{22}$$

$$N_{\max,i} = \sigma_{p,\,\text{sall}} A_i \tag{23}$$

Table 3. Pile forces and the acceptable extreme values.

Symbol	i	$N_{\min,i}$	N_i	$N_{\max,i}$	Acceptable
Equation			(21)	(23)	
Unit	-	MN	MN	MN	
Numerical value	1	0	0,015	0,155	OK
	2	0	0,013	0,155	OK
	3	0	0,435	0,450	OK
	4	0	0,595	0,720	OK

Rotation center

$$\begin{cases} x_0 = -\dfrac{\begin{vmatrix} k_{11} & k_{13} \\ k_{12} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{11}k_{23} - k_{12}k_{13}}{k_{11}k_{22} - k_{12}^2} \\ z_0 = -\dfrac{\begin{vmatrix} k_{12} & k_{13} \\ k_{22} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{12}k_{23} - k_{22}k_{13}}{k_{11}k_{22} - k_{12}^2} \end{cases} \qquad \text{(24a, b)}$$

x_0 = -10,0382 m

z_0 = -1,4847 m

Angle of principal direction (anticlockwise)

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{11} - k_{22}}\right) \qquad (25)$$

= 0,186 rad

= 10,639 °

Principal direction coordinate system (Figure 3)

$$x_{ki} = (z_i - z_0)\sin\phi_0 + (x_i - x_0)\cos\phi_0 \qquad (26)$$

$$z_{ki} = (z_i - z_0)\cos\phi_0 - (x_i - x_0)\sin\phi_0 \qquad (27)$$

$$\alpha_{ki} = \alpha_i - \phi_0 \qquad (28)$$

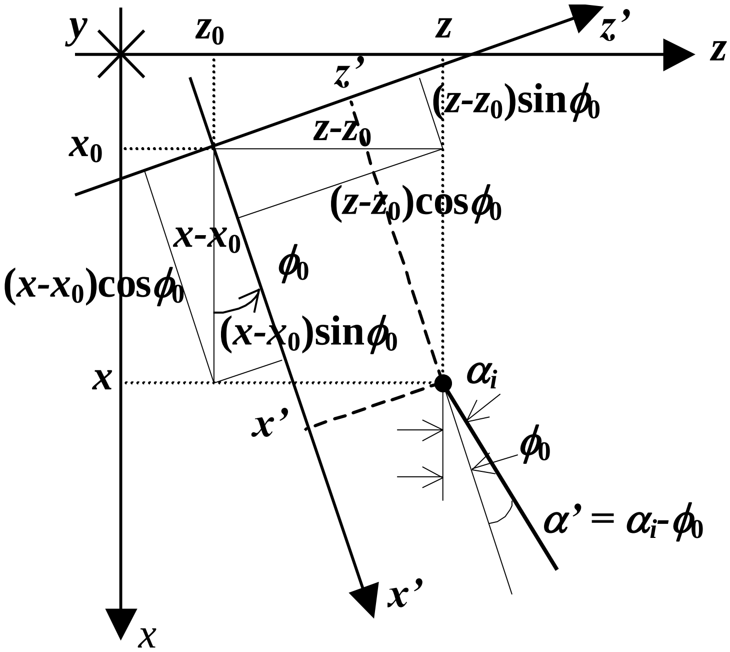


Figure 3.

Table 4. Calculation of the stiffness matrix.

Variable	Row	Number	Coordinates		Angle
Symbol	$i_k = i$	$j_{ki} = j_i$	x_{ki}	z_{ki}	α_{ki}
Equation			(26)	(27)	(28)
Unit	-	-	m	m	°
Numerical value	1	5	9,771	-2,360	-10,639
	2	5	10,140	-0,394	-10,639
	3	3	10,140	-0,394	0,671
	4	1	10,509	1,572	3,398
	⋮				

Variable	Row	Stiffness	Cosines		Arm
Symbol	$i_k = i$	$k_{ki} = k_i$	$p_{xk,i}$	$p_{zk,i}$	$r_{yk,i}$
Equation			(4)	(6)	(7)
Unit	-	MN/m	-	-	m
Numerical value	1	21,700	0,983	-0,185	-0,515
	2	21,700	0,983	-0,185	1,485
	3	348,596	1,000	0,012	-0,513
	4	458,392	0,998	0,059	0,946 …

… Variable	Row	Elements of the stiffness matrix for pile row i					
Symbol	$i_k = i$	k_{11ki}	k_{12ki}	k_{13ki}	k_{22ki}	k_{23ki}	k_{33ki}
Unit	-	MN/m	MN/m	MN	MN/m	MN	MNm
Numerical value	1	104,802	-19,686	-54,952	3,698	10,322	28,813
	2	104,802	-19,686	158,318	3,698	-29,739	239,162
	3	1045,646	12,253	-536,250	0,144	-6,284	275,011
	4	456,782	27,119	432,883	1,610	25,700	410,235
Variable		Elements of the stiffness matrix for the pile group					
Symbol		k_{11k}	$k_{12k}=k_{21k}$	$k_{13k}=k_{31k}$	k_{22k}	$k_{23k}=k_{32k}$	k_{33k}
Unit		MN/m	MN/m	MN	MN/m	MN	MNm
Numerical value		1712,032	0,000	0,000	9,149	0,000	953,222

Elements of the stiffness matrix for the pile group are calculated in Table 4.

$$[K_k]=\begin{bmatrix} k_{11k} & k_{12k} & k_{13k} \\ k_{21k} & k_{22k} & k_{23k} \\ k_{31k} & k_{32k} & k_{33k} \end{bmatrix} \tag{29}$$

$$=\begin{bmatrix} 1712,032 & 0,000 & 0,000 \\ 0,000 & 9,149 & 0,000 \\ 0,000 & 0,000 & 953,222 \end{bmatrix} + \begin{bmatrix} \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN} & \text{MN} & \text{MNm} \end{bmatrix}$$

<u>ANSWER</u>:

Pile forces are acceptable.

Rotation center is

$x_0 =$ -10,0382 m

$z_0 =$ -1,4847 m

Angle of principal direction is

$\phi_0 =$ 0,186 rad

Principle stiffnesses (diagonal main elements of the stiffness matrix) are

$k_{11k} =$ 1 712 MN/m

$k_{22k} =$ 9 MN/m

$k_{33k} =$ 953 MNm

28. PROBLEM

Bending moment

M_y = 1 MNm

is acting on the top of a column founded on piles (Figure 1 and Table 1). Determine the horizontal force $F_{z\,1\text{-}1}$ and moment $M_{y\,1\text{-}1}$ at section 1 - 1!

Table 1. Pile group information.

Variable	Row	Number of piles	Slope	Stiffness	Vertical location	Horizont. location
Symbol	i	j_i	$\tan\alpha_i$	k_i	x_i	z_i
Unit	-	-	-	MN/m	m	m
Numerical value	1	1	-0,250	20,000	0,000	-0,800
	2	1	0,250	20,000	0,000	-0,800
	3	1	-0,250	20,000	0,000	0,800
	4	1	0,250	20,000	0,000	0,800

The width of the square column is

a = 0,600 m

E = 30 000 MN/m^2

Dimensions

L = 6 m

d = 1 m

h = 10 m

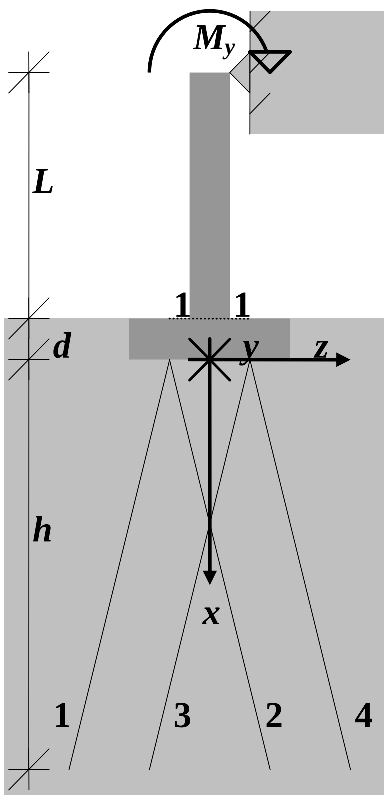


Figure 1.

Elements of the stiffness matrix are calculated in Table 2.

Table 2.

Variable	Row	Number of piles	Vertical location	Horizont. location	Slope	Directio-nal angle	Stiffness
Symbol	i	j_i	x_i	z_i	$\tan\alpha_i$	α_i	k_i
Unit	-	-	m	m	-	°	MN/m
Numerical value	1	1	0,000	-0,800	-0,250	-14,036	20,000
	2	1	0,000	-0,800	0,250	14,036	20,000
	3	1	0,000	0,800	-0,250	-14,036	20,000
	4	1	0,000	0,800	0,250	14,036	20,000

⋮

Variable	Row	Cosines of directional angles		Lever arm
Symbol	i	$p_{x,i}$	$p_{z,i}$	$r_{y,i}$
Unit	-	-	-	m
Numerical value	1	0,970	-0,243	-0,776
	2	0,970	0,243	-0,776
	3	0,970	-0,243	0,776
	4	0,970	0,243	0,776

⋮

Variable	Row	Diagonal main elements of piles			Other elements		
Symbol	i	$k_{11,i}$	$k_{22,i}$	$k_{33,i}$	$k_{12,i}=k_{21,i}$	$k_{13,i}=k_{31,i}$	$k_{23,i}=k_{32,i}$
Unit	-	MN/m	MN/m	MNm	MN/m	MN	MN
Numerical value	1	18,824	1,176	12,047	-4,706	-15,059	3,765
	2	18,824	1,176	12,047	4,706	-15,059	-3,765
	3	18,824	1,176	12,047	-4,706	15,059	-3,765
	4	18,824	1,176	12,047	4,706	15,059	3,765

Variable	Main elements of the pile group			Other elements		
Symbol	k_{11}	k_{22}	k_{33}	$k_{12,i}=k_{21}$	$k_{13,i}=k_{31}$	$k_{23,i}=k_{32}$
Unit	MN/m	MN/m	MNm	MN/m	MN	MN
Numerical value	75,294	4,706	48,188	0,000	0,000	0,000

Angle of principal direction

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{22}-k_{11}}\right) \tag{1}$$
$$= \quad 0{,}000 \text{ rad}$$

Rotation center

$$\begin{cases} x_0 = -\dfrac{\begin{vmatrix} k_{11} & k_{13} \\ k_{12} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{11}k_{23}-k_{12}k_{13}}{k_{11}k_{22}-k_{12}^2} \\ z_0 = -\dfrac{\begin{vmatrix} k_{12} & k_{13} \\ k_{22} & k_{23} \end{vmatrix}}{\begin{vmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{vmatrix}} = -\dfrac{k_{12}k_{23}-k_{22}k_{13}}{k_{11}k_{22}-k_{12}^2} \end{cases} \tag{2a, b}$$

$$\Rightarrow \begin{cases} x_0 = \quad 0{,}000 \text{ m} \\ z_0 = \quad 0{,}000 \text{ m} \end{cases}$$

Stiffness matrix (Table 2)

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{3}$$
$$= \begin{bmatrix} 75{,}294 & 0{,}000 & 0{,}000 \\ 0{,}000 & 4{,}706 & 0{,}000 \\ 0{,}000 & 0{,}000 & 48{,}188 \end{bmatrix} \begin{bmatrix} \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN/m} & \text{MN/m} & \text{MN} \\ \text{MN} & \text{MN} & \text{MNm} \end{bmatrix}$$

Inverse matrix

$$[K]^{-1} = \begin{bmatrix} 0{,}013 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}213 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}021 \end{bmatrix}$$

Equilibrium condition

$$\{f\} = [K]\{\delta\} \tag{4}$$

Displacement vector

$$\{\delta\} = [K]^{-1}\{f\} \tag{5}$$

At the top of the column a force is applied

$$F_z = \quad -1 \text{ MN}$$

So the real support force is

$$H = cF_z \tag{6}$$

Moment at origin due to horizontal force

$$M_{y\,0,Fz} = F_z(L+d) \tag{7}$$

Elements of the force vector at $x = 0$ due to moment ($M_y = 1$) and horizontal force ($F_z = -1$) are shown in Table 3.

Table 3.

	$i = M_y$	$i = F_z$	
$F_{x\,0,i}$	0,000	0,000	MN
$F_{z\,0,i}$	0,000	-1,000	MN
$M_{y\,0,i}$	1,000	-7,000	MNm

Elements of the displacement vector at $x = 0$ due to moment ($M_y = 1$) and horizontal force ($F_z = -1$) are shown in Table 4.

Table 4.

	$i = M_y$	$i = F_z$		
$u_{0,i}$	0,000	0,000	m	vertical displacement
$w_{0,i}$	0,000	-0,213	m	horizontal displacement
$\varphi_{0,i}$	0,021	-0,145	rad	rotation

Bending stiffness of the column

$$D = E\frac{a^4}{12} \tag{8}$$

$$= \quad 324 \text{ MN/m}^2$$

Horizontal displacement at the top of the column due to moment M_y (Figure 2)

$$w_{My} = w_{0,My} + (L+d)\varphi_{0,My} + \frac{M_y L^2}{2D} \tag{9}$$

$$= \quad 0{,}201 \text{ m}$$

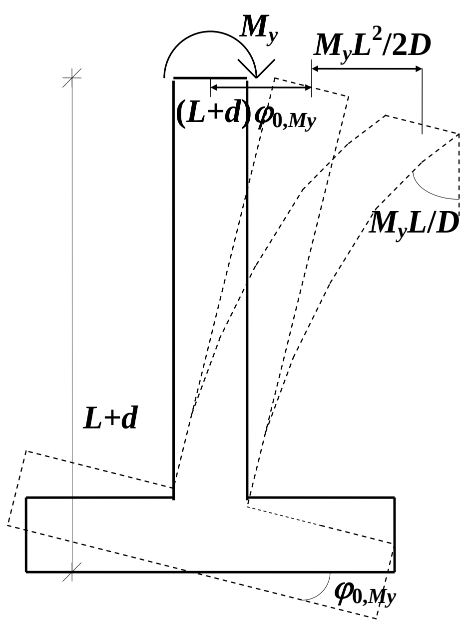


Figure 2.

Horizontal displacement at the top of the column due to horizontal force F_z (Figure 3)

$$w_{Fz} = w_{0,Fz} + (L+d)\varphi_{0,Fz} + \frac{F_z L^3}{3D} \tag{10}$$

$$= \quad -1{,}452 \text{ m}$$

Compatibility condition at the top of the column

$$w = w_{My} + cw_{Fz} = 0 \tag{11}$$

$$\Rightarrow c = \frac{-w_{My}}{w_{Fz}} \tag{12}$$

$$= \quad 0{,}138$$

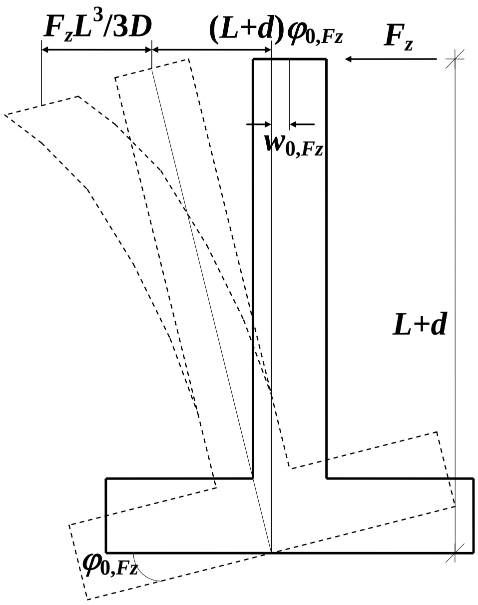


Figure 3.

Horizontal force and moment at section 1 - 1

$$F_{z\,1\text{-}1} = H = cF_z \tag{13}$$

$$= \quad -0{,}138 \text{ MN}$$

$$M_{y\,1\text{-}1} = M_y + HL \tag{14}$$

$$= \quad 0{,}170 \text{ MNm}$$

<u>ANSWER</u>:

Horizontal force and moment at section 1 - 1 are, respectively,

$$F_{z\,1\text{-}1} = \quad -0{,}138 \text{ MN}$$

$$M_{y\,1\text{-}1} = \quad 0{,}170 \text{ MNm}$$

DETERMINING OF THE DEFLECTION AND ROTATION OF CANTILEVER BEAM BY USING MOHR'S METHOD

In Figure A it is presented, how the deflection and rotation of a cantilever beam having constant stiffness due to point load F and point moment M_0 acting at the end of the cantilever are determined.

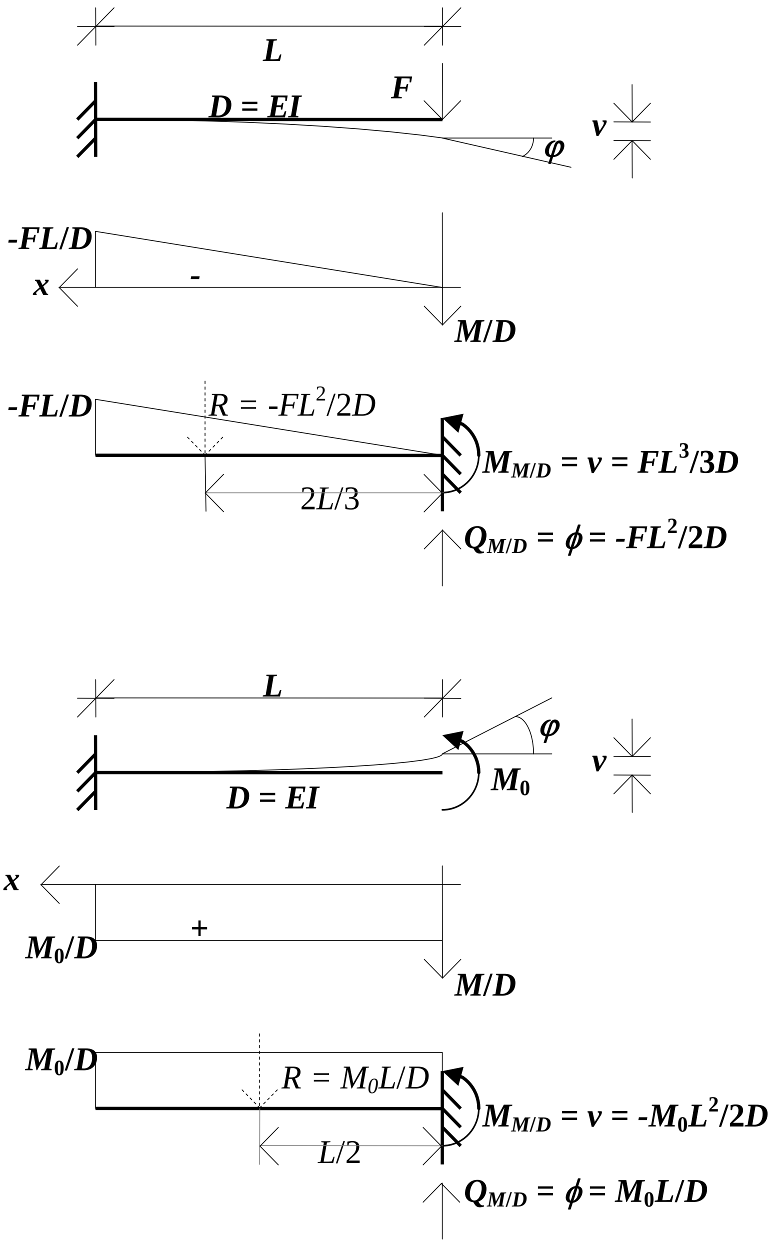


Figure A.

Mohr's method

1) **The bending moments $M(x)$ are divided by the bending stiffnesses $D(x)$ (M/D-distribution curve).**

2) **The boundary conditions of the structure are modified so that the deflections of the original structure correspond to the bending moments of the modified one and, respectively, rotations correspond to shear forces. The modified structure is subjected to M/D-distribution curve.**

3) **Bending moment caused by M/D-load $M_{M/D}(x)$ corresponds to deflection $v(x)$ and, respectively, shear force $Q_{M/D}(x)$ corresponds to rotation $\varphi(x)$.**

Some examples in Finnish: Arvo Ylinen: *Kimmo ja lujuusoppi I*. WSOY. 2 edition. Porvoo 1965. P. 268 - 273.

29. PROBLEM

Determine the area, where vertical force

V = 1 MN

can be situated so that all piles are compressed (core figure)! See Figure 1.

Use system of equilibrium conditions of space pile group!

All piles are in vertical direction and have similar properties (cross-section, same material).

Dimensions are

d = 1,500 m

L = 5,000 m

Table 1. Pile group information.

Symbol	i	x_i	y_i	z_i	L_i
Unit	-	m	m	m	m
Numerical value	1	0,000	0,750	-1,500	10,000
	2	0,000	0,750	0,000	15,000
	3	0,000	0,750	1,500	20,000
	4	0,000	-0,750	-1,500	10,000
	5	0,000	-0,750	0,000	15,000

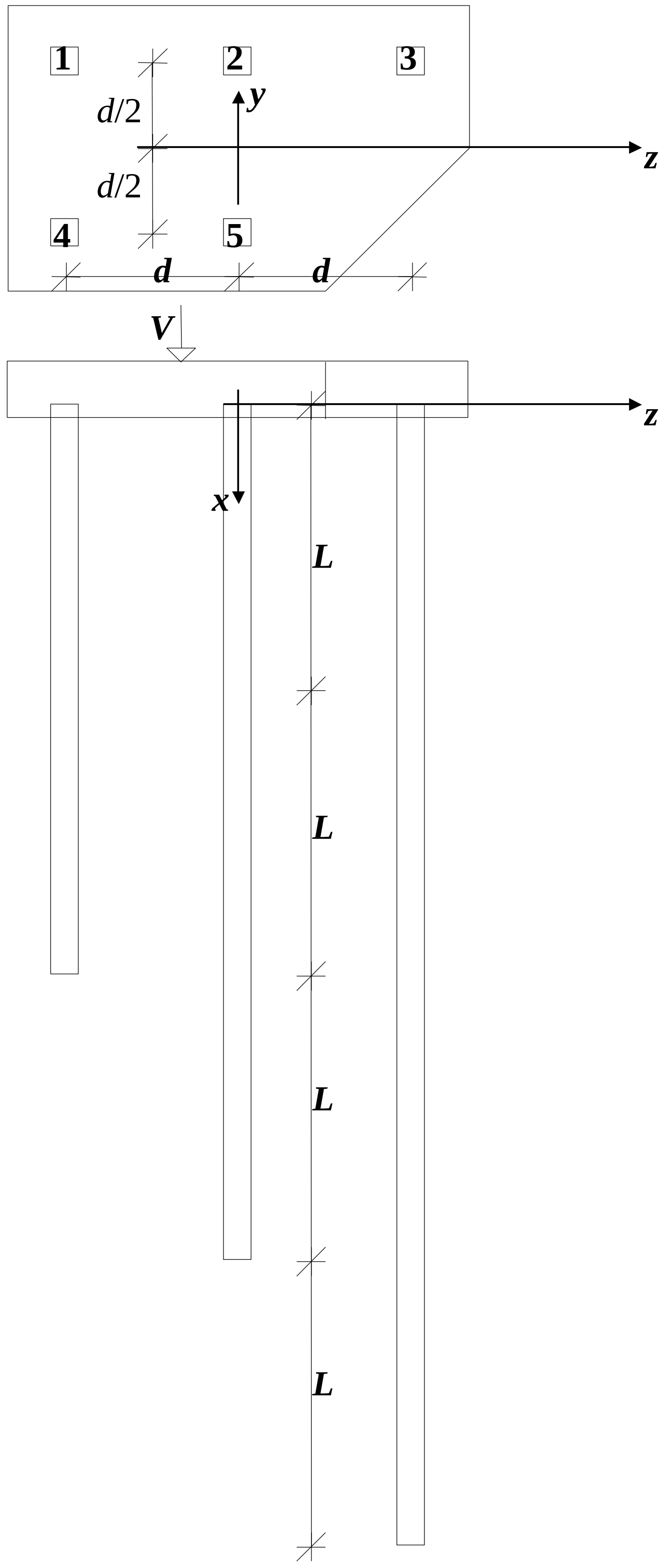


Figure 1.

Because of the fact that now

$$p_{y,i} = \cos\alpha_{i,y} = 0 \quad (1)$$

$$p_{z,i} = \cos\alpha_{i,z} = 0 \quad (2)$$

$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} = 0 \quad (3)$$

we have a pile group in plane. Thus,

$$[K] = \begin{array}{c} \begin{array}{cccccc} 1 & & & & 2 & 3 \\ p_x & p_y & p_z & r_x & r_y & r_z \end{array} \\ \left[\begin{array}{cccccc} \sum kp_x^2 & 0 & 0 & 0 & \sum kp_x r_y & \sum kp_x r_z \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sum kr_y p_x & 0 & 0 & 0 & \sum kr_y^2 & \sum kr_y r_z \\ \sum kr_z p_x & 0 & 0 & 0 & \sum kr_z r_y & \sum kr_z^2 \end{array}\right] \end{array} \begin{array}{ll} p_x & 1 \\ p_y & \\ p_z & \\ r_x & \\ r_y & 2 \\ r_z & 3 \end{array} \quad (4)$$

Relative stiffnesses of the piles

$$k_i = \frac{\dfrac{E_i A_i}{L_i}}{\dfrac{E_3 A_3}{L_3}} \quad (5)$$

$$\Rightarrow k_i = \frac{L_3}{L_i}$$

$k_1 =$ 2,000

$k_2 =$ 1,333

$k_3 =$ 1,000

Centroid

$$y_0 = \frac{\sum k_i y_i}{\sum k_i} \quad (6)$$

$=$ 0,098 m

$$z_0 = \frac{\sum k_i z_i}{\sum k_i} \quad (7)$$

$=$ -0,587 m

Correspondingly,

$$p_{x,i} = \cos\alpha_{i,x} \tag{8}$$

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{9}$$

$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{10}$$

Elements of stiffness matrix are presented in Table 2.

Table 2.

Variable	Pile	Location			Angle	Stiffness
Symbol	i	x_i	y_i	z_i	α_i	k_i
Unit		m	m	m	°	-
Numerical value	1	0,000	0,652	-0,913	0,000	2,000
	2	0,000	0,652	0,587	0,000	1,333
	3	0,000	0,652	2,087	0,000	1,000
	4	0,000	-0,848	-0,913	0,000	2,000
	5	0,000	-0,848	0,587	0,000	1,333
	⋮					

Variable	Pile	Cosines of directional angles			Lever arms		
Symbol	i	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Unit		-	-	-	m	m	m
Numerical value	1	1,000	0,000	0,000	0,000	-0,913	-0,652
	2	1,000	0,000	0,000	0,000	0,587	-0,652
	3	1,000	0,000	0,000	0,000	2,087	-0,652
	4	1,000	0,000	0,000	0,000	-0,913	0,848
	5	1,000	0,000	0,000	0,000	0,587	0,848
	⋮						

$\vdots$

Variable	Pile	Elements of the stiffness matrix of pile row *i*					
Symbol	i	$k_{11,i}$	$k_{12,i}$	$k_{13,i}$	$k_{22,i}$	$k_{23,i}$	$k_{33,i}$
Numerical	1	2,000	-1,826	-1,304	1,667	1,191	0,851
value	2	1,333	0,783	-0,870	0,459	-0,510	0,567
	3	1,000	2,087	-0,652	4,355	-1,361	0,425
	4	2,000	-1,826	1,696	1,667	-1,548	1,438
	5	1,333	0,783	1,130	0,459	0,664	0,958
Unit		MN/m	MN	MN	MNm	MNm	MNm
Numerical value		7,667	0,000	0,000	8,609	-1,565	4,239
Symbol		k_{11}	$k_{12}=k_{21}$	$k_{13}=k_{31}$	k_{22}	$k_{23}=k_{32}$	k_{33}
Variable		Elements of the stiffness matrix of the pile group					

Stiffness matrix

$$[K] = \begin{bmatrix} 7{,}667 & 0{,}000 & 0{,}000 \\ 0{,}000 & 8{,}609 & -1{,}565 \\ 0{,}000 & -1{,}565 & 4{,}239 \end{bmatrix} \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN/m} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

Inverse matrix

$$[K]^{-1} = \begin{bmatrix} 0{,}130 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}125 & 0{,}046 \\ 0{,}000 & 0{,}046 & 0{,}253 \end{bmatrix} \begin{bmatrix} \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{m/N} & \mathrm{m/N} & \mathrm{1/N} \\ \mathrm{1/N} & \mathrm{1/N} & \mathrm{1/Nm} \end{bmatrix}$$

From system of equilibrium condition

$$\{F\} = [K]\{\delta\} \tag{11}$$

the deflection vector

$$\{\delta\} = [K]^{-1}\{F\} \tag{12}$$

$$\Rightarrow \begin{Bmatrix} u \\ \varphi \\ \theta \end{Bmatrix} = [K]^{-1} \begin{Bmatrix} 1 \\ z \\ -y \end{Bmatrix} \quad \begin{matrix} \text{(}x\text{-axial vertical force [MN])} \\ \text{(Moment with respect to } y\text{-axis [MNm])} \\ \text{(Moment with respect to } z\text{-axis [MNm])} \end{matrix} \tag{13}$$

$$\Rightarrow \begin{cases} u = 0{,}000\,y + 0{,}000\,z + 0{,}130 \\ v = -0{,}046\,y + 0{,}125\,z + 0{,}000 \\ \theta = -0{,}253\,y + 0{,}046\,z + 0{,}000 \end{cases}$$

is obtained.

Pile forces

$$N_i = k_i \Delta_i \quad (14)$$

$$= k_i (p_{x,i} u + r_{y,i} \varphi + r_{z,i} \theta) \quad (15)$$

N_1 =	0,414	y	+	-0,287	z	+	0,261	= 0
N_2 =	0,184	y	+	0,057	z	+	0,174	= 0
N_3 =	0,069	y	+	0,230	z	+	0,130	= 0
N_4 =	-0,345	y	+	-0,149	z	+	0,261	= 0
N_5 =	-0,322	y	+	0,149	z	+	0,174	= 0

ANSWER:

Boundary lines of the core figure are shown in Figure 2.

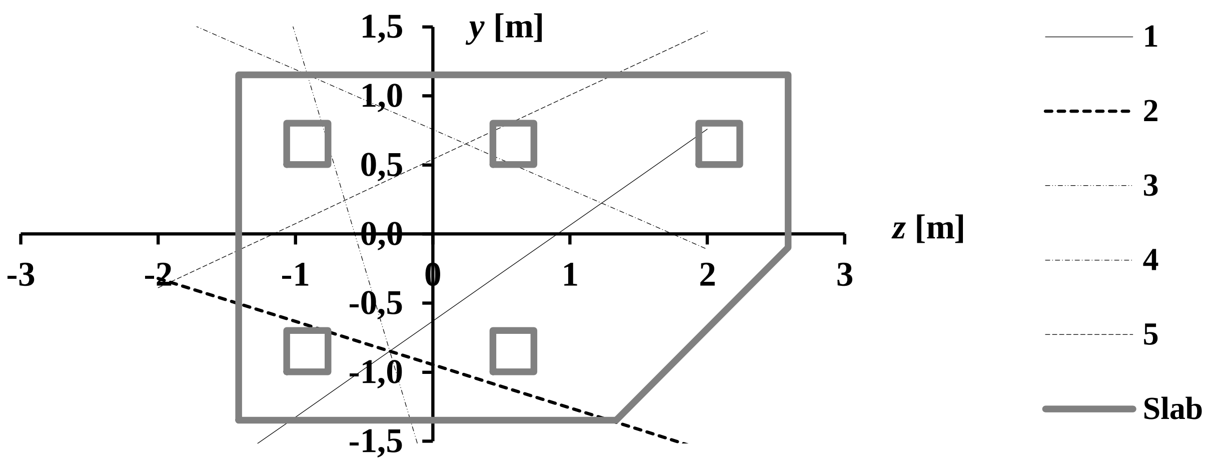


Figure 2.

30. PROBLEM

Determine the pile forces shown in Figure 1.

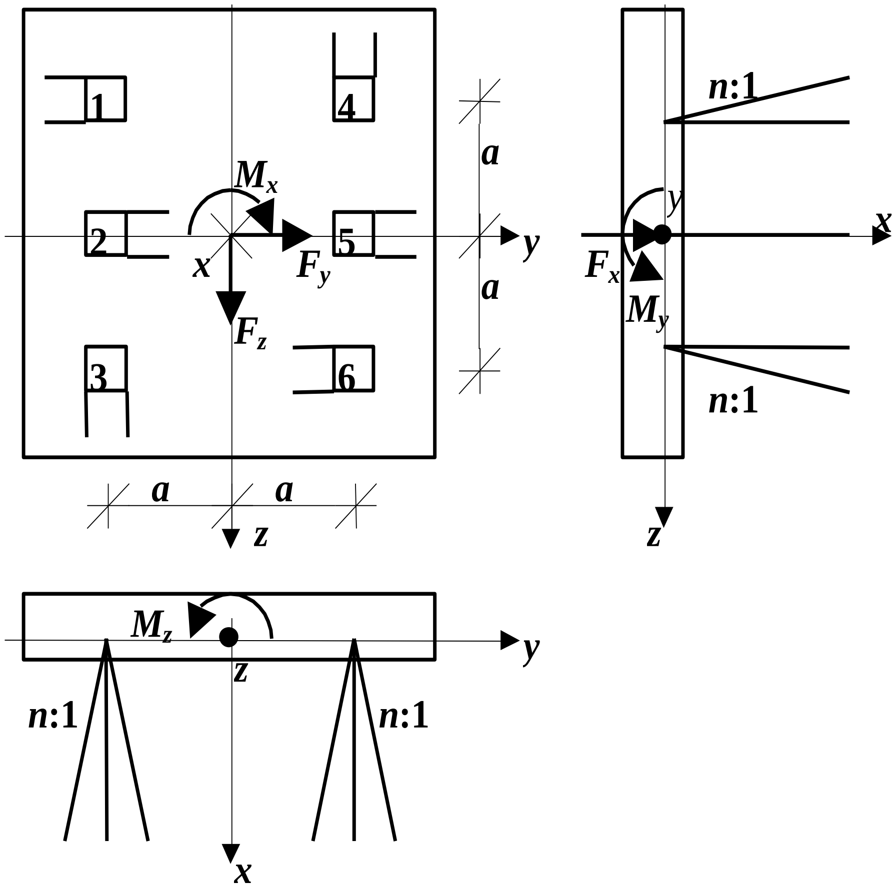


Figure 1.

Elements of the force vector

F_x = 0,000 MN

F_y = 0,000 MN

F_z = 0,000 MN

M_x = 1,000 MNm

M_y = 0,000 MNm

M_z = 0,000 MNm

Slope of diagonal piles

n = 4

All piles have the same relative stiffness EA/L **.**

System of equilibrium condition

$$\{F\} = [K]\{\delta\} \tag{1}$$

where force vector

$$\{F\} = \{F_x \;\; F_y \;\; F_z \;\; M_x \;\; M_y \;\; M_z\}^T \tag{2}$$

and displacement vector

$$\{\delta\} = \{u \;\; v \;\; w \;\; \omega \;\; \varphi \;\; \theta\}^T \tag{3}$$

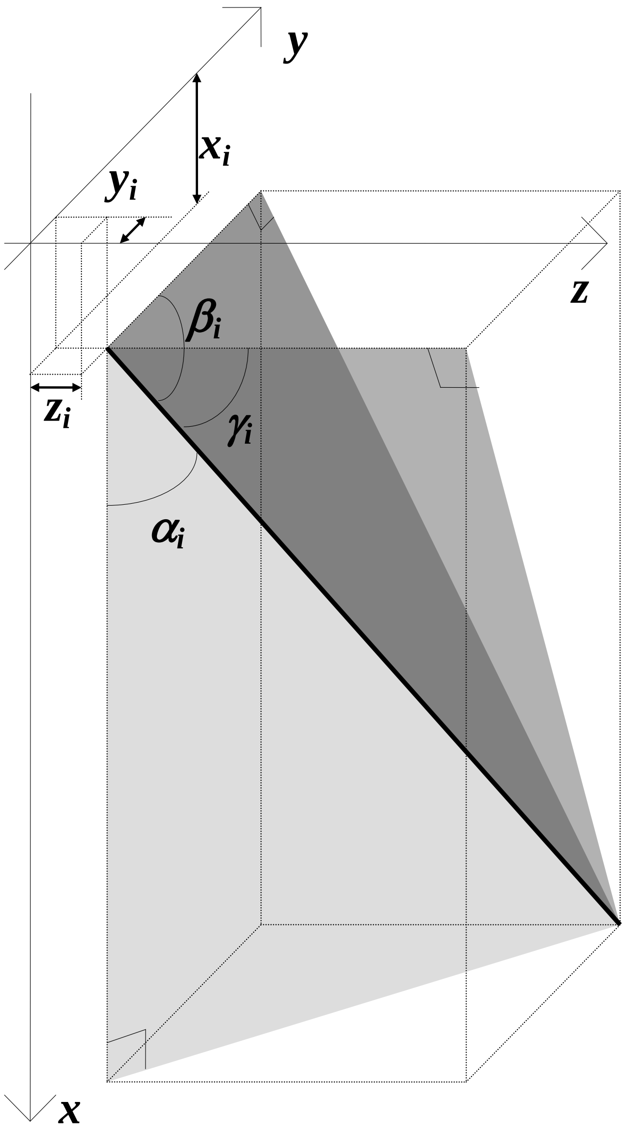


Figure 2.

and stiffness matrix of the system is sum of the stiffness matrixes of the piles

$$[K]=\sum_{i=1}^{n}k_i\begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} & p_{x,i}r_{x,i} & p_{x,i}r_{y,i} & p_{x,i}r_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} & p_{y,i}r_{x,i} & p_{y,i}r_{y,i} & p_{y,i}r_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 & p_{z,i}r_{x,i} & p_{z,i}r_{y,i} & p_{z,i}r_{z,i} \\ r_{x,i}p_{x,i} & r_{x,i}p_{y,i} & r_{x,i}p_{z,i} & r_{x,i}^2 & r_{x,i}r_{y,i} & r_{x,i}r_{z,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{y,i} & r_{y,i}p_{z,i} & r_{y,i}r_{x,i} & r_{y,i}^2 & r_{y,i}r_{z,i} \\ r_{z,i}p_{x,i} & r_{z,i}p_{y,i} & r_{z,i}p_{z,i} & r_{z,i}r_{x,i} & r_{z,i}r_{y,i} & r_{z,i}^2 \end{bmatrix} \tag{4}$$

where

$$p_{x,i} = \cos\alpha_i \tag{5}$$
$$p_{y,i} = \cos\beta_i \tag{6}$$
$$p_{z,i} = \cos\gamma_i \tag{7}$$
$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} \tag{8}$$
$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{9}$$
$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{10}$$

Angles α_i, β_i and γ_i are shown in Figure 2.

