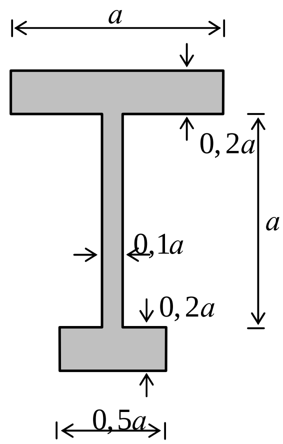
Osa II: Sauvarakenteiden plastiset menetelmät

4. Kimmoisen ideaaliplastisen poikkileikkauksen täysplastinen momentti

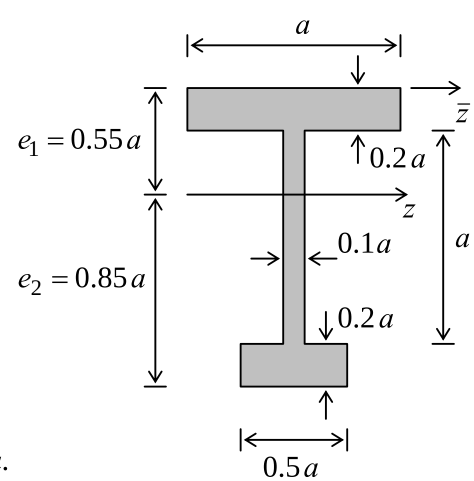
Tehtävä 4.1:

Määritä oheisen homogeenisen poikkileikkauksen (a) myötömomentti, (b) täysplastinen momentti ja (c) muotokerroin. Poikkileikkauksen myötöraja sekä vedossa että puristuksessa on σ_m.



Ratkaisu:

a) Myötömomentti:



Neutraaliakselin paikka:

$$e_1 = \frac{S_{\bar{z}}}{A},$$

missä

$$A = (1 \cdot 0.2 + 0.5 \cdot 0.2 + 0.1 \cdot 1)\,a^2 = 0.40\,a^2,$$

$$S_{\bar{z}} = (1 \cdot 0.2 \cdot 0.1 + 0.5 \cdot 0.2 \cdot 1.3 + 0.1 \cdot 1 \cdot 0.7)\,a^3$$
$$= 0.22\,a^3.$$

$$\Rightarrow \quad e_1 = \frac{S_{\bar{z}}}{A} = \frac{0.22\,a^3}{0.40\,a^2} = 0.55\,a \quad \Rightarrow \quad e_2 = 0.85\,a.$$

Myötöjännitys

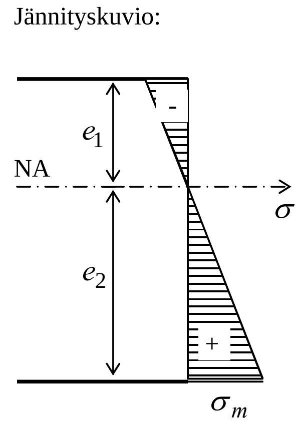
$$\sigma_m = \frac{M_m}{I_z} e_2$$

saavutetaan ensin poikkileikkauksen alareunassa. Jäyhyysmomentti on

$$I_z = \left(\frac{1 \cdot 0.2^3}{12} + 1 \cdot 0.2 \cdot 0.45^2 + \frac{0.1 \cdot 1^3}{12} + 0.1 \cdot 1 \cdot 0.15^2 + \frac{0.5 \cdot 0.2^3}{12} + 0.5 \cdot 0.2 \cdot 0.75^2\right) a^4 = 0.10833\,a^4,$$

joten myötömomentille saadaan

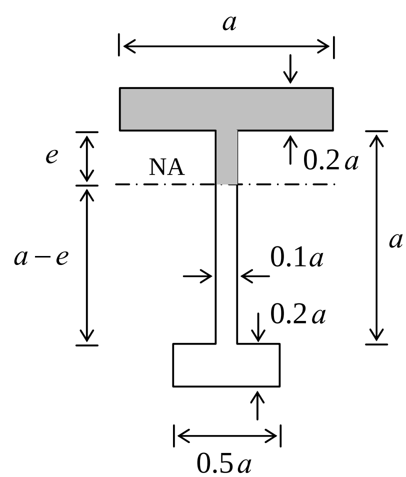
$$\underline{\underline{M_m}} = \frac{I_z}{e_2}\sigma_m = \frac{0.10833a^4}{0.85a}\sigma_m = \underline{\underline{0.1274a^3\sigma_m}}\ .$$



Taivutusvastus on
$W = 0.1274a^3$.

b) Täysplastinen momentti:

Neutraaliakselin paikka:
$N = \sigma_m A_{ala} - \sigma_m A_{ylä} = 0 \quad \Rightarrow$
$A_{ala} = A_{ylä}$,



missä

$$A_{ala} = 0.5 \cdot 0.2a^2 + 0.1a \cdot (a-e)$$
$$= 0.2a^2 - 0.1ae,$$
$$A_{ylä} = 1 \cdot 0.2a^2 + 0.1ae,$$

joten

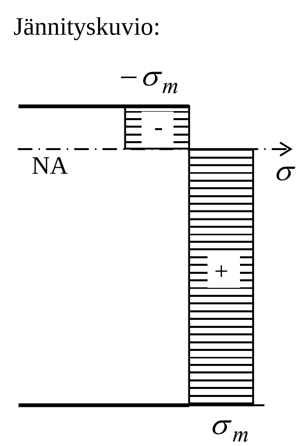
$$0.2a^2 - 0.1ae = 0.2a^2 + 0.1ae \quad \Rightarrow \quad e = 0.$$

Täysplastinen momentti on: $M_p = \sigma_m W_p$, missä plastinen taivutusvastus on:

$$W_p = S_{ala} + |S_{ylä}| = (0.5 \cdot 0.2 \cdot 1.1 + 0.1 \cdot 1 \cdot 0.5 + 1 \cdot 0.2 \cdot 0.1)a^3 = 0.18a^3,$$

joten

$$\underline{\underline{M_p = 0.18a^3\sigma_m}}\ .$$



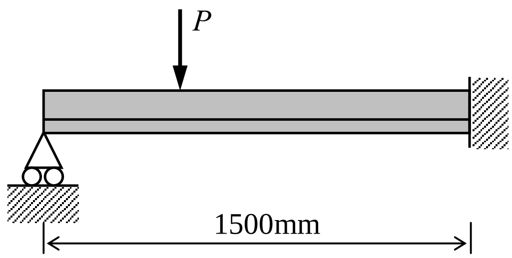
c) Muotokerroin:

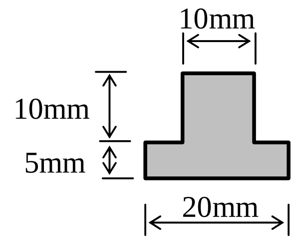
$$\underline{\underline{\phi}} = \frac{W_p}{W} = \frac{0.18}{0.1274} = \underline{\underline{1.413}}.$$

6. Rajakuorman määrittäminen virtuaalisen työn periaatteella

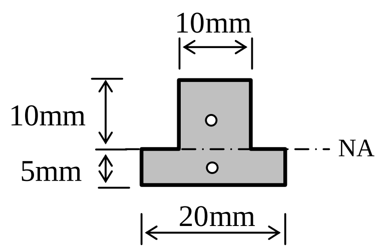
Tehtävä 6.1:

Oheisessa kuvassa on homogeeninen palkki ja sen poikkileikkaus. Määritä palkin täysplastinen momentti, kun materiaalin myötöraja sekä vedolle että puristukselle on 250N/mm^2. Määritä myös kuorman P pienin arvo, jolla rakenne menettää kantokykynsä, kun P voi vaikuttaa mielivaltaisessa kohdassa jännettä.





Ratkaisu:



Neutraaliakseli:

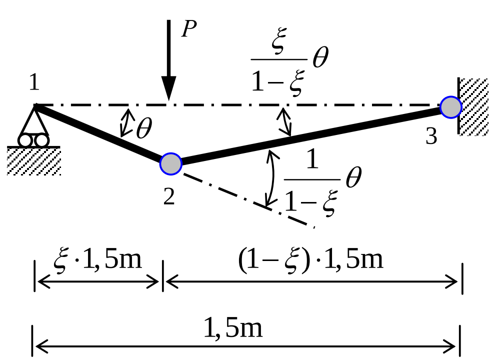
Kuviosta nähdään heti, että $A_1 = A_2$, kun neutraaliakseli NA on laipan yläreunan korkeudella. Saadaan

$$A_1 = 20\text{mm} \cdot 5\text{mm} = 100\text{mm}^2, \quad A_2 = 10\text{mm} \cdot 10\text{mm} = 100\text{mm}^2$$

$$S_1' = A_1 \cdot 2{,}5\text{mm} = 250\text{mm}^3, \quad |S_2'| = A_2 \cdot 5\text{mm} = 500\text{mm}^3$$

$$W_p = S_1' + |S_2'| = 250\text{mm}^3 + 500\text{mm}^3 = 750\text{mm}^3$$

$$M_p = \sigma_m (S_1' + |S_2'|) = 250\text{N/mm}^2 \cdot 750\text{mm}^3 = 187500\text{Nmm} = \underline{\underline{187{,}5\text{Nm}}}$$



Plastisten nivelten virtuaaliset kiertymät:

$$\theta_2 = \frac{1}{1-\xi}\theta, \quad \theta_3 = -\frac{\xi}{1-\xi}\theta$$

Virtuaalisen työn periaate:

$$-W_{\text{int}} = M_p\,|\theta_2| + M_p\,|\theta_3| = M_p\left(\frac{1}{1-\xi} + \frac{\xi}{1-\xi}\right)\theta = \frac{1+\xi}{1-\xi}M_p\theta$$

$$W_{\text{ext}} = P \cdot \xi \cdot 1{,}5\text{m} \cdot \theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Leftrightarrow\; P \cdot \xi \cdot 1{,}5\text{m} \cdot \theta = \frac{1+\xi}{1-\xi}M_p\theta \;\Rightarrow\; P = \frac{1+\xi}{\xi(1-\xi)}\frac{M_p}{1{,}5\text{m}}$$

Rajakuorma on siis

$$P_p = \frac{1+\xi}{\xi(1-\xi)}\frac{187{,}5\text{N}}{1{,}5} = \frac{1+\xi}{\xi(1-\xi)} \cdot 125\text{N}\,,$$

kun kuorma *P* on kohdassa $\xi \cdot 1{,}5\text{m}$.