The elements of the stiffness matrix are presented in Table 1.

Table 1.

Variable	Pile	Number of piles	Vertical location	Horizontal location		Stiffness
Symbol	i	j_i	x_i	y_i	z_i	k_i
Unit	-	-	m	m	m	MN/m
Numerical value	1	1	0,000	-2,000	-2,000	1,000
	2	1	0,000	-2,000	0,000	1,000
	3	1	0,000	-2,000	2,000	1,000
	4	1	0,000	2,000	-2,000	1,000
	5	1	0,000	2,000	0,000	1,000
	6	1	0,000	2,000	2,000	1,000
	⋮					

⋮

Variable	Pile	Slope and directional angles					
Symbol	i	$\tan\alpha_i$	α_i	$\tan\beta_i$	β_i	$\tan\gamma_i$	γ_i
Unit	-	-	°	-	°		°
Numerical value	1	-0,250	-14,036	-4,000	104,036	∞	90,000
	2	0,250	14,036	4,000	75,964	∞	90,000
	3	0,250	14,036	∞	90,000	4,000	75,964
	4	-0,250	-14,036	∞	90,000	-4,000	104,036
	5	0,250	14,036	4,000	75,964	∞	90,000
	6	-0,250	-14,036	-4,000	104,036	∞	90,000

⋮

Variable	Pile	Cosines of directional angles			Lever arms		
Symbol	i	$p_{x,i}$	$p_{y,i}$	$p_{z,i}$	$r_{x,i}$	$r_{y,i}$	$r_{z,i}$
Unit	-	-	-	-	m	m	m
Numerical value	1	0,970	-0,243	0,000	-0,485	-1,940	1,940
	2	0,970	0,243	0,000	0,000	0,000	1,940
	3	0,970	0,000	0,243	-0,485	1,940	1,940
	4	0,970	0,000	-0,243	-0,485	-1,940	-1,940
	5	0,970	0,243	0,000	0,000	0,000	-1,940
	6	0,970	-0,243	0,000	0,485	1,940	-1,940

⋮

Variable	i	$K_{11,i}$	$K_{12,i}$	$K_{13,i}$	$K_{14,i}$	$K_{15,i}$	$K_{16,i}$
Numerical value	1	0,941	-0,235	0,000	-0,471	-1,882	1,882
	2	0,941	0,235	0,000	0,000	0,000	1,882
	3	0,941	0,000	0,235	-0,471	1,882	1,882
	4	0,941	0,000	-0,235	-0,471	-1,882	-1,882
	5	0,941	0,235	0,000	0,000	0,000	-1,882
	6	0,941	-0,235	0,000	0,471	1,882	-1,882
Unit		MN/m	MN/m	MN/m	MN	MN	MN
Numerical value		5,647	0,000	0,000	-0,941	0,000	0,000
Symbol		K_{11}	$K_{12}=K_{21}$	$K_{13}=K_{31}$	$K_{14}=K_{41}$	$K_{15}=K_{51}$	$K_{16}=K_{61}$

⋮

⋮

Symbol	i	$K_{22,i}$	$K_{23,i}$	$K_{24,i}$	$K_{25,i}$	$K_{26,i}$
Numerical value	1	0,059	0,000	0,118	0,471	-0,471
	2	0,059	0,000	0,000	0,000	0,471
	3	0,000	0,000	0,000	0,000	0,000
	4	0,000	0,000	0,000	0,000	0,000
	5	0,059	0,000	0,000	0,000	-0,471
	6	0,059	0,000	-0,118	-0,471	0,471
Unit		MN/m	MN/m	MN	MN	MN
Numerical value		0,235	0,000	0,000	0,000	0,000
Symbol		K_{22}	$K_{23}=K_{32}$	$K_{24}=K_{42}$	$K_{25}=K_{52}$	$K_{26}=K_{62}$

⋮

Symbol	i	$K_{33,i}$	$K_{34,i}$	$K_{35,i}$	$K_{36,i}$
Numerical value	1	0,000	0,000	0,000	0,000
	2	0,000	0,000	0,000	0,000
	3	0,059	-0,118	0,471	0,471
	4	0,059	0,118	0,471	0,471
	5	0,000	0,000	0,000	0,000
	6	0,000	0,000	0,000	0,000
Unit		MN/m	MN	MN	MN
Numerical value		0,118	0,000	0,941	0,941
Symbol		K_{33}	$K_{34}=K_{43}$	$K_{35}=K_{53}$	$K_{36}=K_{63}$

⋮

Symbol	i	$K_{44,i}$	$K_{45,i}$	$K_{46,i}$	$K_{55,i}$	$K_{56,i}$	$K_{66,i}$
Numerical value	1	0,235	0,941	-0,941	3,765	-3,765	3,765
	2	0,000	0,000	0,000	0,000	0,000	3,765
	3	0,235	-0,941	-0,941	3,765	3,765	3,765
	4	0,235	0,941	0,941	3,765	3,765	3,765
	5	0,000	0,000	0,000	0,000	0,000	3,765
	6	0,235	0,941	-0,941	3,765	-3,765	3,765
Unit		MNm	MNm	MNm	MNm	MNm	MNm
Numerical value		0,941	1,882	-1,882	15,059	0,000	22,588
Symbol		K_{44}	$K_{45}=K_{54}$	$K_{46}=K_{64}$	K_{55}	$K_{56}=K_{65}$	K_{66}

Stiffness matrix

$$K = \begin{bmatrix} 5{,}647 & 0{,}000 & 0{,}000 & -0{,}941 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}235 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}118 & 0{,}000 & 0{,}941 & 0{,}941 \\ -0{,}941 & 0{,}000 & 0{,}000 & 0{,}941 & 1{,}882 & -1{,}882 \\ 0{,}000 & 0{,}000 & 0{,}941 & 1{,}882 & 15{,}059 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}941 & -1{,}882 & 0{,}000 & 22{,}588 \end{bmatrix}$$

Elements of the displacement vector

u = 0,531 m

v = 0,000 m

w = 6,375 m

ω = 3,188 rad

φ = -0,797 rad

θ = 0,000 rad

Pile forces

$$N_i = k_i \Delta_i \qquad (11)$$

$$= k_i (p_{x,i}\, u + p_{y,i}\, v + p_{z,i}\, w + r_{x,i}\, \omega + r_{y,i}\, \varphi + r_{z,i}\, \theta) \qquad (12)$$

ANSWER:

Pile forces are

N_1 = 0,515 MN

N_2 = 0,515 MN

N_3 = -1,031 MN

N_4 = -1,031 MN

N_5 = 0,515 MN

N_6 = 0,515 MN

31. PROBLEM

Determine the shear force and bending moment distribution, respectively, and the deflection curve of the massive pile (large diameter), shown in Figure 1, by using three beam elements!

Diameter of the pile

d = 0,700 m

Nominal strength of concrete

K = 45 MN/m^2

Element lengths

a_1 = 3,500 m

a_2 = 3,500 m

a_3 = 3,500 m

Loads

H = 1,500 MN

M = 0,200 MNm

Foundation coefficient

c_a = 40 MN/m^3

c_b = 5 MN/m^3

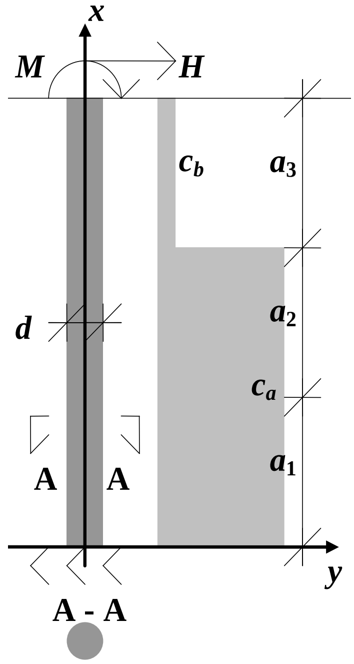


Figure 1.

Equilibrium condition of a beam element (positive directions are shown in Figure 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix}$$

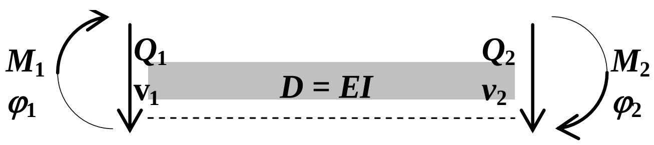


Figure 2.

Spring coefficient at the top of element *i* (Figure 3)

$$k_i = d \cdot \frac{a_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + a_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \qquad (1)$$

k_{01} = ∞

= 1,0E+100 MN/m

k_{02} = ∞

= 1,0E+100 MN/m

k_1 = 98,000 MN/m

k_2 = 55,125 MN/m

k_3 = 6,125 MN/m

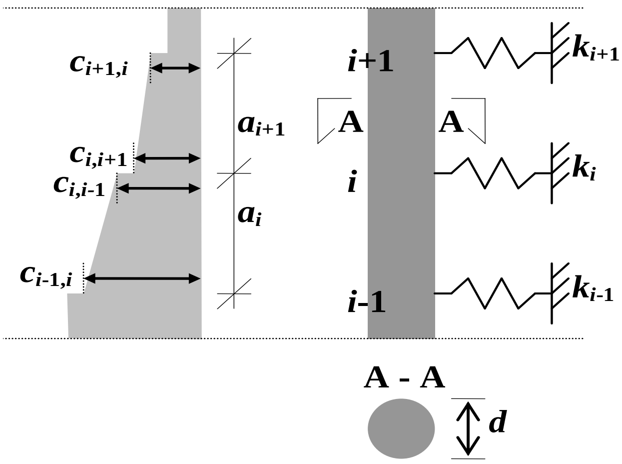


Figure 3.

Modulus of elasticity of concrete

$$E_i = k\sqrt{K_i K_0} \qquad (2)$$

where k = 1 and K_0 = 25·10⁶ MN/m².

E_i = 33 541 MN/m^2

Moment of inertia for a circle cross-section

$$I = \frac{\pi (d/2)^4}{4} \qquad (3)$$

= 0,011786 m^4

Bending stiffness of the pile

$$D = EI \qquad (4)$$

= 395 MNm^2

Stiffness matrix of element *i*

$$K_i^e = \begin{bmatrix} 110{,}641 & 193{,}621 & -110{,}641 & 193{,}621 \\ 193{,}621 & 451{,}783 & -193{,}621 & 225{,}892 \\ -110{,}641 & -193{,}621 & 110{,}641 & -193{,}621 \\ 193{,}621 & 225{,}892 & -193{,}621 & 451{,}783 \end{bmatrix}$$

Corresponding units

$$\left[K_i^e\right] = \begin{bmatrix} \mathrm{MN/m} & \mathrm{MN} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MNm} & \mathrm{MN} & \mathrm{MNm} \\ \mathrm{MN/m} & \mathrm{MN} & \mathrm{MN/m} & \mathrm{MN} \\ \mathrm{MN} & \mathrm{MNm} & \mathrm{MN} & \mathrm{MNm} \end{bmatrix}$$

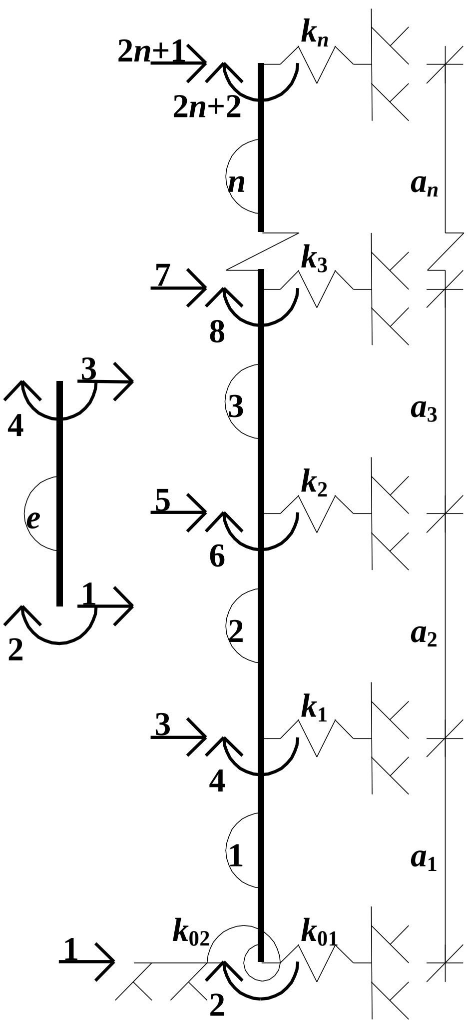


Figure 4.

From the equilibrium condition of the system

$$\{F\} = [K]\{\delta\} \tag{5}$$

the displacement vector

$$\{\delta\} = [K]^{-1}\{F\} \tag{6}$$

is obtained.

System stiffness matrix (Figure 4)

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} k_{11}^1 + k_{01} & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 & 0 & 0 \\ k_{21} = k_{12} & k_{22}^1 + k_{02} & k_{23}^1 & k_{24}^1 & 0 & 0 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 & 0 & 0 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 & 0 & 0 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^2 + k_{11}^3 + k_2 & k_{34}^2 + k_{12}^3 & k_{13}^3 & k_{14}^3 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^2 + k_{22}^3 & k_{23}^3 & k_{24}^3 \\ k_{71} = k_{18} & k_{72} = k_{27} & k_{73} = k_{37} & k_{74} = k_{47} & k_{75} = k_{57} & k_{76} = k_{67} & k_{33}^3 + k_3 & k_{34}^3 \\ k_{81} = k_{18} & k_{82} = k_{28} & k_{83} = k_{38} & k_{84} = k_{48} & k_{85} = k_{58} & k_{86} = k_{68} & k_{87} = k_{78} & k_{44}^3 \end{bmatrix} \tag{8}$$

Numerical values

1,0E+100	193,621	-110,641	193,621	0,000	0,000	0,000	0,000
193,621	1,0E+100	-193,621	225,892	0,000	0,000	0,000	0,000
-110,641	-193,621	319,282	0,000	-110,641	193,621	0,000	0,000
193,621	225,892	0,000	903,567	-193,621	225,892	0,000	0,000
0,000	0,000	-110,641	-193,621	276,407	0,000	-110,641	193,621
0,000	0,000	193,621	225,892	0,000	903,567	-193,621	225,892
0,000	0,000	0,000	0,000	-110,641	-193,621	116,766	-193,621
0,000	0,000	0,000	0,000	193,621	225,892	-193,621	451,783

Corresponding units

MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm

Inverse matrix $[K]^{-1}$

0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,005	0,001	0,003	-0,001	-0,001	-0,001
0,000	0,000	0,001	0,002	0,002	0,000	0,001	0,000
0,000	0,000	0,003	0,002	0,012	0,002	0,017	0,001
0,000	0,000	-0,001	0,000	0,002	0,003	0,011	0,002
0,000	0,000	-0,001	0,001	0,017	0,011	0,076	0,020
0,000	0,000	-0,001	0,000	0,001	0,002	0,020	0,009

Corresponding units

m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm

Loads at the node points

$$F_0 = 0{,}000 \text{ MN}$$
$$M_0 = 0{,}000 \text{ MNm}$$
$$F_1 = 0{,}000 \text{ MN}$$
$$M_1 = 0{,}000 \text{ MNm}$$
$$F_2 = 0{,}000 \text{ MN}$$
$$M_2 = 0{,}000 \text{ MNm}$$
$$F_3 = 1{,}500 \text{ MN}$$
$$M_3 = 0{,}200 \text{ MNm}$$

Node displacements and rotations

$$w_0 = 0{,}000 \text{ m}$$
$$\varphi_0 = 0{,}000 \text{ rad}$$
$$w_1 = -0{,}002 \text{ m}$$
$$\varphi_1 = 0{,}001 \text{ rad}$$
$$w_2 = 0{,}025 \text{ m}$$
$$\varphi_2 = 0{,}018 \text{ rad}$$
$$w_3 = 0{,}118 \text{ m}$$
$$\varphi_3 = 0{,}031 \text{ rad}$$

Force vector of element *i*

$$F_i^{\,e} = K_i^{\,e}\,\delta_i^{\,e} \tag{9}$$

1. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0{,}000 \text{ m} \\ 0{,}000 \\ -0{,}002 \text{ m} \\ 0{,}001 \end{bmatrix} = \begin{bmatrix} 0{,}407 \text{ MN} \\ 0{,}603 \text{ MNm} \\ -0{,}407 \text{ MN} \\ 0{,}822 \text{ MNm} \end{bmatrix}$$

2. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} -0{,}002 \text{ m} \\ 0{,}001 \\ 0{,}025 \text{ m} \\ 0{,}018 \end{bmatrix} = \begin{bmatrix} 0{,}601 \text{ MN} \\ -0{,}822 \text{ MNm} \\ -0{,}601 \text{ MN} \\ 2{,}926 \text{ MNm} \end{bmatrix}$$

3. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_3^e \cdot \begin{bmatrix} 0{,}025 \text{ m} \\ 0{,}018 \\ 0{,}118 \text{ m} \\ 0{,}031 \end{bmatrix} = \begin{bmatrix} -0{,}779 \text{ MN} \\ -2{,}926 \text{ MNm} \\ 0{,}779 \text{ MN} \\ 0{,}200 \text{ MNm} \end{bmatrix}$$

<u>ANSWER</u>:

The shear force and bending moment distribution, respectively, and the deflections at the node points are shown in Table 1. The corresponding curves are shown in Figures 5, 6 and 7, respectively. Positive directions are shown in Figure 8.

Table 1.

x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,407	0,603	0,000
3,500	-0,407	-0,822	-0,002
3,500	-0,601	-0,822	-0,002
7,000	-0,601	-2,926	0,025
7,000	0,779	-2,926	0,025
10,500	0,779	-0,200	0,118
10,500	0	0	

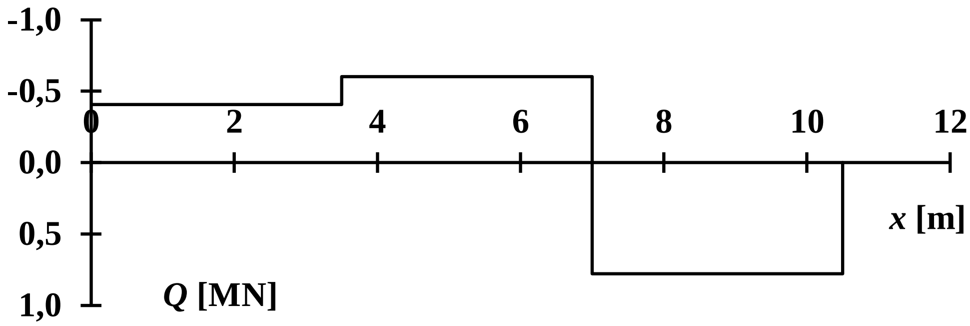


Figure 5. Shear force distribution curve.

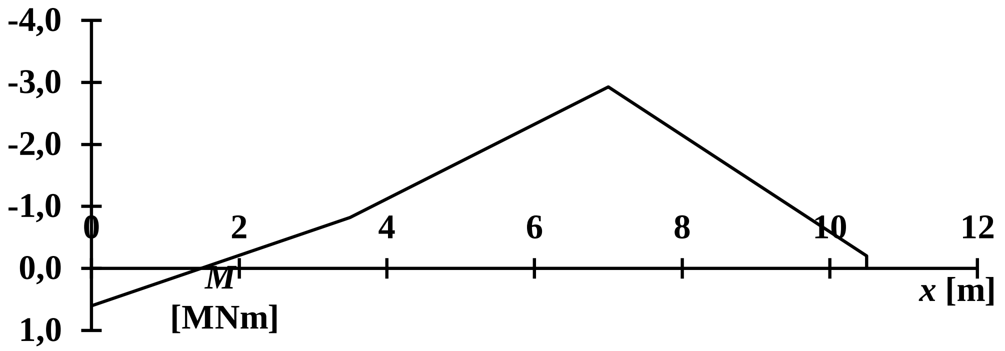


Figure 6. Bending moment distribution curve.

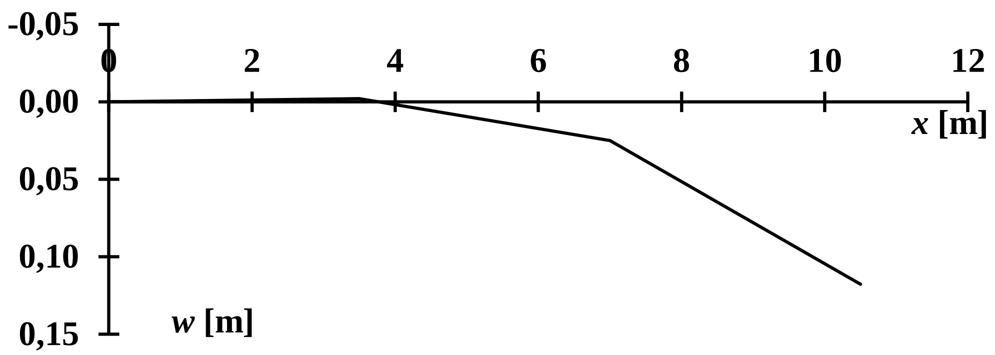


Figure 7. Deflection curve.

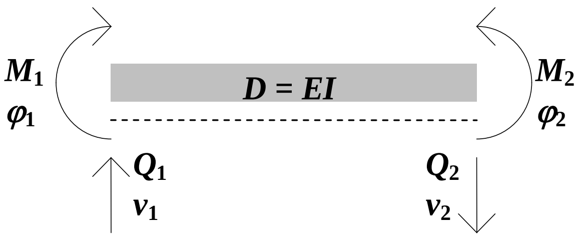


Figure 8. Positive directions (compare to Figure 2).

32. PROBLEM

Determine the shear force and bending moment distribution, respectively, and the deflection curve of the massive pile (large diameter) and column, shown in Figure 1, by using in the system stiffness matrix the three uppermost beam elements out of four!

Diameter of the pile

d = 0,700 m

Nominal strength of concrete

K = 45 MN/m^2

Element lengths

a_1 = 3,500 m

a_2 = 3,500 m

a_3 = 3,500 m

a_4 = 3,500 m

Loads

H = 1,500 MN

M = -5,050 MNm

Foundation coefficient

c_a = 40 MN/m^3

c_b = 5 MN/m^3

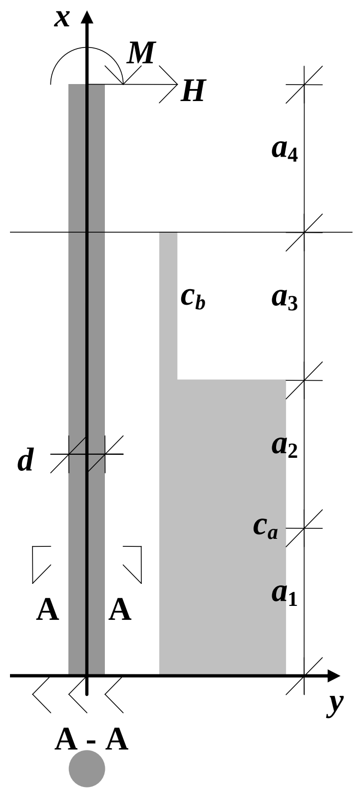


Figure 1.

Equilibrium condition of a beam element (positive directions are shown in Figure 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix}$$

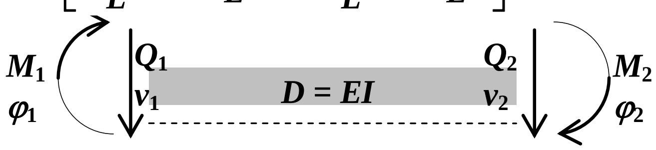


Figure 2.

Spring coefficient at the top of element i (Figure 3)

$$k_i = d \cdot \frac{a_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + a_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \tag{1}$$

k_1 = 98,000 MN/m

k_2 = 55,125 MN/m

k_3 = 6,125 MN/m

k_4 = 0 MN/m

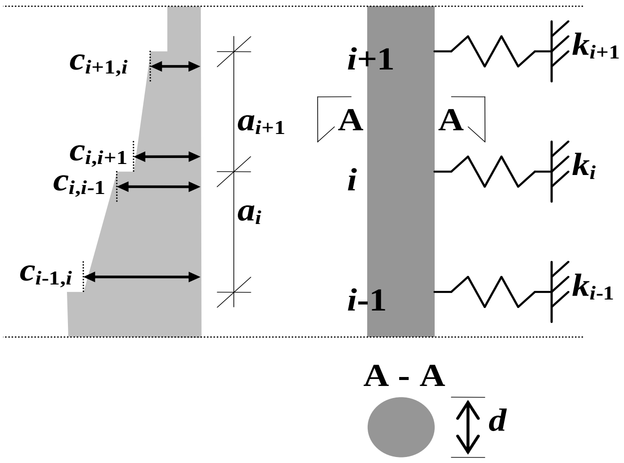


Figure 3.

Modulus of elasticity of concrete

$$E_i = k\sqrt{K_i K_0} \tag{2}$$

where k = 1 and $K_0 = 25 \cdot 10^6$ MN/m^2.

E_i = 33 541 MN/m^2

Moment of inertia for a circle cross-section

$$I = \frac{\pi (d/2)^4}{4} \tag{3}$$

= 0,011786 m^4

Bending stiffness of the pile

$$D = EI \tag{4}$$

= 395,310 MNm^2

Stiffness matrix of element i

$$K_i^e = \begin{bmatrix} 110{,}641 & 193{,}621 & -110{,}641 & 193{,}621 \\ 193{,}621 & 451{,}783 & -193{,}621 & 225{,}892 \\ -110{,}641 & -193{,}621 & 110{,}641 & -193{,}621 \\ 193{,}621 & 225{,}892 & -193{,}621 & 451{,}783 \end{bmatrix}$$

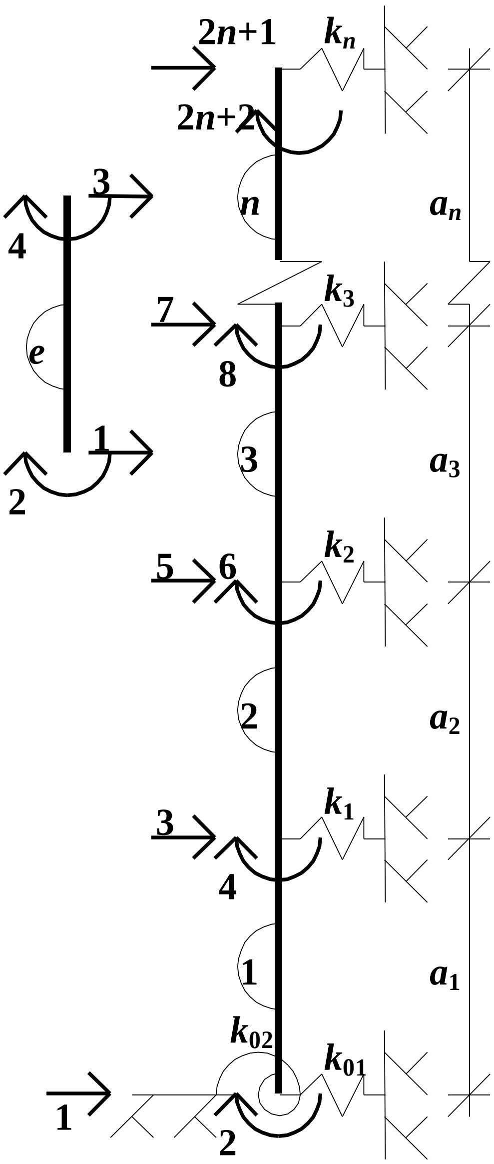


Figure 4.

From the equilibrium condition of the system

$$\{F\} = [K]\{\delta\} \tag{5}$$

the displacement vector

$$\{\delta\} = [K]^{-1}\{F\} \tag{6}$$

is obtained.

System stiffness matrix (Figure 4)

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 & 0 & 0 & 0 & 0 \\ k_{21} = k_{12} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 & 0 & 0 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^2 + k_{11}^3 + k_2 & k_{34}^2 + k_{12}^3 & k_{13}^3 & k_{14}^3 & 0 & 0 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^2 + k_{22}^3 & k_{23}^3 & k_{24}^3 & 0 & 0 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^3 + k_{11}^4 + k_3 & k_{34}^3 + k_{12}^4 & k_{13}^4 & k_{14}^4 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^3 + k_{22}^4 & k_{23}^4 & k_{24}^4 \\ k_{71} = k_{18} & k_{72} = k_{27} & k_{73} = k_{37} & k_{74} = k_{47} & k_{75} = k_{57} & k_{76} = k_{67} & k_{33}^4 + k_4 & k_{34}^4 \\ k_{81} = k_{18} & k_{82} = k_{28} & k_{83} = k_{38} & k_{84} = k_{48} & k_{85} = k_{58} & k_{86} = k_{68} & k_{87} = k_{78} & k_{44}^4 \end{bmatrix} \tag{8}$$

Numerical values

319,282	0,000	-110,641	193,621	0,000	0,000	0,000	0,000
0,000	903,567	-193,621	225,892	0,000	0,000	0,000	0,000
-110,641	-193,621	276,407	0,000	-110,641	193,621	0,000	0,000
193,621	225,892	0,000	903,567	-193,621	225,892	0,000	0,000
0,000	0,000	-110,641	-193,621	227,407	0,000	-110,641	193,621
0,000	0,000	193,621	225,892	0,000	903,567	-193,621	225,892
0,000	0,000	0,000	0,000	-110,641	-193,621	110,641	-193,621
0,000	0,000	0,000	0,000	193,621	225,892	-193,621	451,783

Corresponding units

MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm
MN/m	MN	MN/m	MN	MN/m	MN	MN/m	MN
MN	MNm	MN	MNm	MN	MNm	MN	MNm

Inverse matrix $[K]^{-1}$

0,005	0,001	0,003	-0,001	-0,001	-0,001	-0,005	-0,001
0,001	0,002	0,002	0,000	0,001	0,000	-0,001	0,000
0,003	0,002	0,012	0,002	0,017	0,001	0,019	0,001
-0,001	0,000	0,002	0,003	0,011	0,002	0,019	0,002
-0,001	0,001	0,017	0,011	0,076	0,020	0,145	0,020
-0,001	0,000	0,001	0,002	0,020	0,009	0,052	0,009
-0,005	-0,001	0,019	0,019	0,145	0,052	0,363	0,068
-0,001	0,000	0,001	0,002	0,020	0,009	0,068	0,018

Corresponding units

m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm
m/MN	1/MN	m/MN	1/MN	m/MN	1/MN	m/MN	1/MN
1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm	1/MN	1/MNm

Loads at the node points

$$F_1 = 0{,}000 \text{ MN}$$
$$M_1 = 0{,}000 \text{ MNm}$$
$$F_2 = 0{,}000 \text{ MN}$$
$$M_2 = 0{,}000 \text{ MNm}$$
$$F_3 = 0{,}000 \text{ MN}$$
$$M_3 = 0{,}000 \text{ MNm}$$
$$F_4 = 1{,}500 \text{ MN}$$
$$M_4 = -5{,}050 \text{ MNm}$$

Node displacements and rotations

$$w_1 = -0{,}002 \text{ m}$$
$$\varphi_1 = 0{,}001 \text{ rad}$$
$$w_2 = 0{,}025 \text{ m}$$
$$\varphi_2 = 0{,}018 \text{ rad}$$
$$w_3 = 0{,}118 \text{ m}$$
$$\varphi_3 = 0{,}031 \text{ rad}$$
$$w_4 = 0{,}204 \text{ m}$$
$$\varphi_4 = 0{,}010 \text{ rad}$$

Force vector of element *i*

$$F_i^{\,e} = K_i^{\,e}\,\delta_i^{\,e} \tag{9}$$

1. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0 \text{ m} \\ 0 \\ -0{,}002 \text{ m} \\ 0{,}001 \end{bmatrix} = \begin{bmatrix} 0{,}407 \text{ MN} \\ 0{,}603 \text{ MNm} \\ -0{,}407 \text{ MN} \\ 0{,}822 \text{ MNm} \end{bmatrix}$$

2. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} -0{,}002 \text{ m} \\ 0{,}001 \\ 0{,}025 \text{ m} \\ 0{,}018 \end{bmatrix} = \begin{bmatrix} 0{,}601 \text{ MN} \\ -0{,}822 \text{ MNm} \\ -0{,}601 \text{ MN} \\ 2{,}926 \text{ MNm} \end{bmatrix}$$

3. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_3^e \cdot \begin{bmatrix} 0{,}025 \text{ m} \\ 0{,}018 \\ 0{,}118 \text{ m} \\ 0{,}031 \end{bmatrix} = \begin{bmatrix} -0{,}779 \text{ MN} \\ -2{,}926 \text{ MNm} \\ 0{,}779 \text{ MN} \\ 0{,}200 \text{ MNm} \end{bmatrix}$$

4. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_4^e \cdot \begin{bmatrix} 0{,}118 \text{ m} \\ 0{,}031 \\ 0{,}204 \text{ m} \\ 0{,}010 \end{bmatrix} = \begin{bmatrix} -1{,}500 \text{ MN} \\ -0{,}200 \text{ MNm} \\ 1{,}500 \text{ MN} \\ -5{,}050 \text{ MNm} \end{bmatrix}$$

ANSWER:

The shear force and bending moment distribution, respectively, and the deflections at the node points are shown in Table 1. The corresponding curves are shown in Figures 5, 6 and 7, respectively. Positive directions are shown in Figure 8.

Table 1.

x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,407	0,603	0,000
3,500	-0,407	-0,822	-0,002
3,500	-0,601	-0,822	-0,002
7,000	-0,601	-2,926	0,025
7,000	0,779	-2,926	0,025
10,500	0,779	-0,200	0,118
10,500	1,500	-0,200	0,118
14,000	1,500	5,050	0,204
14,000	0	0	

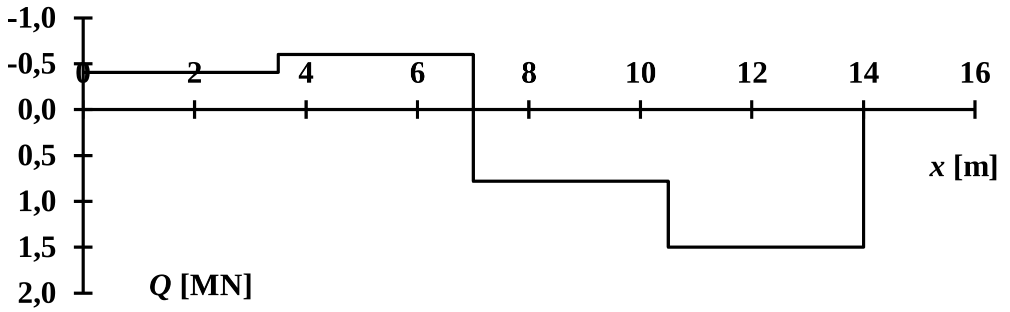


Figure 5. Shear force distribution curve.

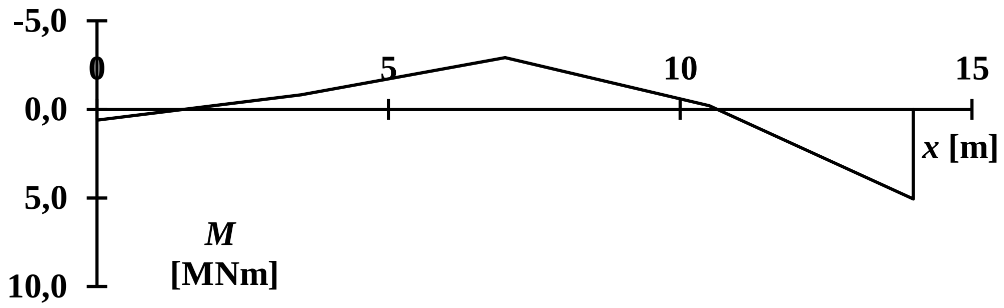


Figure 6. Bending moment distribution curve.

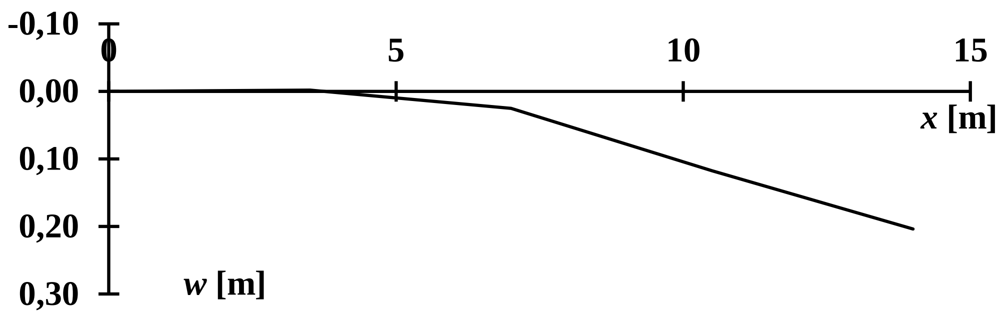


Figure 7. Deflection curve.

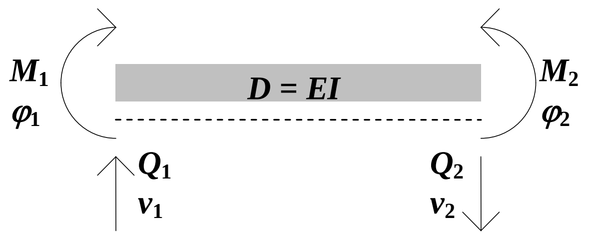


Figure 8. Positive directions (compare to Figure 2).

33. PROBLEM

A horizontal load caused by braking of a vehicle is acting on the top of an intermediate column-type support. The size of the load according to the Finnish Standard is

$$J = 0{,}500 \text{ MN}$$

The intermediate support consists of a massive concrete pile with circular cross-section (1) and an equal concrete column (2) shown in Figure 1. What is displacement a at the top of the column caused by the braking load?

Use force method to solve the problem!

Elasticity of the soil is approximated by a single spring, whose spring coefficient

$$k = 120 \text{ MN/m}$$

The diameters of the pile and the column, respectively, are

$$d_1 = 1{,}200 \text{ m}$$
$$d_2 = 0{,}800 \text{ m}$$

Height

$$h_1 = 10 \text{ m}$$
$$h_2 = 8 \text{ m}$$

Modulus of elasticity of concrete

$$E = 30\,000 \text{ MN/m}^2$$

Bending stiffness of the superstructure is assumed to be infinity.

$$D = \infty$$

The bottom of the pile can be assumed to act as a hinge.

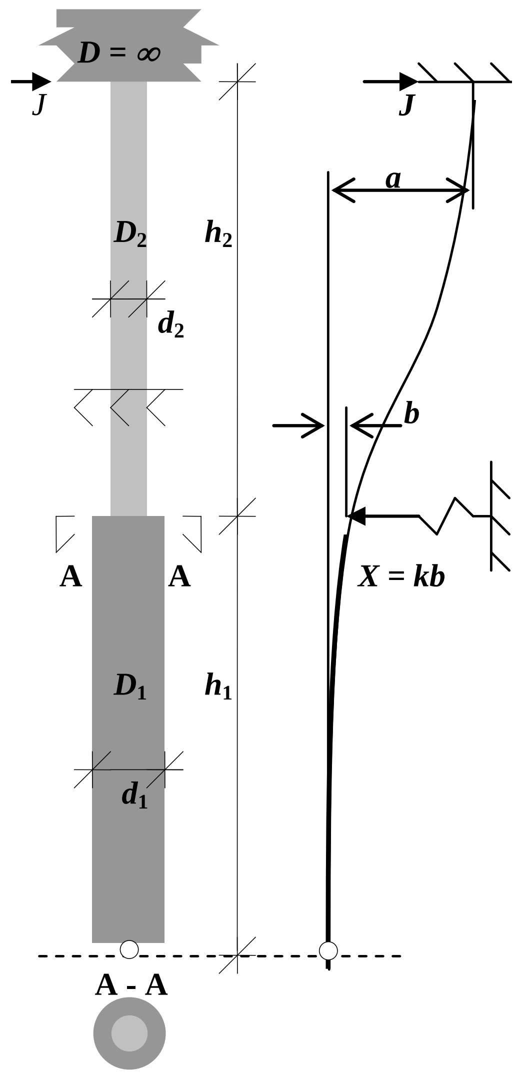


Figure 1.

Bending stiffnesses

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$D_1 =$ 3 054 MNm^2

$D_2 =$ 603 MNm^2

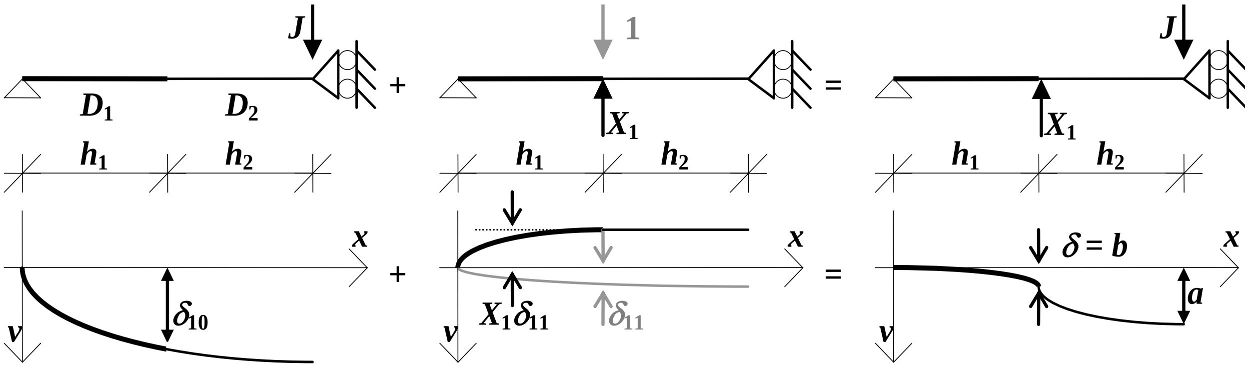


Figure 2.

Equilibrium condition (Figure 2)

$$\delta_{10}\text{-}X_1\delta_{11} = \delta \tag{2}$$

where

$$\delta = \frac{X_1}{k} \tag{3}$$

Bending moment due to point load P = 1 (Figure 3)

$$M_1(x) = \begin{cases} x, & x \in \{0..h_1\} \\ h_1, & x \in \{h_1..h_1 + h_2\} \end{cases} \tag{4}$$

Bending moment due to braking load (Figure 4)

$$M_0(x) = Jx \tag{5}$$

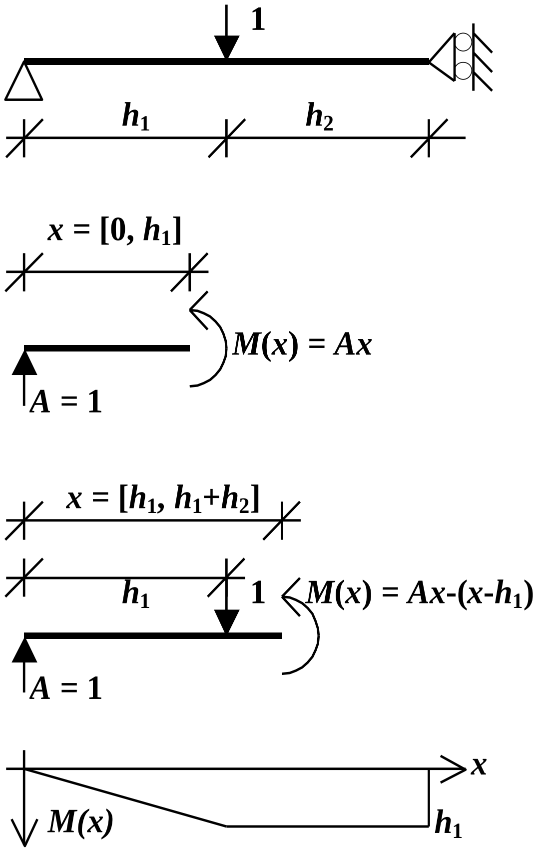


Figure 3.

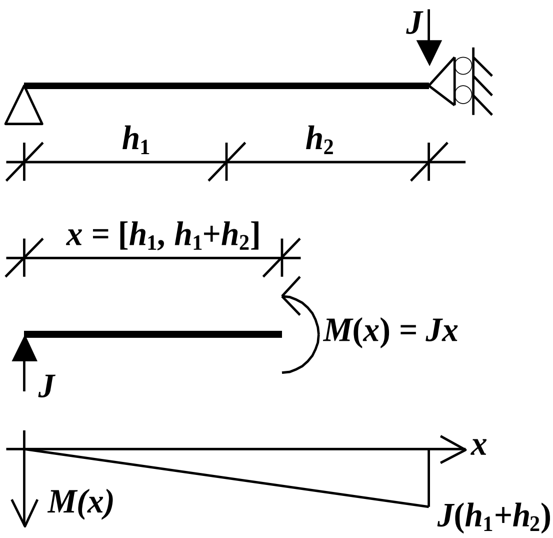


Figure 4.

Displacements

$$\delta_{10} = \int \frac{M_1 M_0}{D} dx \tag{6}$$

$$= \int_0^{h_1} \frac{Jx^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{h_1 Jx}{D_2} dx \tag{7}$$

$$= J\left[\frac{h_1^3}{3D_1} + \frac{h_1 h_2 (2h_1 + h_2)}{2D_2}\right] \tag{8}$$

$$= \quad 0{,}983 \ \mathrm{m}$$

$$\delta_{11} = \int \frac{M_1 M_1}{D} dx \tag{9}$$

$$= \int_0^{h_1} \frac{x^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{h_1^2}{D_2} dx \tag{10}$$

$$= \frac{h_1^3}{3D_1} + \frac{h_1^2 h_2}{D_2} \tag{11}$$

$$= \quad 1{,}435 \ \mathrm{m/MN}$$

Reaction force from Equations 2, 3, 8 and 11

$$X_1 = \frac{\delta_{10}}{\frac{1}{k} + \delta_{11}} \tag{12}$$

$$= \quad 0{,}681 \ \mathrm{MN}$$

Bending moment of the structure (Figure 5)

$$M = M_0 - X_1 M_1 \tag{13}$$

$$= \begin{cases} (J - X_1)x, & x \in \{0 \ldots h_1\} \\ Jx - X_1 h_1, & x \in \{h_1 \ldots h_1 + h_2\} \end{cases} \tag{14}$$

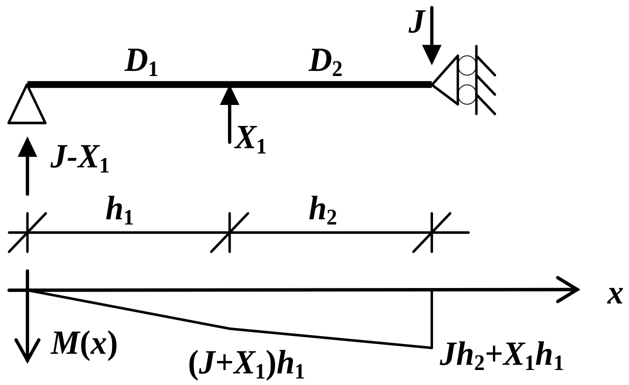


Figure 5.

Bending moment due to load $P = 1$ acting at the top of the column (Figure 6)

$$M_C(x) = x \tag{15}$$

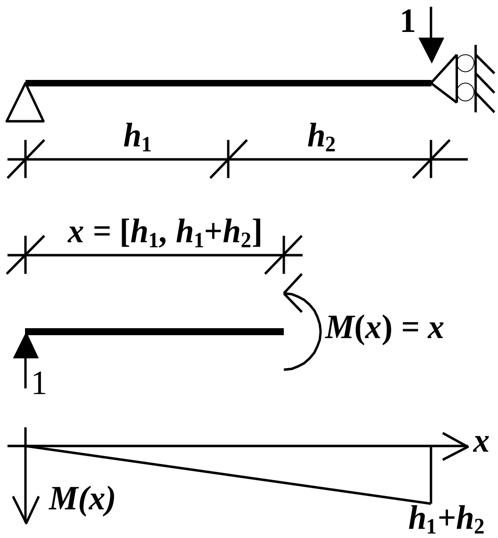


Figure 6.

Displacement at the top of the column

$$a = \delta_1 \tag{16}$$

$$= \int \frac{MM_C}{D} dx \tag{17}$$

$$= \int_0^{h_1} \frac{(J - X_1)x^2}{D_1} dx + \int_{h_1}^{h_1+h_2} \frac{Jx^2 - X_1 h_1 x}{D_2} dx \tag{18}$$

$$= \frac{(J - X_1)h_1^3}{3D_1} + \frac{Jh_2\left(3h_1^2 + 3h_1 h_2 + h_2^2\right)}{3D_2} - \frac{X_1 h_1 h_2 (2h_1 + h_2)}{2D_2} \tag{19}$$

$$= \quad 0{,}051 \text{ m}$$

ANSWER: **Displacement at the top of the column**

a = **0,051 m**

34. PROBLEM

A horizontal load caused by braking of a vehicle is acting on the top of an intermediate column-type support. The size of the load according to the Finnish Standard is

$$J = 0{,}500 \text{ MN}$$

The intermediate support consists of a massive concrete pile with circular cross-section (1) and an equal concrete column (2) shown in Figure 1. What is displacement a at the top of the column caused by the braking load?

Solv the problem by using the expression of the deflections of the cantilever!

Elasticity of the soil is approximated by a single spring, whose spring coefficient

$$k = 120 \text{ MN/m}$$

The diameters of the pile and the column, respectively, are

$$d_1 = 1{,}200 \text{ m}$$

$$d_2 = 0{,}800 \text{ m}$$

Height

$$h_1 = 10 \text{ m}$$

$$h_2 = 8 \text{ m}$$

Modulus of elasticity of concrete

$$E = 30\,000 \text{ MN/m}^2$$

Bending stiffness of the superstructure is assumed to be infinity.

$$D = \infty$$

The bottom of the pile can be assumed to act as a hinge.

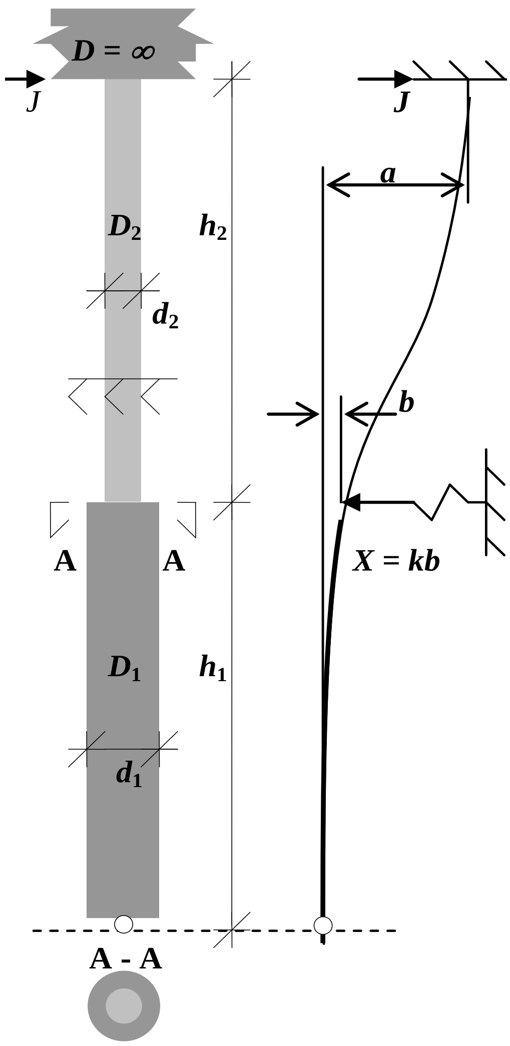


Figure 1.

Bending stiffnesses

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

D_1 = 3 054 MNm^2

D_2 = 603 MNm^2

Equations given in the *Formulary* are applied.

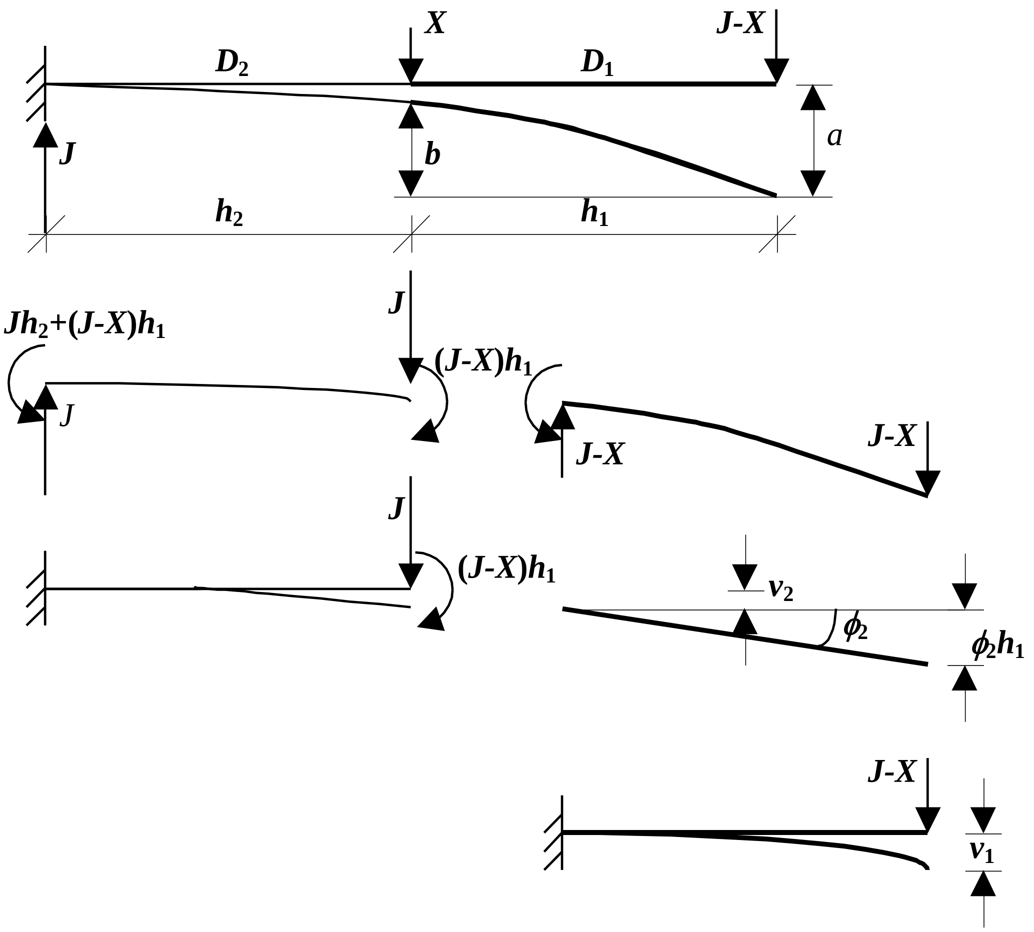


Figure 2.

Deflection at the spring (1) (Figure 2)

$$b = v_1 + \phi_2 h_1 \tag{2}$$

$$= \frac{(J - X)h_1^3}{3D_1} + \left[\frac{Jh_2^2}{2D_2} + \frac{(J - X)h_1 h_2}{D_2}\right]h_1 \tag{3}$$

where the braking load

$$X = kb \tag{4}$$

From Equations 21 and 22

$$b = J\frac{\dfrac{h_1^3}{3D_1} + \dfrac{h_1 h_2^2}{2D_2} + \dfrac{h_1^2 h_2}{D_2}}{1 + \dfrac{kh_1^3}{3D_1} + \dfrac{kh_1^2 h_2}{D_2}} \tag{5}$$

$$= \mathbf{0{,}006\ m}$$

Displacement at the top of the column is sum of spring deflection b and deflection v_2 at the end of cantilever (2)

$$a = b + v_2 \tag{6}$$

$$= b + \frac{Jh_2^3}{3D_2} + \frac{(J - X)h_1 h_2^2}{2D_2} \tag{7}$$

$$= \mathbf{0{,}051\ m}$$

<u>ANSWER</u>: **Displacement at the top of the column**

$$a = \mathbf{0{,}051\ m}$$

E x t r a   S t u d y

Deflection curve, when the origin is at the bottom of the pile (dashed line in Fig. 3)

$$v(x)=\begin{cases}\dfrac{(J-X)h_1^3}{6D_1}\left[2-3\dfrac{x}{h_1}+\left(\dfrac{x}{h_1}\right)^3\right]+\\ \left[\dfrac{Jh_2^2}{2D_2}+\dfrac{(J-X)h_1h_2}{D_2}\right](h_1-x)+a-b, \quad x=[0, \quad h_1]\\ \dfrac{Jh_2^3}{6D_2}\left[2-3\dfrac{x-h_1}{h_2}+\left(\dfrac{x-h_1}{h_2}\right)^3\right]+\dfrac{(J-X)h_1}{2D_2}(h_2-x+h_1)^2,\\ x=[h_1, \quad h_2]\end{cases} \tag{8}$$

where the braking load from Equation 22 is

$$X = 0{,}681 \text{ MN}$$

The deflection curve is mirrored so, that the deflection in origin is zero (continuous line in Fig. 8)

$$v_p(x) = a-v(x) \tag{9}$$

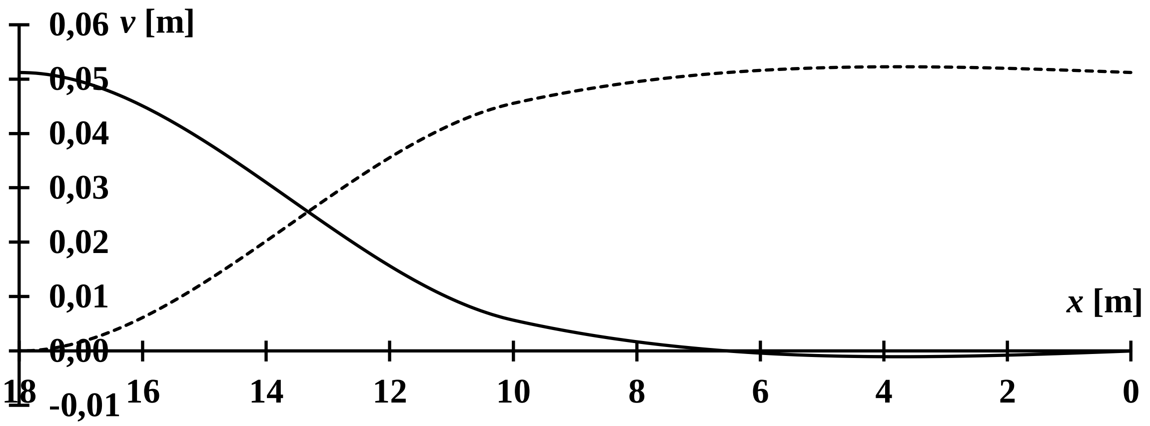


Figure 3.

35. PROBLEM

A horizontal load caused by braking of a vehicle is acting on the top of an intermediate column-type support. The size of the load according to the Finnish Standard is

$$J = 0{,}500 \text{ MN}$$

The intermediate support consists of a massive concrete pile with circular cross-section (1) and an equal concrete column (2) shown in Figure 1. What is displacement a at the top of the column caused by the braking load?

Solv the problem by using Mohr's method!

Elasticity of the soil is approximated by a single spring, whose spring coefficient

$$k = 120 \text{ MN/m}$$

The diameters of the pile and the column, respectively, are

$$d_1 = 1{,}200 \text{ m}$$
$$d_2 = 0{,}800 \text{ m}$$

Height

$$h_1 = 10 \text{ m}$$
$$h_2 = 8 \text{ m}$$

Modulus of elasticity of concrete

$$E = 30\,000 \text{ MN/m}^2$$

Bending stiffness of the superstructure is assumed to be infinity.

$$D = \infty$$

The bottom of the pile can be assumed to act as a hinge.

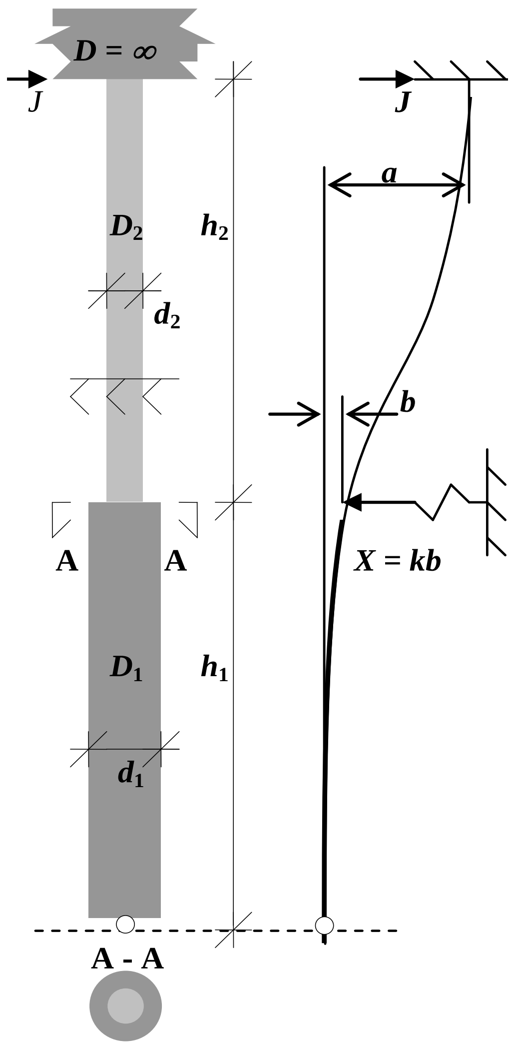


Figure 1.

Bending stiffnesses

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$D_1 =$ 3 054 MNm^2

$D_2 =$ 603 MNm^2

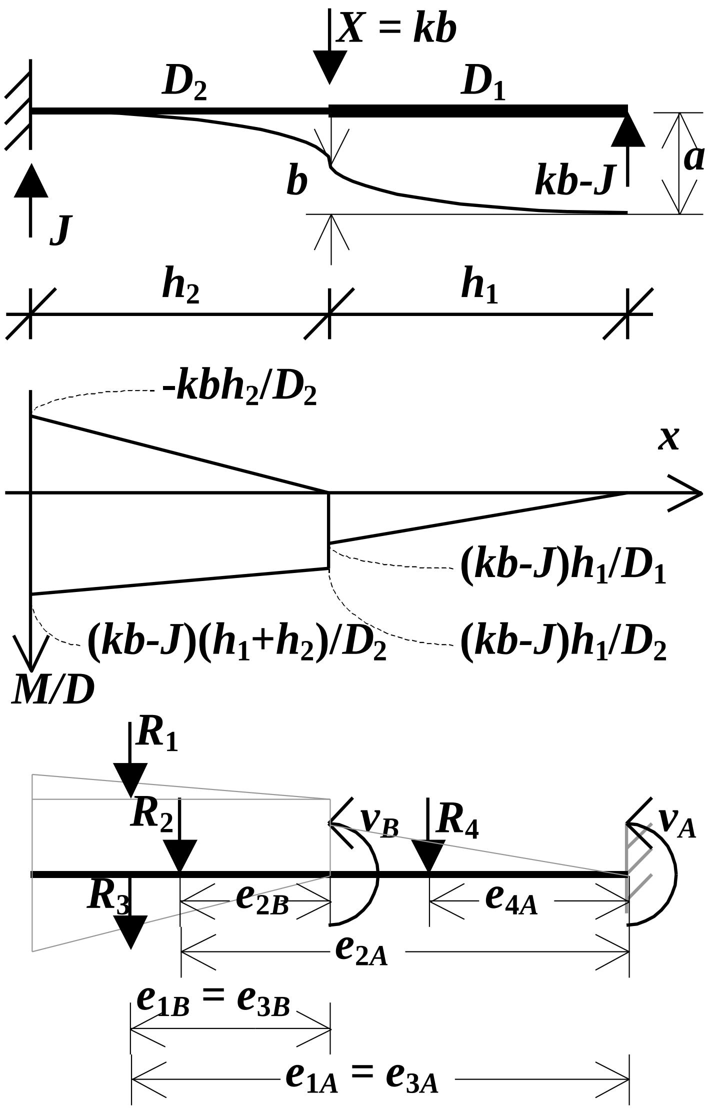


Figure 2.

Deflection v at the spring ($_B$) and at the bottom of the pile ($_A$) (Figure 2)

$$\begin{cases} v_B = a - b = -R_1 e_{1B} - R_2 e_{2B} - R_3 e_{3B} \\ v_A = \quad\quad a = -R_1 e_{1A} - R_2 e_{2A} - R_3 e_{3A} - R_4 e_{4A} \end{cases} \quad \textbf{(28a, b)}$$

By summation

$$-b = R_1(e_{1A} - e_{1B}) + R_2(e_{2A} - e_{2B}) + R_3(e_{3A} - e_{3B}) + R_4 e_{4A} \quad \textbf{(29)}$$

Load resultants

$$R_1 = \frac{kb - J}{2D_2} h_2^2 \quad \textbf{(30)}$$

$$R_2 = \frac{kb - J}{D_2} h_1 h_2 \quad \textbf{(31)}$$

$$R_3 = \frac{-kb}{2D_2} h_2^2 \quad \textbf{(32)}$$

$$R_4 = \frac{kb - J}{2D_1} h_1^2 \quad \textbf{(33)}$$

Distances

$$e_{iA} - e_{iB} = h_1 \quad \textbf{(34)}$$

$$e_{4A} = 2h_1/3 \quad \textbf{(35)}$$

Spring deflection from Equation 29

$$b = J \frac{\dfrac{h_1 h_2^2}{2D_2} + \dfrac{h_1^2 h_2}{D_2} + \dfrac{h_1^3}{3D_1}}{1 + \dfrac{kh_1^2 h_2}{D_2} + \dfrac{kh_1^3}{3D_1}} \quad \textbf{(36)}$$

$$= \quad 0{,}006 \text{ m}$$

Displacement at the top of the column from Equation 28a

$$a = b + \frac{Jh_2^3}{3D_2} - \frac{kb - J}{2D_2} h_1 h_2^2 \quad \textbf{(37)}$$

$$= \quad 0{,}051 \text{ m}$$

<u>ANSWER</u>:

Displacement at the top of the column

$\boldsymbol{a}$ **= 0,051 m**

36. PROBLEM

A horizontal load caused by braking of a vehicle is acting on the top of an intermediate column-type support. The size of the load according to the Finnish Standard is

J = 0,500 MN

The intermediate support consists of a massive concrete pile with circular cross-section (1) and an equal concrete column (2) shown in Figure 1. What is displacement a at the top of the column caused by the braking load?

Solv the problem by using Finite Element Method (FEM)!

Elasticity of the soil is approximated by a single spring, whose spring coefficient

k = 120 MN/m

The diameters of the pile and the column, respectively, are

d_1 = 1,200 m

d_2 = 0,800 m

Height

h_1 = 10 m

h_2 = 8 m

Modulus of elasticity of concrete

E = 30 000 MN/m^2

Bending stiffness of the superstructure is assumed to be infinity.

$D = \infty$

The bottom of the pile can be assumed to act as a hinge.

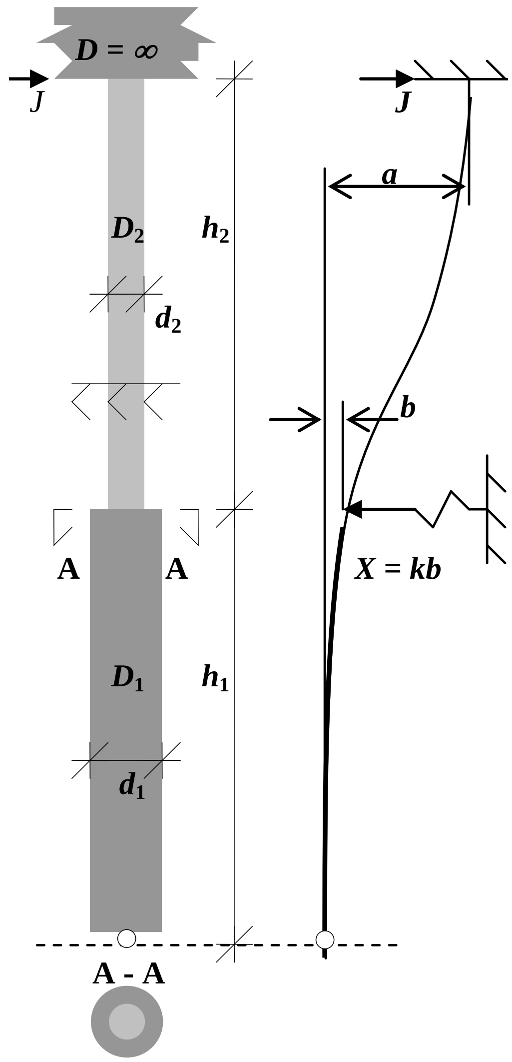


Figure 1.

Bending stiffnesses

$$D_i = E\frac{\pi}{4}\left(\frac{d_i}{2}\right)^4 \tag{1}$$

$D_1 =$ 3 054 MNm^2

$D_2 =$ 603 MNm^2

Spring coefficient

$k_{01} =$ ∞ MN/m

$=$ 1,0E+100 MN/m (at calculation)

$k_1 =$ 120 MN/m

$k_2 =$ 0 MN/m

Equilibrium condition of a beam element (positive directions are shown in Fig. 2)

$$\begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} \tag{38}$$

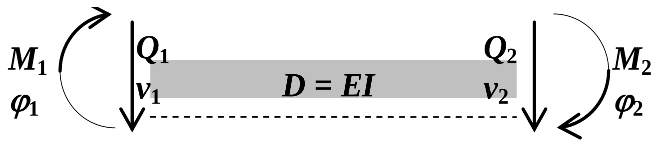


Figure 2.

Stiffness matrix of element 1

$$K_i^e = \begin{bmatrix} 36{,}644 & 183{,}218 & -36{,}644 & 183{,}218 \\ 183{,}218 & 1\,221{,}451 & -183{,}218 & 610{,}726 \\ -36{,}644 & -183{,}218 & 36{,}644 & -183{,}218 \\ 183{,}218 & 610{,}726 & -183{,}218 & 1\,221{,}451 \end{bmatrix}$$

Stiffness matrix of element 2

$$K_i^e = \begin{bmatrix} 14{,}137 & 56{,}549 & -14{,}137 & 56{,}549 \\ 56{,}549 & 301{,}593 & -56{,}549 & 150{,}796 \\ -14{,}137 & -56{,}549 & 14{,}137 & -56{,}549 \\ 56{,}549 & 150{,}796 & -56{,}549 & 301{,}593 \end{bmatrix}$$

System stiffness matrix (Figure 3)

$$K = \begin{bmatrix} k_{11}^1 + k_{01} & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 \\ k_{21} = k_{12} & k_{22}^1 & k_{23}^1 & k_{24}^1 & 0 & 0 \\ k_{31} = k_{13} & k_{32} = k_{23} & k_{33}^1 + k_{11}^2 + k_1 & k_{34}^1 + k_{12}^2 & k_{13}^2 & k_{14}^2 \\ k_{41} = k_{14} & k_{42} = k_{24} & k_{43} = k_{34} & k_{44}^1 + k_{22}^2 & k_{23}^2 & k_{24}^2 \\ k_{51} = k_{15} & k_{52} = k_{25} & k_{53} = k_{35} & k_{54} = k_{45} & k_{33}^2 & k_{34}^2 \\ k_{61} = k_{16} & k_{62} = k_{26} & k_{63} = k_{36} & k_{64} = k_{46} & k_{65} = k_{56} & k_{44}^2 \end{bmatrix}$$

(39)

$$= \begin{bmatrix} 1{,}0E{+}100 & 183{,}218 & -36{,}644 & 183{,}218 & 0{,}000 & 0{,}000 \\ 183{,}218 & 1221{,}451 & -183{,}218 & 610{,}726 & 0{,}000 & 0{,}000 \\ -36{,}644 & -183{,}218 & 170{,}781 & -126{,}669 & -14{,}137 & 56{,}549 \\ 183{,}218 & 610{,}726 & -126{,}669 & 1523{,}044 & -56{,}549 & 150{,}796 \\ 0{,}000 & 0{,}000 & -14{,}137 & -56{,}549 & 14{,}137 & -56{,}549 \\ 0{,}000 & 0{,}000 & 56{,}549 & 150{,}796 & -56{,}549 & 301{,}593 \end{bmatrix}$$

Loads at the node points

$F_1 =$ 0,000 MN

$M_1 =$ 0,000 MNm

$F_2 =$ 0,000 MN

$M_2 =$ 0,000 MNm

$F_3 =$ 0,500 MN

$M_3 =$ -2,192 MNm

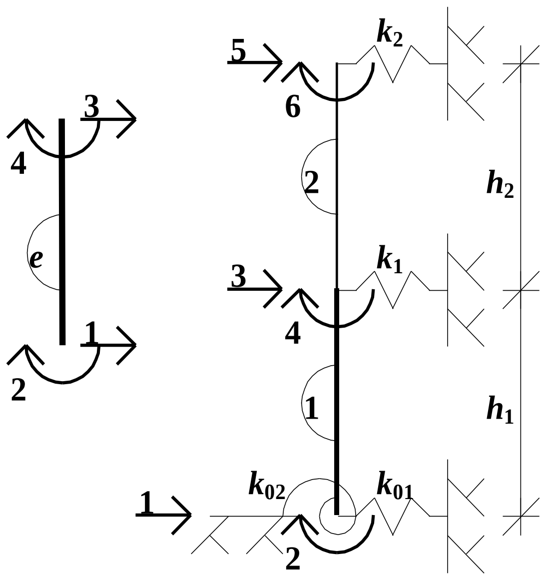


Figure 3.

Node displacements and rotations

$w_1 =$ 0,000 m

$\varphi_1 =$ 0,000 rad

$w_2 =$ 0,006 m

$\varphi_2 =$ 0,003 rad

$w_3 =$ 0,051 m

$\varphi_3 =$ 0,000 rad

ANSWER:

By finding such a value of M_3, that rotation

$$\varphi_3 = 0$$

the unknown displacement

$$a = w_3 \qquad (40)$$

$$= 0{,}051 \text{ m}$$

is obtained.

Extra Study

Force vector of element *i*

$$F_i^{\,e} = K_i^{\,e}\,\delta_i^{\,e} \tag{41}$$

1. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_1^e \cdot \begin{bmatrix} 0{,}000\ \text{m} \\ 0{,}000 \\ 0{,}006\ \text{m} \\ 0{,}003 \end{bmatrix} = \begin{bmatrix} 0{,}181\ \text{MN} \\ 0{,}000\ \text{MNm} \\ -0{,}181\ \text{MN} \\ 1{,}808\ \text{MNm} \end{bmatrix}$$

2. element

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = K_2^e \cdot \begin{bmatrix} 0{,}006\ \text{m} \\ 0{,}003 \\ 0{,}051\ \text{m} \\ 0{,}000 \end{bmatrix} = \begin{bmatrix} -0{,}500\ \text{MN} \\ -1{,}808\ \text{MNm} \\ 0{,}500\ \text{MN} \\ -2{,}192\ \text{MNm} \end{bmatrix}$$

The shear force and bending moment distribution, respectively, and the deflections at the node points are shown in Table 1. The corresponding curves are shown in Figures 5, 6 and 7, respectively. Positive directions are shown in Figure 4.