Pienin rajakuorman arvo:

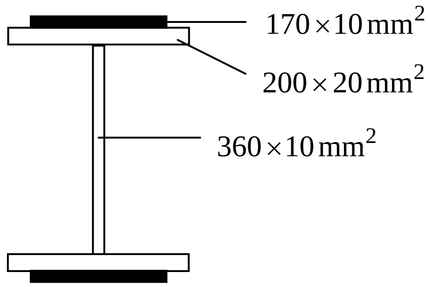
$$\frac{dP_p}{d\xi} = \frac{1+\xi}{\xi(1-\xi)} \cdot 125\text{N} = \frac{\xi(1-\xi)-(1-2\xi)(1+\xi)}{\xi^2(1-\xi)^2} \cdot 125\text{N} = \frac{\xi^2+2\xi-1}{\xi^2(1-\xi)^2} \cdot 125\text{N}$$

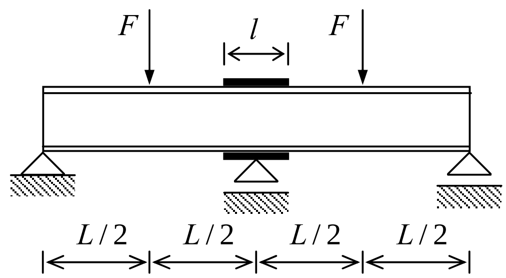
$$\frac{dP_p}{d\xi} = 0 \;\Leftrightarrow\; \xi^2+2\xi-1 = 0 \;\Rightarrow\; \xi = -1 \pm \sqrt{1+1} \;\Rightarrow\; \xi = \sqrt{2}-1$$

$$P_{p,\min} = \frac{\sqrt{2}}{(\sqrt{2}-1)(2-\sqrt{2})} \cdot 125\text{N} \approx \underline{\underline{728{,}6\text{N}}}$$

Tehtävä 6.2:

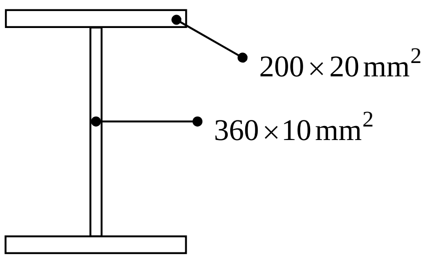
Kuvan mukaista kaksiaukkoista palkkia vahvistetaan keskituen kohdalta $l = 1.5\ \text{m}$ pituisilla peitelevyillä. Määritä palkin rajakuorma. Materiaalin myötöraja on $\sigma_m = 210\,\text{MPa}$. Pituus $L = 10\,\text{m}$.

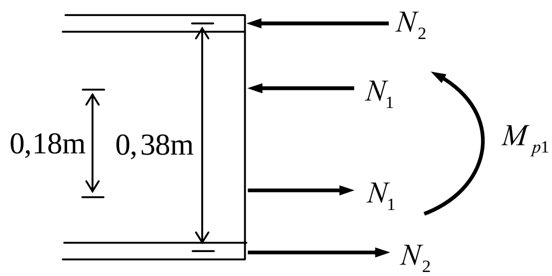




Ratkaisu:

Poikkileikkauksen täysplastinen momentti ilman peitelevyjä:



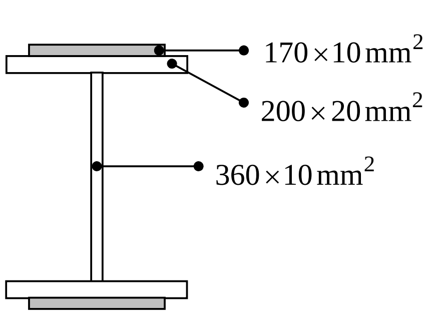


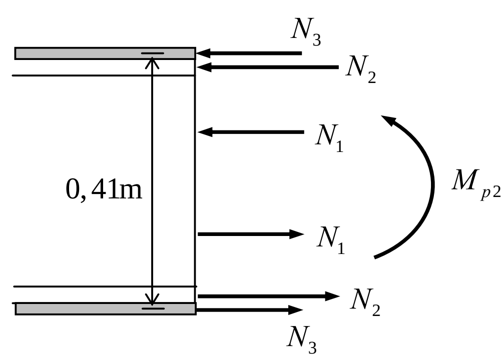
$N_1 = (0{,}18 \cdot 0{,}01 \cdot 210)\text{MN} = 0{,}378\text{MN}$

$N_2 = (0{,}2 \cdot 0{,}02 \cdot 210)\text{MN} = 0{,}84\text{MN}$

$M_{p1} = N_1 \cdot 0{,}18\text{m} + N_2 \cdot 0{,}38\text{m} = \underline{0{,}387\text{MNm}}$

Poikkileikkauksen täysplastinen momentti peitelevyjen kanssa:

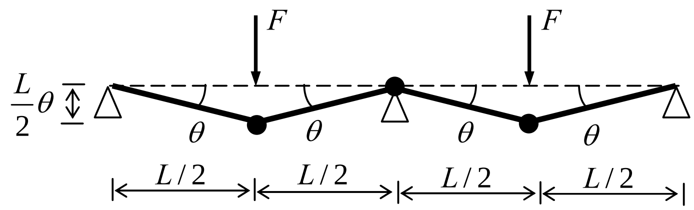




$N_3 = (0{,}17 \cdot 0{,}01 \cdot 210)\text{MN} = 0{,}357\text{MN}$

$M_{p2} = N_1 \cdot 0{,}18\text{m} + N_2 \cdot 0{,}38\text{m} + N_3 \cdot 0{,}41\text{m} = \underline{0{,}534\text{MNm}}$

Mekanismi 1:

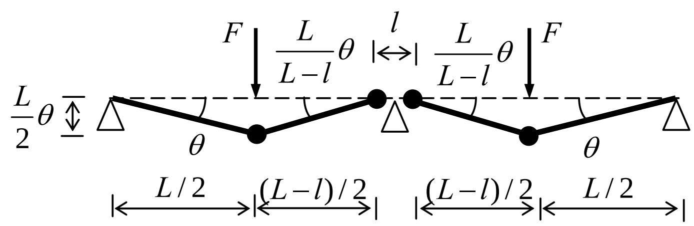


$$-W_{\text{int}} = 2 \cdot (M_{p1} \cdot 2\theta + M_{p2} \cdot \theta) = 2 \cdot (0,387 \cdot 2 + 0,534)\text{MNm} = 2,62\text{MNm} \cdot \theta$$

$$W_{\text{ext}} = 2 \cdot F \cdot \frac{L}{2}\theta = F\theta \cdot 5\text{m}$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; F_{p1} = 0,262\text{MN}$$

Mekanismi 2:



$$-W_{\text{int}} = 2 \cdot [M_{p1} \cdot (1 + \frac{L}{L-l})\theta + M_{p1} \cdot \frac{L}{L-l}\theta]$$

$$= 2 \cdot (0,387 \cdot 2,176 + 0,387 \cdot 1,176)\text{MNm} = 2,57\text{MNm} \cdot \theta$$

$$W_{\text{ext}} = 2 \cdot F \cdot \frac{L}{2}\theta = F\theta \cdot 5\text{m}$$

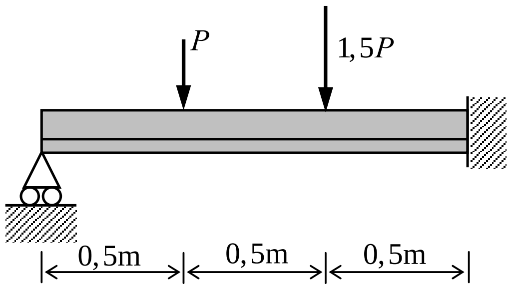
$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; F_{p2} = 0,257\text{MN}$$

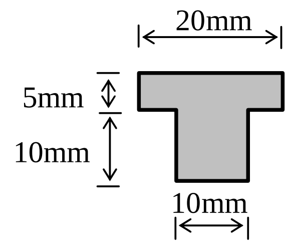
Rajakuorma:

$$F_p = \min\{F_{p1}, F_{p2}\} = \underline{\underline{0,257\text{MN}}}$$

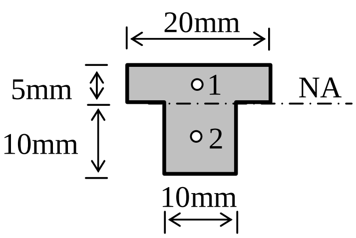
Tehtävä 6.3:

Oheisessa kuvassa on homogeeninen palkki ja sen poikkileikkaus. Määritä palkin täysplastinen momentti, kun materiaalin myötöraja sekä vedolle että puristukselle on 250N/mm^2. Määritä myös kuorman P arvo, jolla rakenne menettää kantokykynsä.





Ratkaisu:



Neutraaliakseli:

Kuviosta nähdään heti, että $A_1 = A_2$, kun neutraaliakseli NA on laipan alareunan korkeudella.

Täysplastinen momentti:

$$A_1 = 20\text{mm} \cdot 5\text{mm} = 100\text{mm}^2, \quad A_2 = 10\text{mm} \cdot 10\text{mm} = 100\text{mm}^2$$

$$S_1' = A_1 \cdot 2{,}5\text{mm} = 250\text{mm}^3, \quad |S_2'| = A_2 \cdot 5\text{mm} = 500\text{mm}^3$$

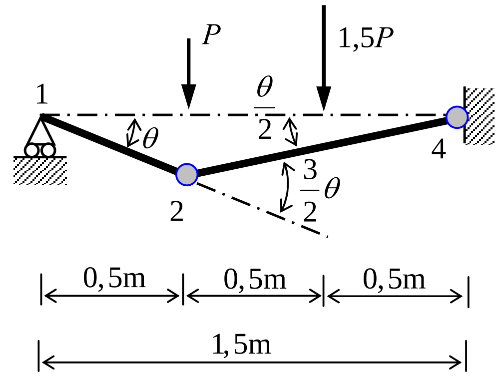
$$W_p = S_1' + |S_2'| = 250\text{mm}^3 + 500\text{mm}^3 = 750\text{mm}^3$$

$$M_p = \sigma_m W_p = 250\text{N/mm}^2 \cdot 750\text{mm}^3 = 187500\text{Nmm} = \underline{\underline{187{,}5\text{Nm}}}$$

Perusmekanismien lukumäärä:

$$p = m - n_s = 3 - 1 = 2$$

Mekanismi 1:



Plastisten nivelten virtuaaliset kiertymät:
$\theta_2 = \frac{3}{2}\theta, \quad \theta_4 = -\frac{\theta}{2}$