Table 1.

x_i [m]	Q_i [MN]	M_i [MNm]	w_i [m]
0,000	-0,181	0,000	0,000
10,000	-0,181	-1,808	0,006
10,000	0,500	-1,808	0,006
18,000	0,500	2,192	0,051
18,000	0	0	

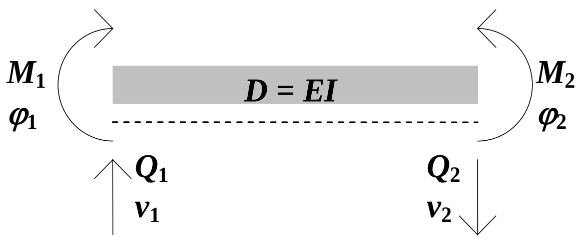


Figure 4. Positive directions (compare to Figure 2).

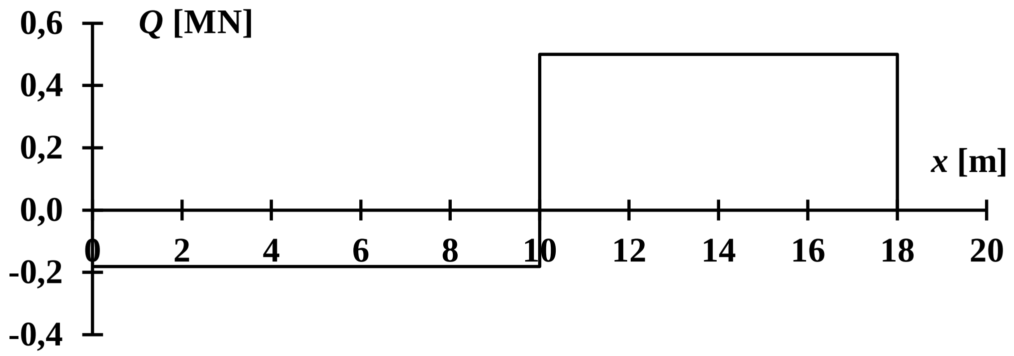


Figure 5. Shear force distribution curve.

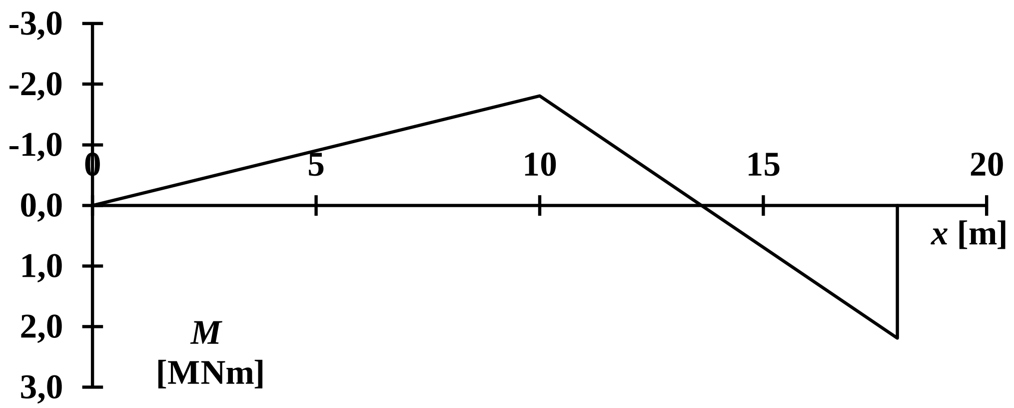


Figure 6. Bending moment distribution curve.

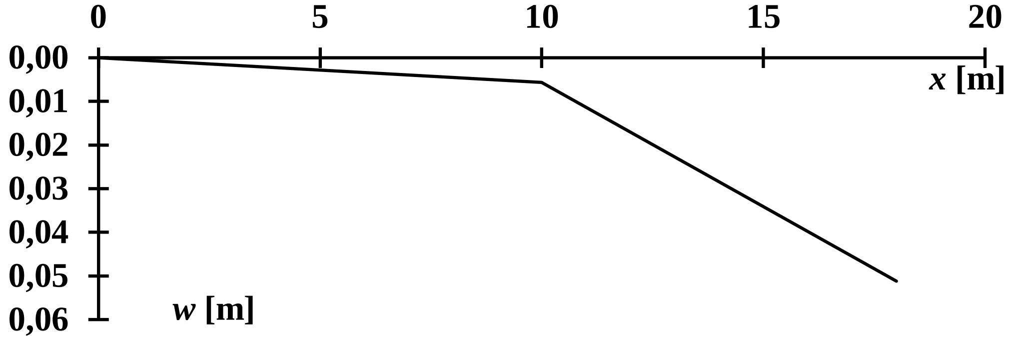


Figure 7. Deflection curve.

37. PROBLEM

The base slab shown in Figure 1 is partly supported on non-yielding rock (support B) and partly on continuous (cases A and B) or non-continuous (case C) elastic (Winkler) foundation, whose foundation coefficient is

$$c = 10 \text{ MN/m}^3$$

Length of the base slab

$$L = 6 \text{ m}$$

A) Determine the area (distance e) so that load

$$F = 1 \text{ MN/m}$$

does not cause any tension at support B!

B) Determine the displacement of the other edge of the base slab (point A) when load F is located so that the reaction force at support B is equal to zero?

C) If the foundation is approximated by

$$n = 7$$

equal springs located as shown in Figure 1, how big are the spring forces when load F is located so that the reaction force at support B is equal to zero?

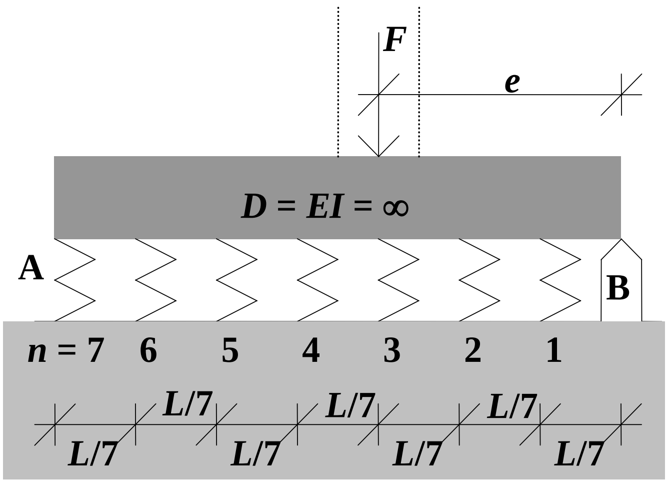


Figure 1.

A)

Stress (Figure 2)

$$\sigma(y) = \frac{F}{A} + \frac{M}{I_z} y \tag{1}$$

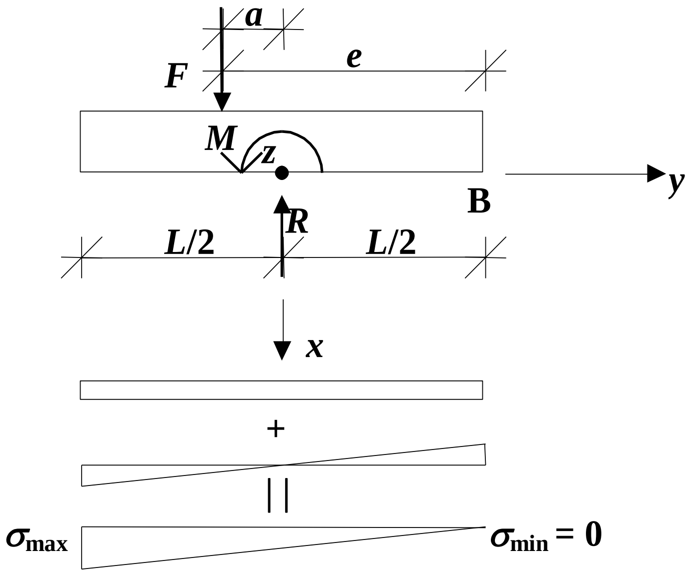


Figure 2.

Base area

$$A = bL \tag{2}$$

where b is the width of the base slab.

Moment of inertia with respect to z-axis

$$I_z = \frac{bL^3}{12} \tag{3}$$

Stress at support B

$$\sigma_B = \frac{F}{A} - \frac{aF}{I_z}\frac{L}{2} = 0 \tag{4}$$

$$\Rightarrow \quad 0 = \frac{F}{bL}\left(1 - \frac{6a}{L}\right) \tag{5}$$

$$\Rightarrow a = \frac{L}{6} \tag{6}$$

a = 1,000 m

Distance

$$e \le \frac{L}{2} + a \tag{7}$$

$\le$ 4,000 m

Stress at support B as function of load location e is shown in Figure 3.

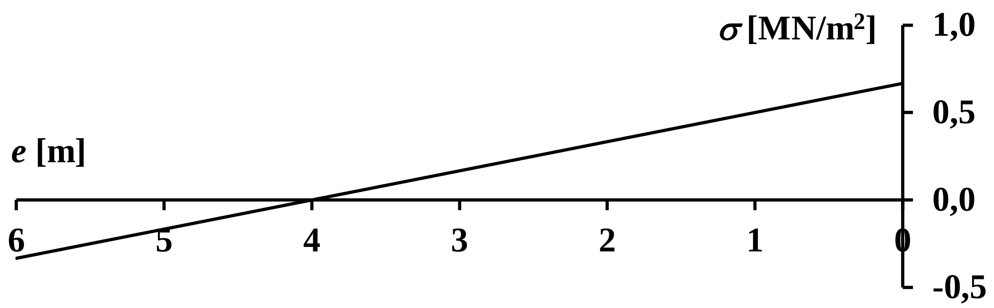


Figure 3.

<u>ANSWER</u>:

Distance

$e \le$ 4,000 m

B)

Pressure under the base slab is the product of foundation coefficient c and deflection v

$$p = cv \tag{8}$$

Resultant of the pressure (Figure 4)

$$R = \frac{1}{2} pL \tag{9}$$

$$\Rightarrow R = \frac{1}{2} cvL \tag{10}$$

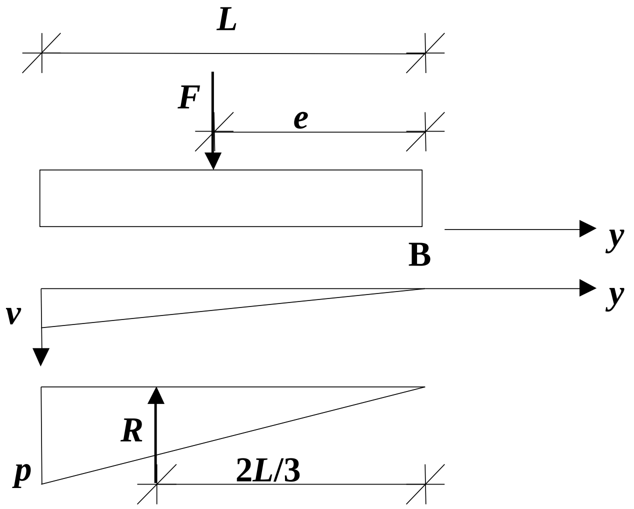


Figure 4.

Bending moment equilibrium condition with respect to support B

$$M_B = 0 \tag{11}$$

$$\Rightarrow \quad 0 = eF - \frac{2}{3} LR \tag{12}$$

$$\Rightarrow \quad 0 = eF - \frac{2}{3} L \frac{1}{2} cvL \tag{13}$$

$$\Rightarrow v(e) = \frac{3F}{cL^2} e \tag{14}$$

$$= \quad 0{,}008 \; e$$

Deflection of the left edge as function of load location e is shown in Figure 5.

When

$$e = \quad 4{,}000 \text{ m}$$

displacement is

$$v = \quad 0{,}033 \text{ m}$$

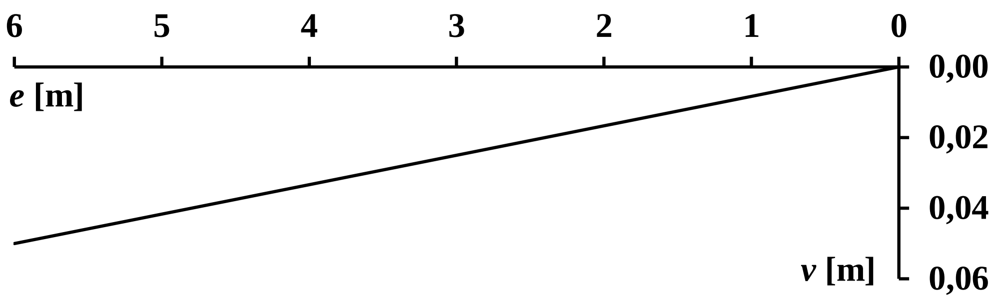


Figure 5.

<u>ANSWER</u>:

Deflection of the left edge

$$v(e) = 0{,}008\ e$$

C)

If spring force at the last spring is T_n, then the force at spring i is (Figure 6)

$$T_i = \frac{i}{n} T_n \qquad (15)$$

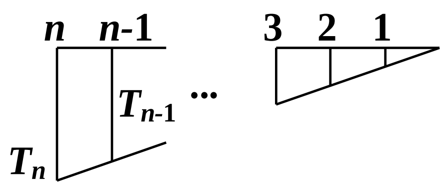


Figure 6.

Equilibrium condition

$$\sum_{i=1}^{n} \frac{i}{n} T_n = F \tag{16}$$

$$\Rightarrow \qquad T_n = \frac{nF}{\sum_{i=1}^{n} i} \tag{17}$$

Force at spring *i*

$$T_i = \frac{iF}{\sum_{i=1}^{n} i} \tag{18}$$

<u>ANSWER</u>:

Spring forces are shown in Figure 7.

F_1	=	0,036	MN/m
F_2	=	0,071	MN/m
F_3	=	0,107	MN/m
F_4	=	0,143	MN/m
F_5	=	0,179	MN/m
F_6	=	0,214	MN/m
F_7	=	0,250	MN/m

Checking

ΣF_i	=	1,000	MN/m

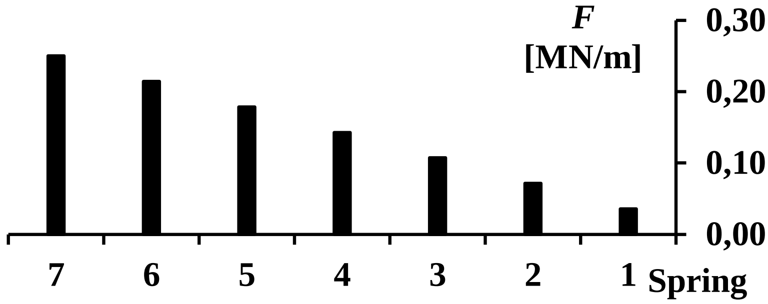


Figure 7.

38. PROBLEM

A concrete slab shown in Figure 1, height h_1 and strength of material K

$h_1 =$ 0,100 m

$K =$ 30 MN/m^2

is founded on a polystyrene layer, height h_2 and modulus of elasticity E_2

$h_2 =$ 0,100 m

$E_2 =$ 15 MN/m^2

on a gravel layer, height h_3 and modulus of elasticity E_3

$h_3 =$ 0,200 m

$E_3 =$ 40 MN/m^2

and on earth, foundation coefficient

$c_4 =$ 5 MN/m^3

Determine the deflection curve and the distribution curves of bottom pressure, bending moment and shear force, respectively, of the floor!
To solve the problem, divide the structure by using five node points as shown in Fig. 1 and suppose, that the deflection curve is linear between two adjacent nodes.
Solve the bending moments at the nodes by using two different ways:

1) **differential equation of bending and**
2) **concentrating the pressure under the structure at the nodes!**

Dimension

$a =$ 2,500 m

Loading per length

$F_1 =$ 0,050 MN/m

$F_2 =$ 0,025 MN/m

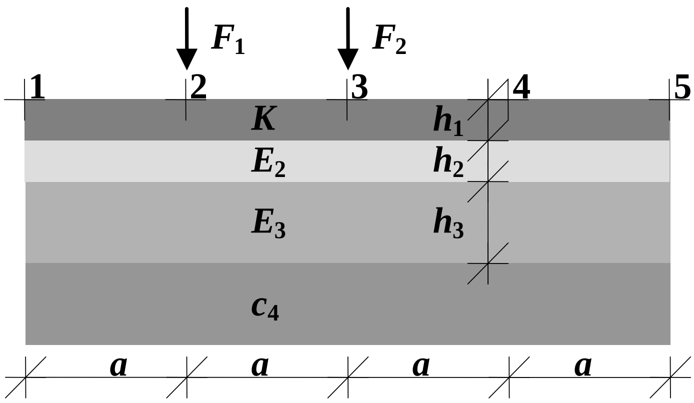


Figure 1.

Resultant of the bottom pressure at node point R_i is obtained from the respective areas of the bottom pressure distribution curve shown in Figure 2.

$$R_i = \begin{cases} \sigma_i \dfrac{dx}{2} + \dfrac{1}{2}\dfrac{\sigma_{i+1} - \sigma_i}{2}\dfrac{dx}{2}, & i = 1 \\ \sigma_i \dfrac{dx}{2} - \dfrac{1}{2}\dfrac{\sigma_i - \sigma_{i-1}}{2}\dfrac{dx}{2} + \sigma_i \dfrac{dx}{2} + \dfrac{1}{2}\dfrac{\sigma_{i+1} - \sigma_i}{2}\dfrac{dx}{2}, & \begin{cases} i \in \{2 \ldots n-1\} \\ i \in N \end{cases} \\ \sigma_i \dfrac{dx}{2} - \dfrac{1}{2}\dfrac{\sigma_i - \sigma_{i-1}}{2}\dfrac{dx}{2}, & i = n \end{cases}$$

(1)

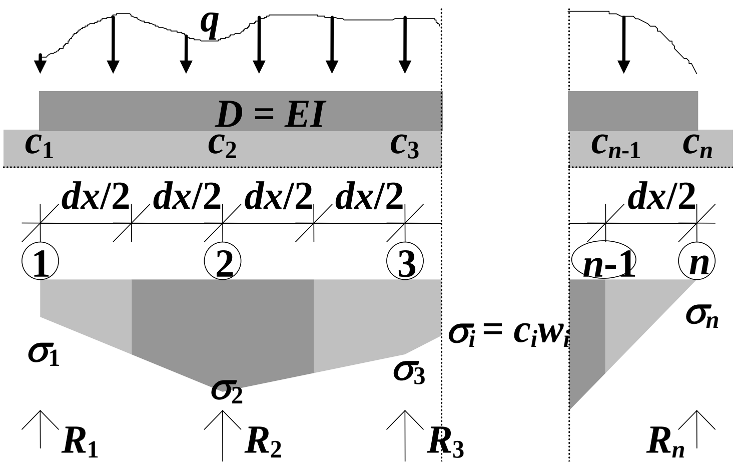


Figure 2.

Because pressure σ_i is a product of foundation coefficient c_i and deflection w_i, then

$$\sigma_i = c_i w_i \qquad (2)$$

Thus the reaction force at node i is

$$R_i = \begin{cases} \dfrac{cdx}{8}(3w_i + w_{i+1}), & i = 1 \\ \dfrac{cdx}{8}(w_{i-1} + 6w_i + w_{i+1}), & i \in \{2 \ldots n-1\}, \quad i \in N \\ \dfrac{cdx}{8}(w_{i-1} + 3w_i), & i = n \end{cases} \qquad (3)$$

when foundation coefficient c is uniform.

Reaction forces (Figure 3)

$$R_1 = \frac{ca}{8}(3w_1 + w_2) \tag{4}$$

$$R_2 = \frac{ca}{8}(w_1 + 6w_2 + w_3) \tag{5}$$

$$R_3 = \frac{ca}{8}(w_2 + 6w_3 + w_4) \tag{6}$$

$$R_4 = \frac{ca}{8}(w_3 + 6w_4 + w_5) \tag{7}$$

$$R_5 = \frac{ca}{8}(w_4 + 3w_5) \tag{8}$$

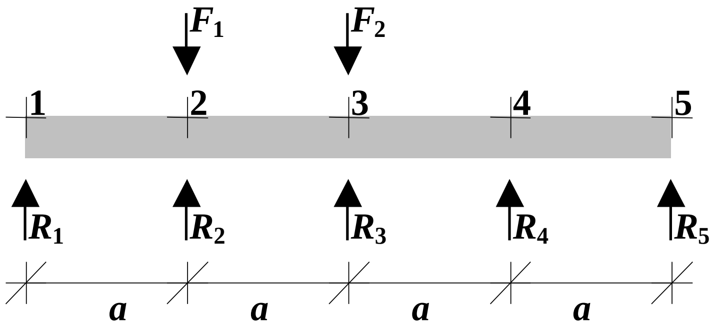


Figure 3.

Derivation of deflection between point *i* and its adjacent points

$$w'_{i-1/2} = \frac{w_i - w_{i-1}}{dx} \tag{9}$$

$$w'_{i+1/2} = \frac{w_{i+1} - w_i}{dx} \tag{10}$$

Second derivation of deflection

$$w''_i = \frac{w'_{i+1/2} - w'_{i-1/2}}{dx} \tag{11}$$

Bending moment

$$M_i = -EI_i w_i'' \tag{12}$$

$$\Rightarrow M_i = -EI_i \frac{w'_{i+1/2} - w'_{i-1/2}}{dx} \tag{13}$$

$$\Rightarrow M_i = -EI_i \frac{w_{i+1} - w_i - w_i + w_{i-1}}{dx^2} \tag{14}$$

$$\Rightarrow M_i = \frac{EI_i}{dx^2}\left(-w_{i-1} + 2w_i - w_{i+1}\right) \tag{15}$$

Bending moment at point 2, 3 and 4, respectively,

$$M_2 = R_1 a = -\frac{EI}{a^2}(w_1 - 2w_2 + w_3) \tag{16}$$

$$M_3 = 2R_5 a + R_4 a = -\frac{EI}{a^2}(w_2 - 2w_3 + w_4) \tag{17}$$

$$M_4 = R_5 a = -\frac{EI}{a^2}(w_3 - 2w_4 + w_5) \tag{18}$$

Equilibrium condition of vertical forces (Figure 3)

$$F_1 + F_2 = R_1 + R_2 + R_3 + R_4 + R_5 \tag{19}$$

Equilibrium condition of moment with respect to point 3

$$2aR_1 + aR_2 = aR_4 + 2aR_5 + aF_1 \tag{20}$$

System of equilibrium conditions (from Equations 16…20)

$$\begin{cases}
\frac{ca^2}{8}(3w_1 + w_2) = -\frac{EI}{a^2}w_1 + 2\frac{EI}{a^2}w_2 - \frac{EI}{a^2}w_3 \\
2\frac{ca^2}{8}(w_4 + 3w_5) + \frac{ca^2}{8}(w_3 + 6w_4 + w_5) = -\frac{EI}{a^2}w_2 + 2\frac{EI}{a^2}w_3 - \frac{EI}{a^2}w_4 \\
\frac{ca^2}{8}(w_4 + 3w_5) = -\frac{EI}{a^2}w_3 + 2\frac{EI}{a^2}w_4 - \frac{EI}{a^2}w_5 \\
F_1 + F_2 = \frac{ca}{8}\left(3w_1 + w_2 + w_1 + 6w_2 + w_3 + w_2 + 6w_3 + w_4 + \right. \\
\qquad \left. + w_3 + 6w_4 + w_5 + w_4 + 3w_5\right) \\
0 = 2\frac{ca^2}{8}(3w_1 + w_2) + \frac{ca^2}{8}(w_1 + 6w_2 + w_3) - \frac{ca^2}{8}(w_3 + 6w_4 + w_5) + \\
\qquad - 2\frac{ca^2}{8}(w_4 + 3w_5) - aF_1
\end{cases} \tag{21a…e}$$

$$\Rightarrow \begin{cases} \left(3\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_1+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_2+\dfrac{EI}{a^2}w_3 = 0 \\ \dfrac{EI}{a^2}w_2+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_3+\left(8\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_4+7\dfrac{ca^2}{8}w_5 = 0 \\ \dfrac{EI}{a^2}w_3+\left(\dfrac{ca^2}{8}-2\dfrac{EI}{a^2}\right)w_4+\left(3\dfrac{ca^2}{8}+\dfrac{EI}{a^2}\right)w_5 = 0 \\ 4\dfrac{ca}{8}w_1+8\dfrac{ca}{8}w_2+8\dfrac{ca}{8}w_3+8\dfrac{ca}{8}w_4+4\dfrac{ca}{8}w_5 = F_1+F_2 \\ 7\dfrac{ca}{8}w_1+8\dfrac{ca}{8}w_2-8\dfrac{ca}{8}w_4-7\dfrac{ca}{8}w_5 = F_1 \end{cases} \quad \text{(22a…e)}$$

Moment of inertia of the slab per length

$$I = h_1^3/12 \quad (23)$$
$$= 0{,}000083 \text{ m}^4/\text{m}$$

Modulus of elasticity of concrete

$$E = k\sqrt{KK_0} \quad (24)$$

where k = 1 and K_0 = 25·10^6 MN/m^2.

$$E = 27\,386 \text{ MN/m}^2$$

Bending stiffness

$$D = EI \quad (25)$$
$$= 2{,}282 \text{ MNm}^2$$

Foundation coefficient (Figure 4; Finnish concrete floor standard BY 31, p. 39)

$$c = \frac{1}{\dfrac{h_2}{E_2}+\dfrac{h_3}{E_3}+\dfrac{1}{c_4}} \quad (26)$$
$$= 4{,}724 \text{ MN/m}^3$$

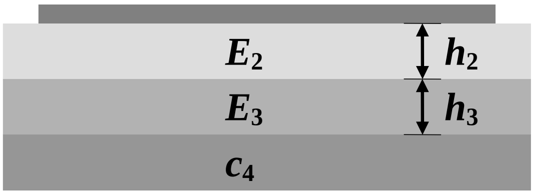


Figure 4.

Matrix

$$[K] = \begin{bmatrix} 11{,}438 & 2{,}961 & 0{,}365 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}365 & 2{,}961 & 29{,}893 & 25{,}837 \\ 0{,}000 & 0{,}000 & 0{,}365 & 2{,}961 & 11{,}438 \\ 5{,}906 & 11{,}811 & 11{,}811 & 11{,}811 & 5{,}906 \\ 10{,}335 & 11{,}811 & 0{,}000 & -11{,}811 & -10{,}335 \end{bmatrix}$$

Inverse matrix

$$[K]^{-1} = \begin{bmatrix} 0{,}112 & -0{,}010 & -0{,}001 & -0{,}001 & -0{,}027 \\ -0{,}102 & 0{,}051 & -0{,}003 & -0{,}009 & 0{,}118 \\ 0{,}049 & -0{,}091 & 0{,}049 & 0{,}104 & -0{,}114 \\ -0{,}003 & 0{,}051 & -0{,}102 & -0{,}009 & 0{,}009 \\ -0{,}001 & -0{,}010 & 0{,}112 & -0{,}001 & 0{,}001 \end{bmatrix}$$

Elements of force vector (sub-indexes refer to Equations 22)

F_a = 0,000 MNm/m
F_b = 0,000 MNm/m
F_c = 0,000 MNm/m
F_d = 0,075 MN/m
F_e = 0,050 MN/m

System of equilibrium conditions

$$\{w\} = [K]^{-1}\{f\} \tag{27}$$

Elements of displacement vector

$$w_1 = -0{,}001413 \text{ m}$$
$$w_2 = 0{,}005194 \text{ m}$$
$$w_3 = 0{,}002139 \text{ m}$$
$$w_4 = -0{,}000279 \text{ m}$$
$$w_5 = 0{,}000004 \text{ m}$$

Shear force

$$Q_i = \frac{M_{i+1} - M_i}{a} \tag{28}$$

<u>ANSWER</u>:

Deflections, bottom pressures, bending moments and shear forces, respectively, at the node points are shown in Table 1 and the corresponding curves in Figures 5…8, respectively.

Table 1.

x_i [m]	w_i [m]	σ_i (2) [MN/m^2]	R_i (3) [MN]	M_i (15) [MNm]	Q_i (28) [MN]
0,000	-0,00141	-0,00667	0,00141	0,00000	0,00141
2,500	0,00519	0,02454	0,04708	0,00353	0,00141
2,500	0,00519	0,02454	0,04708	0,00353	-0,00150
5,000	0,00214	0,01010	0,02620	-0,00023	-0,00150
5,000	0,00214	0,01010	0,02620	-0,00023	-0,00030
7,500	-0,00028	-0,00132	0,00070	-0,00099	-0,00030
7,500	-0,00028	-0,00132	0,00070	-0,00099	0,00039
10,000	0,00000	0,00002	-0,00039	0,00000	0,00039
10,000		0		0	0

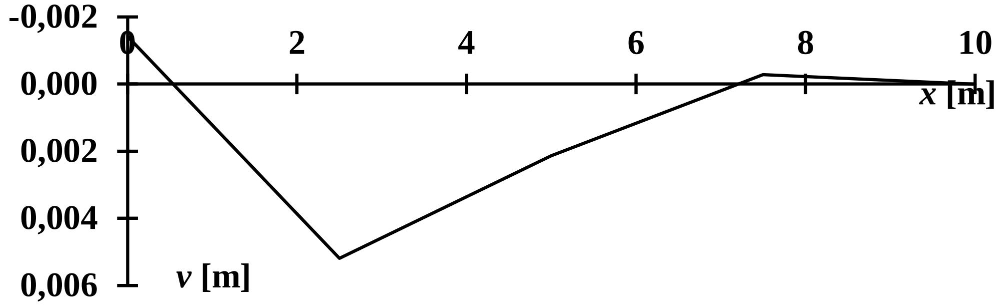


Figure 5. Deflection curve.

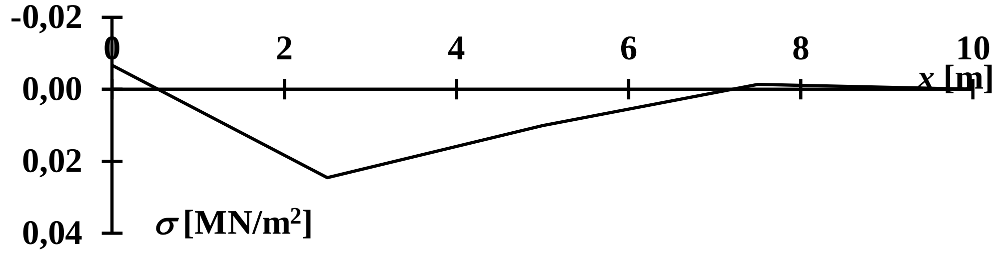


Figure 6. Bottom pressure distribution curve.

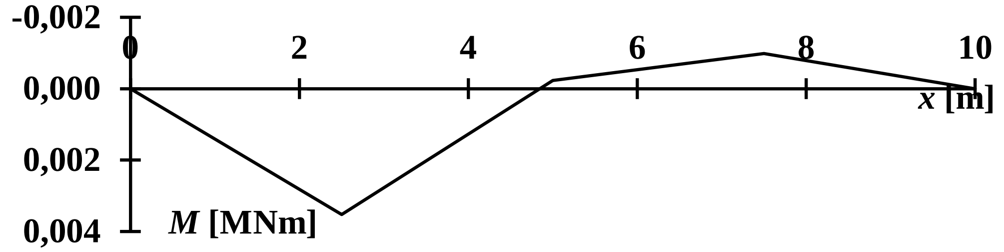


Figure 7. Bending moment distribution curve.

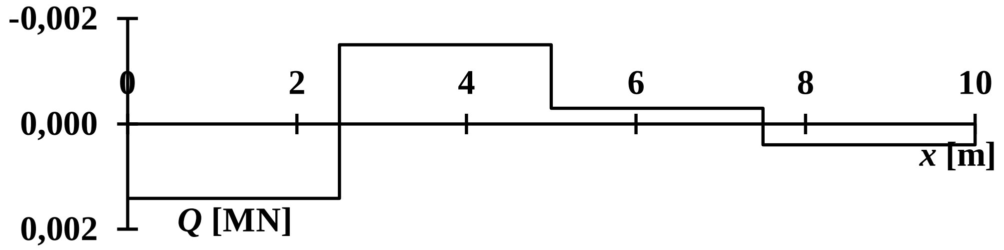


Figure 8. Shear force distribution curve.

39. PROBLEM

Due to temperature variation, the top of a foundation wall shown in Figure 1 is moving laterally

δ = 0,020 m

Determine the bottom pressure as function of the foundation coefficient, when the latter one varies between

c_{min} = 10 MN/m^3

and

c_{max} = 300 MN/m^3

Determine the bottom pressure also in the case when

c_{∞} = ∞ MN/m^3

Vertical force

F_x = 0,300 MN/m

Modulus of elasticity of the wall

E = 15 000 MN/m^2

Dimensions

d = 0,500 m

h = 3,000 m

b = 1,000 m

t = 0,200 m

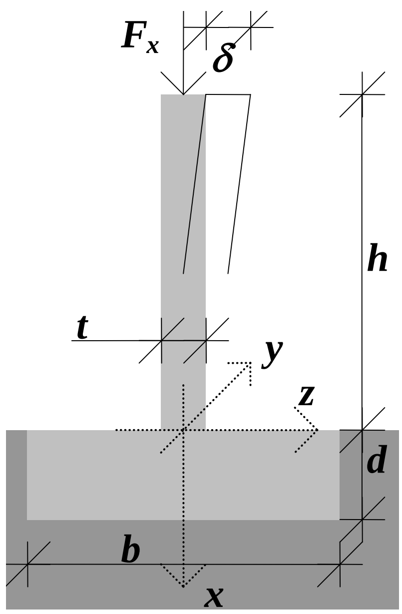


Figure 1.

Resultant of bottom pressure (Figure 2)

$$R = \int \sigma(z) dz \tag{1}$$

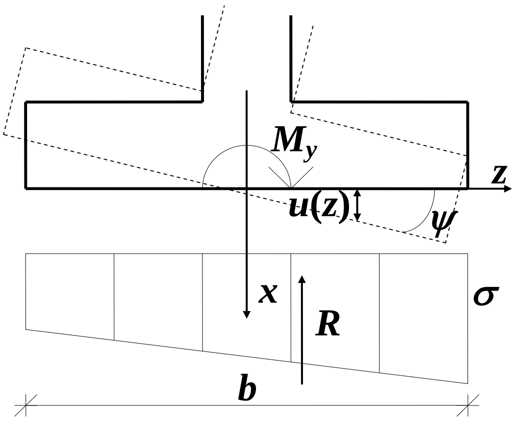


Figure 2.

Bottom pressure

$$\sigma(z) = cu(z) \tag{2}$$

Deflection

$$u(z) = \psi z \tag{3}$$

where ψ is the rotation angle.

Moment with respect to y-axis

$$M_y = \int \sigma(z) z dz \tag{4}$$

$$= c\psi \int z^2 dz \tag{5}$$

Moment of inertia of the bottom cross-section (l) with respect to y-axis

$$I_{y,l} = \int z^2 dz \tag{6}$$

Hence moment

$$M_y(c) = c\,\psi I_{y,l} \tag{7}$$

On the other hand, moment as function of virtual horizontal force H (Figure 3)

$$M_y = H(h+d) \tag{8}$$

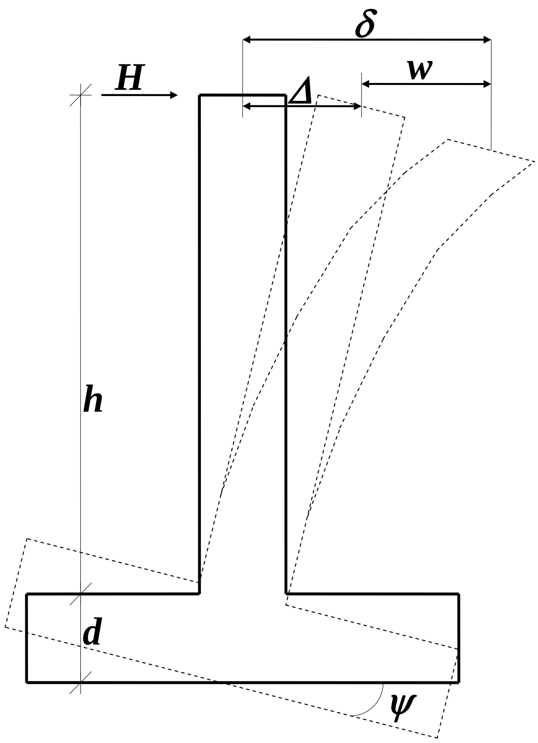


Figure 3.

From Equations 7 and 8

$$\psi = \frac{H(h+d)}{cI_{y,l}} \tag{9}$$

Deflection due to rotation angle at the top of the wall

$$\Delta = (h+d)\psi \tag{10}$$

$$= \frac{H(h+d)^2}{cI_{y,l}} \tag{11}$$

Deflection due to horizontal force at the top of the wall (m)

$$w = \frac{Hh^3}{3D_{y,m}} \tag{12}$$

Total deflection at the top of the wall

$$\delta(c) = \Delta + w \tag{13}$$

$$= \frac{H(h+d)^2}{cI_{y,l}} + \frac{Hh^3}{3D_{y,m}} \tag{14}$$

Horizontal force

$$H(c) = \frac{\delta}{\dfrac{(h+d)^2}{cI_{y,l}} + \dfrac{h^3}{3D_{y,m}}} \tag{15}$$

Bottom pressure

$$\sigma(z) = \frac{F_x}{A_l} + \frac{M_y}{I_{y,l}} z \tag{16}$$

$$= \frac{F_x}{A_l} + \frac{H(h+d)}{I_{y,l}} z \tag{17}$$

$$\Rightarrow \sigma(c,z) = \frac{F_x}{A_l} + \frac{\delta(h+d)}{I_{y,l}\left(\dfrac{(h+d)^2}{cI_{y,l}} + \dfrac{h^3}{3EI_{y,m}}\right)} z \tag{18}$$

$$= \frac{F_x}{A_l} + \frac{\delta(h+d)}{\dfrac{(h+d)^2}{c} + \dfrac{h^3 I_{y,l}}{3EI_{y,m}}} z \tag{19}$$

Bottom area per unit length

$$A_l = b \tag{20}$$

Moment of inertia of the base slab and the wall with respect to y-axis per unit length, respectively,

$$I_{y,l} = \frac{b^3}{12} \tag{21}$$

$$I_{y,m} = \frac{t^3}{12} \tag{22}$$

Bottom pressure

$$\sigma(c,z)=\frac{F_x}{b}+\frac{\delta(h+d)}{\frac{(h+d)^2}{c}+\frac{h^3b^3}{3Et^3}}z \tag{23}$$

When the foundation coefficient approaches infinity

$$\sigma(z)=\frac{F_x}{b}+\frac{3Et^3\delta(h+d)}{h^3b^3}z \tag{24}$$

<u>ANSWER</u>:

The extreme values of the bottom pressure are shown in Table 1 and the corresponding distribution curves are shown in Figure 4. Correspondingly, the stresses at the edges of the slab are shown in Figure 5.