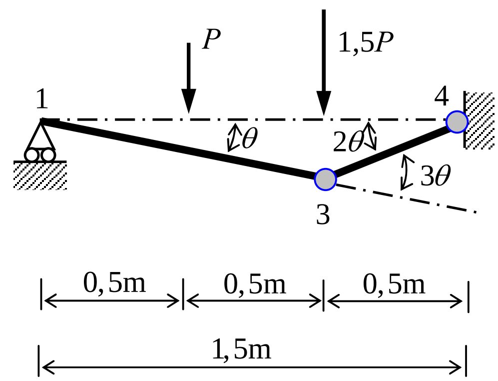
Virtuaalisen työn periaate:

$$-W_{\text{int}} = M_p\,|\theta_2| + M_p\,|\theta_4| = M_p(\frac{3}{2}\theta + \frac{\theta}{2}) = 2M_p\theta$$

$$W_{\text{ext}} = P \cdot 0,5\text{m} \cdot \theta + 1,5P \cdot 0,5\text{m} \cdot \frac{\theta}{2} = 0,875\text{m} \cdot P\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \iff 0,875\text{m} \cdot P\theta = 2M_p\theta \Rightarrow P = \frac{2,2857}{\text{m}} M_p$$

Mekanismi 2:



Plastisten nivelten virtuaaliset kiertymät:
$\theta_3 = 3\theta, \quad \theta_4 = -2\theta$

Virtuaalisen työn periaate:

$$-W_{\text{int}} = M_p\,|\theta_3| + M_p\,|\theta_4| = M_p(3\theta + 2\theta) = 5M_p\theta$$

$$W_{\text{ext}} = P \cdot 0,5\text{m} \cdot \theta + 1,5P \cdot 0,5\text{m} \cdot 2\theta = 2\text{m} \cdot P\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \iff 2\text{m} \cdot P\theta = 5M_p\theta \Rightarrow P = \frac{2,5}{\text{m}} M_p$$

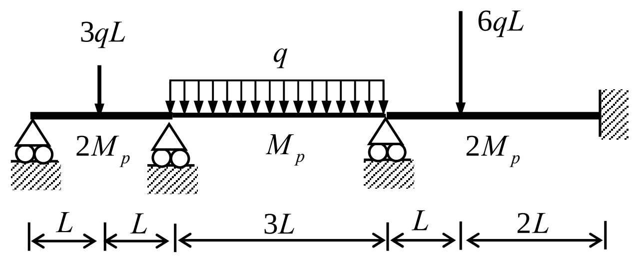
Plastinen rajakuorma:

Mekanismi 1 on sortumismekanismi ja

$$P_p = \frac{2,2857}{\text{m}} M_p = \frac{2,2857}{\text{m}} 187,5\text{Nm} = \underline{\underline{428,6\text{N}}}$$

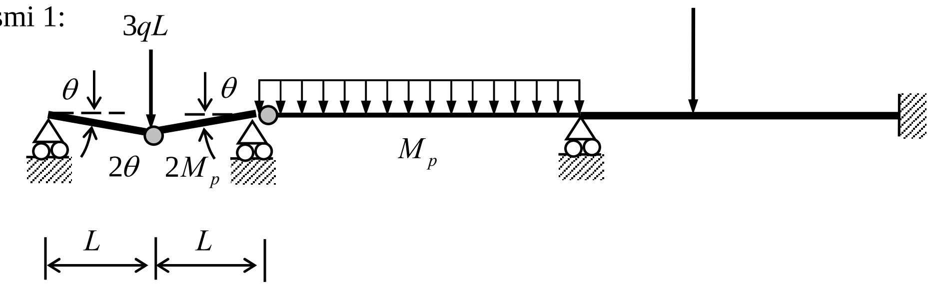
Tehtävä 6.4:

Määritä oheisen jatkuvan palkin plastinen rajakuorma q_p.



Ratkaisu:

Mekanismi 1:

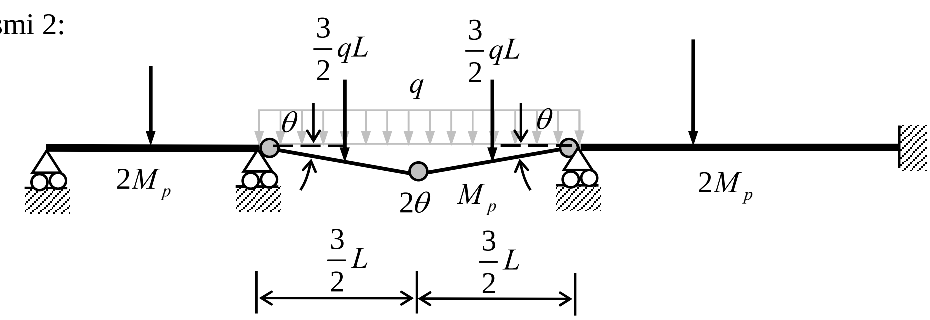


$$-W_{\text{int}} = 2M_p \cdot 2\theta + M_p \cdot \theta = 5M_p\theta, \quad W_{\text{ext}} = 3qL \cdot \theta L = 3qL^2\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; 3qL^2\theta = 5M_p\theta \;\Rightarrow\; \underline{q = \frac{5}{3}\frac{M_p}{L^2}}$$

Huom! Oikean puoleinen plastinen nivel on hieman tuen olikealla puolella, jossa täysplastinen momentti on M_p.

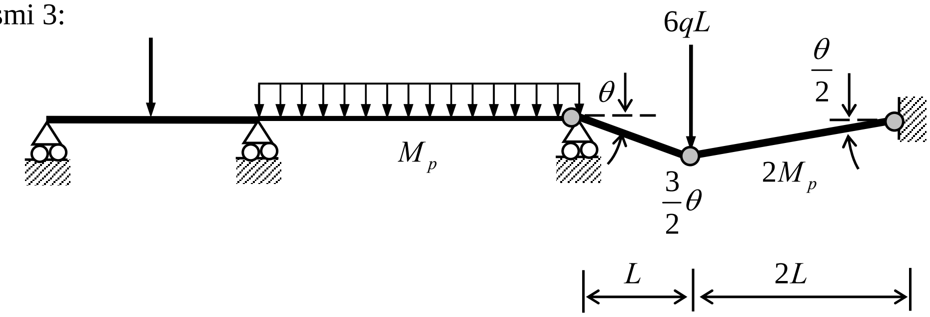
Mekanismi 2:



$$-W_{\text{int}} = M_p \cdot \theta + M_p \cdot 2\theta + M_p \cdot \theta = 4M_p\theta, \quad W_{\text{ext}} = \frac{3}{2}qL \cdot \frac{3}{4}L\theta + \frac{3}{2}qL \cdot \frac{3}{4}L\theta = \frac{9}{4}qL^2\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; \frac{9}{4}qL^2\theta = 4M_p\theta \;\Rightarrow\; \underline{q = \frac{16}{9}\frac{M_p}{L^2}}$$

Mekanismi 3:



$$-W_{\text{int}} = M_p \cdot \theta + 2M_p \cdot \frac{3}{2}\theta + 2M_p \cdot \frac{\theta}{2} = 5M_p\theta, \quad W_{\text{ext}} = 6qL \cdot L\theta = 6qL^2\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; 6qL^2\theta = 5M_p\theta \;\Rightarrow\; \underline{q = \frac{5}{6}\frac{M_p}{L^2}}$$

Huom! Vasemman puoleinen plastinen nivel on hieman tuen vasemmalla puolella, jossa täysplastinen momentti on M_p.

Saadaan siis:

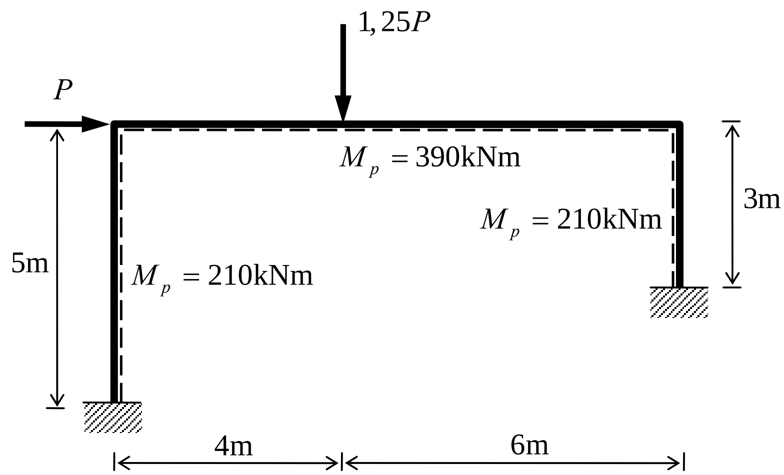
$$q_p = \frac{5}{6}\frac{M_p}{L^2} \approx 0{,}8333\frac{M_p}{L^2}$$

Tässä tapauksessa ei löydy yhdistelmämekanismia, joka antaisi pienemmän arvon q_p:lle.

7. Mekanismimenetelmä

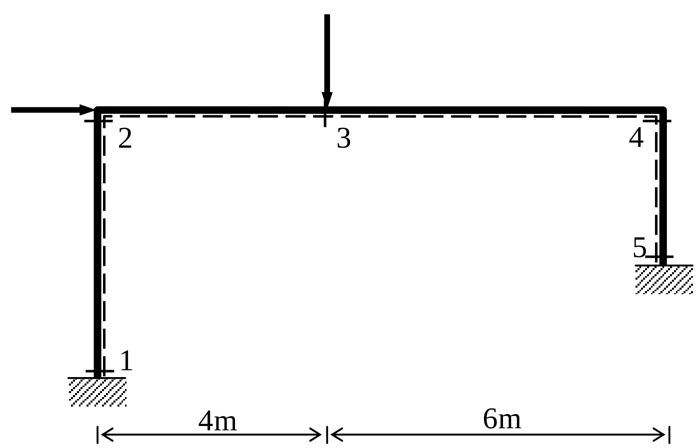
Tehtävä 7.1:

Määritä oheisen kehän plastinen rajakuorma



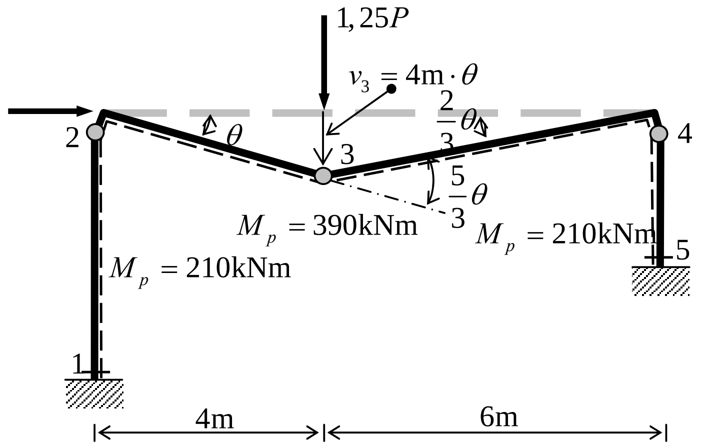
Ratkaisu:

Nivelten paikat ja perusmekanismien lukumäärä:



$p = m - n_s = 5 - 3 = 2$

Mekanismi (a): Palkkimekanismi



Mekanismin siirtymätilaa kuvaavaksi parametriksi otetaan sauvan $2-3$ kaltevuuskulma θ. Geometrisellä tarkastelulla saadaan helposti kuvassa esitetyt kulmat ja nivelen 3 pystysiirtymä v_3 parametrin θ avulla lausuttuna.

Nivelten kulmanmuutokset:

$$\theta_1 = 0,\ \theta_2 = -\theta,\ \theta_3 = \frac{5}{3}\theta,\ \theta_4 = -\frac{2}{3}\theta,\ \theta_5 = 0.$$

Virtuaalisen työn periaate:

$$-W_{\text{int}} = \sum M_{pi}\left|\theta_i\right| = M_{p2}\left|\theta_2\right| + M_{p3}\left|\theta_3\right| + M_{p4}\left|\theta_4\right|$$

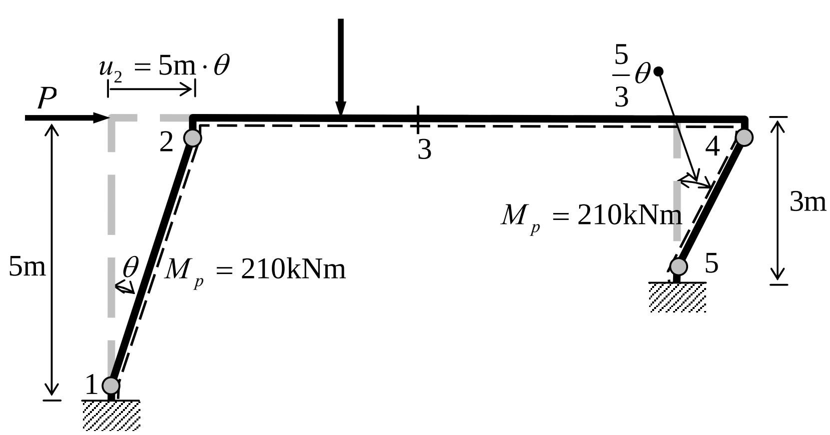
$$= 210\text{kNm}\cdot\theta + 390\text{kNm}\cdot\frac{5}{3}\theta + 210\text{kNm}\cdot\frac{2}{3}\theta = 1000\text{kNm}\cdot\theta$$

$$W_{\text{ext}} = 1,25P\cdot v_3 = 1,25P\cdot 4\text{m}\cdot\theta = 5\text{m}\cdot P\theta$$

$$-W_{\text{int}} = W_{\text{ext}} \iff 1000\text{kNm}\cdot\theta = 5\text{m}\cdot P\theta$$

$$\Rightarrow \underline{P_p^{(a)} = 200\text{kN}}$$

Mekanismi (b): Sivusiirtymämekanismi



Mekanismin siirtymätilaa kuvaavaksi parametriksi otetaan sauvan $1-2$ kaltevuuskulma θ.

Nivelten kulmanmuutokset:

$$\theta_1 = -\theta,\ \theta_2 = \theta,\ \theta_3 = 0,\ \theta_4 = -\frac{5}{3}\theta,\ \theta_5 = \frac{5}{3}\theta.$$

Virtuaalisen työn periaate:

$$\begin{aligned} -W_{\text{int}} &= \sum M_{pi}\left|\theta_i\right| = M_{p1}\left|\theta_1\right| + M_{p2}\left|\theta_2\right| + M_{p4}\left|\theta_4\right| + M_{p5}\left|\theta_5\right| \\ &= 210\text{kNm}\cdot\theta + 210\text{kNm}\cdot\theta + 210\text{kNm}\cdot\frac{5}{3}\theta + 210\text{kNm}\cdot\frac{5}{3}\theta \\ &= 1120\text{kNm}\cdot\theta \end{aligned}$$

$$W_{\text{ext}} = P\cdot u_2 = P\cdot 5\text{m}\cdot\theta = 5\text{m}\cdot P\theta$$

$$-W_{\text{int}} = W_{\text{ext}} \iff 1120\text{kNm}\cdot\theta = 5\text{m}\cdot P\theta$$

$$\Rightarrow \underline{P_p^{(b)} = 224\text{kN}}$$

Mekanismien yhdistely:

Mekanismi	θ_1	θ_2	θ_3	θ_4	θ_5	$-W_{\text{int}}$	W_{ext}	P_p
(a)	0	$-\theta$	$\frac{5}{3}\theta$	$-\frac{2}{3}\theta$	0	$1000\text{kNm}\cdot\theta$	$5\text{m}\cdot P\theta$	200kN
(b)	$-\theta$	θ	0	$-\frac{5}{3}\theta$	$\frac{5}{3}\theta$	$1120\text{kNm}\cdot\theta$	$5\text{m}\cdot P\theta$	224kN
(c)=(a)+(b)	$-\theta$	0	$\frac{5}{3}\theta$	$-\frac{7}{3}\theta$	$\frac{5}{3}\theta$	$1700\text{kNm}\cdot\theta$	$10\text{m}\cdot P\theta$	170kN
$(\text{d}) = (\text{a}) - \frac{2}{5}(\text{b})$	$\frac{2}{5}\theta$	$-\frac{7}{5}\theta$	$\frac{5}{3}\theta$	0	$-\frac{2}{3}\theta$	$1168\text{kNm}\cdot\theta$	$3\text{m}\cdot P\theta$	389,3kN

Yhdistelmämekanismien nivelten kulmanmuutokset saadaan laskemalla perusmekanismien nivelten kulmanmuutosten ao. lineaarikombinaatiot. Yhdistelmämekanismi (c) saatiin hävittämällä nivel 2 ja yhdistelmämekanismi (d) hävittämällä nivel 4. Yhdistelmämekanismien sisäiset virtuaaliset työt lasketaan tavanomaiseen tapaan:

Mekanismi (c):

$$\begin{aligned} W_{\text{int}} &= \sum M_{pi}\left|\theta_i\right| \\ &= 210\text{kNm}\cdot\theta + 390\text{kNm}\cdot\frac{5}{3}\theta + 210\text{kNm}\cdot\frac{7}{3}\theta + 210\text{kNm}\cdot\frac{5}{3}\theta \\ &= 1700\text{kNm}\cdot\theta \end{aligned}$$

Mekanismi (d):

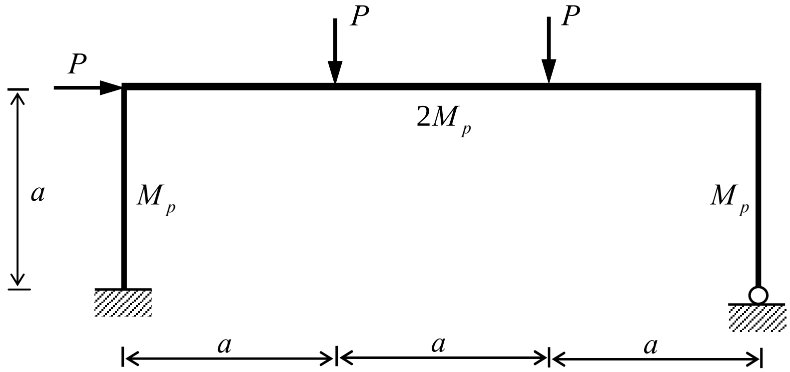
$$\begin{aligned} W_{\text{int}} &= \sum M_{pi} \left|\theta_i\right| \\ &= 210\text{kNm} \cdot \frac{2}{5}\theta + 210\text{kNm} \cdot \frac{7}{5}\theta + 390\text{kNm} \cdot \frac{5}{3}\theta + 210\text{kNm} \cdot \frac{2}{3}\theta \\ &= 1168\text{kNm} \cdot \theta \end{aligned}$$

Yhdistelmämekanismien ulkoiset virtuaaliset työt saadaan laskemalla perusmekanismien virtuaalisten töiden ao. lineaarikombinaatiot. Yhdistelmämekanismien rajakuormat saadaan lopulta ratkaisemalla ao. virtuaalisen työn yhtälöt. Tulokset on merkitty taulukkoon.

Nähdään, että mekanismi (c) antaa pienimmän rajakuorman $\underline{\underline{P_p = 170\text{kN}}}$.

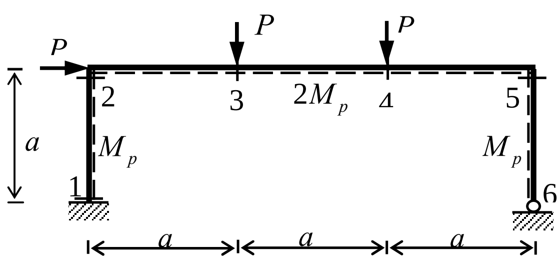
Tehtävä 7.2:

Oheisen kehän vasen tuki on jäykästi kiinnitetty ja oikea on nivelellisesti tuettu. Sen pilarien täysplastinen momentti on M_p ja vaakasauvan $2M_p$. Kuinka suuren kuorman P rakenne kestää.



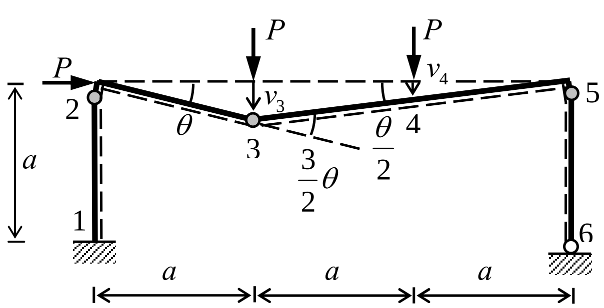
Ratkaisu:

Plastisten nivelten paikat ja perusmekanismien lukumäärä:



$n_s = t + 3r - c - 3 = 5 + 3 \cdot 0 - 0 - 3 = 2$, $p = m - n_s = 5 - 3 = 2$

<u>Palkkimekanismi (a):</u>



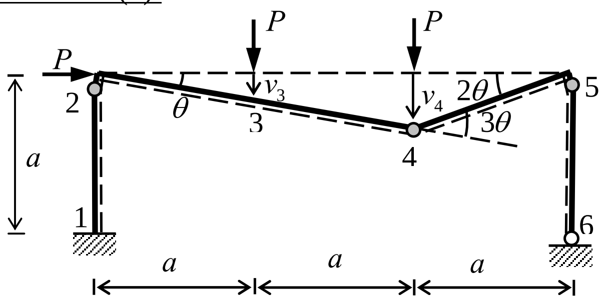
$$\theta_1 = 0,\ \theta_2 = -\theta,\ \theta_3 = \frac{3}{2}\theta,\ \theta_4 = 0,\ \ \theta_5 = -\frac{\theta}{2}$$

$$-W_{\text{int}} = M_p\,|\theta_2| + 2M_p\,|\theta_3| + M_p\,|\theta_5| = M_p(\theta + 2 \cdot \frac{3}{2}\theta + \frac{\theta}{2}) = \frac{9}{2}M_p\theta$$

$$W_{\text{ext}} = Pv_3 + Pv_4 = P \cdot \theta a + P \cdot \frac{\theta}{2} a = \frac{3}{2} Pa\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Leftrightarrow\; \frac{3}{2} Pa\theta = \frac{9}{2} M_p \theta \;\Rightarrow\; \underline{P = 3\frac{M_p}{a}}$$