Table 1. Extreme values of bottom pressure.

		σ_{min} $z = -b/2$ [MN/m^2]	σ_{max} $z = b/2$ [MN/m^2]
c_{min} =	10 MN/m^3	0,273	0,327
c_{max} =	300 MN/m^3	-0,002	0,602
c_{∞} =	∞ MN/m^3	-0,167	0,767

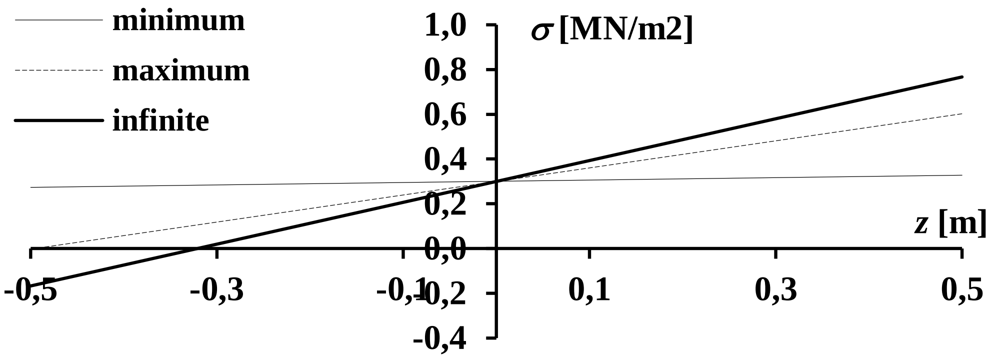


Figure 4. The extreme values of the bottom pressure.

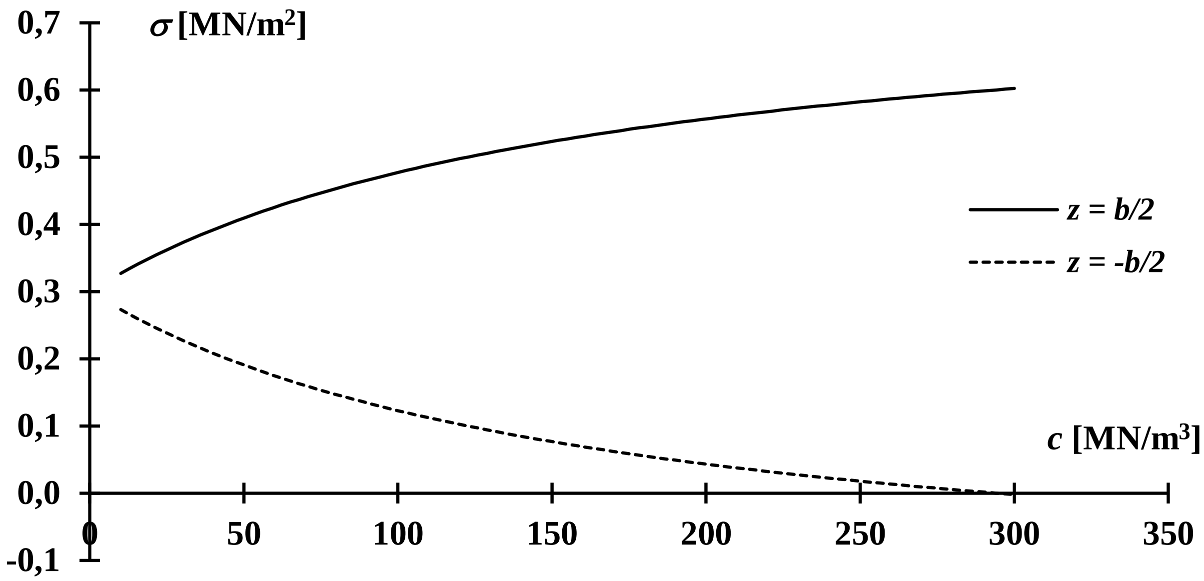


Figure 5. The stresses at the edges of the slab as function of foundation coefficient.

Rak-11.2107 Sillat ja Perustukset

KAAVAKOKOELMA

Rak-11.2107
Sillat ja perustukset
(Bridges and Foundation Structures)

KAAVAKOKOELMA
(FORMULARY)

SISÄLTÖ *(CONTENTS)*

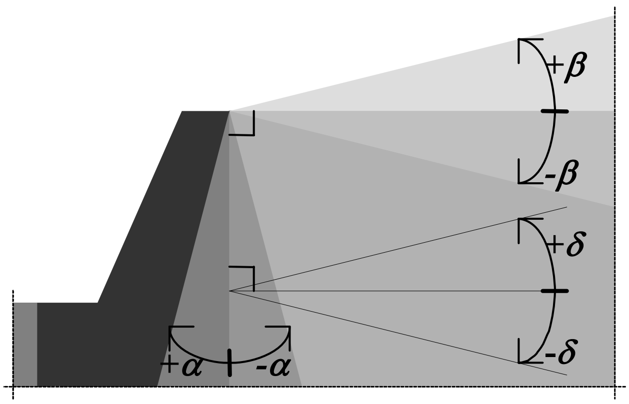
1 MAANPAINE *(EARTH PRESSURE)*

Aktiivisen maanpaineen P_a horisontaalikomponentin P_{ah} maanpaineluku on
(Coefficient of horizontal component P_{ah} of active earth pressure P_a is)

$$K_{ah} = \frac{\cos^2(\varphi + \alpha)}{\cos^2\alpha\left[1 + \sqrt{\frac{\sin(\varphi+\delta)\sin(\varphi-\beta)}{\cos(\alpha-\delta)\cos(\alpha+\beta)}}\right]^2} \tag{1}$$

jossa kuvan 1 merkkisäännöin
(where by using directions shown in Figure 1)

- α on liukupinnan kaltevuuskulma pystytasoon nähden
  (α is the angle of sliding surface with respect to vertical plane),
- β on maanpinnan kaltevuuskulma vaakatasoon nähden
  (β is the angle of earth surface with respect to horizontal plane),
- δ on seinäkitkakulma, kaavat 2 ja 3, ja
  (δ is angle of wall friction, Equations 2 and 3, respectively, and)
- φ on maan sisäinen kitkakulma
  (φ is angle of internal friction).



Kuva 1. Kulmat ja niiden merkit.
*(**Figure 1.** Angles and its directions.)*

Seinäkitkakulma on (*Angle of wall friction is*)

$$\delta = \varphi \tag{2}$$

kun liukupinta ei ole rakenteen ja maan välissä
(when the sliding surface is not between the structure and soil),

$$\delta = \frac{2}{3}\varphi \tag{3}$$

kun liukupinta on teräsrakenteen ja maan välissä, sekä
(when the sliding surface is between the steel structure and soil, and)

$$\delta = \frac{3}{4}\varphi \tag{4}$$

kun liukupinta on betonirakenteen ja maan välissä
(when the sliding surface is between the concrete structure and soil).

Kun (*When*)

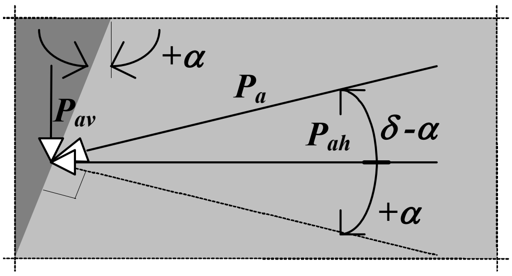
$$\left.\begin{matrix}\alpha \\ \beta \\ \delta\end{matrix}\right\} = \mathbf{0} \tag{5a, b, c}$$

aktiivisen maanpaineen maanpaineluku on
(the coefficient of active earth pressure is)

$$K_a = \tan^2\left(45° - \frac{\varphi}{2}\right) \tag{6}$$

Vertikaalikomponentti P_{av} kuvan 2 mukaan on
(Vertical component P_{av} as shown in Figure 2 is)

$$P_{av} = P_{ah}\tan(\delta - \alpha) \tag{7}$$



Kuva 2. Aktiivinen maanpaine ja sen komponentit.
*(**Figure 2.** Active earth pressure and its components.)*

Lepopaineen maanpaineluku *(Coefficient of earth pressure at rest)*

$$K_{o\beta} = K_o(1 + \sin\beta) \tag{8}$$

jossa *(where)*

$$K_o = 1 - \sin\varphi \tag{9}$$

2 KANTAVUUS *(BEARING CAPACITY)*

2.1 Eurocode EN 1997-1

Kantokyvyn laskenta-arvo avoimissa olosuhteissa
(Design value of bearing capacity in the drained conditions)

$$\frac{R}{A'} = c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{10}$$

jossa *(where)*
- A' on peruslaatan tehokas mitoitusala, kaava 11
  (A' is design effective foundation area, Equation 11),
- c' on tehokkaan koheesion laskenta-arvo
  (c' is design value of cohesion intercept in terms of effective stress),
- q' on maan tehokas mitoituspaine perustamistasolla, kuva 3
  (q' is design effective overburden pressure at the level of the foundation area, Figure 3),
- γ' on perustamistason alapuolinen tehokas tilavuuspaino
  (γ' is design effective unit weight of soil under the foundation area),
- N_c, N_q ja N_γ ovat kantavuuskertoimet, kaavat 12, 13 ja 14
  (N_c, N_q and N_γ are factors of bearing capacity, Equations 12, 13 and 14, respectively),
- b_c, b_q ja b_γ ovat pohjakaltevuuden vaikutuskertoimet, kaavat 16 ja 17
  (b_c, b_q and b_γ are factors for the inclination of the base, Equations 16 and 17, respectively),
- s_c, s_q ja s_γ ovat peruslaatan muodon vaikutuskertoimet, kaavat 18, 19 ja 20 ja
  (s_c, s_q and s_γ are shape factors of the base slab, Equations 18, 19 and 20, respectively, and)
- i_c, i_q ja i_γ ovat kuormitusresultantin kaltevuuden vaikutuskertoimet, kaavat 21, 22 ja 23
  (i_c, i_q and i_γ are inclination factors of the load resultant, Equations 21, 22 and 23, respectively).

Peruslaatan tehokas mitoitusala, kuva 3, on
(The design effective foundation area, Figure 3, is)

$$A' = B' L' \tag{11}$$

jossa *(where)*
- B' on peruslaatan tehokkaan alan pienempi sivumitta ja
  (B' is effective foundation width and)
- L' on peruslaatan tehokkaan alan suurempi sivumitta
  (L' is effective foundation length).

Kantavuuskertoimet ovat *(Factors of the bearing capacity are)*

$$N_c = (N_q - 1)\cot\varphi' \tag{12}$$

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'} \tag{13}$$

$$N_\gamma = 2(N_q - 1)\tan\varphi' \tag{14}$$

jossa φ' on kitkakulman laskenta-arvo. N_γ on voimassa karhealle pohjalle, jossa seinäkitkakulma on
(where φ' is the design value of angle of the internal friction. N_γ is valid for rough base where angle of wall friction is)

$$\delta \geq \frac{\varphi'}{2} \tag{15}$$

Pohjakaltevuuden vaikutuskertoimet ovat *(Factors for the inclination of the base are)*

$$b_c = b_q - \frac{1 - b_q}{N_c \tan\varphi'} \tag{16}$$

$$\left.\begin{array}{l} b_q \\ b_\gamma \end{array}\right\} = (1 - \alpha \tan\varphi')^2 \tag{17a, b}$$

jossa α on perustuksen pohjan kaltevuus vaakatasosta, kuva 3
(where α is the inclination of the foundation base to the horizontal, Figure 3).

Peruslaatan muodon vaikutuskertoimet ovat *(Shape factors of the base slab are)*

$$s_c = \frac{S_q N_q - 1}{N_q - 1} \tag{18}$$

$$s_q = \begin{cases} 1 + \dfrac{B'}{L'}\sin\varphi' \\ 1 + \sin\varphi' \end{cases} \tag{19a, b}$$

$$s_\gamma = \begin{cases} 1 - 0{,}3\dfrac{B'}{L'} \\ 0{,}7 \end{cases} \tag{20a, b}$$

jossa kaava (a) on voimassa suorakaiteelle ja (b) neliölle ja ympyrälle
(where Equation (a) is valid for a rectangular and (b) for a square or circular shape).

Kuormitusresultantin kaltevuuden vaikutuskertoimet ovat
(Inclination factors of the load resultant are)

$$i_c = i_q - \frac{1 - i_q}{N_c \tan\varphi'} \tag{21}$$

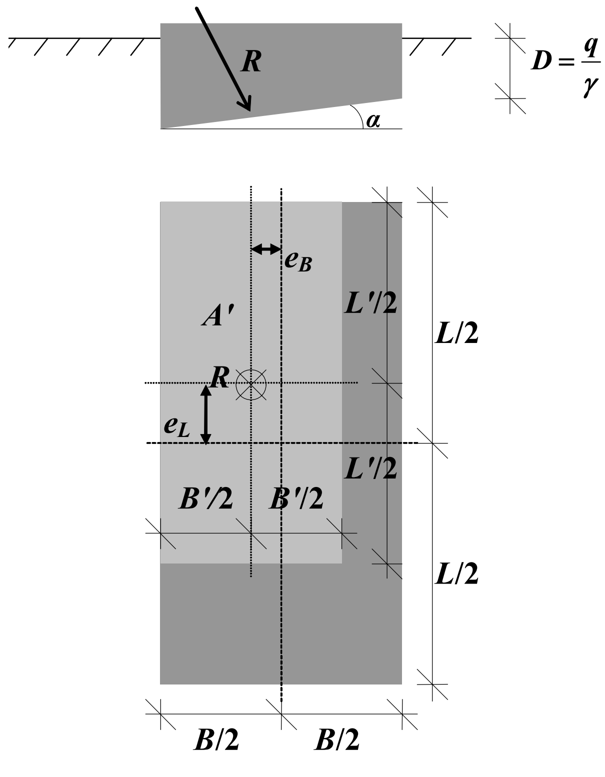
$$i_q = \left(1 - \frac{H}{V + A'c'\cot\varphi'}\right)^m \tag{22}$$

$$i_\gamma = \left(1 - \frac{H}{V + A'c'\cot\varphi'}\right)^{m+1} \tag{23}$$

jossa *(where)*

$$m = \begin{cases} m_B = \dfrac{2 + \dfrac{B'}{L'}}{1 + \dfrac{B'}{L'}} \\ m_\theta = m_L \cos^2\theta + m_B \sin^2\theta \\ m_L = \dfrac{2 + \dfrac{L'}{B'}}{1 + \dfrac{L'}{B'}} \end{cases} \tag{24a, b, c}$$

jossa kaava (a) on voimassa, kun $\boldsymbol{H}$ vaikuttaa $\boldsymbol{B'}$:n suunnassa, (c) on voimassa, kun $\boldsymbol{H}$ vaikuttaa $\boldsymbol{L'}$:n suunnassa ja (b) on voimassa, kun $\boldsymbol{H}$:n vaikutussuunta muodostaa $\boldsymbol{\theta}$-kulman $\boldsymbol{L'}$:n suunnan kanssa.
(where Equation (a) is valid when $\boldsymbol{H}$ *acts in the direction on* $\boldsymbol{B'}$*, (c) is valid when* $\boldsymbol{H}$ *acts in the direction of* $\boldsymbol{L'}$ *and (b) is valid when* $\boldsymbol{H}$ *acts in a direction forming an angle* $\boldsymbol{\theta}$ *with the direction* $\boldsymbol{L'}$*).*



Kuva 3. Merkinnät.
(Figure 3. *Notations.)*

2.2 RIL 121-2004, Pohjarakennusohjeet *(Finnish Standard)*

Kantokyvyn laskenta-arvo *(Design value of bearing capacity)*

$$q_{md} = c_d N_c s_c i_c + \gamma_1' D N_D s_D i_D + \frac{1}{2}\gamma_2' B N_B s_B i_B \tag{25}$$

jossa *(where)*

- c_d on koheesion laskenta-arvo
  (c_d is design value of cohesion),
- D on peruslaatan pienin perustamissyvyys
  (D is the minimum foundation depth of the base slab),
- B on peruslaatan pienempi sivumitta
  (B is the length of the shorter side of base slab),
- N_c, N_D ja N_B ovat kantavuuskertoimet, kaavat 26, 27 ja 28, taulukko 1
  (N_c, N_D and N_B are factors of bearing capacity, Equations 26, 27 and 28, respectively, Table 1),
- s_c, s_D ja s_B ovat peruslaatan muodon vaikutuskertoimet, kaavat 29, 30 ja 31
  (s_c, s_D and s_B are shape factors of the base slab, Equations 29, 30 and 31, respectively),
- i_c, i_D ja i_B ovat kuormitusresultantin kaltevuuden vaikutuskertoimet, kaavat 32, 33 ja 34
  (i_c, i_D and i_B are inclination factors of the load resultant, Equations 32, 33 and 34, respectively),
- γ_1' on perustamistason yläpuolisen maan tehokas tilavuuspaino ja
  (γ_1' is effective unit weight of soil above the foundation level and)
- γ_2' on perustamistason alapuolisen maan tehokas tilavuuspaino
  (γ_2' is effective unit weight of soil under the foundation level).

Kantavuuskertoimet ovat *(Factors of the bearing capacity are)*

$$N_D = \tan^2\left(45° + \frac{\varphi_d}{2}\right) e^{\pi \tan\varphi_d} \tag{26}$$

$$N_c = (N_D - 1)\cot\varphi_d \tag{27}$$

$$N_B = \frac{3}{2}(N_D - 1)\tan\varphi_d \tag{28}$$

jossa φ_d on kitkakulman laskenta-arvo. Likiarvot ovat taulukossa 1.
(where φ_d is the design value of angle of the internal friction. Approximative values are given in Table 1.)

Peruslaatan muodon vaikutuskertoimet ovat *(Shape factors of the base slab are)*

$$s_c = 1 + \frac{1}{5}\frac{B}{L} \tag{29}$$

$$s_D = 1 + \frac{1}{5}\frac{B}{L} \tag{30}$$

$$s_B = 1 - \frac{2}{5}\frac{B}{L} \tag{31}$$

jossa L on peruslaatan suurempi sivumitta *(where L is the longer side length of the base slab).*

Taulukko 1. Kantavuuskertoimet sisäisen kitkakulman funktiona.
*(**Table 1.** Factors of the bearing capacity as function of the angle of internal friction.)*

φ_d [°]	N_c	N_D	N_B
0,0	5,1	1,0	0,0
2,5	5,8	1,3	0,0
5,0	6,5	1,6	0,1
7,5	7,3	2,0	0,2
10,0	8,3	2,5	0,4
12,5	9,5	3,1	0,7
15,0	11,0	3,9	1,2
17,5	12,7	5,0	1,9
20,0	14,8	6,4	2,9
22,5	17,5	8,2	4,5
25,0	20,7	10,7	6,8
27,5	24,8	13,9	10,1
30,0	30,1	18,4	15,1
32,5	37,0	24,6	22,5
35,0	46,1	33,3	33,9
37,5	58,4	45,8	51,6
40,0	75,3	64,2	79,5
42,5	99,2	91,9	124,9
45,0	133,9	134,9	200,8

Kuormitusresultantin kaltevuuden vaikutuskertoimet ovat
(Inclination factors of the load resultant are)

$$i_c = \left(1 - \frac{H_d}{V_d + Ac_d \cot\varphi_d}\right)^2 \tag{32}$$

$$i_D = \left(1 - \frac{H_d}{V_d + Ac_d \cot\varphi_d}\right)^2 \tag{33}$$

$$i_B = \left(1 - \frac{H_d}{V_d + Ac_d \cot\varphi_d}\right)^4 \tag{34}$$

jossa *(where)*

- H_d on vaakakuorman laskenta-arvo
  (H_d is design value of horizontal force),
- V_d on pystykuorman laskenta-arvo ja
  (V_d is design value of vertical force and)

- A on peruslaatan pohjapinta-ala
  (A is bottom area of the base slab).

Epäkeskisessä kuormituksessa, kuva 4, tehdään korvaus
(In the case of eccentric loading, Figure 4, replacement is done)

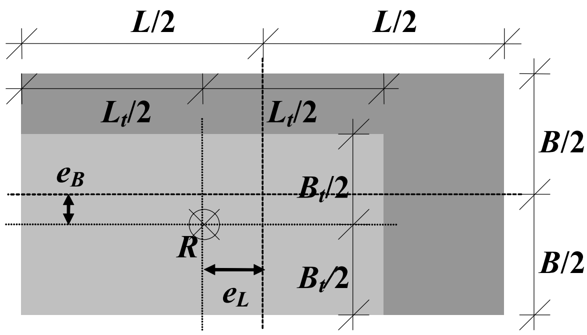
$$L \mapsto L_t = L - 2e_L \tag{35}$$

$$B \mapsto B_t = B - 2e_B \tag{36}$$

$$A \mapsto A_t = L_t B_t \tag{37}$$

jossa *(where)*

- e_L on epäkeskisyys pidemmän sivun suunnassa ja
  (e_L is eccentricity in the direction of the longer side and)
- e_B on epäkeskisyys lyhemmän sivun suunnassa
  (e_B is eccentricity in the direction of the shorter side).



Kuva 4. Peruslaatan toimiva alue epäkeskisessä kuormituksessa.
*(**Figure 4.** Effective area under eccentric loading.)*

3 PAALUTUS *(PILE FOUNDATION)*

Tasapainoyhtälöryhmä *(System of equilibrium conditions)*

$$\{F\} = [K]\{\delta\} \tag{38}$$

3.1 Avaruuspaalutus *(Pile Foundation in Space)*

Yhtälössä 38 voimavektori on *(In Equation 38, the force vector is)*

$$\{F\} = \{F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z\}^{\mathrm{T}} \tag{39}$$

siirtymävektori on *(displacement vector is)*

$$\{\delta\} = \{u \quad v \quad w \quad \omega \quad \varphi \quad \theta\}^{\mathrm{T}} \tag{40}$$

ja jäykkyysmatriisi on *(and stiffness matrix is)*

$$[K] = \sum_{i=1}^{n} [K_i] \tag{41}$$

jossa yhden paalun jäykkyysmatriisi on *(where stiffness matrix of one pile is)*

$$[K_i] = k_i \begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} & p_{x,i}r_{x,i} & p_{x,i}r_{y,i} & p_{x,i}r_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} & p_{y,i}r_{x,i} & p_{y,i}r_{y,i} & p_{y,i}r_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 & p_{z,i}r_{x,i} & p_{z,i}r_{y,i} & p_{z,i}r_{z,i} \\ r_{x,i}p_{x,i} & r_{x,i}p_{y,i} & r_{x,i}p_{z,i} & r_{x,i}^2 & r_{x,i}r_{y,i} & r_{x,i}r_{z,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{y,i} & r_{y,i}p_{z,i} & r_{y,i}r_{x,i} & r_{y,i}^2 & r_{y,i}r_{z,i} \\ r_{z,i}p_{x,i} & r_{z,i}p_{y,i} & r_{z,i}p_{z,i} & r_{z,i}r_{x,i} & r_{z,i}r_{y,i} & r_{z,i}^2 \end{bmatrix} \tag{42}$$

Tällöin n on paalujen lukumäärä ja paalun i jäykkyys on
(Here n is the number of piles. The stiffness of pile i is)

$$k_i = \frac{E_i A_i}{L_i} \tag{43}$$

jossa *(where)*

- E_i on paalun i kimmomoduuli
  (E_i is modulus of elasticity of pile i),
- A_i on paalun i poikkipinta-ala
  (A_i is cross-section area of pile i),
- L_i on paalun i pituus
  (L_i is length of pile i),

suuntakulmien kosinit, kuva 5, ovat *(cosines of directional angles, Figure 5, are)*

$$p_{x,i} = \cos(x_i', x) = \cos\alpha_i \tag{44}$$

$$p_{y,i} = \cos(x_i', y) = \cos\beta_i \tag{45}$$

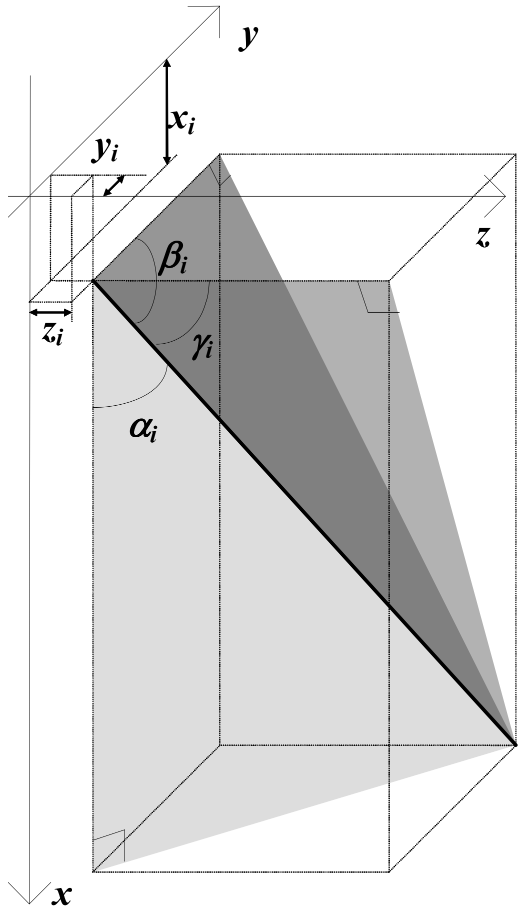
$$p_{z,i} = \cos(x_i', z) = \cos\gamma_i \tag{46}$$

ja momenttivarret ovat *(and lever arms are)*

$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} \tag{47}$$

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{48}$$

$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{49}$$



Kuva 5. Suuntakulmat avaruuskoordinaatistossa.
*(**Figure 5.** Directional angles in space coordinate system.)*

3.2 Tasopaalutus *(Pile Foundation in Plane)*

Yhtälössä 38 voimavektori on *(In Equation 38, the force vector is)*

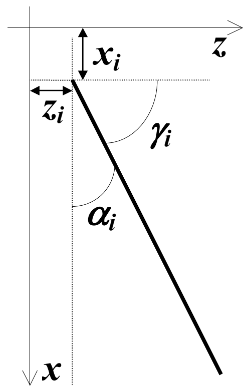
$$\{F\} = \{F_x \quad F_z \quad M_y\}^{\mathrm{T}} \tag{50}$$

siirtymävektori on *(displacement vector is)*

$$\{\delta\} = \{u \quad w \quad \varphi\}^{\mathrm{T}} \tag{51}$$

ja jäykkyysmatriisi **[*K*]** on määritelty kaavassa 41, jossa yhden paalun jäykkyysmatriisi on *(and stiffness matrix* **[*K*]** *is given in Equation 41, where stiffness matrix of one pile is)*

$$[K_i] = k_i \begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{z,i} & p_{x,i}r_{y,i} \\ p_{z,i}p_{x,i} & p_{z,i}^2 & p_{z,i}r_{y,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{z,i} & r_{y,i}^2 \end{bmatrix} \tag{52}$$



Kuva 6. Suuntakulmat tasokoordinaatistossa.
*(**Figure 6.** Directional angles in plane coordinate system.)*

Tällöin *(Here)*
- ***n*** on paalujen lukumäärä
  *(**n** is the number of piles),*
- k_i on paalun ***i*** jäykkyys, kaava 43,
  *(k_i is the stiffness of pile **i**, Equation 43,)*

suuntakulmien kosinit, kuva 6, ovat *(cosines of directional angles, Figure 6, are)*

$$p_{x,i} = \cos\alpha_i \tag{53}$$

$$p_{z,i} = \cos\gamma_i = \sin\alpha_i \tag{54}$$

ja momenttivarsi on *(and lever arm is)*

$$r_{y,i} = z_i\, p_{x,i} - x_i\, p_{z,i} \tag{55}$$

$$\Rightarrow r_{y,i} = z_i \cos\alpha_i - x_i \sin\alpha_i \tag{56}$$

Kiertokeskiö on *(Rotation center is)*

$$\begin{cases} x_0 = -\dfrac{k_{11}k_{23} - k_{13}k_{12}}{k_{11}k_{22} - (k_{12})^2} \\ z_0 = -\dfrac{k_{12}k_{23} - k_{13}k_{22}}{k_{11}k_{22} - (k_{12})^2} \end{cases} \tag{57a, b}$$

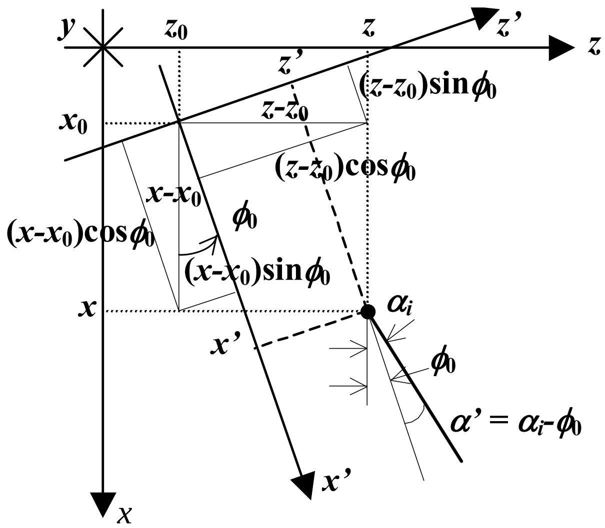
jossa k_{ij} on systeemin jäykkyysmatriisin alkio rivissä i ja sarakkeessa j
(where k_{ij} is stiffness matrix element at row i and column j).

Pääjäykkyyssuunnan kulma on *(Angle of principal direction is)*

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{11} - k_{22}}\right) \tag{58}$$

Pääjäykkyyskoordinaatiston arvot, kuva 7, ovat
(Principal direction coordinate system, Figure 7, is)

$$x' = (z - z_0)\sin\phi_0 + (x - x_0)\cos\phi_0 \tag{59}$$
$$z' = (z - z_0)\cos\phi_0 - (x - x_0)\sin\phi_0 \tag{60}$$
$$\alpha' = \alpha - \phi_0 \tag{61}$$



Kuva 7. Pääjäykkyyskoordinaatisto.
*(**Figure 7.** Principal direction coordinate system.)*

4 ELEMENTTIMENETELMÄ *(FINITE ELEMENT METHOD)*

Palkkielementin ***e*** tasapainoehto, kuva 8, on
*(Equilibrium condition of the beam element **e**, Figure 8, is)*

$$\{F\}^e = [K]^e \{\delta\}^e \tag{62}$$

jossa voimavektori on *(where force vector is)*

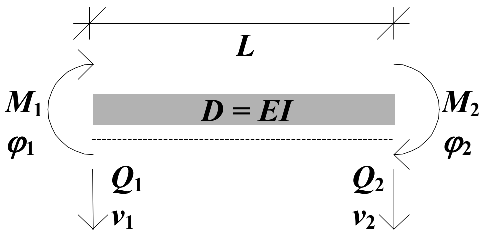
$$\{F\}^e = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} \tag{63}$$

siirtymävektori on *(displacement vector is)*

$$\{\delta\}^e = \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} \tag{64}$$

ja jäykkyysmatriisi on *(and stiffness matrix is)*

$$[K]^e = \begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \tag{65}$$



Kuva 8. Palkkielementin voimasuureet ja muodonmuutokset.
*(**Figure 8.** Forces, bending moments and deformations of the beam element.)*

Jäykkyysmatriisissa $\boldsymbol{L}$ on elementin pituus ja $\boldsymbol{D}$ sen taivutusjäykkyys
*(In stiffness matrix **L** is length and **D** is bending stiffness of the beam):*

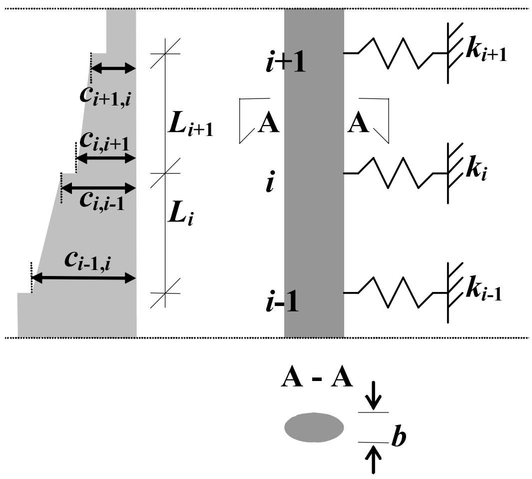
$$\boldsymbol{D} = \boldsymbol{EI} \tag{66}$$

jossa *(where)*
- $\boldsymbol{E}$ on kimmomoduuli ja
  *(**E** is modulus of elasticity and)*
- $\boldsymbol{I}$ on poikkileikkauksen taivutusmomentti
  *(**I** is moment of inertia of the cross-section).*

Kimmoista alustaa vastaava jousivakio elementin $\boldsymbol{i}$ loppupäässä, kuva 9, on
*(Spring coefficient corresponding to elastic foundation at the top of element **i**, Figure 9, is)*

$$k_i = b \cdot \frac{L_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + L_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \tag{67}$$

jossa *(where)*
- $\boldsymbol{b}$ on poikkileikkauksen maanpainetta vastaan kohtisuora projektio
  *(**b** is projection of cross-section against earth pressure),*
- L_n, $n \in \{i, i+1\}$, on elementin $\boldsymbol{n}$ pituus ja
  *(L_n, $n \in \{i, i+1\}$, is length of element **n** and)*
- $c_{n,m}$, $n, m \in \{i-1, i, i+1\}$, on alustavakio pisteessä $\boldsymbol{n}$ pisteen $\boldsymbol{m}$ puolella
  *($c_{n,m}$, $n, m \in \{i-1, i, i+1\}$, is foundation coefficient at point **n** on the side of point **m**).*



Kuva 9. Jousivakio.
*(**Figure 9.** Spring coefficient.)*

5 DIFFERENSSIMENETELMÄ *(DIFFERENCE METHOD)*

Taivutusmomentin differenssiyhtälö pisteessä ***i*** pisteen ***i*-1** puolella, kuva 10, on
*(Difference equation of bending moment at point **i** on the side of point **i-1**, Figure 10, is)*

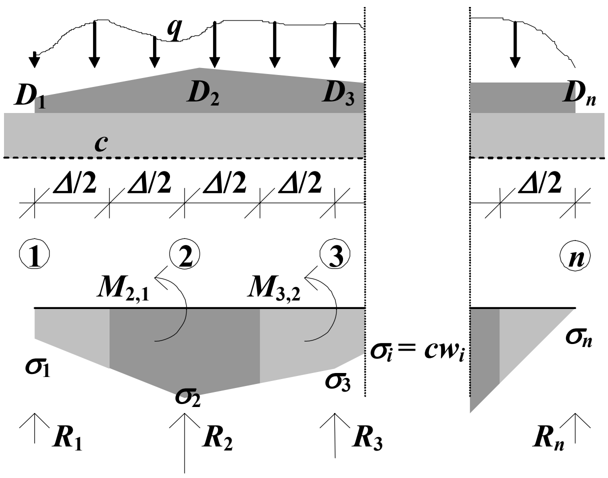
$$M_{i,i-1} = -\frac{D_i}{\Delta^2}\left(w_{i-1} - 2w_i + w_{i+1}\right) \tag{68}$$

jossa *(where)*
- Δ on osan pituus
  (Δ is length of the element),
- w_j, $j \in \{i-1, i, i+1\}$, on pisteen ***j*** taipuma
  *(w_j, $j \in \{i-1, i, i+1\}$, is deflection at point **j**)*

ja taivutusjäykkyys pisteessä ***i*** on *(and bending stiffness at point **i** is)*

$$D_i = EI_i \tag{69}$$

jossa *(where)*
- ***E*** on kimmomoduuli ja
  *(**E** is modulus of elasticity and)*
- I_i on poikkileikkauksen taivutusmomentti pisteessä ***i***
  *(I_i is moment of inertia of the cross-section at point **i**).*



Kuva 10. Taivutusmomentti ja tukireaktio.
*(**Figure 10.** Bending moment and reaction force.)*

Tukireaktion differenssiyhtälö pisteessä ***i***, kuva 10, on
(Difference equation of reaction force at point ***i****, Figure 10, is)*

$$\boldsymbol{R_i} = \begin{cases} \dfrac{\boldsymbol{bc\Delta}}{\mathbf{8}}(\mathbf{3}\boldsymbol{w_i} + \boldsymbol{w_{i+1}}), & \boldsymbol{i} = \mathbf{1} \\ \dfrac{\boldsymbol{bc\Delta}}{\mathbf{8}}(\boldsymbol{w_{i-1}} + \mathbf{6}\boldsymbol{w_i} + \boldsymbol{w_{i+1}}), & \boldsymbol{i} \in \{\mathbf{2} \ldots \boldsymbol{n} - \mathbf{1}\}, \quad \boldsymbol{i} \in \boldsymbol{N} \\ \dfrac{\boldsymbol{bc\Delta}}{\mathbf{8}}(\boldsymbol{w_{i-1}} + \mathbf{3}\boldsymbol{w_i}), & \boldsymbol{i} = \boldsymbol{n} \end{cases} \qquad (70a, b, c)$$

jossa *(where)*

- ***b*** on palkin tai laatan leveys kohtisuorassa kuvan 10 kuvatasoa vastaan
  *(**b** is width of the beam or slab perpendicular to the plane of Figure 10),*
- ***c*** on vakioksi otaksuttu alustaluku ja
  *(**c** is uniform foundation coefficient and)*
- ***n*** on pisteiden lukumäärä
  *(**n** is the number of node points).*