<u>Palkkimekanismi (b):</u>



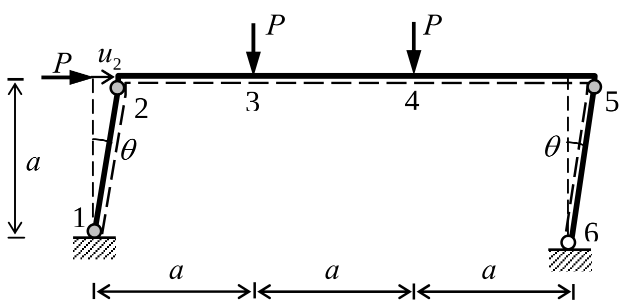
$$\theta_1 = 0,\; \theta_2 = -\theta,\; \theta_3 = 0,\; \theta_4 = 3\theta,\; \theta_5 = -2\theta$$

$$-W_{\text{int}} = M_p\,|\theta_2| + 2M_p\,|\theta_4| + M_p\,|\theta_5| = M_p(2\theta + 2\cdot 3\theta + \theta) = 9M_p\theta$$

$$W_{\text{ext}} = Pv_3 + Pv_4 = P\cdot\theta a + P\cdot 2\theta a = 3Pa\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Leftrightarrow\; 3Pa\theta = 9M_p\theta \;\Rightarrow\; \underline{P = 3\frac{M_p}{a}}$$

<u>Sivusiirtymämekanismi (c):</u>



$$\theta_1 = -\theta,\; \theta_2 = +\theta,\; \theta_3 = 0,\; \theta_4 = 0,\; \theta_5 = -\theta$$

$$-W_{\text{int}} = M_p\,|\theta_1| + M_p\,|\theta_2| + M_p\,|\theta_5| = M_p(\theta + \theta + \theta) = 3M_p\theta$$

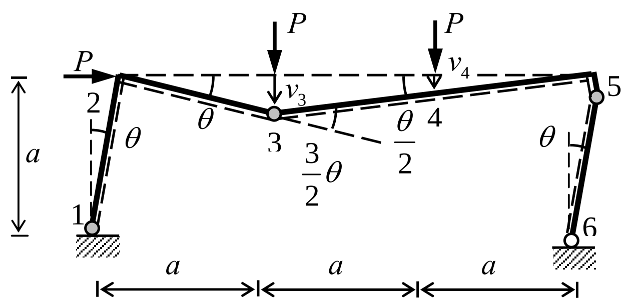
$$W_{\text{ext}} = Pu_2 = P\cdot\theta a = Pa\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \;\Leftrightarrow\; Pa\theta = 3M_p\theta \;\Rightarrow\; \underline{P = 3\frac{M_p}{a}}$$

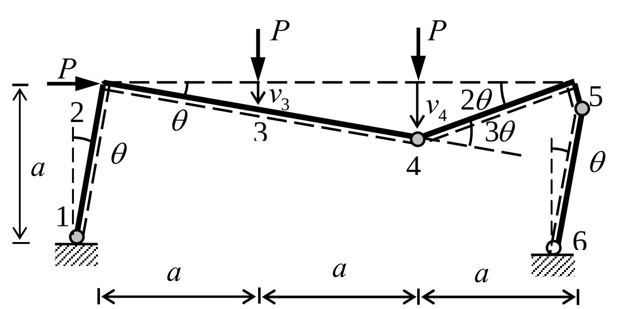
Mekanismien yhdistely:

Mekanismi	θ_1	θ_2	θ_3	θ_4	θ_5	W_{ext}	W_{int}	P
(a)	0	$-\theta$	$\frac{3}{2}\theta$	0	$-\frac{\theta}{2}$	$\frac{3}{2}Pa\theta$	$\frac{9}{2}M_p\theta$	$3\frac{M_p}{a}$
(b)	0	$-\theta$	0	3θ	-2θ	$3Pa\theta$	$9M_p\theta$	$3\frac{M_p}{a}$
(c)	$-\theta$	θ	0	0	$-\theta$	$Pa\theta$	$3M_p\theta$	$3\frac{M_p}{a}$
(d) = (a) + (c)	$-\theta$	0	$\frac{3}{2}\theta$	0	$-\frac{3}{2}\theta$	$\frac{5}{2}Pa\theta$	$\frac{11}{2}M_p\theta$	$\underline{2{,}2\frac{M_p}{a}}$
(e) = (b) + (c)	$-\theta$	0	0	3θ	-3θ	$4Pa\theta$	$10M_p\theta$	$2{,}5\frac{M_p}{a}$

Mekanismi (d):



Mekanismi (e):



Mekanismi (d) antaa pienimmän kuorman P:

Taivutusmomentti pisteessä 2:
Virtuaalinen siirtymätila mekanismi (c): $\theta_1 = -\theta,\ \theta_2 = \theta,\ \theta_3 = 0,\ \theta_4 = 0,\ \ \theta_5 = -\theta$

$$-W_{\text{int}} = M_1\theta_1 + M_2\theta_2 + M_5\theta_5 = -M_p(-\theta) + M_2\theta + (-M_p)(-\theta) = (M_2 + 2M_p)\theta$$

$$W_{\text{ext}} = Pu_2 = P\cdot\theta a = \frac{11}{5}\frac{M_p}{a}\cdot\theta a = \frac{11}{5}M_p\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \ \Rightarrow\ \frac{11}{5}M_p\theta = (M_2 + 2M_p)\theta \ \Leftrightarrow\ \underline{M_2 = \frac{1}{5}M_p}$$

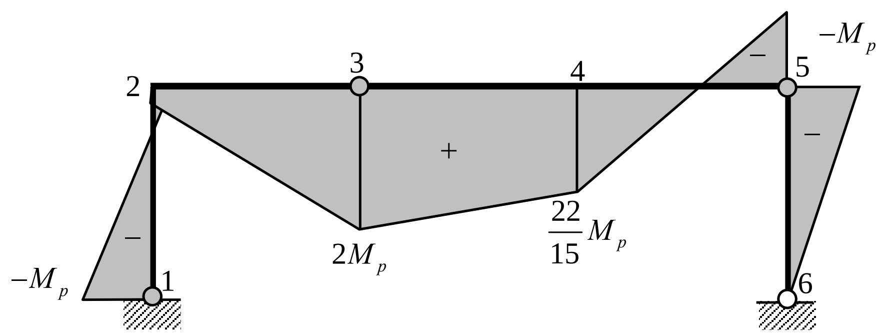
Taivutusmomentti pisteessä 4:
Virtuaalinen siirtymätila mekanismi (b): $\theta_1 = 0,\ \theta_2 = -\theta,\ \theta_3 = 0,\ \theta_4 = 3\theta,\ \ \theta_5 = -2\theta$

$$-W_{\text{int}} = M_2\theta_2 + M_4\theta_4 + M_5\theta_5 = \frac{1}{5}M_p\theta + M_4 3\theta + (-M_p)(-2\theta) = (3M_4 + \frac{11}{5}M_p)\theta$$

$$W_{\text{ext}} = Pv_3 + Pv_4 = \frac{11}{5}\frac{M_p}{a}\cdot\theta a + \frac{11}{5}\frac{M_p}{a}\cdot 2\theta a = \frac{33}{5}M_p\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \ \Rightarrow\ \frac{33}{5}M_p\theta = (3M_4 + \frac{11}{5}M_p)\theta \ \Leftrightarrow\ \underline{M_4 = \frac{22}{15}M_p}$$

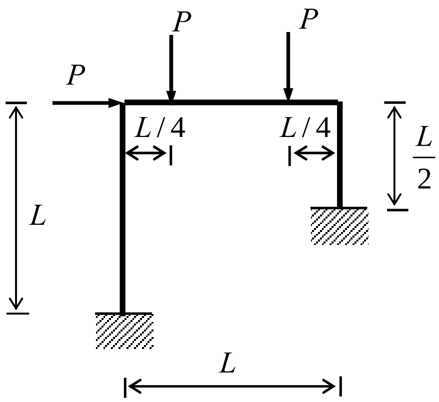
Sortumismekanismin taivutusmomenttikuvio:



Tulos: $\underline{\underline{P_p = 2,2\frac{M_p}{a}}}$

Tehtävä 7.3:

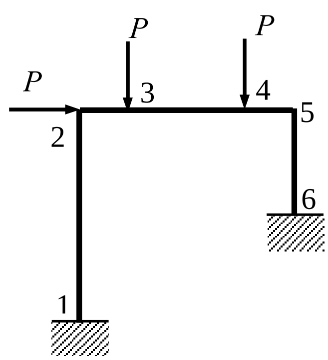
Oheisen kehän täysplastinen momentti on M_p. Kuinka suuren kuorman P rakenne kestää.