6 PINNAN GEOMETRISET SUUREET *(GEOMETRIC QUANTITIES OF AREA)*

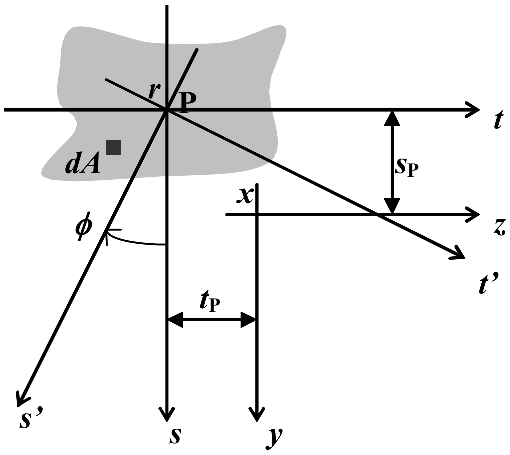
6.1 Yleinen muoto *(General Shape)*

Pinnan pinta-ala, kuva 11, on *(Area, Figure 11, is)*

$$A = \int dA \tag{71}$$

jossa ***dA*** on pinta-alkion ala *(where **dA** is differential area)*

$$dA = dydz \tag{72}$$



Kuva 11. Pinta.
*(**Figure 11.** Area.)*

Staattiset momentit mielivaltaisten ***y***- ja ***z***-akselien suhteen ovat vastaavassa järjestyksessä *(Static moments of inertia with respect to arbitrary **y**- and **z**-axis are, respectively,)*

$$S_y = \int z dA \tag{73}$$

$$S_z = \int y dA \tag{74}$$

Pinnan painopisteen **P** koordinaatit ovat *(Coordinates of centroid **P** are)*

$$y_P = \frac{S_z}{A} \tag{75}$$

$$z_P = \frac{S_y}{A} \tag{76}$$

Painopistekoordinaatit ovat *(Centroid coordinates are)*

$$s = y - y_P \tag{77}$$

$$t = z - z_P \tag{78}$$

Pinnan jäyhyysmomentit mielivaltaisten $\boldsymbol{y}$- ja $\boldsymbol{z}$-akselien suhteen ovat vastaavassa järjestyksessä
(Moments of inertia with respect to arbitrary $\boldsymbol{y}$*- and* $\boldsymbol{z}$*-axis, respectively, are)*

$$I_y = \int z^2 dA \tag{79}$$

$$I_z = \int y^2 dA \tag{80}$$

Vastaavasti tulomomentti on *(Correspondingly, product of inertia is)*

$$I_{yz} = \int yz dA \tag{81}$$

Jäyhyys- [Steinerin sääntö] ja tulomomentit siirretyssä koordinaatistossa ovat
(Moments [Steiner's rule] and Product of inertia at parallel coordinate system are)

$$I_y = I_s + At_{\mathrm{P}}^2 \tag{82}$$

$$I_z = I_t + As_{\mathrm{P}}^2 \tag{83}$$

$$I_{yz} = I_{st} + As_{\mathrm{P}} t_{\mathrm{P}} \tag{84}$$

jossa I_s ja I_t ovat jäyhyysmomentit ja I_{st} on tulomomentti painopisteakseleiden suhteen ja koordinaatit yhtälöistä 77 ja 78 ovat
(where I_s *and* I_t *are moments of inertia and* I_{st} *is product of inertia with respect to centroid axes, respectively, and coordinates from Equations 77 and 78 are)*

$$s_{\mathrm{P}} = -y_{\mathrm{P}} \tag{85}$$

$$t_{\mathrm{P}} = -z_{\mathrm{P}} \tag{86}$$

Jäyhyyssäteet painopisteakseleiden suhteen ovat
(Radii of gyration with respect to centroid axes, respectively, are)

$$i_s = \sqrt{\frac{I_s}{A}} \tag{87}$$

$$i_t = \sqrt{\frac{I_t}{A}} \tag{88}$$

Pääjäyhyysmomentit ovat *(Principal moments of inertia are)*

$$I_{s'} = I_t \sin^2\phi + I_s \cos^2\phi + I_{st} \sin(2\phi) \tag{89}$$

$$I_{t'} = I_t \cos^2\phi + I_s \sin^2\phi - I_{st} \sin(2\phi) \tag{90}$$

jossa pääjäyhyyssuunnan kulma ***s***- ja ***s'***-akselin välillä on
*(where angle of principal direction between **s**- and **s'**-axis is)*

$$\phi = \frac{1}{2}\arctan\frac{2I_{st}}{I_s - I_t} \tag{91}$$

Pääjäyhyyssuunnan koordinaatit ovat *(Coordinates of principal directions are)*

$$s' = s\cos\phi - t\sin\phi \tag{92}$$
$$t' = s\sin\phi + t\cos\phi \tag{93}$$

Pääjäyhyyssäteet ovat *(Principal radii of gyration, respectively, are)*

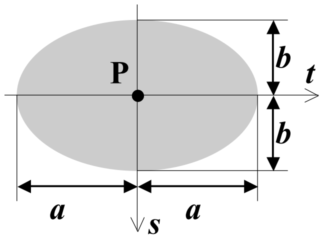
$$i_{s'} = \sqrt{\frac{I_{s'}}{A}} \tag{94}$$
$$i_{t'} = \sqrt{\frac{I_{t'}}{A}} \tag{95}$$

6.2 Ellipsi *(Ellipse)*

Jäyhyysmomentti pääjäyhyysakselin ***t*** suhteen, kuva 12, on
*(Moment of inertia with respect to principal axis **t**, Figure 12, is)*

$$I_t = \frac{\pi a b^3}{4} \tag{96}$$

jossa *(where)*
- ***a*** on puolet ***t***-akselin suuntaisesta ellipsin pääakselista 2***a*** ja
  *(**a** is half of principal axis 2**a** parallel to **t**-axis and)*
- ***b*** on puolet ***t***-akselia vastaan kohtisuorasta ellipsin pääakselista 2***b***
  *(**b** is half of principal axis 2**b** perpendicular to **t**-axis).*



Kuva 12. Ellipsi.
*(**Figure 12.** Ellipse.)*

6.3 Kolmio *(Triangle)*

Jäyhyysmomentit painopisteen **P** kautta kulkevien ***s***- ja ***t***-akselien suhteen, kuva 13a, ovat
*(Moment of inertia with respect to **s**- and **t**-axis going through the centroid **P**, respectively, Figure 13a, are)*

$$\boldsymbol{I_s = \frac{bh^3}{36}} \tag{97}$$

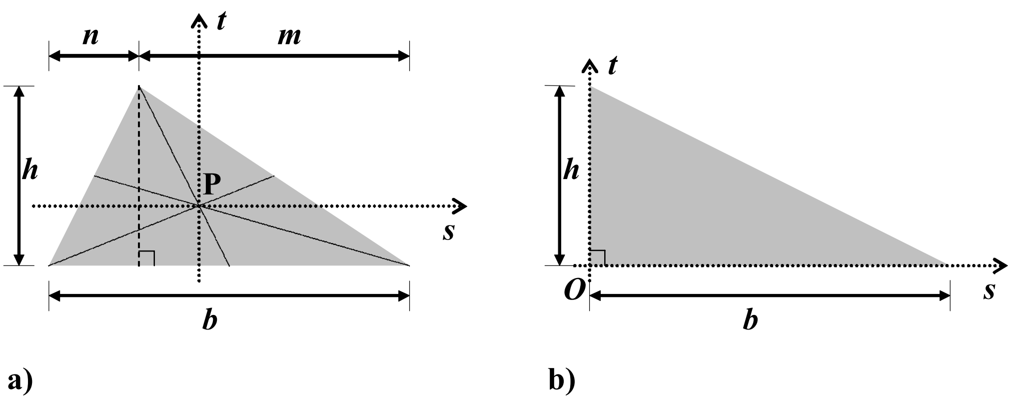
$$\boldsymbol{I_t = \frac{h}{36}\left(n^3 + 2nmb + m^3\right)} \tag{98}$$

jossa *(where)*

- ***n*** on ensimmäisen suorakulmaisen osakolmion ***s***-akselin suuntaisen sivun pituus
  *(**n** is side length parallel to **s**-axis of the first right-angled sub-triangle),*
- ***m*** on toisen suorakulmaisen osakolmion ***s***-akselin suuntaisen sivun pituus
  *(**m** is side length parallel to **s**-axis of the second right-angled sub-triangle),*
- ***h*** on kolmion korkeus ***t***-akselin suunnassa
  *(**h** is height parallel to **t**-axis)*

ja *(and)*

$$\boldsymbol{b = n + m} \tag{99}$$



Kuva 13. Kolmio.
*(**Figure 13.** Triangle.)*

Suorakulmaisen kolmion tulomomentti kulmapisteen ***O*** suhteen, kuva 13b, on
*(Product of inertia of the right-angled triangle with respect to point **O**, Figure 13b, is)*

$$\boldsymbol{I_{st} = \frac{b^2h^2}{24}} \tag{100}$$

jossa *(where)*

- ***b*** on kanta ja
  *(**b** is base and)*
- ***h*** on korkeus
  *(**h** is height).*

7 PINNAN JÄYKKYYSSUUREET *(STIFFNESS QUANTITIES OF AREA)*

Ei-homogeenisen materiaalin tapauksessa saadaan edellisen luvun pinnan geometrisiä suureita vastaavat suureet korvaamalla
(In the case of nonhomogeneous material, the quantities corresponding to geometric quantities of area discussed in previous chapter are obtained by replacing)

$$dA \mapsto E(y,z)dA \tag{101}$$

jossa E on kimmomoduuli *(where E is modulus of elasticity).*

Materiaaliltaan homogeenisia osia sisältävän liittorakenteen tapauksessa aksiaalijäykkyys on
(Axial stiffness for composite structure having homogenous material parts is)

$$C = \sum_{i=1}^{n} E_i A_i \tag{102}$$

jossa n on materiaali- tai osakomponenttien i lukumäärä
(where n is the number of material or separate components i).

Vastaavasti kimmomoduulilla kerrottu staattinen momentti on
(Correspondingly, statical moment multiplied by modulus of elasticity is)

$$\Gamma = \sum_{i=1}^{n} E_i S_i \tag{103}$$

ja taivutusjäykkyys on *(and bending stiffness is)*

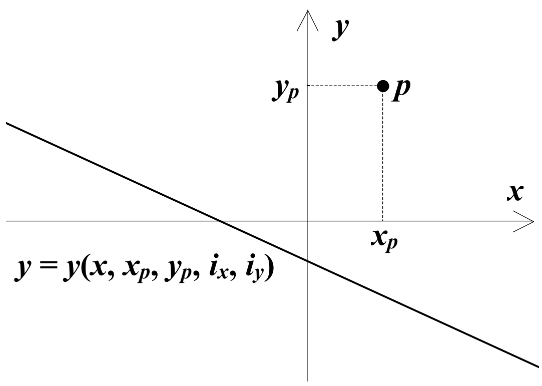
$$D = \sum_{i=1}^{n} E_i I_i \tag{104}$$

8 SYDÄNKUVIO *(CORE FIGURE)*

Sydänkuviota rajoittavan suoran yhtälö, eli voimavaikutuspistettä ***p*** vastaava neutraaliakseli, pinnan pääjäykkyyskoordinaatistossa, kuva 14, on
*(Equation of core figure boundary line, or neutral axis corresponding to load acting at point **p**, in principal direction coordinate system of area, Figure 14, is)*

$$\mathbf{1} + \frac{x_p}{i_y^2}x + \frac{y_p}{i_x^2}y = \mathbf{0} \tag{105}$$

jossa *(where)*
- x_p on pinnan “kuperan kärkipisteen” x-koordinaatti
  (x_p is x-coordinate of “convex corner point” of area),
- y_p on pinnan “kuperan kärkipisteen” y-koordinaatti
  (y_p is y-coordinate of “convex corner point” of area),
- i_x on pinnan jäyhyyssäde pääjäykkyysakselin x suhteen ja
  (i_x is radius of gyration with respect to principal x-axis)
- i_y on pinnan jäyhyyssäde pääjäykkyysakselin y suhteen
  (i_y is radius of gyration with respect to principal y-axis).



Kuva 14. Voimavaikutuspistettä vastaava neutraaliakseli.
*(**Figure 14.** Neutral axis corresponding to load acting point.)*

9 TAIPUMAVIIVA *(DEFLECTION CURVE)*

Seuraavissa kaavoissa ***a*** ja ***b*** määrittävät pistevoiman sijainnin, ***L*** on sauvan pituus ja ***D*** sen taivutusjäykkyys
*(To the following equations **a** and **b** specifies the location of the point load, **L** is length and **D** is bending stiffness of the beam):*

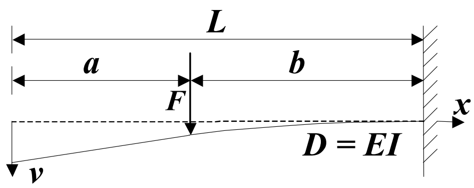
$$\boldsymbol{D = EI} \tag{106}$$

jossa *(where)*
- ***E*** on kimmomoduuli ja
  *(**E** is modulus of elasticity and)*
- ***I*** on poikkileikkauksen taivutusmomentti
  *(**I** is moment of inertia of the cross-section).*

9.1 Uloke *(Cantilever)*

Taipumaviiva pistevoimasta ***F***, kuva 15, on
*(Deflection curve due to point load **F**, Figure 15, is)*

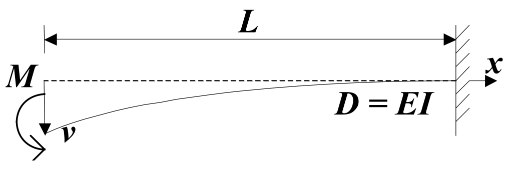
$$v = \begin{cases} \dfrac{Fb^2L}{6D}\left(3 - \dfrac{b}{L} - 3\dfrac{x}{L}\right), & x \in \{0...a\} \\ \dfrac{Fb^3}{6D}\left[2 - 3\dfrac{x-a}{b} + \left(\dfrac{x-a}{b}\right)^3\right], & x \in \{a...L\} \end{cases} \tag{107a, b}$$



Kuva 15. Pistevoiman ***F*** kuormittama uloke.
*(**Figure 15.** Cantilever loaded by point load **F**.)*

Taipumaviiva pistemomentista ***M*** ulokkeen päässä, kuva 16, on
*(Deflection curve due to point moment **M** at the end of the cantilever, Figure 16, is)*

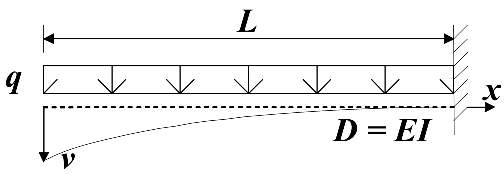
$$v = \frac{M}{2D}(L - x)^2 \tag{108}$$



Kuva 16. Päästä pistemomentin ***M*** kuormittama uloke.
*(**Figure 16.** Cantilever loaded by point moment **M** at the end.)*

Taipumaviiva koko ulokkeelle tasan jakautuneesta viivavoimasta ***q***, kuva 17, on
*(Deflection curve due to uniformly distributed line load **q** acting over the whole cantilever, Figure 17, is)*

$$v = \frac{qL^4}{24D}\left[3 - 4\frac{x}{L} + \left(\frac{x}{L}\right)^4\right] \tag{109}$$

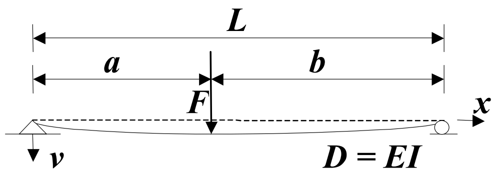


Kuva 17. Koko pituudella tasaisen viivavoiman ***q*** kuormittama uloke.
*(**Figure 17.** Cantilever loaded by uniformly distributed line load **q** acting over the whole length.)*

9.2 Yksinkertainen palkki[1] *(Simple Beam)*[2]

Taipumaviiva pistevoimasta ***F***, kuva 18, on
*(Deflection curve due to point load **F**, Figure 18, is)*

$$v = \begin{cases} \dfrac{Fb}{6DL}\, x\left[a(L+b) - x^2\right], & x \in \{0 \ldots a\} \\ \dfrac{Fa}{6DL}(L-x)\left[b(L+a) - (L-x)^2\right], & x \in \{a \ldots L\} \end{cases} \tag{110 a, b}$$

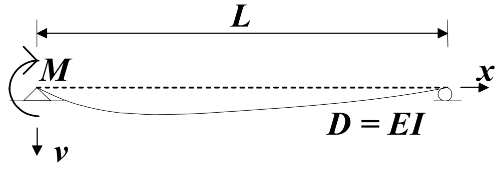


Kuva 18. Pistevoiman ***F*** kuormittama yksinkertainen palkki.
*(**Figure 18.** Simple beam loaded by point load **F**.)*

Taipumaviiva pistemomentista ***M*** palkin päässä, kuva 19, on
*(Deflection curve due to point moment **M** at the end, Figure 19, is)*

$$v = \frac{ML^2}{6D}\left(2\frac{x}{L} - 3\frac{x^2}{L^2} + \frac{x^3}{L^3}\right) \tag{111}$$

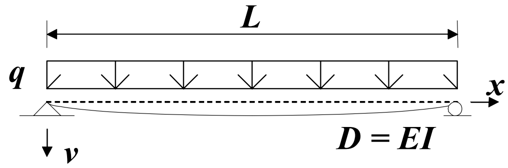
[1] Vapaasti tuettu yksiaukkoinen palkki.
[2] *(Simply supported single span beam.)*



Kuva 19. Päästä pistemomentin ***M*** kuormittama yksinkertainen palkki.
*(**Figure 19.** Simple beam loaded by point moment **M** at the end.)*

Taipumaviiva koko jänteelle tasan jakautuneesta viivavoimasta ***q***, kuva 20, on
*(Deflection curve due to uniformly distributed line load **q** acting over the whole span, Figure 20, is)*

$$v = \frac{qL^4}{24D}\left[\frac{x}{L} - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^4\right] \tag{112}$$



Kuva 20. Koko pituudella tasaisen viivavoiman ***q*** kuormittama yksinkertainen palkki.
*(**Figure 20.** Simple beam loaded by uniformly distributed line load **q** acting over the whole length.)*

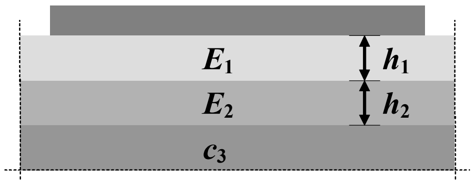
10 ALUSTALUKU *(FOUNDATION COEFFICIENT)*

Kolmen alustakerroksen, kuva 21, alustaluku on[3]
(Foundation coefficient for triple layer foundation, Figure 21, is)[4]

$$c = \frac{1}{\frac{h_1}{E_1} + \frac{h_2}{E_2} + \frac{1}{c_3}} \tag{113}$$

jossa *(where)*

- h_i, $i \in \{1, 2\}$, on kerroksen i korkeus
  (h_i, $i \in \{1, 2\}$, is height of layer i),
- E_i, $i \in \{1, 2\}$, on kerroksen i kimmomoduuli ja
  (E_i, $i \in \{1, 2\}$, is modulus of elasticity of layer i and)
- c_3 on perusmaan alustaluku
  (c_3 is foundation coefficient for base soil).



Kuva 21. Alustakerrokset.
*(**Figure 21.** Foundation layers.)*

[3] BY 31 Betonilattiat.
[4] *(BY 31 Finnish standard for concrete floors.)*

11 BETONIN KIMMOMODUULI *(MODULUS OF ELASTICITY OF CONCRETE)*

Betonin kimmomoduuli tavanomaiselle runkoaineelle on
(Modulus of elasticity of concrete with ordinary aggregate is)

$$\boldsymbol{E = k\sqrt{KK_0}} \tag{114}$$

jossa ***K*** on betonin kuutio- eli nimellislujuus ja vakio
*(where **K** is cubic strength or nominal strength of concrete and constant)*

$$\boldsymbol{K_0 = 25 \cdot 10^6 \ \frac{\mathrm{MN}}{\mathrm{m}^2}}$$

ja dimensioton kerroin *(and dimensionless coefficient)*

$$\boldsymbol{k = \min\begin{cases} \rho_c/\rho_0 \\ 1 \end{cases}} \tag{115}$$

jossa $\boldsymbol{\rho_c}$ on betonin tiheys ja vakio *(where $\boldsymbol{\rho_c}$ is density of concrete and constant)*

$$\boldsymbol{\rho_0 = 2400 \ \frac{\mathrm{kg}}{\mathrm{m}^3}}$$

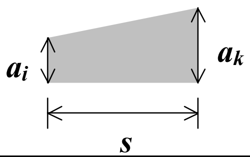
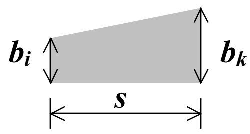
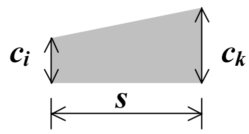
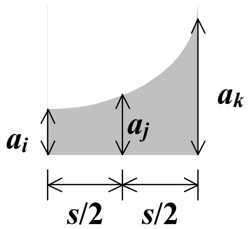
12 TULON INTEGRAALI *(INTEGRAL OF PRODUCT)*

Kolmen funktion tulon integraali taulukon 2 tapauksille
(Integral of product of three functions in the cases of Table 2)

$$F_a = s\left[\frac{1}{6}\left(a_i b_i c_i + a_k b_k c_k\right) + \frac{1}{6}\left(a_i + a_k\right)\left(b_i c_i + b_k c_k\right) + \right. \\ \left. - \frac{1}{12}\left(a_i + a_k\right)\left(b_i - b_k\right)\left(c_i - c_k\right)\right] \tag{116}$$

$$F_b = s\left[\frac{1}{6}\left(a_i b_i c_i + a_k b_k c_k\right) + \frac{1}{3}a_j\left(b_i c_i + b_k c_k\right) + \right. \\ \left. - \frac{1}{60}\left(a_i + 8a_j + a_k\right)\left(b_i - b_k\right)\left(c_i - c_k\right)\right] \tag{117}$$

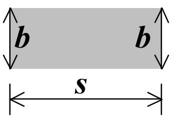
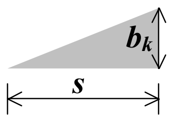
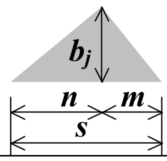
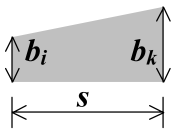
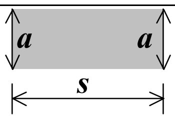
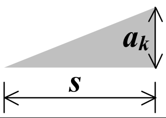
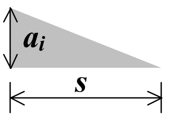
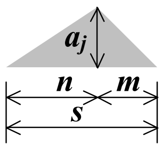
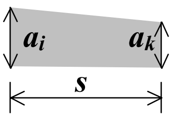
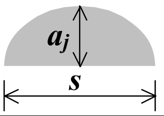
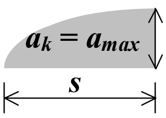
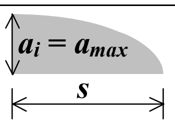
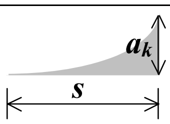
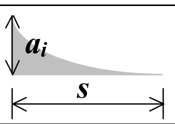
Taulukko 2. ***(Table 2.)***

<table>
<tr><th>$\int_0^s a(x)b(x)c(x)dx$</th><th>$a(x)$</th><th>$b(x)$</th><th>$c(x)$</th></tr>
<tr><td>F_a</td><td>
</td><td rowspan="2">
</td><td rowspan="2">
</td></tr>
<tr><td>F_b</td><td>

a(x) on toisen asteen polynomi.
(a(x) is polynome of second order).</td></tr>
</table>

Kahden funktion tulon integraaleja eri tapauksille on esitetty taulukossa 3.
(Integrals of product of two functions in several cases are presented in Table 3.)

Taulukko 3. ***(Table 3.)***

$\int_0^s a(x)b(x)dx$	b_1 	b_2 	b_3 	b_4 
a_1 	sab	$\frac{1}{2}sab_k$	$\frac{1}{2}sab_j$	$\frac{1}{2}sa(b_i+b_k)$
a_2 	$\frac{1}{2}sa_kb$	$\frac{1}{3}sa_kb_k$	$\frac{1}{6}(s+n)a_kb_j$	$\frac{1}{6}sa_k(b_i+2b_k)$
a_3 	$\frac{1}{2}sa_ib$	$\frac{1}{6}sa_ib_k$	$\frac{1}{6}(s+m)a_ib_j$	$\frac{1}{6}sa_i(2b_i+b_k)$
a_4 	$\frac{1}{2}sa_jb$	$\frac{1}{6}(s+n)a_jb_k$	$\frac{1}{3}sa_jb_j$	$\frac{1}{6}[(s+m)b_i+$ $+(s+n)b_k]a_j$
a_5 	$\frac{1}{2}s(a_i+a_k)b$	$\frac{1}{6}s(a_i+2a_k)b_k$	$\frac{1}{6}[(s+m)a_i+$ $+(s+n)a_k]\cdot b_j$	$\frac{1}{6}s[a_i(2b_i+b_k)+$ $+a_k(b_i+2b_k)]$
a_6 [5] 	$\frac{2}{3}sa_jb$	$\frac{1}{3}sa_jb_k$	$\frac{s^2+nm}{3s}a_jb_j$	$\frac{1}{3}sa_j(b_i+b_k)$
a_7 	$\frac{2}{3}sa_kb$	$\frac{5}{12}sa_kb_k$	$\frac{5s^2-ms-m^2}{12s}a_kb_j$	$\frac{1}{12}sa_k(3b_i+5b_k)$
a_8 	$\frac{2}{3}sa_ib$	$\frac{1}{4}sa_ib_k$	$\frac{5s^2-ns-n^2}{12s}a_ib_j$	$\frac{1}{12}sa_i(5b_i+3b_k)$
a_9 	$\frac{1}{3}sa_kb$	$\frac{1}{4}sa_kb_k$	$\frac{s^2+ns+n^2}{12s}a_kb_j$	$\frac{1}{12}sa_k(b_i+3b_k)$
a_{10} 	$\frac{1}{3}sa_ib$	$\frac{1}{12}sa_ib_k$	$\frac{s^2+ms+m^2}{12s}a_ib_j$	$\frac{1}{12}sa_i(3b_i+b_k)$

[5] a_ι, $\iota\in\{6, 7, 8, 9, 10\}$, on toisen asteen polynomi *(a_ι, $\iota\in\{6, 7, 8, 9, 10\}$, is polynome of second order).*

13 MOMENTTIMENETELMÄ JA KULMANMUUTOSMENETELMÄ *(MOMENT METHOD AND SLOPE-DEFLECTION METHOD)*

13.1 Momenttimenetelmä *(Moment Method)*

Sauvanpääkiertymät kuvan 22 merkkisäännöin ovat
(Rotations at corners by using directions shown in Figure 22 are)

$$\begin{cases} \varphi_{ij} = \alpha_{ij} M_{ij} - \beta_{ij} M_{ji} + \psi_{ij} + \alpha_{ij}^0 \\ \varphi_{ji} = \alpha_{ji} M_{ji} - \beta_{ji} M_{ij} + \psi_{ji} + \alpha_{ji}^0 \end{cases} \tag{118a, b}$$

jossa M_{ij} ja M_{ji} ovat sauvanpäämomentit *(where M_{ij} and M_{ji} are corner moments).*

Sauvavakiot tasajäykän sauvan tapauksessa ovat
(Member coefficients in the case of constant flexural rigidity are)

$$\left.\begin{matrix} \alpha_{ij} \\ \alpha_{ji} \end{matrix}\right\} = \frac{l_{ij}}{3D_{ij}} \tag{119}$$

$$\left.\begin{matrix} \beta_{ij} \\ \beta_{ji} \end{matrix}\right\} = \frac{l_{ij}}{6D_{ij}} \tag{120}$$

jossa l_{ij} on sauvan pituus *(where l_{ij} is length of the beam).*

Sauvan taivutusjäykkyys on *(Bending stiffness of the beam is)*

$$D_{ij} = E_{ij} I_{ij} \tag{121}$$

jossa *(where)*

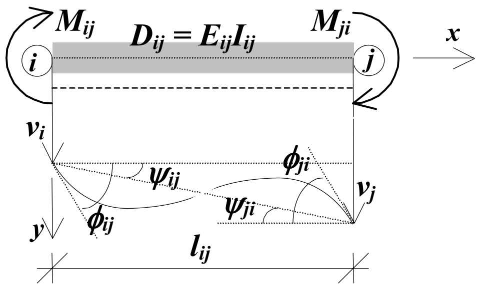
- E_{ij} on kimmomoduuli ja
  (E_{ij} is modulus of elasticity and)
- I_{ij} on poikkileikkauksen taivutusjäyhyys
  (I_{ij} is moment of inertia of the cross-section).

Siirtymästä aiheutuvat kiertymät ovat *(Slope rotations are)*

$$\left.\begin{matrix} \psi_{ij} \\ \psi_{ji} \end{matrix}\right\} = \frac{v_j - v_i}{l_{ij}} \tag{122}$$

jossa v_i ja v_j ovat sauvaa vastaan kohtisuorat siirtymät sauvan päissä i ja j
(where v_i and v_j are displacements perpendicular to the beam at the ends i and j).

α^0_{ij} ja α^0_{ji} ovat kuormituksesta aiheutuvat kiertymät, jotka tasajäykälle sauvalle eri kuormitustapauksissa on annettu taulukossa 4.
(α^0_{ij} and α^0_{ji} are rotations at corners caused by loading and are given in Table 4 in different loading cases, when flexural rigidity is constant.)



Kuva 22. Momentti- ja kulmanmuutosmenetelmän sauvanpäämomentit ja kiertymät.
*(**Figure 22.** Corner moments and rotations of the moment method and of the slope-deflection method.)*

13.2 Kulmanmuutosmenetelmä *(Slope-Deflection Method)*

Sauvanpäämomentit kuvan 22 merkkisäännöin ovat
(Corner moments by using directions shown in Figure 22 are)

$$\begin{cases} M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + M_{Kij} \\ M_{ji} = a_{ji}\varphi_{ji} + b_{ji}\varphi_{ij} - c_{ji}\psi_{ji} + M_{Kji} \end{cases} \qquad (123a, b)$$

jossa φ_{ij} ja φ_{ji} ovat sauvanpääkiertymät *(where φ_{ij} and φ_{ji} are rotations at the corners).*

Sauvavakiot tasajäykän sauvan tapauksessa ovat
(Member coefficients in the case of constant flexural rigidity are)

$$\left.\begin{matrix} a_{ij} \\ a_{ji} \end{matrix}\right\} = \frac{4D_{ij}}{l_{ij}} \qquad (124)$$

$$\left.\begin{matrix} b_{ij} \\ b_{ji} \end{matrix}\right\} = \frac{2D_{ij}}{l_{ij}} \qquad (125)$$

$$\left.\begin{matrix} c_{ij} \\ c_{ji} \end{matrix}\right\} = \frac{6D_{ij}}{l_{ij}} \qquad (126)$$

jossa l_{ij} on sauvan pituus. Sauvan taivutusjäykkyys on määritelty kaavassa 121.
(where l_{ij} is length of the beam. Bending stiffness of the beam is given in Equation 121.)

Siirtymästä aiheutuvat kiertymät on määritelty kaavassa 122.
(Slope rotations are given in Equation 122.)

M_{Kij} ja M_{Kji} ovat kuormituksesta aiheutuvat sauvanpäämomentit, jotka tasajäykälle sauvalle eri kuormitustapauksissa on annettu taulukossa 4.
(M_{Kij} and M_{Kji} are corner moments caused by loading and are given in Table 4 in different loading cases, when flexural rigidity is constant.)

Jos j-päässä on nivel, on sauvanpäämomentti *(If end j is pinned, the corner moment is)*

$$M_{ij} = a_{ij}^{0}\varphi_{ij} - c_{ij}^{0}\psi_{ij} + M_{Kij}^{0} \tag{127a}$$

Vastaavasti, jos nivel on i-päässä, on *(Respectively, if end i is pinned, is)*

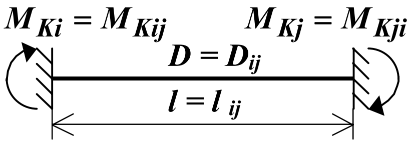
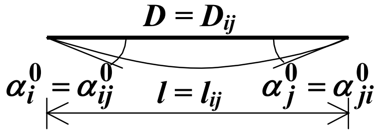
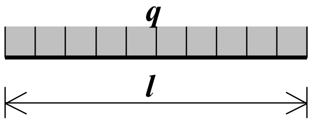
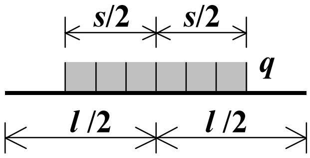
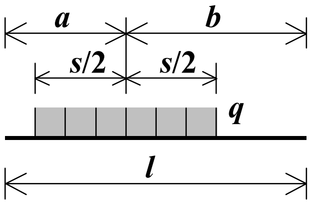
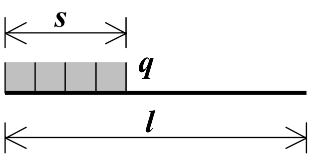
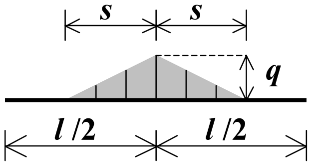
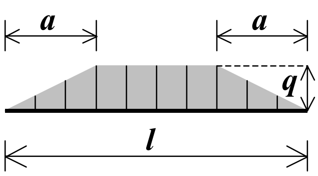
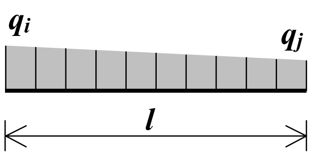
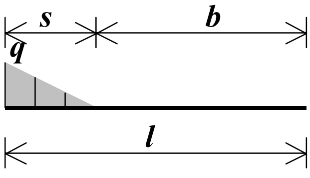
$$M_{ji} = a_{ji}^{0}\varphi_{ji} - c_{ji}^{0}\psi_{ji} + M_{Kji}^{0} \tag{127b}$$

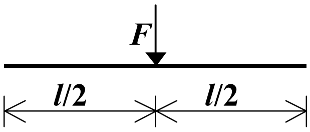
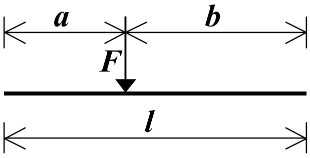
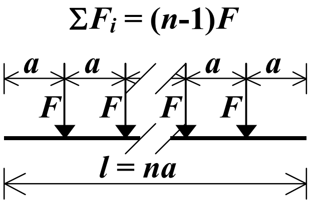
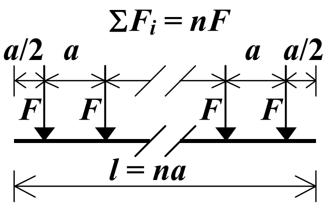
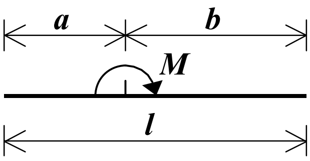
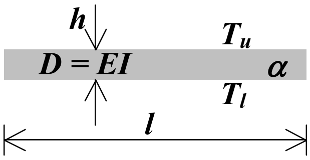
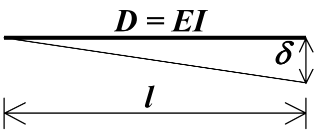
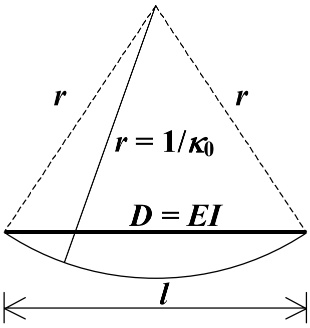
Näissä sauvavakiot tasajäykän sauvan tapauksessa ovat
(Here member coefficients in the case of constant flexural rigidity are)

$$\left.\begin{array}{l} a_{ij}^{0} \\ a_{ji}^{0} \\ c_{ij}^{0} \\ c_{ji}^{0} \end{array}\right\} = \frac{3D_{ij}}{l_{ij}} \tag{128}$$

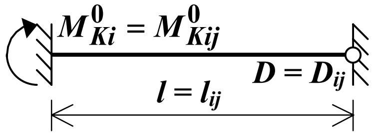
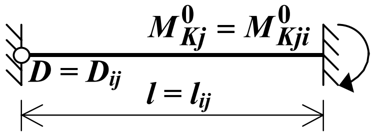
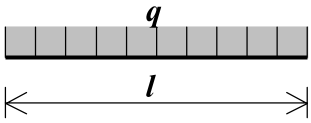
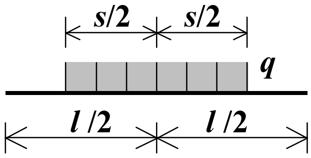
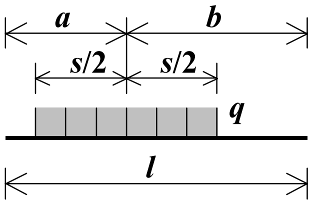
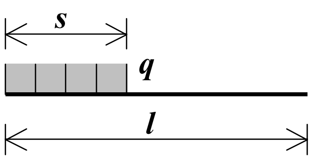
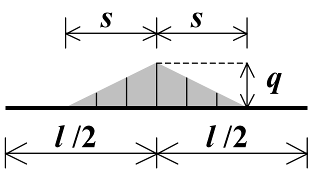
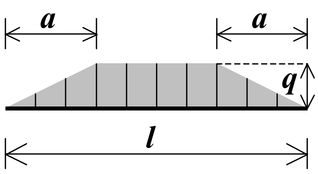
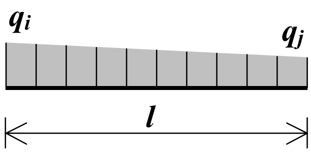
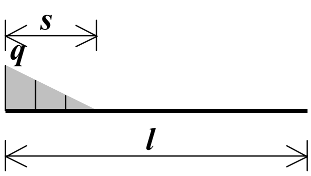
M_{Kij}^{0} ja M_{Kji}^{0} ovat kuormituksesta aiheutuvat sauvanpäämomentit, jotka tasajäykälle sauvalle eri kuormitustapauksissa on annettu taulukossa 5.
(M_{Kij}^{0} and M_{Kji}^{0} are corner moments caused by loading and are given in Table 5 in different loading cases, when flexural rigidity is constant.)