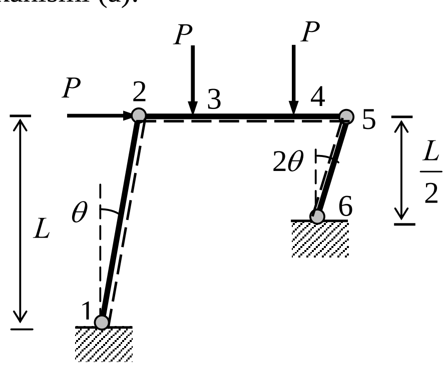
Ratkaisu:

Perusmekanismit:

Nivelten paikat:



Mekanismi (a):

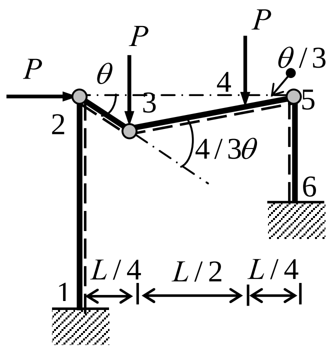


$\theta_1 = -\theta,\ \theta_2 = \theta,\ \theta_5 = -2\theta\ \ \theta_6 = 2\theta$

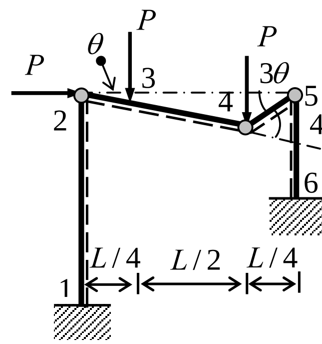
$-W_{\text{int}} = M_p(\theta + \theta + 2\theta + 2\theta) = 6M_p\theta$

$W_{\text{ext}} = PL\theta$

Mekanismi (b):



Mekanismi (c):



$\theta_2 = -\theta,\ \theta_3 = 4/3\theta,\ \theta_5 = -\theta/3$

$-W_{\text{int}} = M_p(\theta + 4/3\theta + \theta/3) = 8/3 M_p\theta$

$W_{\text{ext}} = P \cdot L/4 \cdot \theta + P \cdot L/4 \cdot \theta/3 = PL\theta/3$

$\theta_2 = -\theta,\ \theta_4 = 4\theta,\ \theta_5 = -3\theta$

$-W_{\text{int}} = M_p(\theta + 4\theta + 3\theta) = 8M_p\theta$

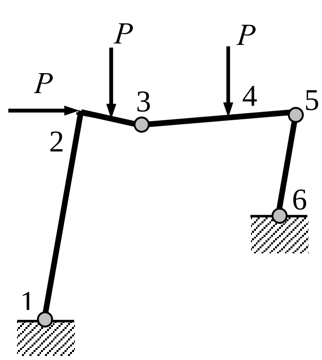
$W_{\text{ext}} = P \cdot L/4 \cdot \theta + P \cdot L/4 \cdot 3\theta = PL\theta$

Mekanismien yhdistely:

Mekanismi	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	$-W_{\text{int}}$	W_{ext}	P
(a)	$-\theta$	θ			-2θ	2θ	$6M_p\theta$	$PL\theta$	$6M_p/L$
(b)		$-\theta$	$4\theta/3$		$-\theta/3$		$8/3M_p\theta$	$PL\theta/3$	$8M_p/L$
(c)		$-\theta$		4θ	-3θ		$8M_p\theta$	$PL\theta$	$8M_p/L$
(d)=(a)+(b)	$-\theta$		$4\theta/3$		$-7\theta/3$	2θ	$20/3M_p\theta$	$4/3PL\theta$	$5M_p/L$
(e)=(a)+(c)	$-\theta$			4θ	-5θ	2θ	12θ	$2PL\theta$	$6M_p/L$

Mekanismi (d) antaa rajakuorman
$P_p = 5M_p/L,$
mikäli myötöehtoa ei ylitetä.

Mekanismi (d)=(a)+(b):



Myötöehdon tarkistus:

Taivutusmomentti pisteessä 2: Otetaan virtuaaliseksi siirtymätilaksi mekanismi (a).

$-W_{\text{int}} = (-M_p)(-\theta) + M_2\theta + (-M_p)(-2\theta) + M_p \cdot 2\theta = (5M_p + M_2)\theta,\ W_{\text{ext}} = P_p L\theta = 5M_p\theta$

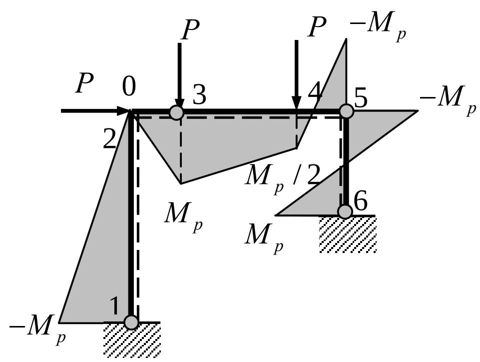
$W_{\text{int}} + W_{\text{ext}} = 0 \;\Rightarrow\; \underline{M_2 = 0}$

Taivutusmomentti pisteessä 4: Otetaan virtuaaliseksi siirtymätilaksi mekanismi (c).

$-W_{\text{int}} = \overbrace{M_2}^{0}(-\theta) + M_4 \cdot 4\theta + (-M_p)(-3\theta) = (4M_4 + 3M_p)\theta,\ W_{\text{ext}} = P_p L\theta = 5M_p\theta$

$W_{\text{int}} + W_{\text{ext}} = 0 \;\Rightarrow\; \underline{M_4 = M_p / 2}$

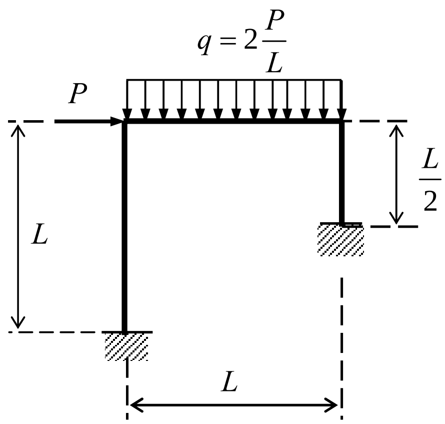
Taivutusmomenttikuvio:



$\Rightarrow$ Myötöehto $|M| \le M_p$ toteutui!

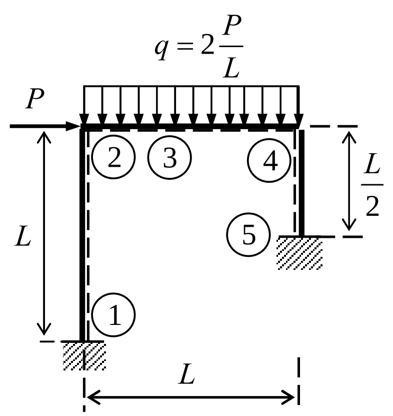
Tehtävä 7.4:

Oheisen kehän täysplastinen momentti on M_p. Kuinka suuren kuorman P rakenne kes-tää.



Ratkaisu:

Perusmekanismien lukmäärä:
$p = m - n_s = 5 - 3 = 2$



Mekanismi (a):

$\theta_1 = -\theta$

$\theta_2 = +\theta$

$\theta_3 = 0$

$\theta_4 = -2\theta$

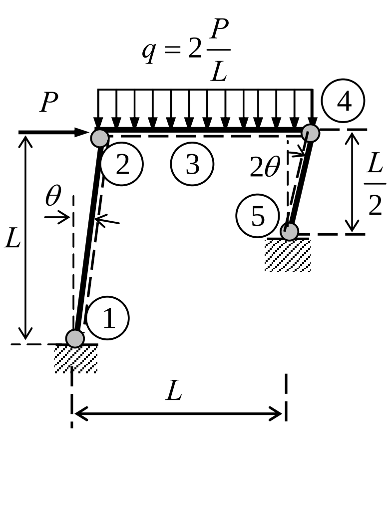
$\theta_5 = +2\theta$

$$-W_{\text{int}} = M_p(|\theta_1| + |\theta_2| + |\theta_3| + |\theta_4| + |\theta_5|)$$

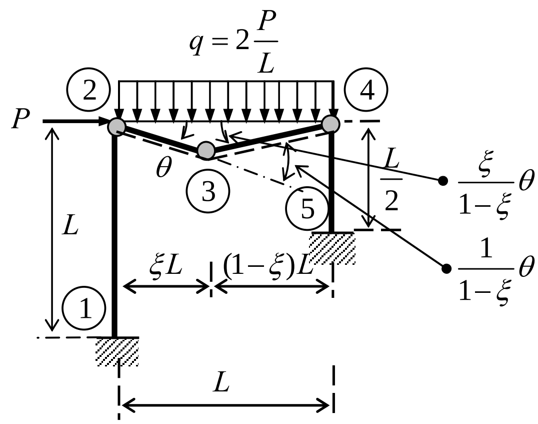
$$= M_p(\theta + \theta + 2\theta + 2\theta) = 6M_p\theta$$

$$W_{\text{ext}} = P \cdot \theta L = PL\theta$$

$$W_{\text{ext}} = -W_{\text{int}} \Rightarrow PL\theta = 6M_p\theta \Rightarrow P = 6\frac{M_p}{L}$$



Mekanismi (b):



$$\theta_1 = 0$$
$$\theta_2 = -\theta$$
$$\theta_3 = +\frac{1}{1-\xi}\theta$$
$$\theta_4 = -\frac{\xi}{1-\xi}\theta$$
$$\theta_5 = 0$$
$$-W_{\text{int}} = M_p(|\theta_1|+|\theta_2|+|\theta_3|+|\theta_4|+|\theta_5|)$$
$$= M_p(\theta + \frac{1}{1-\xi}\theta + \frac{\xi}{1-\xi}\theta) = \frac{2}{1-\xi}M_p\theta$$
$$W_{\text{ext}} = qL\cdot\frac{1}{2}\xi L\theta = 2\frac{P}{L}L\cdot\frac{1}{2}\xi L\theta = P\xi L\theta$$

Symmetrian takia $\xi = 1/2$

$$W_{\text{ext}} = -W_{\text{int}} \;\Rightarrow\; \frac{1}{2}PL\theta = 4M_p\theta \Rightarrow\; P = 8\frac{M_p}{L}$$

Mekanismi	θ_1	θ_2	θ_3	θ_4	θ_5	$-W_{\text{int}}$	W_{ext}
(a)	$-\theta$	$+\theta$	0	-2θ	$+2\theta$	$6M_p\theta$	$PL\theta$
(b)	0	$-\theta$	$\frac{1}{1-\xi}\theta$	$-\frac{\xi}{1-\xi}\theta$	0	$\frac{2}{1-\xi}M_p\theta$	$P\xi L\theta$
(a)+(b)	$-\theta$	0	$\frac{1}{1-\xi}\theta$	$-\frac{2-\xi}{1-\xi}\theta$	$+2\theta$	$2\frac{3-2\xi}{1-\xi}M_p\theta$	$(1+\xi)PL\theta$

Mekanismi (a)+(b):

$$-W_{\text{int}} = M_p(|\theta_1|+|\theta_2|+|\theta_3|+|\theta_4|+|\theta_5|) = M_p(\theta + \frac{1}{1-\xi}\theta + \frac{2-\xi}{1-\xi}\theta + 2\theta) = 2\frac{3-2\xi}{1-\xi}M_p\theta$$
$$W_{\text{ext}} = PL\theta + P\xi L\theta = (1+\xi)PL\theta$$
$$W_{\text{ext}} = -W_{\text{int}}$$
$$\Rightarrow\; (1+\xi)PL\theta = 2\frac{3-2\xi}{1-\xi}M_p\theta \Rightarrow\; P = 2\frac{3-2\xi}{1-\xi^2}\frac{M_p}{L}$$