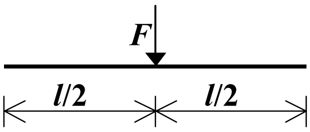
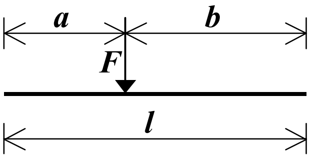
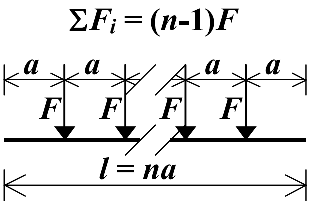
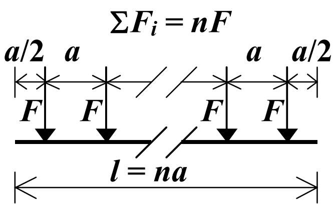
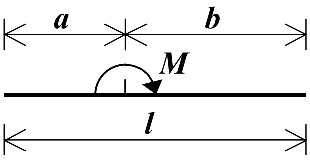
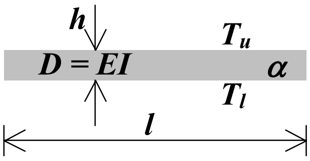
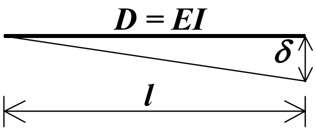
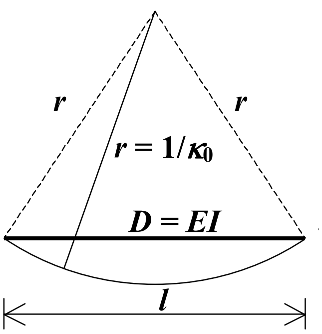
Taulukko 4. ***(Table 4.)***

Kuormitus *(Loading)*		
1 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-ql^2}{12}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{ql^3}{24D}$
2 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-qs}{24l}(3l^2 - s^2)$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{qs}{48D}(3l^2 - s^2)$
3 	$M_{Ki} = \frac{-qs}{12l^2}\left[12ab^2 + s^2(l-3b)\right]$ $M_{Kj} = \frac{qs}{12l^2}\left[12a^2b + s^2(l-3a)\right]$	$\alpha_i^0 = \frac{qbs}{24Dl}\left[4a(b+l) - s^2\right]$ $\alpha_j^0 = \frac{-qas}{24Dl}\left[4b(a+l) - s^2\right]$
4 	$M_{Ki} = \frac{-qs^2}{12l^2}\left[2l(3l-4s) + 3s^2\right]$ $M_{Kj} = \frac{qs^3}{12l^2}(4l - 3s)$	$\alpha_i^0 = \frac{qs^2}{24Dl}(2l - s)^2$ $\alpha_j^0 = \frac{-qs^2}{24Dl}(2l^2 - s^2)$
5 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-qs}{24l}(3l^2 - 2s^2)$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{qs}{48D}(3l^2 - 2s^2)$
6 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-q}{12l}\left[l^3 - a^2(2l - a)\right]$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{q}{24D}\left[l^3 - a^2(2l - a)\right]$
7 	$M_{Ki} = -l^2\left(\frac{q_i}{20} + \frac{q_j}{30}\right)$ $M_{Kj} = l^2\left(\frac{q_i}{30} + \frac{q_j}{20}\right)$	$\alpha_i^0 = \frac{l^3}{D}\left(\frac{q_i}{45} + \frac{7q_j}{360}\right)$ $\alpha_j^0 = \frac{-l^3}{D}\left(\frac{7q_i}{360} + \frac{q_j}{45}\right)$
8 	$M_{Ki} = \frac{-qs^2}{60l^2}(10bl + 3s^2)$ $M_{Kj} = \frac{qs^3}{60l^2}(5b + 2s)$	$\alpha_i^0 = \frac{qs^2}{360Dl}\left[5b(4l+s) + 8s^2\right]$ $\alpha_j^0 = \frac{-qs^2}{360Dl}\left[10b(l+s) + 7s^2\right]$

9		$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-Fl}{8}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{Fl^2}{16D}$
10		$M_{Ki} = \frac{-Fab^2}{l^2}$ $M_{Kj} = \frac{Fa^2b}{l^2}$	$\alpha_i^0 = \frac{Fab}{6Dl}(b+l)$ $\alpha_j^0 = \frac{-Fab}{6Dl}(a+l)$
11		$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-Fl}{12} \cdot \frac{n^2-1}{n}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{Fl^2}{24D} \cdot \frac{n^2-1}{n}$
12		$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{-Fl}{24} \cdot \frac{2n^2+1}{n}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{Fl^2}{48D} \cdot \frac{2n^2+1}{n}$
13		$M_{Ki} = \frac{Mb}{l}\left(2-3\frac{b}{l}\right)$ $M_{Kj} = \frac{Ma}{l}\left(2-3\frac{a}{l}\right)$	$\alpha_i^0 = \frac{Ml}{6D}\left(3\frac{b^2}{l^2}-1\right)$ $\alpha_j^0 = \frac{Ml}{6D}\left(3\frac{a^2}{l^2}-1\right)$
14		$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = \frac{\alpha(T_u-T_l)D}{h}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{-\alpha(T_u-T_l)l}{2h}$
15		$\left.\begin{matrix}M_{Ki}\\ M_{Kj}\end{matrix}\right\} = \frac{-6D\delta}{l^2}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{\delta}{l}$
16		$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\} = -D\kappa_0$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\} = \frac{\kappa_0 l}{2}$

Taulukko 5. ***(Table 5.)***

	Kuormitus *(Loading)*	 $M^0_{Ki} = M^0_{Kij}$, $D = D_{ij}$, $l = l_{ij}$	 $M^0_{Kj} = M^0_{Kji}$, $D = D_{ij}$, $l = l_{ij}$
1	 q; l	$\dfrac{-ql^2}{8}$	$\dfrac{ql^2}{8}$
2	 $s/2$, $s/2$; q; $l/2$, $l/2$	$\dfrac{-qs}{16l}(3l^2 - s^2)$	$\dfrac{qs}{16l}(3l^2 - s^2)$
3	 a, b; $s/2$, $s/2$; q; l	$\dfrac{-qbs}{8l^2}(4a^2 + 8ab - s^2)$	$\dfrac{qas}{8l^2}(4b^2 + 8ab - s^2)$
4	 s; q; l	$\dfrac{-qs^2}{8l^2}(2l - s)^2$	$\dfrac{qs^2}{8l^2}(2l^2 - s^2)$
5	 s, s; q; $l/2$, $l/2$	$\dfrac{-qs}{16l}(3l^2 - 2s^2)$	$\dfrac{qs}{16l}(3l^2 - 2s^2)$
6	 a, a; q; l	$\dfrac{-q}{8l}\left[l^3 - a^2(2l - a)\right]$	$\dfrac{q}{8l}\left[l^3 - a^2(2l - a)\right]$
7	 q_i, q_j; l	$-l^2\left(\dfrac{q_i}{15} + \dfrac{7q_j}{120}\right)$	$l^2\left(\dfrac{7q_i}{120} + \dfrac{q_j}{15}\right)$
8	 s; q; l	$\dfrac{-qs^2}{120l^2}(20l^2 - 15ls + 3s^2)$	$\dfrac{qs^2}{120l^2}(10l^2 - 3s^2)$

9		$\dfrac{-3Fl}{16}$	$\dfrac{3Fl}{16}$
10		$\dfrac{-Fab}{2l^2}(b+l)$	$\dfrac{Fab}{2l^2}(a+l)$
11		$\dfrac{-Fl}{8}\cdot\dfrac{n^2-1}{n}$	$\dfrac{Fl}{8}\cdot\dfrac{n^2-1}{n}$
12		$\dfrac{-Fl}{16}\cdot\dfrac{2n^2+1}{n}$	$\dfrac{Fl}{16}\cdot\dfrac{2n^2+1}{n}$
13		$\dfrac{M}{2}\left(1-3\dfrac{b^2}{l^2}\right)$	$\dfrac{M}{2}\left(1-3\dfrac{a^2}{l^2}\right)$
14		$\dfrac{3\alpha(T_u-T_l)D}{2h}$	$\dfrac{-3\alpha(T_u-T_l)D}{2h}$
15		$\dfrac{-3D\delta}{l^2}$	$\dfrac{3D\delta}{l^2}$
16		$-\dfrac{3}{2}D\kappa_0$	$\dfrac{3}{2}D\kappa_0$

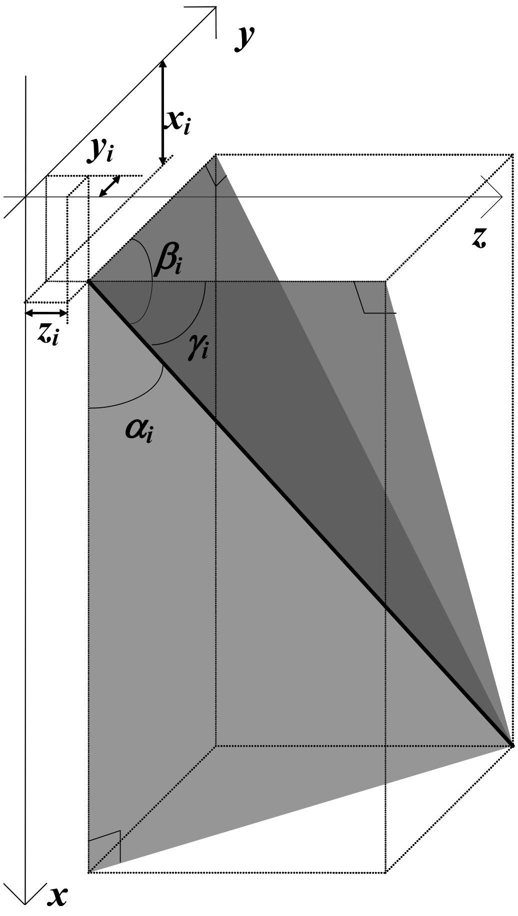
Rak-11.2107 Sillat ja Perustukset

FORMULARY

Aalto University
School of Engineering
Department of Civil and Structural Engineering
R. S. 20110111

Rak-11.2107
Bridges and Foundation Structures

FORMULARY



CONTENTS

1 EARTH PRESSURE

1.1 Active Earth Pressure

Coefficient of horizontal component P_{ah} of active earth pressure P_a is

$$K_{ah} = \frac{\cos^2(\varphi+\alpha)}{\cos^2\alpha\left[1+\sqrt{\dfrac{\sin(\varphi+\delta)\sin(\varphi-\beta)}{\cos(\alpha-\delta)\cos(\alpha+\beta)}}\right]^2} \tag{1}$$

where by using directions shown in Figure 1

- α is the angle of sliding surface with respect to vertical plane,
- β is the angle of earth surface with respect to horizontal plane,
- δ is angle of wall friction, Equations 2, 3 and 4, respectively, and
- φ is angle of internal friction.

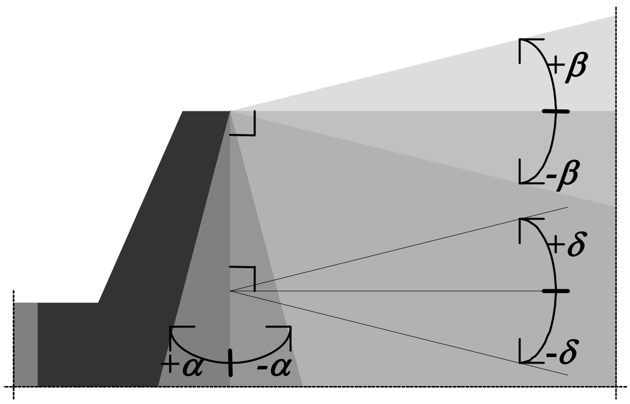


Figure 1. Angles and its directions.

Angle of wall friction is

$$\delta = \varphi \tag{2}$$

when the sliding surface is not between the structure and soil,

$$\delta = \frac{2}{3}\varphi \tag{3}$$

when the sliding surface is between the steel structure and soil, and

$$\delta = \frac{3}{4}\varphi \tag{4}$$

when the sliding surface is between the concrete structure and soil.

When

$$\left.\begin{array}{l}\alpha\\ \beta\\ \delta\end{array}\right\}=0 \tag{5a, b, c}$$

the coefficient of active earth pressure is

$$K_a = \tan^2\left(45° - \frac{\varphi}{2}\right) \tag{6}$$

Vertical component P_{av} as shown in Figure 2 is

$$P_{av} = P_{ah}\tan(\delta - \alpha) \tag{7}$$

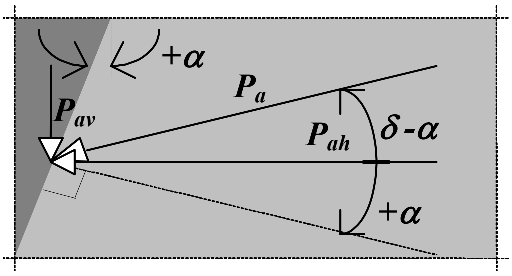


Figure 2. Active earth pressure and its components.

1.2 Earth Pressure at Rest

Coefficient of earth pressure at rest

$$K_{o\beta} = K_o(1 + \sin\beta) \tag{8}$$

where

$$K_o = 1 - \sin\varphi \tag{9}$$

2 SOIL PARAMETERS

Unit weight and internal friction angle of coarse soil based on grain size are obtained from Table 1.

Table 1. Estimation of coarse soil based on grain size.[1]

Soil		Unit weight of soil γ above the ground water level [kN/m^3]	Unit weight of soil γ under the ground water level [kN/m^3]	Internal friction angle ϕ [°]
Fine sand $d_{10} \leq 0,06$	Loose Normal Tight	15...17 16...18	9... 11	30 33 36
Sand $d_{10} > 0,06$	Loose Normal Tight	16...18 17...19	10... 12	32 35 38
Gravel	Loose Normal Tight	17...19 18...20	10... 12	34 37 40
Moraine	Very loose Loose Normal Tight	16...19 17...20 18...21 19...23	10...12 10...12 11...13 11...14	...34 ...36 ...38 ...40
Compressed filling under the foundation[2]	Blast stones Crushed stone Gravel	15...18 19...22 18...21	9...11 11...13 11...13	45 42 40

[1] Finnish Road Administration: Finnish standard of bridge foundation engineering. TIEL 2172068-99. Helsinki 1999. 71 s. ISBN 951-726-583-2. Table 1. p. 9.

[2] To use these values it is required, that the work and materials fulfil the requirements of the Finnish specifications for bridges (2.7.1.2/24/).

3 BEARING CAPACITY

3.1 Eurocode EN 1997-1

Design value of bearing capacity in the drained conditions

$$\frac{R}{A'} = c' N_c b_c s_c i_c + q' N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \tag{10}$$

where

- A' is design effective foundation area, Equation 11,
- c' is design value of cohesion intercept in terms of effective stress,
- q' is design effective overburden pressure at the level of the foundation area, Figure 3,
- γ' is design effective unit weight of soil under the foundation area,
- N_c, N_q and N_γ are factors of bearing capacity, Equations 12, 13 and 14, respectively,
- b_c, b_q and b_γ are factors for the inclination of the base, Equations 16 and 17, respectively,
- s_c, s_q and s_γ are shape factors of the base slab, Equations 18, 19 and 20, respectively, and
- i_c, i_q and i_γ are inclination factors of the load resultant, Equations 21, 22 and 23, respectively.

The design effective foundation area, Figure 3, is

$$A' = B' L' \tag{11}$$

where

- B' is effective foundation width and
- L' is effective foundation length.

Factors of the bearing capacity are

$$N_c = (N_q - 1)\cot\varphi' \tag{12}$$

$$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right)e^{\pi\tan\varphi'} \tag{13}$$

and

$$N_\gamma = 2(N_q - 1)\tan\varphi' \tag{14}$$

where φ' is the design value of angle of the internal friction. N_γ is valid for rough base where angle of wall friction is

$$\delta \geq \frac{\varphi'}{2} \tag{15}$$

Factors for the inclination of the base are

$$b_c = b_q - \frac{1 - b_q}{N_c \tan \varphi'} \tag{16}$$

and

$$\left.\begin{matrix} b_q \\ b_\gamma \end{matrix}\right\} = (1 - \alpha \tan \varphi')^2 \tag{17a, b}$$

where α is the inclination of the foundation base to the horizontal, Figure 3.

Shape factors of the base slab are

$$s_c = \frac{s_q N_q - 1}{N_q - 1} \tag{18}$$

$$s_q = \begin{cases} 1 + \dfrac{B'}{L'} \sin \varphi' \\ 1 + \sin \varphi' \end{cases} \tag{19a, b}$$

and

$$s_\gamma = \begin{cases} 1 - 0{,}3 \dfrac{B'}{L'} \\ 0{,}7 \end{cases} \tag{20a, b}$$

where Equations a are valid for a rectangular and Equations b for a square or circular shape.

Inclination factors of the load resultant are

$$i_c = i_q - \frac{1 - i_q}{N_c \tan \varphi'} \tag{21}$$

$$i_q = \left(1 - \frac{H}{V + A' c' \cot \varphi'}\right)^m \tag{22}$$

and

$$i_\gamma = \left(1 - \frac{H}{V + A' c' \cot \varphi'}\right)^{m+1} \tag{23}$$

where

$$m = \begin{cases} m_B = \dfrac{2 + \dfrac{B'}{L'}}{1 + \dfrac{B'}{L'}} \\ m_\theta = m_L \cos^2\theta + m_B \sin^2\theta \\ m_L = \dfrac{2 + \dfrac{L'}{B'}}{1 + \dfrac{L'}{B'}} \end{cases} \qquad \text{(24a, b, c)}$$

where Equation (a) is valid when $\boldsymbol{H}$ acts in the direction on $\boldsymbol{B'}$, (c) is valid when $\boldsymbol{H}$ acts in the direction of $\boldsymbol{L'}$ and (b) is valid when $\boldsymbol{H}$ acts in a direction forming an angle $\boldsymbol{\theta}$ with the direction $\boldsymbol{L'}$.

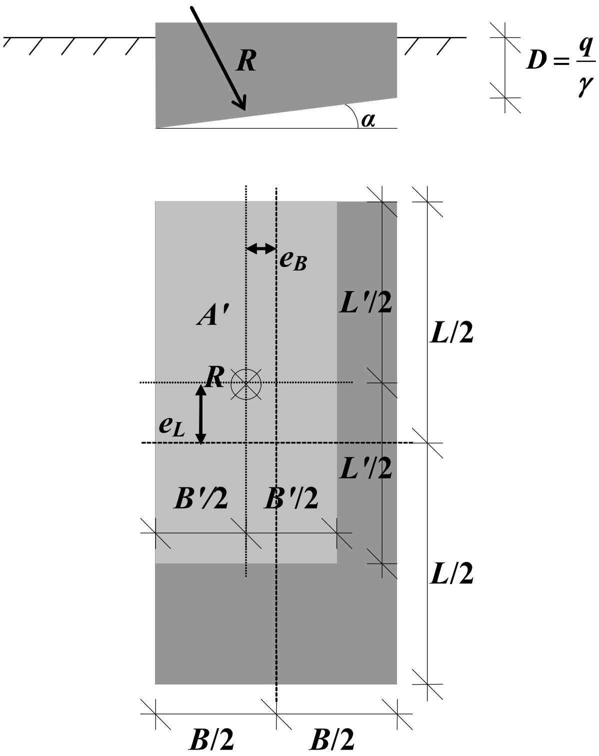


Figure 3. Notations.

3.2 Finnish Standard RIL 121-2004

Design value of bearing capacity

$$q_{md} = c_d N_c s_c i_c + \gamma_1' D N_D s_D i_D + \frac{1}{2}\gamma_2' B N_B s_B i_B \tag{25}$$

where

- c_d is design value of cohesion,
- D is the minimum foundation depth of the base slab,
- B is the length of the shorter side of base slab,
- N_c, N_D and N_B are factors of bearing capacity, Equations 26, 27 and 28, respectively, Table 2,
- s_c, s_D and s_B are shape factors of the base slab, Equations 29, 30 and 31, respectively,
- i_c, i_D and i_B are inclination factors of the load resultant, Equations 32, 33 and 34, respectively,
- γ_1' is effective unit weight of soil above the foundation level and
- γ_2' is effective unit weight of soil under the foundation level.

Factors of the bearing capacity are

$$N_D = \tan^2\left(45° + \frac{\varphi_d}{2}\right)e^{\pi \tan\varphi_d} \tag{26}$$

$$N_c = (N_D - 1)\cot\varphi_d \tag{27}$$

and

$$N_B = \frac{3}{2}(N_D - 1)\tan\varphi_d \tag{28}$$

where φ_d is the design value of angle of the internal friction. Approximative values are given in Table 2.

Shape factors of the base slab are

$$s_c = 1 + \frac{1}{5}\frac{B}{L} \tag{29}$$

$$s_D = 1 + \frac{1}{5}\frac{B}{L} \tag{30}$$

and

$$s_B = 1 - \frac{2}{5}\frac{B}{L} \tag{31}$$

where L is the longer side length of the base slab.

Table 2. Factors of the bearing capacity as function of the angle of internal friction.

φ_d [°]	N_c	N_D	N_B
0,0	5,1	1,0	0,0
2,5	5,8	1,3	0,0
5,0	6,5	1,6	0,1
7,5	7,3	2,0	0,2
10,0	8,3	2,5	0,4
12,5	9,5	3,1	0,7
15,0	11,0	3,9	1,2
17,5	12,7	5,0	1,9
20,0	14,8	6,4	2,9
22,5	17,5	8,2	4,5
25,0	20,7	10,7	6,8
27,5	24,8	13,9	10,1
30,0	30,1	18,4	15,1
32,5	37,0	24,6	22,5
35,0	46,1	33,3	33,9
37,5	58,4	45,8	51,6
40,0	75,3	64,2	79,5
42,5	99,2	91,9	124,9
45,0	133,9	134,9	200,8

Inclination factors of the load resultant are

$$\boldsymbol{i_c} = \left(\mathbf{1} - \frac{\boldsymbol{H_d}}{\boldsymbol{V_d} + \boldsymbol{A c_d} \,\mathbf{cot}\, \boldsymbol{\varphi_d}}\right)^{\mathbf{2}} \tag{32}$$

$$\boldsymbol{i_D} = \left(\mathbf{1} - \frac{\boldsymbol{H_d}}{\boldsymbol{V_d} + \boldsymbol{A c_d} \,\mathbf{cot}\, \boldsymbol{\varphi_d}}\right)^{\mathbf{2}} \tag{33}$$

and

$$\boldsymbol{i_B} = \left(\mathbf{1} - \frac{\boldsymbol{H_d}}{\boldsymbol{V_d} + \boldsymbol{A c_d} \,\mathbf{cot}\, \boldsymbol{\varphi_d}}\right)^{\mathbf{4}} \tag{34}$$

where

- $\boldsymbol{H_d}$ is design value of horizontal force,
- $\boldsymbol{V_d}$ is design value of vertical force and
- $\boldsymbol{A}$ is bottom area of the base slab.

In the case of eccentric loading, Figure 4, replacement is done

$$\boldsymbol{L} \quad \mapsto \quad \boldsymbol{L_t} = \boldsymbol{L} - \mathbf{2}\boldsymbol{e_L} \tag{35}$$

$$\boldsymbol{B} \quad \mapsto \quad \boldsymbol{B_t} = \boldsymbol{B} - \mathbf{2}\boldsymbol{e_B} \tag{36}$$

and

$$\boldsymbol{A} \quad \mapsto \quad \boldsymbol{A_t} = \boldsymbol{L_t B_t} \tag{37}$$

where

- $\boldsymbol{e_L}$ is eccentricity in the direction of the longer side and
- $\boldsymbol{e_B}$ is eccentricity in the direction of the shorter side.

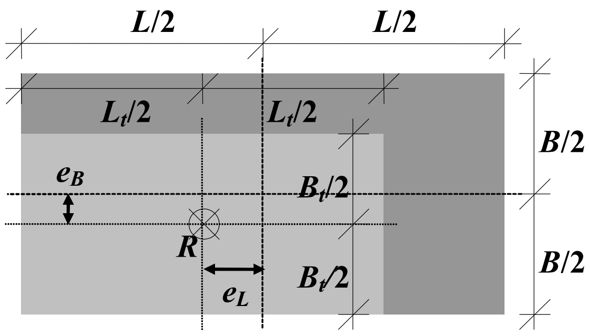


Figure 4. Effective area under eccentric loading.

4 PILE FOUNDATION

System of equilibrium conditions

$$\{F\} = [K]\{\delta\} \tag{38}$$

4.1 Pile Foundation in Space

In Equation 38, the force vector is

$$\{F\} = \{F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z\}^{\mathrm{T}} \tag{39}$$

displacement vector is

$$\{\delta\} = \{u \quad v \quad w \quad \omega \quad \varphi \quad \theta\}^{\mathrm{T}} \tag{40}$$

and stiffness matrix is

$$[K] = \sum_{i=1}^{n} [K_i] \tag{41}$$

where stiffness matrix of one pile is

$$[K_i] = k_i \begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} & p_{x,i}r_{x,i} & p_{x,i}r_{y,i} & p_{x,i}r_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} & p_{y,i}r_{x,i} & p_{y,i}r_{y,i} & p_{y,i}r_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 & p_{z,i}r_{x,i} & p_{z,i}r_{y,i} & p_{z,i}r_{z,i} \\ r_{x,i}p_{x,i} & r_{x,i}p_{y,i} & r_{x,i}p_{z,i} & r_{x,i}^2 & r_{x,i}r_{y,i} & r_{x,i}r_{z,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{y,i} & r_{y,i}p_{z,i} & r_{y,i}r_{x,i} & r_{y,i}^2 & r_{y,i}r_{z,i} \\ r_{z,i}p_{x,i} & r_{z,i}p_{y,i} & r_{z,i}p_{z,i} & r_{z,i}r_{x,i} & r_{z,i}r_{y,i} & r_{z,i}^2 \end{bmatrix} \tag{42}$$

Here n is the number of piles. The stiffness of pile i is

$$k_i = \frac{E_i A_i}{L_i} \tag{43}$$

where

- E_i is modulus of elasticity of pile i,
- A_i is cross-section area of pile i,
- L_i is length of pile i,

cosines of directional angles, Figure 5, are

$$p_{x,i} = \cos(x_i', x) = \cos\alpha_i \tag{44}$$

$$p_{y,i} = \cos(x_i', y) = \cos\beta_i \tag{45}$$

and

$$p_{z,i} = \cos(x_i^{\prime}, z) = \cos \gamma_i \tag{46}$$

and lever arms are

$$r_{x,i} = y_i p_{z,i} - z_i p_{y,i} \tag{47}$$

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{48}$$

and

$$r_{z,i} = x_i p_{y,i} - y_i p_{x,i} \tag{49}$$

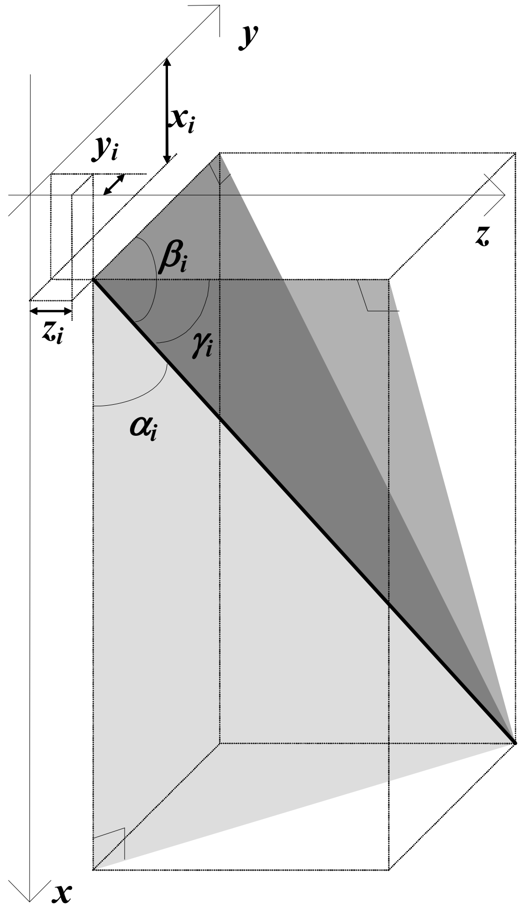


Figure 5. Directional angles in space coordinate system.

4.2 Pile Foundation in Plane

In Equation 38, the force vector is

$$\{F\} = \{F_x \quad F_z \quad M_y\}^{\mathrm{T}} \tag{50}$$

displacement vector is

$$\{\delta\} = \{u \quad w \quad \varphi\}^{\mathrm{T}} \tag{51}$$

and stiffness matrix **[*K*]** is given in Equation 41, where stiffness matrix of one pile is

$$[K_i] = k_i \begin{bmatrix} p_{x,i}^2 & p_{x,i}p_{z,i} & p_{x,i}r_{y,i} \\ p_{z,i}p_{x,i} & p_{z,i}^2 & p_{z,i}r_{y,i} \\ r_{y,i}p_{x,i} & r_{y,i}p_{z,i} & r_{y,i}^2 \end{bmatrix} \tag{52}$$

Here
- n is the number of piles,
- k_i is the stiffness of pile i, Equation 43,

cosines of directional angles, Figure 6, are

$$p_{x,i} = \cos\alpha_i \tag{53}$$

and

$$p_{z,i} = \cos\gamma_i = \sin\alpha_i \tag{54}$$

and lever arm is

$$r_{y,i} = z_i p_{x,i} - x_i p_{z,i} \tag{55}$$

$$\Rightarrow r_{y,i} = z_i \cos\alpha_i - x_i \sin\alpha_i \tag{56}$$

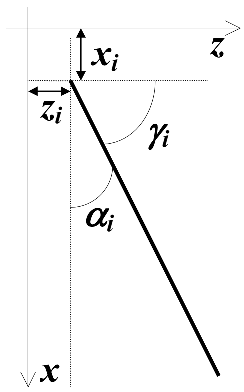


Figure 6. Directional angles in plane coordinate system.

Rotation center is

$$\begin{cases} x_0 = -\dfrac{k_{11}k_{23} - k_{13}k_{12}}{k_{11}k_{22} - (k_{12})^2} \\ z_0 = -\dfrac{k_{12}k_{23} - k_{13}k_{22}}{k_{11}k_{22} - (k_{12})^2} \end{cases} \qquad (57a, b)$$

where k_{ij} is stiffness matrix element at row i and column j.

Angle of principal direction is

$$\phi_0 = \frac{1}{2}\arctan\left(\frac{2k_{12}}{k_{11} - k_{22}}\right) \qquad (58)$$

In principal direction coordinate system, Figure 7, the coordinates are

$$x' = (z - z_0)\sin\phi_0 + (x - x_0)\cos\phi_0 \qquad (59)$$

$$z' = (z - z_0)\cos\phi_0 - (x - x_0)\sin\phi_0 \qquad (60)$$

and angle

$$\alpha' = \alpha - \phi_0 \qquad (61)$$

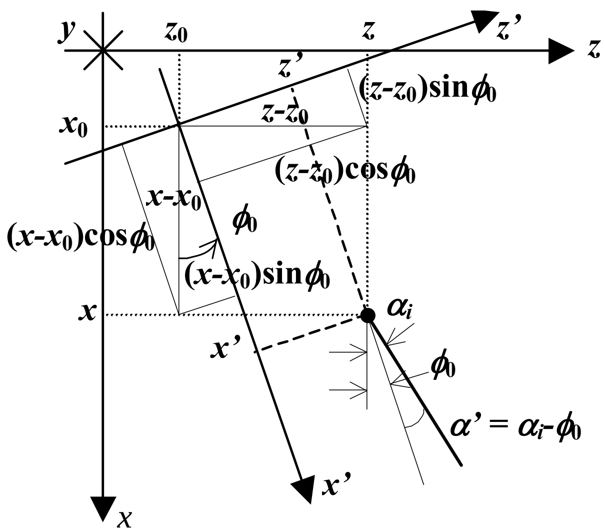


Figure 7. Principal direction coordinate system.

5 FINITE ELEMENT METHOD

Equilibrium condition of the beam element ***e***, Figure 8, is

$$\{F\}^e = [K]^e\{\delta\}^e \tag{62}$$

where force vector is

$$\{F\}^e = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} \tag{63}$$

displacement vector is

$$\{\delta\}^e = \begin{Bmatrix} v_1 \\ \varphi_1 \\ v_2 \\ \varphi_2 \end{Bmatrix} \tag{64}$$

and stiffness matrix is

$$[K]^e = \begin{bmatrix} \frac{12D}{L^3} & \frac{6D}{L^2} & -\frac{12D}{L^3} & \frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{4D}{L} & -\frac{6D}{L^2} & \frac{2D}{L} \\ -\frac{12D}{L^3} & -\frac{6D}{L^2} & \frac{12D}{L^3} & -\frac{6D}{L^2} \\ \frac{6D}{L^2} & \frac{2D}{L} & -\frac{6D}{L^2} & \frac{4D}{L} \end{bmatrix} \tag{65}$$

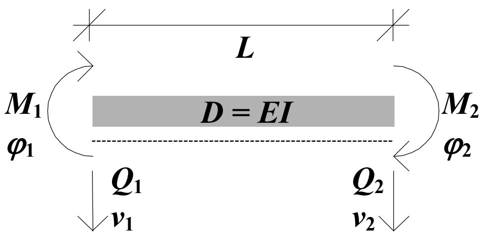


Figure 8. Forces, bending moments and deformations of the beam element.

In stiffness matrix ***L*** is length and ***D*** is bending stiffness of the beam

$$D = EI \tag{66}$$

where

- E is modulus of elasticity and
- I is moment of inertia of the cross-section.

Spring coefficient corresponding to elastic foundation at the top of element i, Figure 9, is

$$k_i = b \cdot \frac{L_i\left(c_{i-1,i} + 3c_{i,i-1}\right) + L_{i+1}\left(3c_{i,i+1} + c_{i+1,i}\right)}{8} \tag{67}$$

where

- b is projection of cross-section against earth pressure,
- L_n, $n \in \{i, i+1\}$, is length of element n and
- $c_{n,m}$, $n, m \in \{i-1, i, i+1\}$, is foundation coefficient at point n on the side of point m.

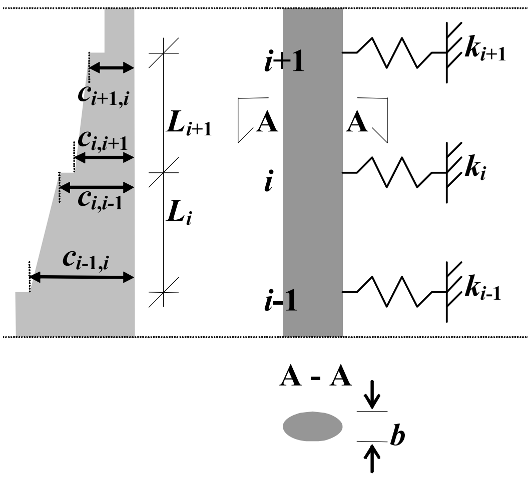


Figure 9. Spring coefficient.

6 DIFFERENCE METHOD

Difference equation of bending moment at point ***i*** on the side of point ***i*-1**, Figure 10, is

$$M_{i,i-1} = -\frac{D_i}{\Delta^2}\left(w_{i-1} - 2w_i + w_{i+1}\right) \tag{68}$$

where

- Δ is length of the element,
- w_j, $j \in \{i-1, i, i+1\}$, is deflection at point j and
- D_i and bending stiffness at point i (Equation 66).

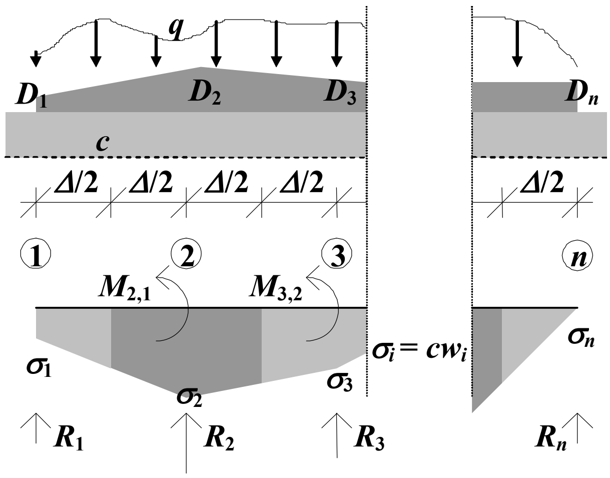


Figure 10. Bending moment and reaction force.

Difference equation of reaction force at point ***i***, Figure 10, is

$$R_i = \begin{cases} \dfrac{bc\Delta}{8}(3w_i + w_{i+1}), & i = 1 \\ \dfrac{bc\Delta}{8}(w_{i-1} + 6w_i + w_{i+1}), & i \in \{2 \ldots n-1\}, \quad i \in N \\ \dfrac{bc\Delta}{8}(w_{i-1} + 3w_i), & i = n \end{cases} \tag{69a, b, c}$$

where

- b is width of the beam or slab perpendicular to the plane of Figure 10,
- c is uniform foundation coefficient and
- n is the number of node points.