Minimointi:

$$\frac{dP}{d\xi} = 2\frac{-2(1-\xi^2)-(-2\xi)(3-2\xi)}{(1-\xi^2)^2}\frac{M_p}{L} = 2\frac{-2+6\xi-2\xi^2}{(1-\xi^2)^2}\frac{M_p}{L} = 0$$
$$\Rightarrow\; \xi^2 - 3\xi + 1 = 0 \;\Rightarrow\; \xi_{1,2} = \frac{3}{2}\pm\sqrt{\frac{9}{4}-1} = \frac{3\pm\sqrt{5}}{2} \Rightarrow\; \xi = \frac{3-\sqrt{5}}{2} \approx 0,381$$
$$P_p = 2\frac{3-2\xi}{1-\xi^2}\frac{M_p}{L} = 2\frac{3-2\cdot\frac{3-\sqrt{5}}{2}}{1-(\frac{3-\sqrt{5}}{2})^2}\frac{M_p}{L} = \underline{\underline{\frac{8\sqrt{5}}{6\sqrt{5}-10}\frac{M_p}{L} \approx 5,236\frac{M_p}{L}.}}$$

Taivutusmomentin tarkistus pisteessä 2:

Otetaan virtuaaliseksi siirtymätilaksi mekanismi (a), jolloin saadaan

$$-W_{\text{int}} = \overbrace{M_1}^{-M_p}\overbrace{\theta_1}^{-\theta} + M_2\overbrace{\theta_2}^{+\theta} + \overbrace{M_3}^{M_p}\overbrace{\theta_3}^{0} + \overbrace{M_4}^{-M_p}\overbrace{\theta_4}^{-2\theta} + \overbrace{M_5}^{+M_p}\overbrace{\theta_5}^{+2\theta} = M_p\theta + M_2\theta + 2M_p\theta + 2M_p\theta$$

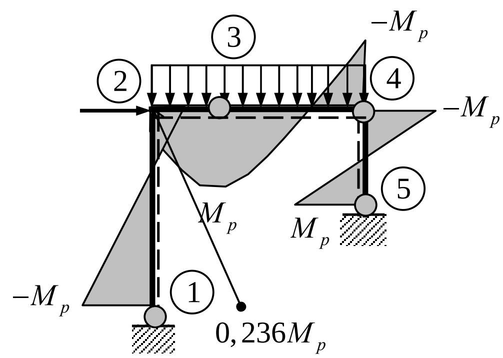
$$= (M_2 + 5M_p)\theta$$

$$W_{\text{ext}} = P_p \cdot \theta L = 5,236\frac{M_p}{L}\cdot\theta L = 5,236 M_p\theta$$

$$-W_{\text{int}} = W_{\text{ext}} \;\Rightarrow\; (M_2 + 5M_p)\theta = 5,236 M_p\theta$$

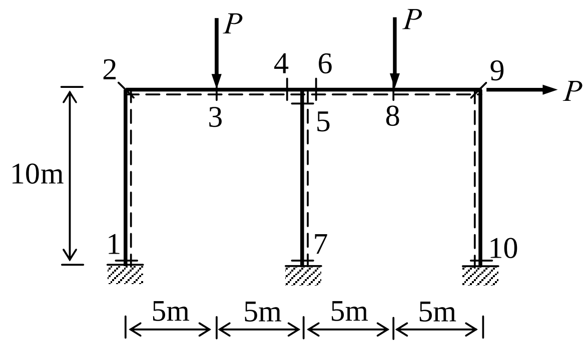
$$\Rightarrow\; M_2 = (5,236 - 5)M_p = \underline{\underline{0,236 M_p < M_p}}.$$

M-kuvio:

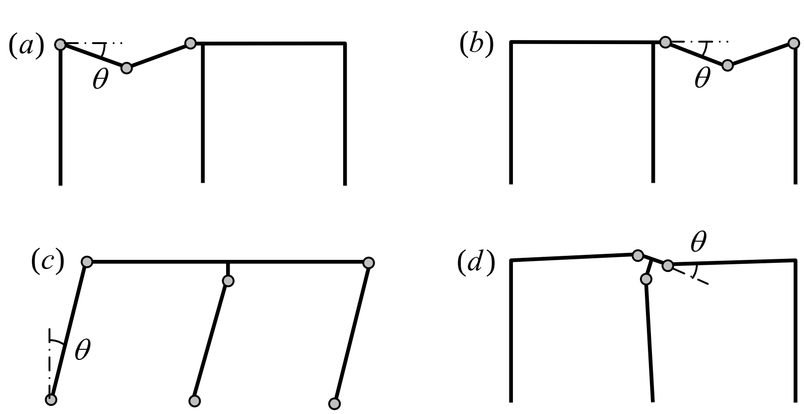


Tehtävä 7.5:

Oheisen kehän kaikkien sauvojen täysplastinen momentti on 250kNm. Määritä sen plastinen rajakuorma P_p.

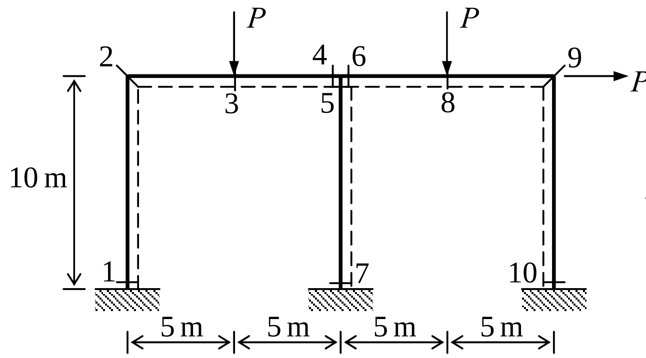


Ohje: Valitse perusmekanismit seuraavasti:



Muodosta seuraavat yhdistelmämekanismit:
(*e*)=(*a*)+(*c*), (*f*)=(*b*)+(*c*), (*g*)=(*f*)+(*d*) ja (*h*)=(*g*)+(*a*).

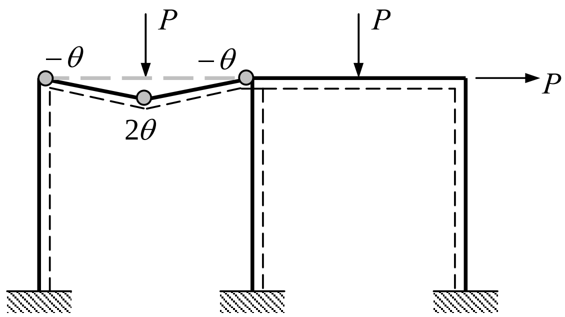
Ratkaisu:



$M_p = 250\,\text{kNm}$

Perusmekanismien lukumäärä: $p = m - n_s = 10 - 6 = 4$.

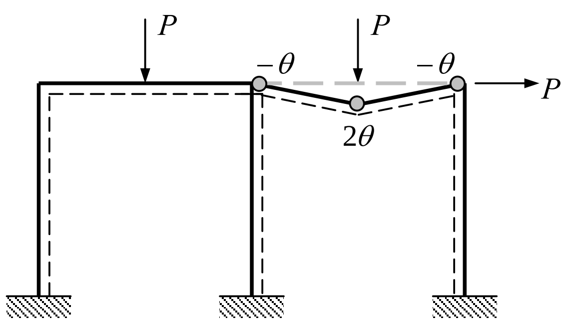
a) Palkkimekanismi.



$\theta_2 = -\theta,$
$\theta_3 = 2\theta,$
$\theta_4 = -\theta,$
$v_3 = 5\,\text{m}\cdot\theta,$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_2| + |\theta_3| + |\theta_4|) = 4M_p\theta,$$
$$W_{\text{ext}} = P\cdot v_3 = 5\,\text{m}\,P\theta,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad \Rightarrow \quad \underline{\underline{P_p^{(a)}}} = \frac{4M_p}{5\,\text{m}} = \frac{4\cdot 250\,\text{kNm}}{5\,\text{m}} = \underline{\underline{200\,\text{kN}}}.$$

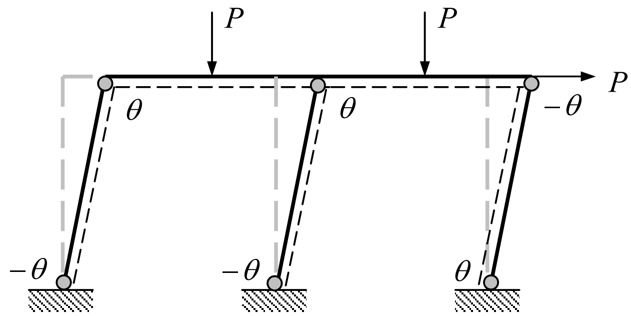
b) Palkkimekanismi.



$\theta_6 = -\theta,$
$\theta_8 = 2\theta,$
$\theta_9 = -\theta,$
$v_8 = 5\,\text{m}\cdot\theta,$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_6| + |\theta_8| + |\theta_9|) = 4M_p\theta,$$
$$W_{\text{ext}} = P\cdot v_8 = 5\,\text{m}\,P\theta,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad \Rightarrow \quad \underline{\underline{P_p^{(b)}}} = \frac{4M_p}{5\,\text{m}} = \frac{4\cdot 250\,\text{kNm}}{5\,\text{m}} = \underline{\underline{200\,\text{kN}}}.$$

c) Sivusiirtymämekanismi.



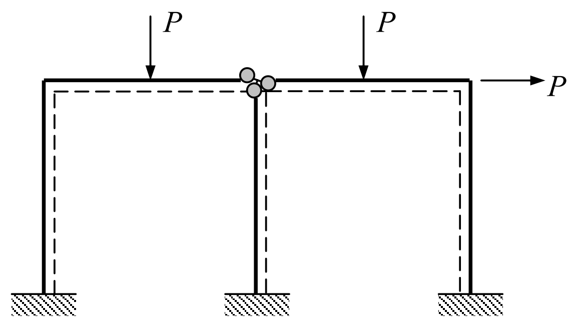
$$\theta_1 = -\theta,\quad \theta_2 = \theta,\quad \theta_5 = \theta,\quad \theta_7 = -\theta,\quad \theta_9 = -\theta,\quad \theta_{10} = \theta,\quad u_9 = 10\,\text{m}\cdot\theta,$$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_1| + |\theta_2| + |\theta_5| + |\theta_7| + |\theta_9| + |\theta_{10}|) = 6M_p\theta\,,$$

$$W_{\text{ext}} = P\cdot u_9 = 10\,\text{m}\,P\theta\,,$$

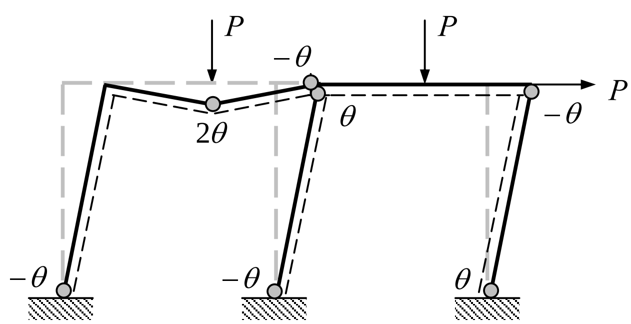
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad \underline{\underline{P_p^{(c)}}} = \frac{6M_p}{10\,\text{m}} = \frac{6\cdot 250\,\text{kNm}}{10\,\text{m}} = \underline{\underline{150\,\text{kN}}}\,.$$

d) Nurkkamekanismi.



$$\theta_4 = -\theta,\quad \theta_5 = -\theta,\quad \theta_6 = \theta.$$

e) Yhdistelmämekanismi a + c.



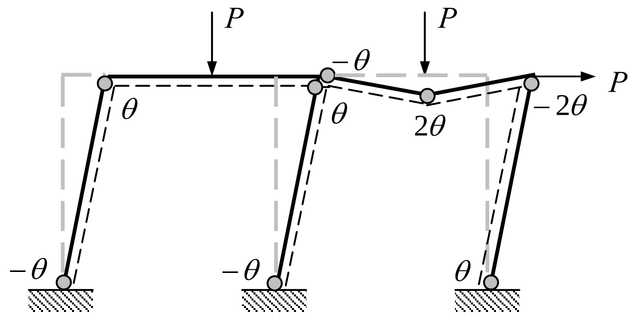
$$\theta_1 = -\theta,\quad \theta_3 = 2\theta,\quad \theta_4 = -\theta,\quad \theta_5 = \theta,\quad \theta_7 = -\theta,\quad \theta_9 = -\theta,\quad \theta_{10} = \theta,$$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_1| + |\theta_3| + |\theta_4| + |\theta_5| + |\theta_7| + |\theta_9| + |\theta_{10}|) = 8M_p\theta\,,$$

$$W_{\text{ext}} = P\cdot v_3 + P\cdot u_9 = (5\,\text{m} + 10\,\text{m})P\theta = 15\,\text{m}\,P\theta\,,$$

$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad \underline{\underline{P_p^{(e)}}} = \frac{8M_p}{15\,\text{m}} = \frac{8\cdot 250\,\text{kNm}}{15\,\text{m}} = \underline{\underline{133.3\,\text{kN}}}\,.$$

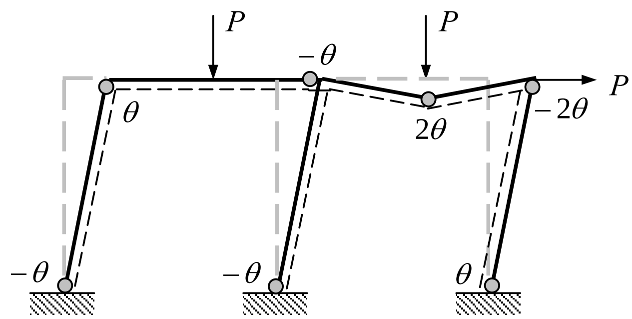
f) Yhdistelmämekanismi b + c.



$\theta_1 = -\theta,$
$\theta_2 = \theta,$
$\theta_5 = \theta,$
$\theta_6 = -\theta,$
$\theta_7 = -\theta,$
$\theta_8 = 2\theta,$
$\theta_9 = -2\theta,$
$\theta_{10} = \theta,$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_1|+|\theta_2|+|\theta_5|+|\theta_6|+|\theta_7|+|\theta_8|+|\theta_9|+|\theta_{10}|) = 10M_p\theta\,,$$
$$W_{\text{ext}} = P\cdot v_8 + P\cdot u_9 = (5\,\text{m}+10\,\text{m})P\theta = 15\,\text{m}\,P\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad \underline{\underline{P_p^{(f)}}} = \frac{10M_p}{15\,\text{m}} = \frac{10\cdot 250\,\text{kNm}}{15\,\text{m}} = \underline{\underline{166.7\,\text{kN}}}\,.$$

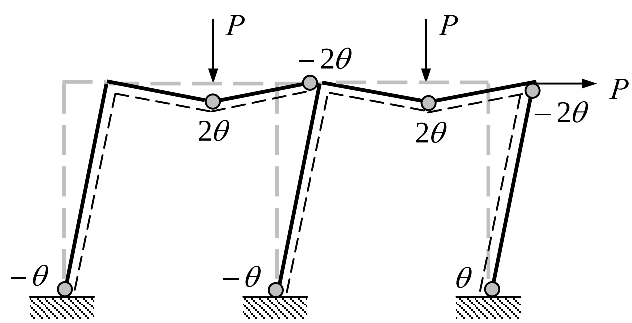
g) Yhdistelmämekanismi f + d.



$\theta_1 = -\theta,$
$\theta_2 = \theta,$
$\theta_4 = -\theta,$
$\theta_7 = -\theta,$
$\theta_8 = 2\theta,$
$\theta_9 = -2\theta,$
$\theta_{10} = \theta,$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_1|+|\theta_2|+|\theta_4|+|\theta_7|+|\theta_8|+|\theta_9|+|\theta_{10}|) = 9M_p\theta\,,$$
$$W_{\text{ext}} = P\cdot v_8 + P\cdot u_9 = (5\,\text{m}+10\,\text{m})P\theta = 15\,\text{m}\,P\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad \underline{\underline{P_p^{(g)}}} = \frac{9M_p}{15\,\text{m}} = \frac{9\cdot 250\,\text{kNm}}{15\,\text{m}} = \underline{\underline{150\,\text{kN}}}\,.$$

h) Yhdistelmämekanismi a + g.



$\theta_1 = -\theta,$
$\theta_3 = 2\theta,$
$\theta_4 = -2\theta,$
$\theta_7 = -\theta,$
$\theta_8 = 2\theta,$
$\theta_9 = -2\theta,$
$\theta_{10} = \theta,$

$$-W_{\text{int}} = \sum M_{pi}|\theta_i| = M_p(|\theta_1| + |\theta_3| + |\theta_4| + |\theta_7| + |\theta_8| + |\theta_9| + |\theta_{10}|) = 11M_p\theta\,,$$
$$W_{\text{ext}} = P\cdot v_3 + P\cdot v_8 + P\cdot u_9 = (5\,\text{m} + 5\,\text{m} + 10\,\text{m})P\theta = 20\,\text{m}\,P\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad \underline{\underline{P_p^{(h)}}} = \frac{11M_p}{20\,\text{m}} = \frac{11\cdot 250\,\text{kNm}}{20\,\text{m}} = \underline{\underline{137.5\,\text{kN}}}\,.$$

Taulukko perus- ja yhdistelmämekanismeista:

Mekan.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	$\frac{-W_{\text{int}}}{M_p\theta}$	$\frac{W_{\text{ext}}}{\text{m}P\theta}$	$\frac{P}{\text{kN}}$
a	0	$-\theta$	2θ	$-\theta$	0	0	0	0	0	0	4	5	200
b	0	0	0	0	0	$-\theta$	0	2θ	$-\theta$	0	4	5	200
c	$-\theta$	θ	0	0	θ	0	$-\theta$	0	$-\theta$	θ	6	10	150
d	0	0	0	$-\theta$	$-\theta$	θ	0	0	0	0	3	-	-
e=a+c	$-\theta$	0	2θ	$-\theta$	θ	0	$-\theta$	0	$-\theta$	θ	8	15	133.3
f=b+c	$-\theta$	θ	0	0	θ	$-\theta$	$-\theta$	2θ	-2θ	θ	10	15	166.7
g=f+d	$-\theta$	θ	0	$-\theta$	0	0	$-\theta$	2θ	-2θ	θ	9	15	150
h=a+g	$-\theta$	0	2θ	-2θ	0	0	$-\theta$	2θ	-2θ	θ	11	20	137.5

Yhdistelmämekanismi e antaa pienimmän kuorman arvon joten rakenteen rajakuorma on: $P_p = 133.3\,\text{kN}$. Täysplastiset momentit yhdistelmämekanismin e nivelissä 1, 4, 7 ja 9 on $-M_p$ ja nivelissä 3, 5 ja 10 M_p. Tarkistetaan vielä, ettei myötöehtoa $M_p \le |M|$ rikota mekanismin e muissa kriittisissä kohdissa (2, 6 ja 8).

Määritetään taivutusmomentti M_2 mekanismin a avulla.

$$-W_{\text{int}} = M_2\overbrace{\theta_2}^{-\theta} + \overbrace{M_3}^{M_p}\overbrace{\theta_3}^{2\theta} + \overbrace{M_4}^{-M_p}\overbrace{\theta_4}^{-\theta} = -M_2\theta + 3M_p\theta\,,$$
$$W_{\text{ext}} = P_p\cdot v_3 = 133.3\,\text{kN}\cdot 5\text{m}\,\theta = 666.5\,\text{kNm}\,\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad M_2 = (-666.5 + 3\cdot 250)\,\text{kNm} = 83.5\,\text{kNm} \le M_p. \quad \text{OK!}$$

Määritetään taivutusmomentti M_8 mekanismin g avulla.

$$-W_{\text{int}} = \overbrace{M_1}^{-M_p}\overbrace{\theta_1}^{-\theta} + \overbrace{M_2}^{83.5\,\text{kNm}}\overbrace{\theta_2}^{\theta} + \overbrace{M_4}^{-M_p}\overbrace{\theta_4}^{-\theta} + \overbrace{M_7}^{-M_p}\overbrace{\theta_7}^{-\theta} + M_8\overbrace{\theta_8}^{2\theta} + \overbrace{M_9}^{-M_p}\overbrace{\theta_9}^{-2\theta} + \overbrace{M_{10}}^{M_p}\overbrace{\theta_{10}}^{\theta}\,,$$
$$= 6M_p\theta + 83.5\,\text{kNm}\,\theta + 2M_8\theta,$$
$$W_{\text{ext}} = P_p\cdot 15\,\text{m}\,\theta = 1999.5\,\text{kNm}\,\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad M_8 = \frac{1}{2}(1999.5 - 83.5 - 6\cdot 250)\,\text{kNm} = 208\,\text{kNm} \le M_p. \quad \text{OK!}$$

Määritetään taivutusmomentti M_6 mekanismin b avulla.

$$-W_{\text{int}} = M_6\overbrace{\theta_6}^{-\theta} + \overbrace{M_8}^{208\,\text{kNm}}\overbrace{\theta_8}^{2\theta} + \overbrace{M_9}^{-M_p}\overbrace{\theta_9}^{-\theta} = -M_6\theta + 416\,\text{kNm}\,\theta + M_p\theta\,,$$
$$W_{\text{ext}} = P_p\cdot 5\text{m}\,\theta = 666.5\,\text{kNm}\,\theta\,,$$
$$-W_{\text{int}} = W_{\text{ext}} \quad\Rightarrow\quad M_6 = (-666.5 + 416 + 250)\,\text{kNm} = -0.5\,\text{kNm} \le M_p. \quad \text{OK!}$$

M – kuvio / kNm :