7 GEOMETRIC QUANTITIES OF AREA

7.1 General Shape

Area, Figure 11, is

$$A = \int dA \tag{70}$$

where dA is differential area

$$dA = dydz \tag{71}$$

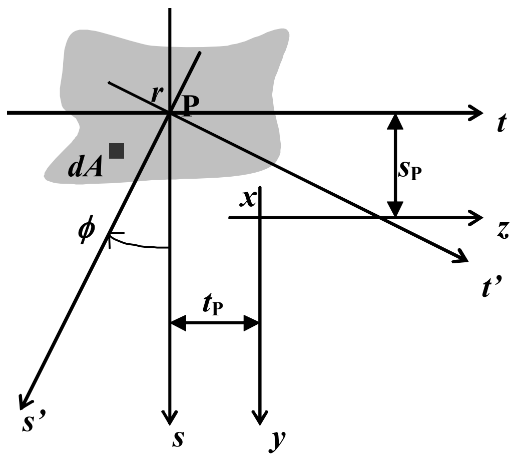


Figure 11. Area.

Static moments of inertia with respect to arbitrary y- and z-axis are, respectively,

$$S_y = \int z dA \tag{72}$$

and

$$S_z = \int y dA \tag{73}$$

Coordinates of centroid **P** are

$$y_P = \frac{S_z}{A} \tag{74}$$

and

$$z_P = \frac{S_y}{A} \tag{75}$$

Centroid coordinates are

$$s = y - y_P \tag{76}$$

and

$$t = z - z_{\mathrm{P}} \tag{77}$$

Moments of inertia with respect to arbitrary ***y***- and ***z***-axis, respectively, are

$$I_y = \int z^2 dA \tag{78}$$

and

$$I_z = \int y^2 dA \tag{79}$$

Correspondingly, product of inertia is

$$I_{yz} = \int yz dA \tag{80}$$

Moments and Product of inertia at parallel coordinate system are (Steiner's rule)

$$I_y = I_s + At_{\mathrm{P}}^2 \tag{81}$$

$$I_z = I_t + As_{\mathrm{P}}^2 \tag{82}$$

$$I_{yz} = I_{st} + As_{\mathrm{P}}t_{\mathrm{P}} \tag{83}$$

where I_s and I_t are moments of inertia and I_{st} is product of inertia with respect to centroid axes, respectively, and coordinates from Equations 76 and 77 are

$$s_{\mathrm{P}} = -y_{\mathrm{P}} \tag{84}$$

and

$$t_{\mathrm{P}} = -z_{\mathrm{P}} \tag{85}$$

Radii of gyration with respect to centroid axes, respectively, are

$$i_s = \sqrt{\frac{I_s}{A}} \tag{86}$$

and

$$i_t = \sqrt{\frac{I_t}{A}} \tag{87}$$

Principal moments of inertia are

$$I_{s'} = I_t \sin^2 \phi + I_s \cos^2 \phi + I_{st} \sin(2\phi) \tag{88}$$

and

$$I_{t'} = I_t \cos^2 \phi + I_s \sin^2 \phi - I_{st} \sin(2\phi) \tag{89}$$

where angle of principal direction between ***s***- and ***s***'-axis is

$$\phi = \frac{1}{2} \arctan \frac{2I_{st}}{I_s - I_t} \tag{90}$$

Coordinates of principal directions are

$$s' = s \cos\phi - t \sin\phi \tag{91}$$

and

$$t' = s \sin\phi + t \cos\phi \tag{92}$$

Principal radii of gyration, respectively, are

$$i_{s'} = \sqrt{\frac{I_{s'}}{A}} \tag{93}$$

and

$$i_{t'} = \sqrt{\frac{I_{t'}}{A}} \tag{94}$$

7.2 Ellipse

Moment of inertia with respect to principal axis ***t***, Figure 12, is

$$I_t = \frac{\pi a b^3}{4} \tag{95}$$

where
- ***a*** is half of principal axis 2***a*** parallel to ***t***-axis and
- ***b*** is half of principal axis 2***b*** perpendicular to ***t***-axis.

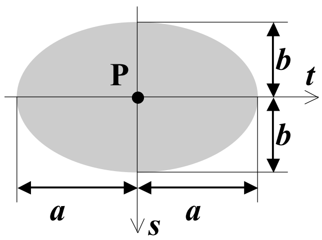


Figure 12. Ellipse.

7.3 Triangle

Moment of inertia with respect to ***s***- and ***t***-axis going through the centroid **P**, respectively, Figure 13a, are

$$\boldsymbol{I_s = \frac{bh^3}{36}} \tag{96}$$

and

$$\boldsymbol{I_t = \frac{h}{36}\left(n^3 + 2nmb + m^3\right)} \tag{97}$$

where
- ***n*** is side length parallel to ***s***-axis of the first right-angled sub-triangle,
- ***m*** is side length parallel to ***s***-axis of the second right-angled sub-triangle,
- ***h*** is height parallel to ***t***-axis

and

$$\boldsymbol{b = n + m} \tag{98}$$

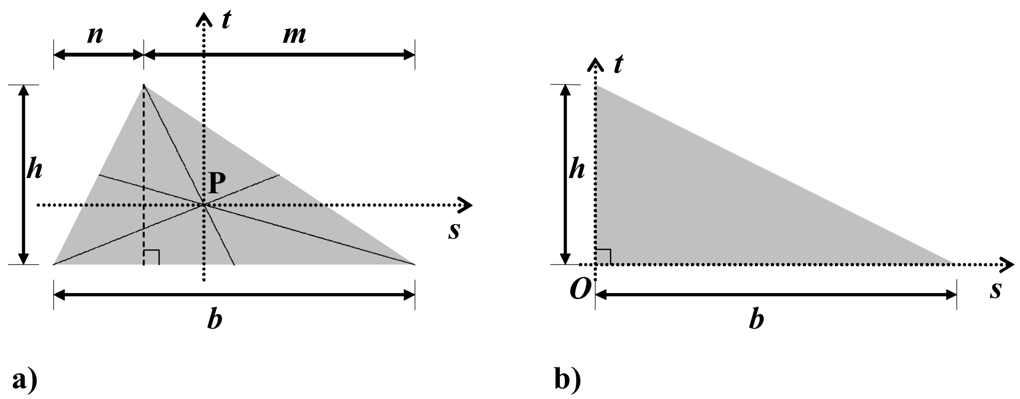


Figure 13. Triangle.

Product of inertia of the right-angled triangle with respect to point ***O***, Figure 13b, is

$$\boldsymbol{I_{st} = \frac{b^2h^2}{24}} \tag{99}$$

where
- ***b*** is base and
- ***h*** is height.

8 STIFFNESS QUANTITIES OF AREA

In the case of nonhomogeneous material, the quantities corresponding to geometric quantities of area discussed in previous chapter are obtained by replacing

$$dA \mapsto E(y,z)dA \tag{100}$$

where E is modulus of elasticity.

Axial stiffness for composite structure having homogenous material parts is

$$C = \sum_{i=1}^{n} E_i A_i \tag{101}$$

where n is the number of material or separate components i.

Correspondingly, statical moment multiplied by modulus of elasticity is

$$\Gamma = \sum_{i=1}^{n} E_i S_i \tag{102}$$

and bending stiffness is

$$D = \sum_{i=1}^{n} E_i I_i \tag{103}$$

9 CORE FIGURE

Equation of core figure boundary line, or neutral axis corresponding to load acting at point $\boldsymbol{p}$, in principal direction coordinate system of area, Figure 14, is

$$1+\frac{x_p}{i_y^2}x+\frac{y_p}{i_x^2}y=0 \tag{104}$$

where

- x_p is x-coordinate of “convex corner point” of area,
- y_p is y-coordinate of “convex corner point” of area,
- i_x is radius of gyration with respect to principal x-axis and
- i_y is radius of gyration with respect to principal y-axis.

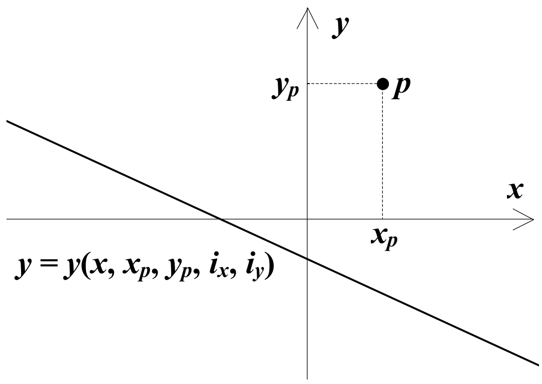


Figure 14. Neutral axis corresponding to load acting point.

10 DEFLECTION CURVE

10.1 Cantilever

Deflection curve due to point load $\boldsymbol{F}$, Figure 15, is

$$v = \begin{cases} \dfrac{Fb^2L}{6D}\left(3 - \dfrac{b}{L} - 3\dfrac{x}{L}\right), & x \in \{0...a\} \\ \dfrac{Fb^3}{6D}\left[2 - 3\dfrac{x-a}{b} + \left(\dfrac{x-a}{b}\right)^3\right], & x \in \{a...L\} \end{cases} \tag{105a, b}$$

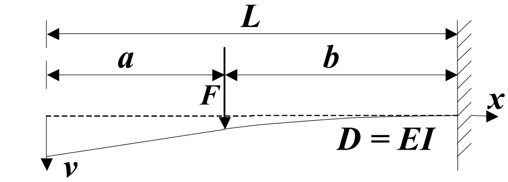


Figure 15. Cantilever loaded by point load $\boldsymbol{F}$.

Deflection curve due to point moment $\boldsymbol{M}$ at the end of the cantilever, Figure 16, is

$$v = \frac{M}{2D}(L - x)^2 \tag{106}$$

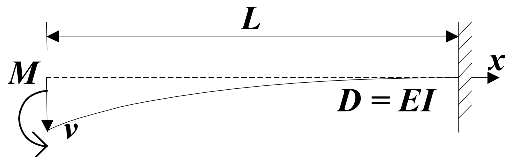


Figure 16. Cantilever loaded by point moment $\boldsymbol{M}$ at the end.

Deflection curve due to uniformly distributed line load $\boldsymbol{q}$ acting over the whole cantilever, Figure 17, is

$$v = \frac{qL^4}{24D}\left[3 - 4\frac{x}{L} + \left(\frac{x}{L}\right)^4\right] \tag{107}$$

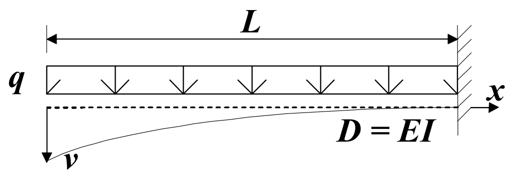


Figure 17. Cantilever loaded by uniformly distributed line load $\boldsymbol{q}$ acting over the whole length.

10.2 Simple Beam[3]

Deflection curve due to point load $\boldsymbol{F}$, Figure 18, is

$$v = \begin{cases} \dfrac{Fb}{6DL}x\left[a(L+b)-x^2\right], & x \in \{0...a\} \\ \dfrac{Fa}{6DL}(L-x)\left[b(L+a)-(L-x)^2\right], & x \in \{a...L\} \end{cases} \tag{108 a, b}$$

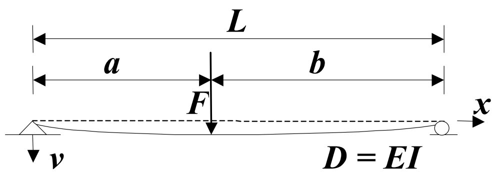


Figure 18. Simple beam loaded by point load ***F***.

Deflection curve due to point moment $\boldsymbol{M}$ at the end, Figure **19**, is

$$v = \frac{ML^2}{6D}\left(2\frac{x}{L} - 3\frac{x^2}{L^2} + \frac{x^3}{L^3}\right) \tag{109}$$

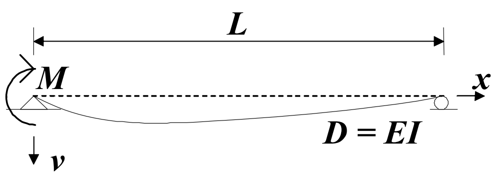


Figure 19. Simple beam loaded by point moment ***M*** at the end.

Deflection curve due to uniformly distributed line load $\boldsymbol{q}$ acting over the whole span, Figure 20, is

$$v = \frac{qL^4}{24D}\left[\frac{x}{L} - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^4\right] \tag{110}$$

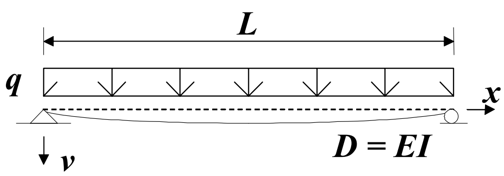


Figure 20. Simple beam loaded by uniformly distributed line load ***q*** acting over the whole length.

[3] Simply supported single span beam.

11 FOUNDATION COEFFICIENT

Foundation coefficient for triple layer foundation, Figure 21, is[4]

$$\boldsymbol{c} = \frac{1}{\dfrac{h_1}{E_1} + \dfrac{h_2}{E_2} + \dfrac{1}{c_3}} \tag{111}$$

where
- h_i, $i \in \{1, 2\}$, is height of layer i,
- E_i, $i \in \{1, 2\}$, is modulus of elasticity of layer i and
- c_3 is foundation coefficient for base soil.

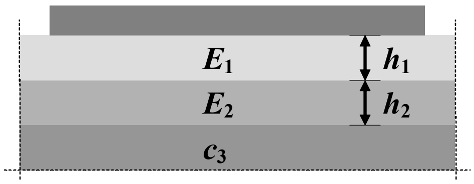


Figure 21. Foundation layers.

[4] BY 31 Finnish standard for concrete floors.

12 MODULUS OF ELASTICITY OF CONCRETE

12.1 Eurocode EN 1992-1-1

Modulus of elasticity of concrete is

$$E = k_o \left(\frac{f_{cm}}{f_{cmo}} \right)^{0,3} \tag{112}$$

where f_{cm} is mean value of cylinder compressive strength of concrete and constant

$$f_{cmo} = 10 \ \frac{\mathrm{MN}}{\mathrm{m}^2}$$

and

$$k_0 = 22 \cdot 10^3 \ \frac{\mathrm{MN}}{\mathrm{m}^2}$$

12.2 Finnish Standard B4

Modulus of elasticity of concrete with ordinary aggregate is

$$E = k \sqrt{K K_0} \tag{113}$$

where K is cubic strength or nominal strength of concrete and constant

$$K_0 = 25 \cdot 10^6 \ \frac{\mathrm{MN}}{\mathrm{m}^2}$$

and dimensionless coefficient

$$k = \min \begin{cases} \rho_c / \rho_0 \\ 1 \end{cases} \tag{114}$$

where ρ_c is density of concrete and constant

$$\rho_0 = 2400 \ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

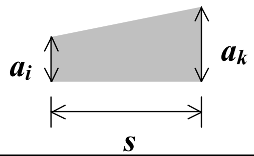
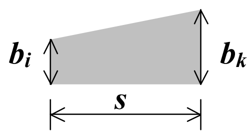
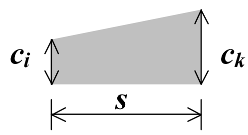
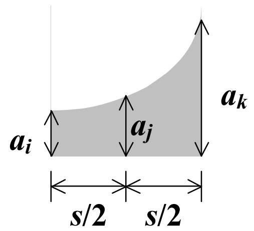
13 INTEGRAL OF PRODUCT

Integral of product of three functions in the cases of Table 3

$$F_a = s\left[\frac{1}{6}(a_i b_i c_i + a_k b_k c_k) + \frac{1}{6}(a_i + a_k)(b_i c_i + b_k c_k) + \\ -\frac{1}{12}(a_i + a_k)(b_i - b_k)(c_i - c_k)\right] \tag{115}$$

$$F_b = s\left[\frac{1}{6}(a_i b_i c_i + a_k b_k c_k) + \frac{1}{3}a_j(b_i c_i + b_k c_k) + \\ -\frac{1}{60}(a_i + 8a_j + a_k)(b_i - b_k)(c_i - c_k)\right] \tag{116}$$

Table 3.

<table>
<tr><th>$\int_0^s a(x)b(x)c(x)dx$</th><th>$a(x)$</th><th>$b(x)$</th><th>$c(x)$</th></tr>
<tr><td>F_a</td><td>
</td><td rowspan="2">
</td><td rowspan="2">
</td></tr>
<tr><td>F_b</td><td>

$a(x)$ is polynome of second order.</td></tr>
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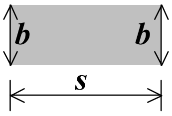
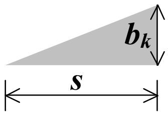
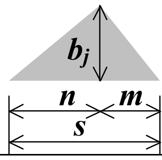
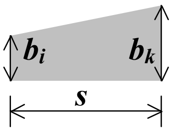
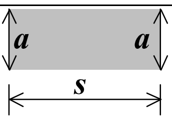
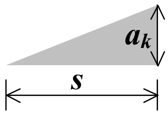
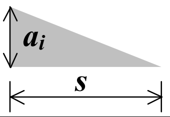
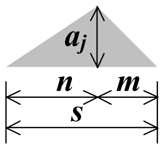
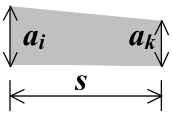
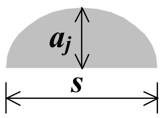
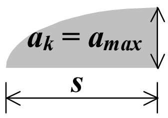
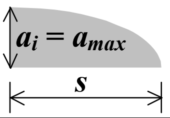
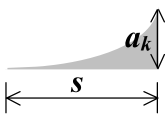
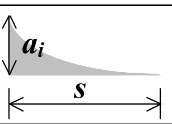
Integrals of product of two functions in several cases are presented in Table 4.

Table 4.

$\int_0^s a(x)b(x)dx$	b_1 	b_2 	b_3 	b_4 
a_1 	sab	$\frac{1}{2}sab_k$	$\frac{1}{2}sab_j$	$\frac{1}{2}sa(b_i+b_k)$
a_2 	$\frac{1}{2}sa_kb$	$\frac{1}{3}sa_kb_k$	$\frac{1}{6}(s+n)a_kb_j$	$\frac{1}{6}sa_k(b_i+2b_k)$
a_3 	$\frac{1}{2}sa_ib$	$\frac{1}{6}sa_ib_k$	$\frac{1}{6}(s+m)a_ib_j$	$\frac{1}{6}sa_i(2b_i+b_k)$
a_4 	$\frac{1}{2}sa_jb$	$\frac{1}{6}(s+n)a_jb_k$	$\frac{1}{3}sa_jb_j$	$\frac{1}{6}[(s+m)b_i+$ $+(s+n)b_k]a_j$
a_5 	$\frac{1}{2}s(a_i+a_k)b$	$\frac{1}{6}s(a_i+2a_k)b_k$	$\frac{1}{6}[(s+m)a_i+$ $+(s+n)a_k]\cdot b_j$	$\frac{1}{6}s[a_i(2b_i+b_k)+$ $+a_k(b_i+2b_k)]$
a_6 [5] 	$\frac{2}{3}sa_jb$	$\frac{1}{3}sa_jb_k$	$\frac{s^2+nm}{3s}a_jb_j$	$\frac{1}{3}sa_j(b_i+b_k)$
a_7 [5] 	$\frac{2}{3}sa_kb$	$\frac{5}{12}sa_kb_k$	$\frac{5s^2-ms-m^2}{12s}a_kb_j$	$\frac{1}{12}sa_k(3b_i+5b_k)$
a_8 [5] 	$\frac{2}{3}sa_ib$	$\frac{1}{4}sa_ib_k$	$\frac{5s^2-ns-n^2}{12s}a_ib_j$	$\frac{1}{12}sa_i(5b_i+3b_k)$
a_9 [5] 	$\frac{1}{3}sa_kb$	$\frac{1}{4}sa_kb_k$	$\frac{s^2+ns+n^2}{12s}a_kb_j$	$\frac{1}{12}sa_k(b_i+3b_k)$
a_{10} [5] 	$\frac{1}{3}sa_ib$	$\frac{1}{12}sa_ib_k$	$\frac{s^2+ms+m^2}{12s}a_ib_j$	$\frac{1}{12}sa_i(3b_i+b_k)$

[5] a_ι, $\iota\in\{6, 7, 8, 9, 10\}$, is polynome of second order.

14 MOMENT METHOD AND SLOPE-DEFLECTION METHOD

14.1 Moment Method

Rotations at corners by using directions shown in Figure 22 are

$$\begin{cases} \varphi_{ij} = \alpha_{ij} M_{ij} - \beta_{ij} M_{ji} + \psi_{ij} + \alpha_{ij}^{0} \\ \varphi_{ji} = \alpha_{ji} M_{ji} - \beta_{ji} M_{ij} + \psi_{ji} + \alpha_{ji}^{0} \end{cases} \tag{117a, b}$$

where M_{ij} and M_{ji} are corner moments.

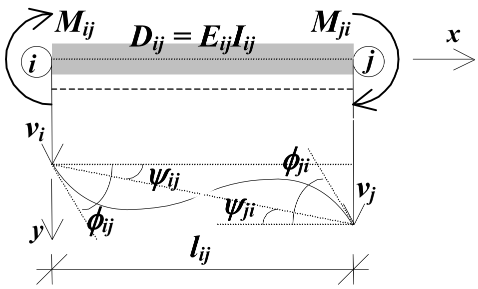


Figure 22. Corner moments and rotations of the moment method and of the slope-deflection method.

Member coefficients in the case of constant flexural rigidity are

$$\left.\begin{matrix} \alpha_{ij} \\ \alpha_{ji} \end{matrix}\right\} = \frac{l_{ij}}{3D_{ij}} \tag{118}$$

$$\left.\begin{matrix} \beta_{ij} \\ \beta_{ji} \end{matrix}\right\} = \frac{l_{ij}}{6D_{ij}} \tag{119}$$

where l_{ij} is length of the beam.

Bending stiffness of the beam is

$$D_{ij} = E_{ij} I_{ij} \tag{120}$$

where
- E_{ij} is modulus of elasticity and
- I_{ij} is moment of inertia of the cross-section.

Slope rotations are

$$\left.\begin{matrix} \psi_{ij} \\ \psi_{ji} \end{matrix}\right\} = \frac{v_j - v_i}{l_{ij}} \tag{121}$$

where v_i and v_j are displacements perpendicular to the beam at the ends i and j.

α_{ij}^{0} and α_{ji}^{0} are rotations at corners caused by loading and are given in Table 5 in different loading cases, when flexural rigidity is constant.

14.2 Slope-Deflection Method

Corner moments by using directions shown in Figure 22 are

$$\begin{cases} M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + M_{Kij} \\ M_{ji} = a_{ji}\varphi_{ji} + b_{ji}\varphi_{ij} - c_{ji}\psi_{ji} + M_{Kji} \end{cases} \tag{122a, b}$$

where φ_{ij} and φ_{ji} are rotations at the corners.

Member coefficients in the case of constant flexural rigidity are

$$\left.\begin{matrix} a_{ij} \\ a_{ji} \end{matrix}\right\} = \frac{4D_{ij}}{l_{ij}} \tag{123}$$

$$\left.\begin{matrix} b_{ij} \\ b_{ji} \end{matrix}\right\} = \frac{2D_{ij}}{l_{ij}} \tag{124}$$

and

$$\left.\begin{matrix} c_{ij} \\ c_{ji} \end{matrix}\right\} = \frac{6D_{ij}}{l_{ij}} \tag{125}$$

where l_{ij} is length of the beam. Bending stiffness of the beam is given in Equation 120.

Slope rotations are given in Equation 121.

M_{Kij} and M_{Kji} are corner moments caused by loading and are given in Table 5 in different loading cases, when flexural rigidity is constant.

If end j is pinned, the corner moment is

$$M_{ij} = a_{ij}^{0}\varphi_{ij} - c_{ij}^{0}\psi_{ij} + M_{Kij}^{0} \tag{126a}$$

Respectively, if end i is pinned, is

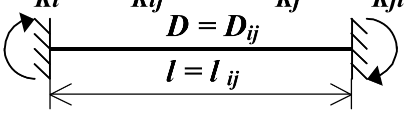
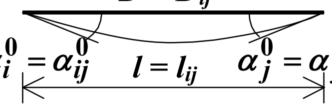
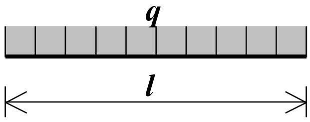
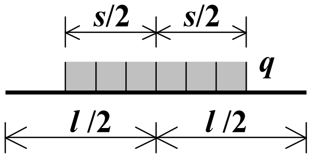
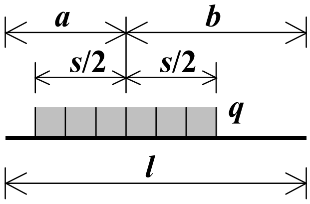
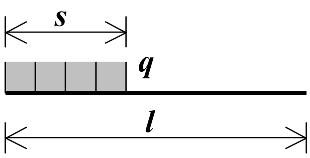
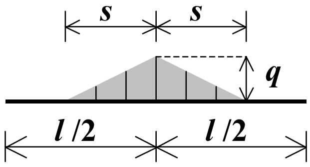
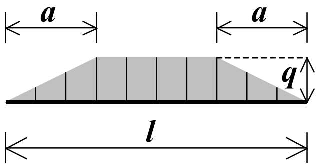
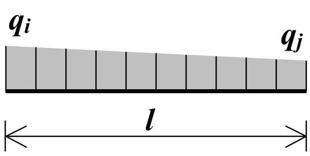
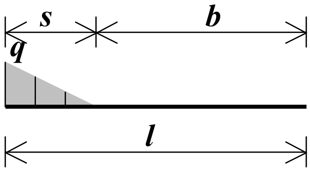
$$M_{ji} = a_{ji}^{0}\varphi_{ji} - c_{ji}^{0}\psi_{ji} + M_{Kji}^{0} \tag{126b}$$

Here member coefficients in the case of constant flexural rigidity are

$$\left.\begin{matrix} a_{ij}^{0} \\ a_{ji}^{0} \\ c_{ij}^{0} \\ c_{ji}^{0} \end{matrix}\right\} = \frac{3D_{ij}}{l_{ij}} \tag{127}$$

M_{Kij}^{0} and M_{Kji}^{0} are corner moments caused by loading and are given in Table 6 in different loading cases, when flexural rigidity is constant.

Table 5.

Loading	$M_{Ki}=M_{Kij}$ $M_{Kj}=M_{Kji}$ $D=D_{ij}$ $l=l_{ij}$ 	$D=D_{ij}$ $\alpha_i^0=\alpha_{ij}^0$ $l=l_{ij}$ $\alpha_j^0=\alpha_{ji}^0$ 
1 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-ql^2}{12}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{ql^3}{24D}$
2 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-qs}{24l}(3l^2-s^2)$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{qs}{48D}(3l^2-s^2)$
3 	$M_{Ki}=\frac{-qs}{12l^2}\left[12ab^2+s^2(l-3b)\right]$ $M_{Kj}=\frac{qs}{12l^2}\left[12a^2b+s^2(l-3a)\right]$	$\alpha_i^0=\frac{qbs}{24Dl}\left[4a(b+l)-s^2\right]$ $\alpha_j^0=\frac{-qas}{24Dl}\left[4b(a+l)-s^2\right]$
4 	$M_{Ki}=\frac{-qs^2}{12l^2}\left[2l(3l-4s)+3s^2\right]$ $M_{Kj}=\frac{qs^3}{12l^2}(4l-3s)$	$\alpha_i^0=\frac{qs^2}{24Dl}(2l-s)^2$ $\alpha_j^0=\frac{-qs^2}{24Dl}(2l^2-s^2)$
5 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-qs}{24l}(3l^2-2s^2)$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{qs}{48D}(3l^2-2s^2)$
6 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-q}{12l}\left[l^3-a^2(2l-a)\right]$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{q}{24D}\left[l^3-a^2(2l-a)\right]$
7 	$M_{Ki}=-l^2\left(\frac{q_i}{20}+\frac{q_j}{30}\right)$ $M_{Kj}=l^2\left(\frac{q_i}{30}+\frac{q_j}{20}\right)$	$\alpha_i^0=\frac{l^3}{D}\left(\frac{q_i}{45}+\frac{7q_j}{360}\right)$ $\alpha_j^0=\frac{-l^3}{D}\left(\frac{7q_i}{360}+\frac{q_j}{45}\right)$
8 	$M_{Ki}=\frac{-qs^2}{60l^2}(10bl+3s^2)$ $M_{Kj}=\frac{qs^3}{60l^2}(5b+2s)$	$\alpha_i^0=\frac{qs^2}{360Dl}\left[5b(4l+s)+8s^2\right]$ $\alpha_j^0=\frac{-qs^2}{360Dl}\left[10b(l+s)+7s^2\right]$

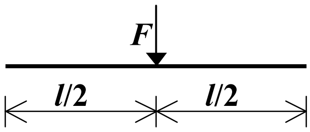
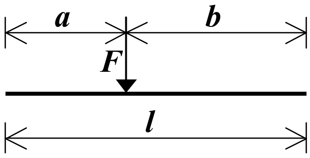
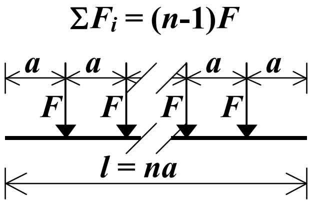
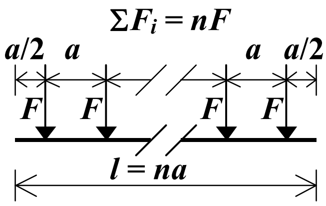
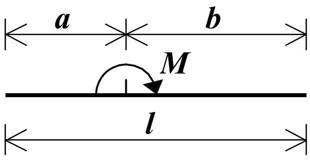
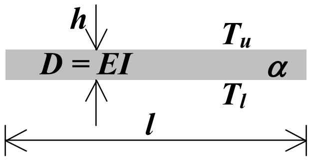
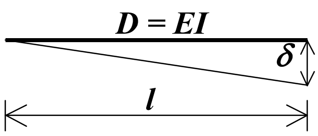
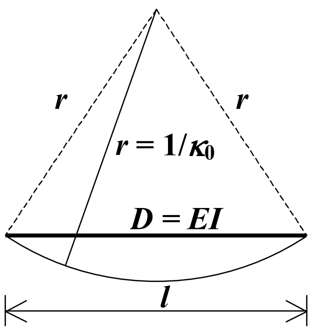
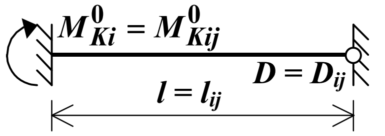
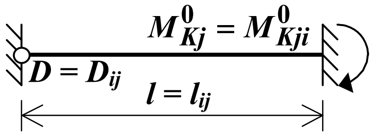
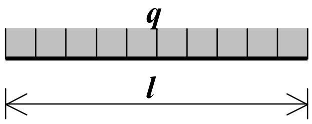
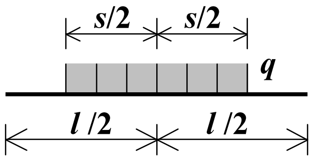
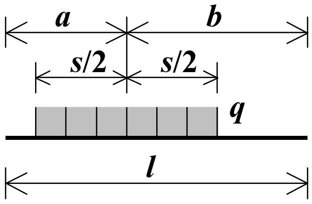
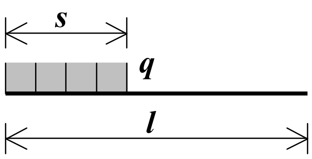
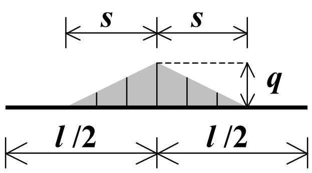
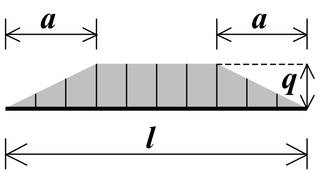
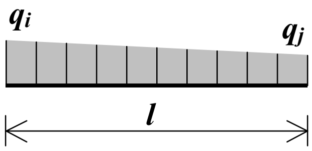
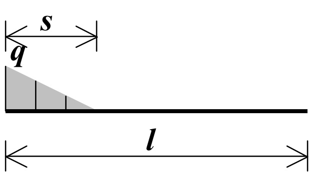
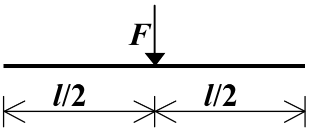
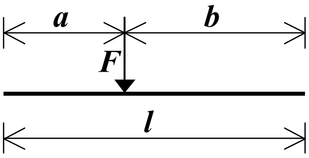
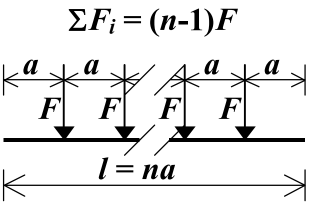
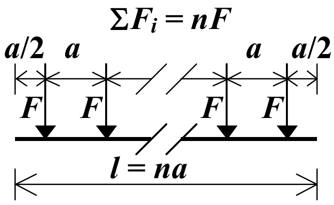
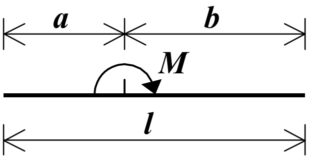
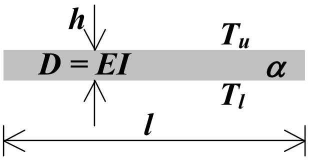
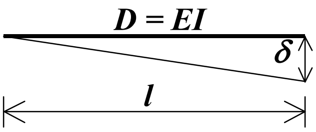
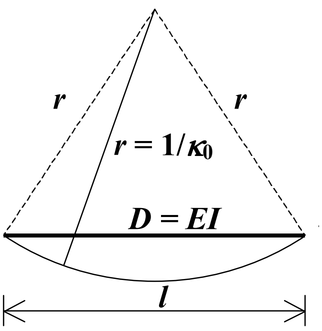
9 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-Fl}{8}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{Fl^2}{16D}$
10 	$M_{Ki}=\frac{-Fab^2}{l^2}$ $M_{Kj}=\frac{Fa^2b}{l^2}$	$\alpha_i^0=\frac{Fab}{6Dl}(b+l)$ $\alpha_j^0=\frac{-Fab}{6Dl}(a+l)$
11 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-Fl}{12}\cdot\frac{n^2-1}{n}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{Fl^2}{24D}\cdot\frac{n^2-1}{n}$
12 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{-Fl}{24}\cdot\frac{2n^2+1}{n}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{Fl^2}{48D}\cdot\frac{2n^2+1}{n}$
13 	$M_{Ki}=\frac{Mb}{l}\left(2-3\frac{b}{l}\right)$ $M_{Kj}=\frac{Ma}{l}\left(2-3\frac{a}{l}\right)$	$\alpha_i^0=\frac{Ml}{6D}\left(3\frac{b^2}{l^2}-1\right)$ $\alpha_j^0=\frac{Ml}{6D}\left(3\frac{a^2}{l^2}-1\right)$
14 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=\frac{\alpha(T_u-T_l)D}{h}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{-\alpha(T_u-T_l)l}{2h}$
15 	$\left.\begin{matrix}M_{Ki}\\ M_{Kj}\end{matrix}\right\}=\frac{-6D\delta}{l^2}$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{\delta}{l}$
16 	$\left.\begin{matrix}M_{Ki}\\ -M_{Kj}\end{matrix}\right\}=-D\kappa_0$	$\left.\begin{matrix}\alpha_i^0\\ -\alpha_j^0\end{matrix}\right\}=\frac{\kappa_0 l}{2}$

Table 6.

Loading		
1 	$\frac{-ql^2}{8}$	$\frac{ql^2}{8}$
2 	$\frac{-qs}{16l}(3l^2 - s^2)$	$\frac{qs}{16l}(3l^2 - s^2)$
3 	$\frac{-qbs}{8l^2}(4a^2 + 8ab - s^2)$	$\frac{qas}{8l^2}(4b^2 + 8ab - s^2)$
4 	$\frac{-qs^2}{8l^2}(2l - s)^2$	$\frac{qs^2}{8l^2}(2l^2 - s^2)$
5 	$\frac{-qs}{16l}(3l^2 - 2s^2)$	$\frac{qs}{16l}(3l^2 - 2s^2)$
6 	$\frac{-q}{8l}\left[l^3 - a^2(2l - a)\right]$	$\frac{q}{8l}\left[l^3 - a^2(2l - a)\right]$
7 	$-l^2\left(\frac{q_i}{15} + \frac{7q_j}{120}\right)$	$l^2\left(\frac{7q_i}{120} + \frac{q_j}{15}\right)$
8 	$\frac{-qs^2}{120l^2}(20l^2 - 15ls + 3s^2)$	$\frac{qs^2}{120l^2}(10l^2 - 3s^2)$

9 	$\dfrac{-3Fl}{16}$	$\dfrac{3Fl}{16}$
10 	$\dfrac{-Fab}{2l^2}(b+l)$	$\dfrac{Fab}{2l^2}(a+l)$
11 	$\dfrac{-Fl}{8}\cdot\dfrac{n^2-1}{n}$	$\dfrac{Fl}{8}\cdot\dfrac{n^2-1}{n}$
12 	$\dfrac{-Fl}{16}\cdot\dfrac{2n^2+1}{n}$	$\dfrac{Fl}{16}\cdot\dfrac{2n^2+1}{n}$
13 	$\dfrac{M}{2}\left(1-3\dfrac{b^2}{l^2}\right)$	$\dfrac{M}{2}\left(1-3\dfrac{a^2}{l^2}\right)$
14 	$\dfrac{3\alpha(T_u-T_l)D}{2h}$	$\dfrac{-3\alpha(T_u-T_l)D}{2h}$
15 	$\dfrac{-3D\delta}{l^2}$	$\dfrac{3D\delta}{l^2}$
16 	$-\dfrac{3}{2}D\kappa_0$	$\dfrac{3}{2}D\kappa_0$